CERN-TH.5684/90
LBL-29189
UCB-90/24

ELECTROWEAK PHASE TRANSITION AND DARK MATTER ABUNDANCE

S. DIMOPOULOS^{*,a,b}, R. ESMAILZADEH^c, L. HALL^d AND N. TETRADIS^{*,a,b}

^a *Theory Division, CERN
CH-1211 Geneva 23, Switzerland*

^b *Department of Physics, Boston University, MA 02215 Boston, U.S.A.*

^c *Center for Particle Astrophysics, University
of California, CA 94720 Berkeley, U.S.A.*

^d *Department of Physics, University of California, CA 94720 Berkeley, California*

ABSTRACT

If the critical temperature of the electroweak phase transition is sufficiently small, it can change the final abundances of heavy dark matter particles, by keeping them massless for a long time. Recent experimental bounds on the Higgs mass from LEP imply that this is not the case in the minimal standard model.

* Permanent address: Department of Physics, Stanford University, CA 94305 Stanford, U.S.A.

1. *Introduction.* One of the most important features of the hot big bang cosmology is that it allows the calculation of relic particle abundances from first principles. This successfully yields the temperature of the microwave background, but is frustrated in the case of baryons and electrons by our ignorance of the fundamental description of the origin of the baryon asymmetry. If the cosmic dark matter is composed of exotic heavy particles then this abundance can also be computed [1, 2]. As the temperature of the hot plasma falls beneath the mass of the dark matter particle the abundance will decrease by annihilation in order to follow the equilibrium Boltzmann distribution. At some point, the annihilation reaction rate, which is dropping exponentially with temperature, will fall beneath the expansion rate of the universe, which drops only as a power of temperature. Subsequent to this freezeout temperature, T_f , the comoving abundance of a stable species decreases very slowly and asymptotically reaches a constant value.

What is the origin of the mass of the exotic dark matter particle? Since all known heavy elementary particles acquire mass by electroweak symmetry breaking, it is plausible that this is the origin of the mass of the dark matter species. This may change the above picture of relic particle freezeout and consequently the relic abundance. Just prior to the electroweak symmetry breaking critical temperature T_c the dark matter particles X were massless. If the phase transition is second order then $m_X(T)/T$ grows beneath T_c . The requirement that the usual Lee-Weinberg picture yields a good approximation to the relic abundance is that the phase transition is essentially over by the time the temperature drops to T_f . This requires not only that $T_c > T_f$, but that $m_X(T)$ becomes close to the zero temperature mass m_X^0 (say within 10%) at some $T > T_f$.

However, we can imagine alternative situations. What happens if T_c is beneath the usual freezeout temperature T_f ? What happens if, at $T = T_f$, $m_X(T)$ is nowhere near its final value and $\dot{m}_X/m_X$ is rapidly changing? In either of these cases freezeout will occur during the second-order phase transition. The relic abundance for a given mass will not be given by the Lee-Weinberg value. There are three reasons for this:

- a) During the phase transition the expansion of the universe is not given by the usual expression used at T_f . This is because the effective number of relativistic degrees of freedom is varying and also the vacuum energy is varying.
- b) The mass $m_X(T)$ is varying on the same time scale as the Hubble parameter, and hence the condition for freezeout may be altered.
- c) The value of the mass today, m_X^0 , may be significantly larger than the value of the mass at the freezeout during the phase transition.

In this paper we show that such modifications to the freezeout picture do not occur in the standard model. We do this by explicitly calculating T_c and showing that it is never low enough to modify the usual picture even when there is a very heavy dark matter particle acquiring its mass from the electroweak phase transition.

2. *The critical temperature.* The electroweak phase transition temperature depends on the parameters of the standard model which appear in the effective potential for the Higgs field ϕ . At zero temperature and at the one-loop level the effective potential is given by* [3, 4]:

$$V^0(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4 + B\phi^4\left[\log\left(\frac{\phi^2}{\sigma^2}\right) - \frac{25}{6}\right] \quad (1)$$

where σ is an arbitrary scale and

$$B = \frac{3}{64}\left(\frac{e^2}{4\pi}\right)^2 \frac{2 + \sec^4\theta_w}{\sin^4\theta_w} - \frac{1}{16\pi^2} \sum_i g_i^4 \quad (2)$$

exhibits the contributions of gauge-boson loops and loops of fermions with Yukawa couplings g_i . We shall constrain $g_i \leq 1$, in order to be able to trust the one-loop result. For the moment we shall omit scalar loops and we shall consider only one dark matter heavy fermion contributing to B . The potential of equation (1) has

* Throughout this paper we impose the following renormalization conditions: $d^4V/d\phi^4|_{\phi=\sigma} = \lambda$, $d^2V/d\phi^2|_{\phi=\sigma} = \mu^2$.

a minimum at a value $\phi_{\min}$, which is set by experiment at $\phi_{\min} = 250$ GeV. We choose $\sigma = \phi_{\min}$, which implies:

$$\sigma^2 = \frac{\mu^2}{B - \frac{\lambda}{6B} + \frac{41}{3}} \quad (3)$$

The relation for the zero-temperature Higgs mass is:

$$(m_H^0)^2 = 2B\sigma^2 \left(\frac{\lambda}{6B} - \frac{32}{3} \right) \quad (4)$$

The above constraints eliminate two of the free parameters. For convenience we choose the remaining ones to be the Higgs mass m_H^0 and the Yukawa couplings g_i .

The temperature-dependent contribution to the effective potential [5, 6] is equal to the free energy of the particles, whose mass depends on the Higgs expectation value. Therefore for each bosonic degree of freedom the contribution is:

$$V_b^T(\phi, T) = \frac{T}{2\pi^2} \int_0^\infty p^2 dp \log[1 - \exp(-\frac{1}{T} \sqrt{p^2 + m_b^2(\phi)})] \quad (5)$$

and for each fermionic degree of freedom:

$$V_f^T(\phi, T) = -\frac{T}{2\pi^2} \int_0^\infty p^2 dp \log[1 + \exp(-\frac{1}{T} \sqrt{p^2 + m_f^2(\phi)})] \quad (6)$$

The full effective potential for the standard model is given by the expression:

$$V(\phi, T) = V^0(\phi) + 6V_W^T + 3V_Z^T + 4V_X^T \quad (7)$$

The masses of the W -boson, the Z -boson and the heavy fermion X are the tree

level ϕ -dependent terms:

$$m_W = \frac{e\phi}{2\sin\theta} \quad (8)$$

$$m_Z = \frac{e\phi}{2\sin\theta \cos\theta} \quad (9)$$

$$m_X = g_X \phi \quad (10)$$

For small values of the Higgs mass (a few GeV) and in the absence of heavy fermions (the so-called Coleman-Weinberg limit) the phase transition is driven by radiative corrections and is of first order. In this case, it is possible that extreme supercooling may occur before QCD effects eliminate the barrier separating the two minima of the effective potential [7]. However, recent bounds on the Higgs mass [8] set $m_H^0 \geq 24$ GeV, thus implying a second-order phase transition[†]. In Fig. 1 we plot the effective potential at different values of the temperature, for $m_H^0 = 70$ GeV and $g_X = 0.2$. The transition from the unbroken to the broken phase occurs at a critical temperature T_c . Performing an expansion of the effective potential for small values of ϕ , we find:

$$T_c^2 = \frac{-\mu^2}{\frac{1}{15} \frac{e^2}{4\pi^2} (2 + \frac{1}{\cos^2\theta}) + \frac{g_X^2}{6}} \quad (11)$$

The response time of the Higgs expectation value to the evolution of the minimum of the potential (whose timescale is of order $V/\dot{V} \sim t_{\text{microsec}}$) is inversely proportional to the Higgs decay width. It is, therefore, a very good approximation to assume that the Higgs expectation value traces the minimum of the potential. This allows us to calculate the temperature dependence of the mass of the dark matter fermion. In Fig. 2 we plot $m_X = m_X(T)$ for $m_H^0 = 70$ GeV and $g_X = 0.2$. The mass remains zero until the temperature becomes equal to T_c and then rises quickly to its zero temperature value.

[†] In principle, the phase transition is technically always first order, but for large Higgs masses it is very weakly so and can be effectively considered second order. [9]

For large Higgs masses the inclusion of the scalar loops will modify the picture presented above [6, 10]. For $|\mu^2| \leq \sigma^2$ the zero-temperature contribution can be written in a simplified form. The estimation of the temperature-dependent contribution is problematic [6], because the tree-level mass for the Higgs:

$$m_H^2 = \mu^2 + \frac{\lambda}{2}\phi^2 \quad (12)$$

to be substituted in the one-loop expression for the free energy, is negative for small ϕ values. The resolution of the problem lies in the inclusion of higher loop contributions to the effective potential. However, those contributions [6] do not seem to modify the one-loop expression for T_c , as long as $\lambda \leq 1$.

The conclusions of the above discussion can be summarized as follows. For $0 \leq m_H^0 \leq 150$ GeV and $0 \leq g_X \leq 1$, the critical temperature is given by the expression:

$$T_c^2 = \frac{-\mu^2}{\frac{1}{16} \frac{e^2}{\sin^2 \theta_w} (2 + \frac{1}{\cos^2 \theta_w}) + \frac{g_X^2}{6} + \frac{\lambda}{24}} \quad (13)$$

and equations (3), (4) have to be satisfied, with B given by:

$$B = \frac{3}{64} \left(\frac{e^2}{4\pi} \right)^2 \frac{2 + \sec^4 \theta_w}{\sin^4 \theta_w} - \frac{1}{16\pi^2} g_X^4 + \frac{1}{64\pi^2} \frac{\lambda^2}{4} \quad (14)$$

The presence of more than one heavy fermion leads to the replacement of g_X^2 and g_X^4 by $\sum_i g_i^2$ and $\sum_i g_i^4$ in equations (13) and (14) respectively.

3. Analysis and conclusions. The condition for the usual Lee-Weinberg freeze-out analysis to apply, even when the dark matter particle obtains its mass from a second-order electroweak phase transition, can be stated as follows. It is sufficient that $m_X(T)$ becomes close to m_X^0 at temperatures T above the usual freeze-out temperature T_f . For definiteness we define T' by $m_X(T') = 0.9 m_X^0$, and require $T' > T_f$. From Fig. 2 it can be seen that, for the case $g_X = 0.2$ and $m_H^0 = 70$ GeV, T' is not much less than T_c because the vev turns on rapidly. In fact, we find

that for all perturbative values of the parameters $T'/T_c > 1/3$. Hence the phase transition does not alter the Lee-Weinberg abundance if $T_c/3 > T_f = m_X^0/25$, that is if:

$$\frac{T_c}{m_X^0} = \frac{1}{g_X} \left(\frac{(m_H^0)^2}{(m_t^0)^2 + \frac{(m_X^0)^2}{3} + \frac{(m_H^0)^2}{4} + \frac{16}{3} B \sigma^2} \right)^{\frac{1}{2}} > \frac{1}{8} \quad (15)$$

where we have dropped the W and Z contributions. We now briefly explain why this condition is always satisfied.

The physical Higgs boson mass m_H^0 is known to be larger than 24 GeV [8]. This excludes the Coleman-Weinberg [3] case, which gives a first-order transition. For $m_H^0 > 24$ GeV the radiative B term can be neglected if $g_X, g_t \ll 1$. If the Higgs mass is very large and dominates the denominator, (15) becomes $g_X < 16$, which is clearly true; while if the top quark mass dominates the denominator it becomes $g_X < 8m_H/m_t$, which is also true (we took g_X, g_t small). An alternative possibility is to take g_X or g_t very large so that they dominate both numerator (via B) and denominator. In this case the left-hand side of (15) becomes $\frac{\sqrt{3/2}}{g_X} \simeq 0.4$, so that (15) is again satisfied.

These arguments are supported by a numerical evaluation of T_c/m_X^0 . In Fig. 3 we plot T_c/m_X^0 as a function of g_X for different Higgs and top quark masses. The lowest values are obtained for large g_X and are favoured by large top quark masses. At this point we should remark that T_c in expression (13) was calculated by demanding that the second derivative of the effective potential with respect to ϕ is zero at the origin. For a given value of the Higgs mass, there is an upper bound on the mass of the heavy fermions of the theory, from the requirement that the effective potential is bound from below [11]. We have used top quark masses such that, for a given Higgs mass, vacuum instability is avoided for all values of g_X . Fig. 3 clearly indicates that $\frac{T_c}{m_X^0} > 0.4$, for any value of the free parameters. Our result does not change for a large number of new particles e.g. if the dark matter particle is in an N -plet under some symmetry. In the large g_X limit both

numerator and denominator of the left-hand side of (15) are proportional to N , which therefore cancels.

We have studied the possibility that dark matter particles acquire their mass at the second-order electroweak phase transition of the one-Higgs doublet, perturbatively coupled standard model. We conclude that such particles do not freeze out during the phase transition, but after it is completed. Consequently their relic abundance is not substantially altered from the usual Lee-Weinberg value.

We have three final comments:

- a) The only way to avoid this conclusion is to add both heavy fermions and bosons to the standard model in such a way that their contributions to B cancel. This requires a fine tuning and we consider it extremely unlikely.
- b) Freezeout during a phase transition may be possible in theories with many Higgs. Conditions similar to that of (15) must be met. The case where g_X dominates both numerator and denominator is unlikely to differ from the one-Higgs model: there is no small parameter to justify a small value of T_c/m_X^0 . Interesting new effects may occur if there is more than one phase transition and the dark matter acquires mass from that of lowest T_c and smallest $v \ll v = 250$ GeV. Such effects would require a very light Higgs. The experimental constraint on this light Higgs is much weaker than that for the standard Higgs boson, however, it will involve a fine tuning.
- c) Our analysis is clearly inapplicable to the case in which the standard model is strongly coupled; neither would it apply to dynamical electroweak symmetry breaking. In the case of technicolour models the phase transition is expected to be first-order. For a first-order phase transition bubble growth may sweep dark matter particles into dense piles, enhancing annihilation [12].

After this work was completed we became aware of a preprint by K. Enqvist, K. Kainulainen, J. Maalampi and T. Weiler [13], where similar issues are discussed. These authors, however, do not discuss the dependence of the critical temperature on the dark matter particle mass, which is crucial for our results.

Acknowledgements: S.D. acknowledges support from the NSF under Grant PHY 89/17438. R.E. acknowledges support from the NSF Office of Science and Technology Centers, under cooperative agreement AST-8809616. L.H. is partially supported by an NSF Presidential Young Investigator Award and in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098, and in part by the NSF under grant PHY85-15857. N.T. acknowledges support from the DOE under the grant number DE-FG03-88ER40472 and the NSF under Grant PHY 89/17438. S.D. and N.T. would like to thank Boston University and CERN for their hospitality.

REFERENCES

1. B. W. Lee and S. Weinberg, Phys. Rev. Lett. 39 (1977) 165.
2. M. I. Vysotskii, A. D. Dolgov and Ya. B. Zel'dovich, JETP Lett. 26 (1977) 188.
3. S. Coleman and E. Weinberg, Phys. Rev. D 7 (1973) 1888.
4. A. Guth and E. Weinberg, Phys. Rev. Lett. 45 (1980) 1131.
5. R. Jackiw, Phys. Rev. D 9 (1974) 1686.
6. L. Dolan and R. Jackiw, Phys. Rev. D 9 (1974) 3320.
7. E. Witten, Nucl. Phys. B 177 (1981) 477.
8. The ALEPH Collaboration, CERN-EP/90-16 (1990).
9. M. Sher, Physics Reports 179 (1989) 273.
10. J. Iliopoulos, C. Itzykson and A. Martin, Rev. Mod. Phys. 47 (1975) 165.
11. H. D. Politzer and S. Wolfram, Phys. Lett. 82B (1979) 242; Phys. Lett. 83B (1979) 421.
12. E. Witten, Phys. Rev. D 30 (1984) 272.
13. K. Enqvist, K. Kainulainen, J. Maalampi and J. Weiler, HU-TFT-89-58, VAND-TH-89-12 (1989).

FIGURE CAPTIONS

- 1) The temperature-dependent effective potential as a function of the Higgs expectation value at different temperatures, for $m_H^0 = 70$ GeV and $g_X = 0.2$.
- 2) The mass of a fermion with Yukawa coupling $g_X = 0.2$ as a function of the temperature, for $m_H^0 = 70$ GeV.
- 3) $\frac{T_f}{m_X}$ as a function of the Yukawa coupling g_X for different Higgs and top quark masses.

FIGURE 1

FIGURE 2

FIGURE 3