STRONG INTERMITTENCY IN MOMENTUM SPACE

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ABSTRACT

It is shown that projections of 3-dimensional intermittent distribution in momentum space on rapidity and/or azimuthal angle can lead to a drastic reduction of intermittency parameters and to deviations from the power law behaviour at very small intervals. Existing data suggests much larger 3-dimensional intermittency exponents than those presently observed in rapidity spectra.

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Recently, several experiments ¹⁻⁶ have observed approximate power law behaviour of normalised factorial moments predicted by the hypothesis of intermittency⁷. The relation

$$F_q(\delta) = F_q(\Delta)(\Delta/\delta)^{1/q} \quad (1)$$

where $F_q(\delta)$ is the normalized factorial moment of rank q and δ is the interval considered, was observed in the region $1 < \delta < 0.1$ both in rapidity and azimuthal angle. Deviations from (1) are seen at smaller values of δ , where the dependence of $F_q(\delta)$ on δ seems to flatten off. Furthermore, the measured intermittency exponents are small, particularly for the moments of low rank¹⁻⁶. It is thus generally believed that

- (a) intermittent behaviour holds down only to rapidity intervals of the order of 0.1, and
- (b) it is a rather weak effect, representing a minor correction to the otherwise perfectly smooth distributions.

In the present note we would like to point out that these statements may turn out to be premature and that more complete measurements are needed to determine the actual size and range of fluctuations in particle spectra. Moreover, as we argue below, the existing data on intermittency in joint rapidity and azimuthal angle spectra can be interpreted as evidence that the effect may be much stronger when measured in full 3-dimensional momentum space.

Our argument follows the observation made recently by Och⁸ who analysed the 2-dimensional α -model⁷ and found that its one-dimensional projection does not show intermittency at small intervals. When applied to the reality of multiparticle physics, Och's argument implies that intermittency parameters derived from the analysis of rapidity distributions may be substantially underestimated. Indeed, correlation functions in rapidity space are only projections (integrated over transverse momenta) of the full correlation functions in momentum space. Since any projection procedure is likely to remove, at least partly, the fluctuations of the intermittent system, one may worry that the measurements of intermittency derived from projected distributions can substantially reduce the effect. In this paper we attempt to estimate the size of this reduction.

To describe intermittency in a multiparticle system, we consider inclusive distributions exhibiting power law singularity in the difference of 4-momenta of the particles

$$Q_{ij}^2 = |(p_i^\mu - p_j^\mu)^2| \quad (2)$$

Since the factorial moments are integrals of the inclusive multiparticle distributions

$$F_q(\delta) = \int_\delta \frac{d^3 p_1}{E_1} \dots \int_\delta \frac{d^3 p_q}{E_q} \rho_q(p_1, \dots, p_q) / \left\{ \int_\delta \frac{d^3 p}{E} \rho_1(p) \right\}^q \quad (3)$$

power law singularity in Q_{ij} implies the power law (1) of factorial moments evaluated at small intervals of Q , i.e. intermittency.

Using (3) and the assumed singular form of multiparticle distributions we studied the factorial moments F_2 and F_3 projected on rapidity and/or azimuthal angle. The inclusive two- and three-particle distributions were taken in the form

$$\begin{aligned} \rho_2(p_1, p_2) &= \rho_1(p_1) \rho_1(p_2) Q_{12}^{-\alpha} \\ \rho_3(p_1, p_2, p_3) &= \rho_1(p_1) \rho_1(p_2) \rho_1(p_3) \{ Q_{12}^{-\alpha} Q_{23}^{-\alpha} + \text{perm.} \} \end{aligned} \quad (4)$$

with the single particle distribution⁹

$$\rho_1(p) = \exp \left\{ -6.25 \sqrt{m_\pi^2 + p_\perp^2} \right\} \quad (5)$$

The results are shown in the Figures 1 and 2 where the moments F_2 and F_3 are plotted versus the corresponding bin sizes in double-logarithmic scale. All curves are calculated for $\alpha = 0.2$ and are normalized to 1 at $\delta = 1$. One sees that the effects of projection are rather dramatic:

- (a) the observed slopes in rapidity are reduced by more than an order of magnitude, as compared to the actual slope of the assumed power law singularity in momentum space;[†]
- (b) the dependence on the bin size of the "projected" factorial moments flattens off at small intervals, closely resembling the experimental observations.

Another important point seen in Figs. 1 and 2 is that the slope in the plot of F_q versus $(\delta y \delta \varphi)$ is significantly larger than that of the one-dimensional curves δy or $\delta \varphi$ in agreement with experimental observations^{1,3-5}. This implies that

- (a) the experimentally observed differences between slopes in one-dimensional and two-dimensional analyses can be explained without any reference to jet physics¹⁰, and
- (b) the data on $(\delta y \delta \varphi)$ intermittency supports the hypothesis of strong intermittency in 3-dimensional momentum space.

In Figs. 1 and 2 we show also the calculated rapidity dependence of F_2 and F_3 for a transverse momentum cut of 0.15 GeV. One sees that the slopes are larger than those of factorial moments integrated over all transverse momenta. We consider this observation as a further confirmation of the hypothesis of strong intermittency in momentum space, although the calculated effect is weaker than that observed recently by the NA22 Collaboration⁵.

We thus conclude that the existing data confirms the relevance of the Och's observation that fluctuations in one-dimensional distributions are strongly reduced in comparison to those in three dimensions. One may thus expect rather strong intermittency effects once the multidimensional spectra are analysed in high statistics experiments. *

We would like to close this paper with the following comments:

- (i) The particular form (4) of the multiparticle distributions was taken because of its simplicity. We have checked that other similar forms (e.g. adopting the ansatz (4) for inclusive correlation functions instead of inclusive distributions¹²) lead to qualitatively similar results, provided the distributions are singular in Q_{ij} .
- (ii) It should be pointed out that if singularity in multiparticle inclusive distributions were factorizable, e.g. in rapidity, azimuthal angle and/or transverse momentum differences, the effects we are discussing should disappear. Thus the experimental

[†] The observed slopes for F_3 are approximately twice as large as those for F_2 . This is a straightforward consequence of using the same parameter α for ρ_3 and ρ_2 in Eq. (4).

* High statistics is an important requirement because of the "empty bin effect" which reduces observed fluctuations at small intervals and limited statistics.¹¹

observation of stronger intermittency in joint $(\delta y, \delta\varphi)$ distributions, as compared to δy and $\delta\varphi$ provides important evidence that such a factorization does not take place and that we can expect even stronger intermittency in 3-dimensional spectra.

(iii) The results presented in Figs. 1 and 2 show clearly that the expected intermittency effects strongly depend on the variables in which they are measured. Thus only by analysing intermittency in several different projections may one have a chance to arrive at a quantitative description of the phenomenon. In this context we would like to point out that our calculations indicate that intermittency observed in $(\delta y, \delta\varphi)$ binning should depend not only on the value of the product $\delta y\delta\varphi$ but separately on δy and $\delta\varphi$. It would be interesting to verify this observation in the data.

Note: As we were finishing writing this paper we received the preprint MPI-PAE/PTh 26/90 by W. Ochs which included conclusions similar to ours.

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REFERENCES

- 1- R. Holynski *et al.*, Phys. Rev. Lett. **62** (1989) 733; Cracow preprint INP 1456/ph, June 1989. B. Wosiek, Report presented at the Cracow-Munich Seminar, April 1990 and private communication.
- 2- I. Derado, P. Jancso, N. Schmitz and P. Stopa, MPI Munich preprint (1990) and private communication.
- 3- W. Braunschweig *et al.*, (TASSO Coll.), Phys. Lett. **B231** (1989) 548.
- 4- B. Buschbeck and P. Lipa (UA1 Coll.), Mod. Phys. Lett. **A4**, (1989) 1871; B. Buschbeck, report at Santa Fe Workshop (March 1990), preprint HEPY-PUB 535/90.
- 5- I.V. Ajinenko *et al.*, (EHS/NA22 Coll.), Phys. Lett. **B222** (1989) 306 and **B235** (1990)373; W. Kittel, Report at Santa Fe Workshop (March 1990).
- 6- E. Stenlund (EMU01 Coll.), talk presented at Quark Matter '90, Menton (May 1990).
- 7- A. Bialas and R. Peschanski, Nucl. Phys **B273** (1986) 703; Nucl. Phys. **B308** (1988) 857.
- 8- W. Ochs, report at Santa Fe Workshop (March 1990) and private communication.
- 9- M. Adamus *et al* (EHS-NA22 Coll.), Zeit. f. Phys. **C39** (1988) 311.
- 10- W. Ochs and J. Wosiek, Phys. Lett. **B214** (1988) 617.

- 11- B. Buschbeck, W. Kittel and P. Lipa, private communication.
- 12- P. Carruthers and I. Sarcevic, Phys. Rev. Lett. **63** (1989) 1562.

FIGURE CAPTIONS

- Fig. 1- Factorial moments F_2 versus the bin size for r different projected distributions.
- Fig. 2- Factorial moments F_3 versus the bin size for r different projected distributions.

FIGURE 1

FIGURE 2