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THE VACUUM ABHORS TOP BAGS

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ABSTRACT

The Higgs field near a very heavy top quark or any other heavy fermion is expected to be significantly deformed. By computing explicit solutions of the classical equations of motion for a spherically symmetric configuration without gauge fields, we show that in the standard model this cannot happen without violating either vacuum stability or perturbation theory at energies very close to the top quark mass.

The standard picture in models with spontaneous symmetry breaking involves a vacuum expectation value (vev) for the Higgs field(s) which corresponds to the minimum of the effective potential and is constant over the whole space. However, it is expected that the presence of particles with strong coupling to the Higgs field can modify this picture. A significant deviation from the constant Higgs vev can occur in regions of non-zero particle density. Variations of this idea have been discussed in the past, in the context of strong interactions [1, 2]. In these works, the role of the Higgs field is played by an auxiliary field σ to which ordinary quarks or nucleons couple strongly. Bag-like configurations, of fixed fermion number, with the quarks confined in a finite volume, are shown to have lower energy than the unconfined state. The detailed dynamics depend on the relative strength of the σ -field self coupling and the σ -quark interaction, which drive the phenomenon. For large values of the σ -quark coupling, the total energy reaches an asymptotic value, which is a function of the σ field self coupling. Various numerical realizations of the above scenario, with technical discussions, exist in the literature [3].

The purpose of this letter is to discuss the possibility of a similar configuration in the electroweak sector of the standard model. Recent searches for the top quark [4] put a lower bound on its mass of 89 GeV and imply a large Yukawa coupling to the Higgs. It is plausible that a significant deviation from the constant Higgs vev occurs around a top quark or other heavier fermion, which lowers the physical mass and has observable phenomenological implications (form factors in the decay width, production, scattering cross-sections etc.)

We address the problem in the formalism of refs. [2] and [3], to which the reader is referred for a detailed discussion. We are interested in solving the classical equations of motion in the one-fermion sector. (The classical approximation will be sufficient for our conclusions. We shall comment on the effect of radiative corrections and the modifications induced by the effective action formalism.) We seek solutions which are not translationally invariant, but rather concentrate around a small region of non-zero fermion density. The boson field is described by a coherent state, while for the fermion we make an expansion in a localized

basis, which is related to the plane-wave expansion through a Bogoliubov transformation. We obtain numerical solutions for the state of zero momentum, while states of non-zero momentum can be constructed by a finite-momentum projection. Since we intuitively expect that the Higgs field dynamics drive the bag formation, we shall simplify the problem by choosing a gauge field independent ansatz [5]. Moreover, we assume a time-independent, spherically symmetric configuration to be the lowest energy state.

With the above assumptions, the lagrangian for our model becomes

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - g\sigma)\psi + \frac{1}{2}(\partial\sigma)^2 - \lambda(\sigma^2 - v^2)^2 \quad (1)$$

where ψ is the top quark field, σ the Higgs field and $v = 250$ GeV. Variation of the classical action leads to the classical equations of motion. In the notation of ref. [2], we write the Dirac spinor as

$$\psi = \begin{pmatrix} i\frac{\psi_1}{r}\chi_{1/2,m} \\ \frac{\psi_2}{r}\frac{\partial}{\partial r}\chi_{1/2,m} \end{pmatrix} \quad (2)$$

with $\chi_{1/2,m}$ a two-component spinor. This leads to the following set of coupled differential equations, to be solved numerically:

$$\frac{d^2\sigma}{dr^2} + \frac{2}{r}\frac{d\sigma}{dr} - 4\lambda\sigma(\sigma^2 - v^2) = g\frac{\psi_1^2 - \psi_2^2}{r} \quad (3)$$

$$\frac{d\psi_1}{dr} = \frac{1}{r}\psi_1 + (E_\psi + g\sigma)\psi_2 \quad (4)$$

$$\frac{d\psi_2}{dr} = \frac{1}{r}\psi_2 - (E_\psi - g\sigma)\psi_1. \quad (5)$$

The integration of eqs. (3), (4) and (5), with normalization to unit fermion number, was performed with the aid of the program COLSYS [6]. The mass of the

configuration includes the fermion energy E_ψ and the energy of the Higgs field:

$$m_{\text{bag}} = E_\psi + \int d^3r \left[\frac{1}{2} (\partial\sigma)^2 + \lambda(\sigma^2 - v^2)^2 \right]. \quad (6)$$

This is to be compared with the mass of a plane wave excitation in the absence of a bag:

$$m_{\text{wave}} = gv. \quad (7)$$

In Fig. 1 we plot a solution with $g = 3$, $m_H = 40$ GeV and $m_{\text{bag}} = 2.96v$. The formation of a bag is favored by large values of the Yukawa coupling g and small values of the self-coupling λ (and therefore light Higgs masses). Fig. 2 exhibits the dependence of the depth of the bag on the free parameters of the theory. Fig. 3 compares the mass of the plane wave and the bag solutions, as a function of the Yukawa coupling for $m_H = 40$ GeV. For $g \rightarrow \infty$, m_{bag}/v reaches an asymptotic value of 5.5 in this case. We expect the existence of an m_H -dependent upper limit to m_{bag} to be a generic feature of the classical solutions. For $m_H^2/8v^2 \gg 1$ and $g \gg 1$ we reproduce the results of the SLAC bag [2], with the fermion confined in the bag surface. The two lines in Fig. 3 do not merge continuously at small g . They intersect at a small angle, and there exist bag solutions with slightly higher mass than the plane wave solutions. Bag configurations do not exist for $g < 2.3$. The range of the parameter space for which bag configurations do exist is depicted in Fig. 4. The curve in the lower right corner of the graph corresponds to the values of g and m_H for which the bag is 1% lighter than the plane wave solution:

$$\frac{m_{\text{wave}} - m_{\text{bag}}}{m_{\text{wave}}} = 1\%. \quad (8)$$

The sliver of parameter space, in which the relative binding of the bag configurations goes from 1% to zero, then slightly unbound bags exist, and finally no bags can form, practically coincides with the location of the 1% line.

With these values in mind, we turn to the severe constraints imposed by the renormalization group on the values of the standard model's unknown parameters. One problematic situation involves the divergence of running Yukawa couplings, which indicates the appearance of a non-perturbative regime in the Higgs sector [7]. Divergence of the self-coupling λ would imply the triviality of the $\lambda\phi^4$ theory [8]. Also, large Yukawa couplings for fixed self-coupling yield an effective Higgs potential unbound from below, leading to vacuum instability [9]. These complications may be avoided by the appearance of new physics at a higher scale Λ . For various values of Λ the allowed regions in the (g, m_H) plane [10] lie within the curves enclosing the origin in Fig. 4. Our solutions clearly lie in a region for which new physics appears at a scale of order one TeV, where its contributions to the effective action become large.

The nearly vertical lines in Fig. 4 are the vacuum stability bounds, which are the most important for the parameter range of interest to us. The point here is that the scale Λ of new physics (to avoid vacuum instability with $\phi > \Lambda$) is equivalent to the energy scale E at which λ becomes negative. E can be derived from the renormalization group equations. The one-loop β -functions are dominated by the top quark contributions. We have [7, 10]:

$$\frac{dg}{d[\log(E/M)]} = \frac{9}{32\pi^2} g^3 \quad (9)$$

$$\frac{d\lambda}{d[\log(E/M)]} = -\frac{3}{4\pi^2} g^4. \quad (10)$$

The scale M , at which the running couplings take the values indicated in Fig. 4, is identified with the mass of the plane wave excitation: $M = m_{\text{wave}} = gv$. The running λ turns negative at an energy E satisfying:

$$\frac{E}{gv} = \exp \left[\frac{4\pi^2}{3} \frac{\lambda(M = gv)}{g^4(M = gv)} \right]. \quad (11)$$

For all points of the 1% line this happens for $1 \leq E/gv \leq 1.01$. The appearance of

new physics at such a low scale has serious implications for the form and even the existence of our solutions. Thus, results obtained in this framework are invalid.

In conclusion, we have considered the possibility of the deformation of the Higgs field around a heavy top quark, in the context of the standard model. By constructing explicit numerical solutions of the classical equations of motion we have demonstrated that this should not be expected, unless new physics with unknown effects appears at a scale very close to the top quark mass.

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5. Strictly speaking, neglecting the gauge fields is not allowable, since Gauss' law is a constraint, not an equation of motion. The essence of our assumption is that including the gauge fields will not alter significantly the features of our solutions.
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FIGURE CAPTIONS

- 1) A bag solution with $g = 3$, $m_H = 40$ GeV and $m_{bag} = 2.96v$. We plot the dimensionless, rescaled quantities: $\tilde{r} = rv$, $\tilde{\sigma} = \frac{\sigma}{v}$, $\tilde{\psi}_1 = \frac{\psi_1}{v/\sqrt{2}}$, $\tilde{\psi}_2 = \frac{\psi_2}{v/\sqrt{2}}$, and fermion density $\tilde{\rho} = \frac{\psi_1^2 + \psi_2^2}{v^2}$.
- 2) $\tilde{\sigma} = \frac{\sigma}{v}$ as a function of $\tilde{r} = rv$ for (a): $g = 3$, $m_H = 40$ GeV, $m_{bag} = 2.97v$, (b): $g = 4$, $m_H = 40$ GeV, $m_{bag} = 3.80v$, (c): $g = 5$, $m_H = 40$ GeV, $m_{bag} = 4.43v$ and (d): $g = 5$, $m_H = 400$ GeV, $m_{bag} = 4.95v$.
- 3) The bag mass m_{bag} and the plane wave mass m_{wave} as a function of the Yukawa coupling for $m_H = 40$ GeV.
- 4) Parameter space for bag solutions. Bags are (classically) energetically favored in the region to the right of the rightmost curve, with $\frac{m_{wave} - m_{bag}}{m_{wave}} > 1\%$. The

allowable parameter space (assuming various scales of new physics) lies within the curves enclosing the origin.

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4