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NLLjet:
A MONTE CARLO CODE FOR e^+e^- QCD JETS
INCLUDING NEXT-TO-LEADING-ORDER TERMS

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ABSTRACT

NLLjet, a Monte Carlo generator for the parton shower in e^+e^- annihilation has been developed. Since it includes the next-to-leading-order contribution, it has a potential to determine the QCD Λ parameter by analysing e^+e^- jets.

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1. INTRODUCTION

The study of hadron jets has many aspects, but the most important goal is to check that it is described by QCD and to determine the fundamental parameters of QCD. The quark mass and the parameter for QCD-vacuum (θ -vacuum) do not play any important role in high-energy perturbative QCD [1]. There is only one parameter: the coupling constant of QCD, $\alpha_s = g^2/4\pi$, or equivalently the QCD Λ parameter.

The standard strategy for simulating hadronic events in the e^+e^- annihilation is divided into two steps. First we generate a set of partons (quarks and gluons) according to the perturbative QCD calculation, and then we convert them into observable particles using a hadronization model. Although the whole process should, of course, be described by the QCD Lagrangian, the lack of enough knowledge regarding the solution of QCD in the soft sector forces us to employ a two-step treatment. For the moment, Q_0 denotes the typical energy scale for separating the two steps.

Although the first step has a reliable basis – the perturbative QCD – we need to approximate since we cannot sum all perturbative diagrams. There are two methods of doing this: i) the matrix element method (ME) and ii) with the parton shower (PS) [2]. In the ME method, we calculate the matrix element by means of conventional perturbation. Because of the increasing technical difficulty, the calculation is done only up to order α_s^2 (and tree calculation of order α_s^3). In PS, using the method based on the renormalization group equation, we sum up the contributions of dominant terms from all order diagrams. The PS that keeps only the leading terms is called PS-LL (leading logarithmic approximation), and the one that keeps up to next-to-leading terms is called PS-NLL (next-to-leading logarithmic approximation).

In order to determine the QCD Λ , the model must satisfy two criteria. The first is that the model be constructed on the basis of the approximation that is capable of determining Λ . The second requirement is that the effect of the second

step be minimized. To remove the dependence on hadronization, we take Q_0 as small as possible. In other words, Q_0 must be about 1 GeV, which is the scale of lower limit since the onset of the non-perturbative effect appears around 1 GeV.

According to the first criterion, PS-LL is not appropriate for the determination of QCD Λ , since in LL one cannot fix the scheme [1, 3]. The second criterion suggests that the ME method is not suitable since, Q_0 , which corresponds to the minimum invariant mass of a pair of partons in ME, is much larger than 1 GeV owing to the mass singularity. Around 1 GeV, the difference between the strong coupling constant calculated in the LL order form and that calculated in the NLL form is not negligible. This fact is also a shortcoming of PS-LL. Summarizing above the argument: among methods at hand, only PS-NLL has the potential for determining QCD Λ .

The development of the QCD parton shower has a long history. The first important work was the invention of the ‘Jet calculus’ by Konishi, Ukawa and Veneziano in 1979 [4]. Soon afterwards, Odorico and others formulated the method of PS-LL, and they also developed programs for generating parton shower in e^+e^- annihilation and in other processes [5]. In 1984, following the results of the study of the soft gluon, Marchesini and Webber introduced into PS-LL the concept of angular ordering [6]. This helped to answer the question of how to separate the overlapping phase spaces of evolving partons. Taking these developments into account, Sjöstrand and Bengtsson released the program LUND version 6 for e^+e^- annihilation, which incorporated ME and PS with their string fragmentation [7]. In LUND version 6, they also introduced into PS-LL the matrix element for $e^+e^- \rightarrow q\bar{q}g$, in order to reproduce the hard three-jet events. The introduction of the three-jet cross-section was unavoidable because the LL approximation is not correct in the large transverse momentum region. However, the theoretical status was not clear, since the three-jet cross-section is only a portion of the contribution in NLL order.

The theoretical framework for going beyond the LL was built by Kalinowski,

Konishi, Scharbach and Taylor in 1981 [8]. They also calculated the quantities in NLL for the quark sector. The calculation of those for the gluon sector was done by Gunion, Kalinowski and Szymanowski in 1984 [9], which meant that all parts were ready for the construction of a Monte Carlo program based on the PS-NLL method.

In 1986, authors had shown how to construct a Monte Carlo model for PS-NLL, which is not just a trivial assembly of existing parts but includes a detailed consideration of the kinematics [10,11]. In the conventional perturbative QCD analysis, unobserved partons are integrated out, whereas in the Monte Carlo approach we must devise a formulation for determining the momenta of all partons. Based on our formulation, we have developed a Monte Carlo program for e^+e^- jets, called NLLjet. It should be noted that this program generates only the four-momenta of partons and that it does not include the function for hadronization. As already noted, we expect that here the model-dependence of hadronization is minimized. Employing the information given in Appendices, one can easily generate $e^+e^- \rightarrow \text{hadrons}$, connecting NLLjet with a hadronization model.

The purpose of this paper is to show how our formulation is implemented in NLLjet. So we concentrate on the technical aspects and the kinematical details. Practical information for the user is also provided.

In the next section, our formulation for NLLjet is briefly reviewed. Sections 3 to 11 give a description of the parts of NLLjet presented in Section 2. Appendix A summarizes the basic notation used throughout this paper. Appendix B explains the program modules of NLLjet, and a description of common areas is given in Appendix C. The control parameters are explained in Appendix D, which is important for the user. In Appendix E, we show some examples.

2. FORMULATION of NLLjet

The formulation of the parton shower beyond the LL is presented in Ref. [8]. Using the off-shell unitarity equation of Polyakov for e^+e^- annihilation, these authors write all the matrix elements for $e^+e^- \rightarrow \text{partons}$ diagrammatically. The next step is to impose a projection operator on each propagator in order to separate the term with mass singularity from the finite term. Here the choice of gauge is essential. In a physical gauge, the appearance of a pair of the same propagators is responsible for the mass singularity. A diagram that contains an interference-type subdiagram gives less logarithms originating from mass singularity, and such a non-leading part is contracted into an effective vertex. After repeated application of the procedure, the remaining diagrams have the following property: the effective vertices are finite and the diagrams are diagonal, which allows us to simulate the parton shower as a stochastic branching process. The basic ingredients of the PS-NLL are the following four items.

1) The effective propagator in the NLL. This corresponds to the non-branching probability (non-emission probability) Π_{NE} , which is calculated by integrating the Altarelli-Parisi function in NLL order. The non-branching probability, i.e., the Sudakov factor [12], is given by

$$\Pi_{NE}^q(Q^2, Q_1^2) = \exp \left[- \int_{Q_1^2}^{Q^2} \frac{dz}{z} \int_{z_-}^{z_+} d\bar{x} \frac{\alpha_s(\bar{K}^2)}{2\pi} P_{qq}(x, K^2) \right] ,$$

$$\Pi_{NE}^g(Q^2, Q_1^2) = \exp \left\{ - \int_{Q_1^2}^{Q^2} \frac{dz}{z} \int_{z_-}^{z_+} d\bar{x} \frac{\alpha_s(\bar{K}^2)}{2\pi} \left[\frac{1}{2} P_{gg}(x, K^2) + \sum_q P_{gq}(x, K^2) \right] \right\} , \quad (2.1)$$

where kinematical boundary $z_{\pm}$ is given in Section 3. In our formulation, $\bar{K}^2$ is $\mathbf{p}_T^2 = x(1-x)K^2$ [14]. The Altarelli-Parisi function $P_{ab}(x)$ gives inclusive

probability for $a \rightarrow b + \text{anything}$. The explicit form in LL is [13]

$$\begin{aligned} P_{q\bar{q}}(z) &= C_F \frac{1+z^2}{1-z}, \\ P_{gq}(z) &= T_R [z^2 + (1-z)^2], \\ P_{gg}(z) &= 2C_A \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right]. \end{aligned} \quad (2.2)$$

Those in NLL order are more complex; they depend on K^2 and they are found in Ref. [10]. $\Pi_{NE}^A(Q^2, Q_1^2)$ ($a = q$ or g) gives the probability that a parton a of virtuality $= Q^2$ turns to one of virtuality $= Q_1^2$ without branching.

2) Two-body and three-body effective vertices. In the NLL, vertex functions for two-body branching, $V^{(2)}(x_1, x_2)$ ($x_1 + x_2 = 1$), include higher-order terms. The following channels exist:

$$\begin{aligned} q &\rightarrow q + g, \\ g &\rightarrow g + g, \\ g &\rightarrow q + \bar{q}. \end{aligned}$$

Vertex functions for three-body branching, $V^{(3)}(x_1, x_2, x_3)$ ($x_1 + x_2 + x_3 = 1$), generate new processes which are absent in the LL, i.e.

$$\begin{aligned} q &\rightarrow q + g + g, \\ q &\rightarrow q + q' + \bar{q}', \\ g &\rightarrow g + g + g, \\ g &\rightarrow g + q + \bar{q}. \end{aligned}$$

To denote the branchings $p(x=1) \rightarrow a(x_1) + b(x_2)$ and $p(x=1) \rightarrow a(x_1) + b(x_2) + c(x_3)$, we write $V_{ab}^{(2)}(x_1, x_2)$ and $V_{abc}^{(3)}(x_1, x_2, x_3)$ since the parton p is unique for a given set of daughter partons. The last argument is sometimes omitted in the following.

3) Effective vertices that couple to the electroweak current. In the present order, $q\bar{q}$ and $q\bar{q}g$ are produced as the primary objects. In the lowest order, the $q\bar{q}g$

cross-section is

$$\frac{d\sigma}{dz_1 dz_2} = \sigma_0 \frac{\alpha_s}{2\pi} C_F \frac{z_1^2 + z_2^2}{(1-z_1)(1-z_2)}, \quad (2.3)$$

where σ_0 is the lowest-order cross-section for $e^+e^- \rightarrow q\bar{q}$. We have also included the order α_s^2 cross-section calculated by Gottschalk and Shatz [15].

4) Kinematics. This includes the angular ordering [6, 14], the double-cascade scheme [11], separation of jets, choice of variables, and so on.

The evolution of the parton shower is recorded by a pair of variables (z, Q^2) in IMF; z is always conserved at each branching. The magnitude of p_T is determined by z and Q^2 through p^- momentum conservation. The rotational angle of p_T around the z -axis is denoted by ϕ . In NLLjet, there is no dynamical condition for ϕ , and it is determined at random after the evolution has finished. In the following, we sometimes denote the momentum of a parton as a triplet of variables (z, Q^2, p_T) .

First we generate partons at the primary vertex, which couples to the electroweak current in e^+e^- annihilation. There are two types of primary vertex: $q\bar{q}$ and $q\bar{q}g$. One of two channels is selected at random, as described in Section 5. In the $q\bar{q}$ primary vertex, both the quark and the antiquark get $z = 1$ and $p_T = 0$. In the $q\bar{q}g$ primary vertex, the light-cone fractions of three partons z_i and the transverse momentum of the quark p_T are determined according to a distribution. The transverse momentum of the antiquark is always 0 and that of the gluon is $-p_T$. The virtuality of a parton is not determined here. Details regarding these points will be found in Sections 5, 6, and 7.

After generation of the primary vertex, we develop the parton shower iteratively.

- a) There is a parton that has the potential to branch. It has a definite light-cone fraction but its virtuality is not yet determined. The virtuality of the parton is less than Q_{max}^2 by kinematical relations.

b) Using the Sudakov factor, we determine its virtuality Q^2 in the range of $Q_{\max}^2 > Q^2 > Q_0^2$. If $Q^2 < Q_0^2$, then it cannot branch, it gets virtuality $Q^2 = Q_0^2$, and it is marked inactive. If the determined virtuality is greater than Q_0^2 , then it can branch and will proceed to the next step.

c) Using the Altarelli-Parisi function $P(x)$, we determine the light-cone fraction, x ($x_- < x < x_+$), of the leading daughter parton. For the branching of a gluon, we first choose one of two channels, $g + \text{anything}$ or $q + \bar{q}$, by the ratio

$$\int_{x_-}^{x_+} dx \frac{\alpha_s(\bar{K}^2)}{2\pi} \frac{1}{2} P_{gg}(x) : \int_{x_-}^{x_+} dx \frac{\alpha_s(\bar{K}^2)}{2\pi} \sum_q P_{gq}(x) ,$$

and determine x .

d) In steps b) and c), Q^2 and x are determined in the phase space which is allowed kinematically. However, a part of the phase space is not allowed by the condition based on perturbative QCD. Here we check whether the condition holds or not. If the condition is broken, then we discard the branching, we set the maximum virtuality of the parton to be its temporal value Q^2 and we return to step b).

e) The parton now branches. The remaining job is to select the branching channels and, if the selected channel is a three-body one, we also determine the light-cone fraction of the second daughter parton. This step is carried out as follows. When the branching parton is a quark, there are three channels, $q + g$, $q + g + g$, and $q + q' + \bar{q}'$. The channel is selected by the ratio

$$V_{gg}^{(2)}(x_1) : \int V_{ggg}^{(3)}(x_1, x_2) dx_2 : \int V_{qqg}^{(3)}(x_1, x_2) dx_2 . \quad (2.4)$$

Actually the last channel is more complicated since there are cases for $q = q'$ and $q \neq q'$. If a three-body channel, e.g. the ggg -channel, is selected, then we determine x_2 using $V_{ggg}^{(3)}(x_1, x_2)$ fixing x_1 . When the branching parton

is a gluon and it branches into $g + \text{anything}$ in step b), there are three channels, $g + g$, $g + g + g$, and $g + q + \bar{q}$. The procedure is the same as that for a quark. For $g \rightarrow q\bar{q}$, it just enters into the mode. Now the branching has finished: the mother parton acquires its virtuality. Daughter partons have a definite light-cone fraction and they enter step a) iteratively.

The kinematics for two- and three-body branchings is given in Sections 3 and 4. Additional constraints in step d), i.e., the angular ordering and the double-cascade scheme, are described in Sections 8 and 9.

After the evolution, we determine the partons' four-momenta from their triplet variables (x, Q^2, p_T) . The determination of p^+ and p^- (and p_T^2 for $g\bar{q}g$) in CMF for partons at the primary vertex is based on equations in Sections 6 and 7. From these values, the p^+ , p^- , and p_T^2 of all remaining partons in CMF are determined by equations in Sections 3 and 4. The energy and the p_z are given by Eq. (A.2), and the rotational angle ϕ is selected at random.

Finally, since we develop the parton shower taking the z -axis as the direction of IMF, we rotate the system of partons according to the coupling of the primary quark to the electroweak current.

3. TWO-BODY BRANCHING

We consider the branching

$$p(x=1, Q^2, 0) \rightarrow p_1(x, Q_1^2, p_T) + p_2(1-x, Q_2^2, -p_T) .$$

Here we normalize the light-cone fractions by that of the mother parton p . In general, the transverse momentum of p is not zero. We can write the transverse momenta of p_1 and p_2 as

$$\begin{aligned} p_{T1} &= xp_T + k_T , \\ p_{T2} &= (1-x)p_T - k_T . \end{aligned} \quad (3.1)$$

The following equation does not depend on p_T , and the Eqs.(3.1) are used only when the four-momenta of partons are reconstructed. With the above understanding, the relative transverse momentum k_T is written as p_T in the rest of this section. The squared transverse momentum is obtained by

$$p_T^2 = x(1-x) \left(Q^2 - \frac{Q_1^2}{x} - \frac{Q_2^2}{1-x} \right). \quad (3.2)$$

This equation causes the phase spaces for p_1 and p_2 to overlap. The allowed region of x is as follows:

$$\begin{aligned} x_+ &> x > x_- , \\ x_{\pm} &= (1/2)(1 + \epsilon_1 - \epsilon_2 \pm \sqrt{D}) , \\ D &= 1 - 2(\epsilon_1 + \epsilon_2) + (\epsilon_1 - \epsilon_2)^2 . \end{aligned} \quad (3.3)$$

Here $D > 0$ is the condition for the existence of the allowed region:

$$\begin{aligned} q \rightarrow q + g & \quad 1 - 2(\epsilon_q + \epsilon_0) + (\epsilon_q - \epsilon_0)^2 > 0 , \\ g \rightarrow g + g & \quad 1 - 4\epsilon_0 > 0 , \\ g \rightarrow q + \bar{q} & \quad 1 - 4\epsilon_q > 0 , \end{aligned} \quad (3.4)$$

and

$$\begin{aligned} 2p_1 p_2 &= \frac{1-x}{x} Q_1^2 + \frac{x}{1-x} Q_2^2 + \frac{p_T^2}{x(1-x)} , \\ (p_1 + p_2)^2 &= \frac{Q_1^2}{x} + \frac{Q_2^2}{1-x} + \frac{p_T^2}{x(1-x)} , \end{aligned}$$

or

$$y_{12} = \frac{\epsilon_1}{x} + \frac{\epsilon_2}{1-x} + \frac{\kappa}{x(1-x)} . \quad (3.5)$$

The allowed maximum virtuality is

$$\begin{aligned} Q_{1 \max}^2 &= x \left(Q^2 - \frac{Q_2^2}{1-x} \right) \\ Q_{2 \max}^2 &= (1-x) \left(Q^2 - \frac{Q_1^2}{x} \right) . \end{aligned} \quad (3.6)$$

When the virtuality of the sister parton is not yet determined, it is taken to be Q_0^2 in order to calculate the allowed maximum virtuality.

4. THREE-BODY BRANCHING

We consider the branching

$$p(x=1, Q^2, 0) \rightarrow p_1(x_1, Q_1^2, p_{T1}) + p_2(x_2, Q_2^2, p_{T2}) + p_3(x_3, Q_3^2, p_{T3}) ,$$

where $x_1 + x_2 + x_3 = 1$ and $p_{T1} + p_{T2} + p_{T3} = 0$. Here we normalize the light-cone fractions by that of the mother parton p . In general, the transverse momentum of p is not zero. We can write the transverse momenta of p_i ($i = 1, 2, 3$) as

$$p_{Ti} = x_i p_T + k_{Ti} , \quad (4.1)$$

where $k_{T1} + k_{T2} + k_{T3} = 0$. The following equation does not depend on p_T , and Eq.(4.1) is used only when the four-momenta of partons are reconstructed. With the above understanding, the relative transverse momentum k_{Ti} is written as p_{Ti} in the rest of this section. We have

$$Q^2 = p^+ p^- = P \sum_i p_i^- = \sum_i \frac{Q_i^2 + p_{Ti}^2}{x_i} . \quad (4.2)$$

The allowed maximum virtuality is

$$Q_{i \max}^2 = x_i \left(Q^2 - \frac{Q_j^2}{x_j} - \frac{Q_k^2}{x_k} \right) \quad (i \neq j, k) . \quad (4.3)$$

When the virtuality of the sister parton is not yet determined, it is taken to be Q_0^2 in order to calculate the allowed maximum virtuality. On the other hand, fixing x_1 , the allowed range of x_2 is

$$\begin{aligned} x_2 &= y(1-x_1) , \\ x_3 &= 1 - x_1 - x_2 , \\ y_+ &> y > y_- , \\ y_{\pm} &= (1/2)(1 \pm \sqrt{D}) , \\ D &= 1 - \frac{4x_1 Q_{2 \min}^2}{(1-x_1)(x_1 Q^2 - Q_1^2)} . \end{aligned} \quad (4.4)$$

In Eq. (4.4), we use the fact that the minimum virtuality of p_2 is equal to that

of p_3 . Three-body branching is possible when the following condition holds:

$$x_1 Q^2 > Q_1^2 \text{ and } D > 0 \quad (4.5)$$

$$\begin{aligned} q \rightarrow q + g + g & \quad Q_1^2 \min = Q_q^2, \quad Q_2^2 \min = Q_0^2, \\ q \rightarrow q + \bar{q} + q & \quad Q_1^2 \min = Q_q^2, \quad Q_2^2 \min = Q_q^2, \\ g \rightarrow g + g + g & \quad Q_1^2 \min = Q_0^2, \quad Q_2^2 \min = Q_0^2, \\ g \rightarrow g + \bar{q} + q & \quad Q_1^2 \min = Q_q^2, \quad Q_2^2 \min = Q_q^2. \end{aligned} \quad (4.6)$$

In the case of two-body branching, the transverse momenta are determined by the virtuality and by the light-cone fraction [Eq.(3.2)], leaving a trivial freedom for the rotation around the z -axis. For three-body branching, we have only Eq.(4.2) and $p_{T1} + p_{T2} + p_{T3} = 0$. We determine p_{Ti} at random by the uniform distribution in the phase space, $\delta(p_{T1} + p_{T2} + p_{T3}) d^2 p_{T1} d^2 p_{T2} d^2 p_{T3}$. The phase space is proportional to

$$d\tilde{p}_{T1}^2 d\tilde{p}_{T2}^2 d\tilde{\phi}_1 d\tilde{\phi}_2 \quad (4.7)$$

We have the following relations:

$$\begin{aligned} p_{T1} &= (x_2 + x_3) \tilde{p}_{T1}, \\ p_{T2} &= -x_2 \tilde{p}_{T1} + \tilde{p}_{T2}, \\ p_{T3} &= -x_3 \tilde{p}_{T1} - \tilde{p}_{T2}, \end{aligned} \quad (4.8)$$

$$\tilde{Q}^2 = Q^2 - \sum_{i=1}^3 \frac{Q_i^2}{x_i} = \frac{x_2 + x_3}{x_1} \tilde{p}_{T1}^2 + \frac{x_2 + x_3}{x_2 x_3} \tilde{p}_{T2}^2. \quad (4.9)$$

We generate $\tilde{p}_{T1}^2$, $\tilde{\phi}_1$, and $\tilde{\phi}_2$ at random for $0 < \tilde{p}_{T1}^2 < [x_1/(x_2 + x_3)] \tilde{Q}^2$, and determine $\tilde{p}_{T2}^2$ using Eq. (4.9); we then determine p_{Ti} with Eq. (4.8).

5. PRIMARY VERTEX

The term ‘primary vertex’ stands for the process that initiates the parton shower in the e^+e^- annihilation. In NLL order, it is either $q\bar{q}$ or $q\bar{q}g$. Each parton at the primary vertex has the potential to cascade. The multiplicity of partons in a generated event has no relation to its type of primary vertex. In general, the event that starts from $q\bar{q}$ has large thrust and the one from $q\bar{q}g$ has small thrust. The mixture of $q\bar{q}$ and $q\bar{q}g$ can be determined in the next-to-leading order. There is a parameter δ to define these two types of primary vertex. Although intermediate quantities depend on δ , observables do not have a strong dependence on δ for the moderate variation of δ . This is because we consider *all* terms in NLL.

We write the differential cross-section for $e^+e^- \rightarrow q\bar{q}g$ as $d\sigma(z_1, z_2)/dz_1 dz_2$, where z_1 and z_2 are the energy fractions of the quark and the antiquark, respectively. We define the phase space $S(\delta)$, where the invariant mass of $q + g$ and of $\bar{q} + g$ is greater than δs :

$$S(\delta) = (1 - \delta > z_1 > 0 \text{ and } 1 - \delta > z_2 > 0) \quad (5.1)$$

The above equation assumes that partons are massless. To avoid double counting of the phase space, the virtuality of the quark and that of the antiquark in the $q\bar{q}$ primary vertex is restricted to be less than δs . The $q\bar{q}$ and $q\bar{q}g$ primary vertices are generated in proportion to the corresponding cross-sections.

$$\begin{aligned} \sigma(q\bar{q}) &= \sigma(\text{total}) - \sigma(q\bar{q}g), \\ \sigma(q\bar{q}g) &= \int_{S(\delta)} \frac{d\sigma(z_1, z_2)}{dz_1 dz_2} dz_1 dz_2. \end{aligned} \quad (5.2)$$

The part of the $q\bar{q}g$ cross-section outside $S(\delta)$, in which the mass singularity exists, is absorbed into the next-to-leading order of the Altarelli-Parisi function $P_{qg}^{(1)}$. The ratio $\sigma(q\bar{q}) : \sigma(q\bar{q}g)$ and the P_{qg} (or the Sudakov factor, which is the

integral of $P_{q\bar{q}}$ depend on δ , but the sum of both contributions does not depend on δ , at least in the inclusive limit. This is the old well-known fact: that the combination $C_n^{(1)} - 2\gamma_n^{(1)}/\beta_0$ is independent of the renormalization scheme. In practice, allowed window for δ is not large. If we take δ to be very small, $\sigma(q\bar{q}g)$ dominates over $\sigma(q\bar{q})$, which breaks the perturbation. If we take δ to be very large, most of generated events start from $q\bar{q}$, and the generated distribution is not good in the low-thrust region. Our standard value is $\delta = 0.1$.

6. THE $q\bar{q}$ PRIMARY VERTEX

We consider $e^+e^- \rightarrow q(p_1) + \bar{q}(p_2)$. The infinite momentum is determined as follows:

$$\eta = \frac{P}{W} = (1/2) \left[1 + \epsilon_1 - \epsilon_2 + \sqrt{1 - 2(\epsilon_1 + \epsilon_2) + (\epsilon_1 - \epsilon_2)^2} \right] . \quad (6.1)$$

It should be noted that $1 > \eta > 0$, and that $\eta = 1$ when $Q_2^2 = 0$. The inside of the square root in Eq. (6.1) is positive in the region bounded by a parabola that goes through $(\epsilon_1, \epsilon_2) = (0, 1)$, $(1/4, 1/4)$, $(1, 0)$. The four-momenta of p_1 and p_2 are

$$\begin{aligned} p_1^+ &= \eta W, & p_1^- &= (\epsilon_1/\eta)W, & p_{T1} &= 0, \\ p_2^+ &= (1-\eta)W, & p_2^- &= (1-\epsilon_1/\eta)W, & p_{T2} &= 0. \end{aligned} \quad (6.2)$$

As discussed in the previous section, there is restriction on virtuality as

$$\delta s > Q_1^2, Q_2^2. \quad (6.3)$$

If $\delta < 1/4$, the inside of the square root in Eq. (6.1) is positive.

7. THE $q\bar{q}g$ PRIMARY VERTEX

We consider $e^+e^- \rightarrow q(p_1) + \bar{q}(p_2) + g(p_3)$. The transverse momenta of p_1 and p_3 are p_T and $-p_T$. The infinite momentum is determined as follows:

$$\begin{aligned} \eta &= \frac{P}{W} = (1/2) \left[1 + y - \epsilon_2 + \sqrt{1 - 2(y + \epsilon_2) + (y - \epsilon_2)^2} \right], \\ y &= y_{13} = \frac{(p_1 + p_3)^2}{s} = \frac{\epsilon_1}{x_1} + \frac{\epsilon_3}{1-x_1} + \frac{\kappa}{x_1(1-x_1)}. \end{aligned} \quad (7.1)$$

It should be noted that $1 > y > 0$ and $1 > \eta > 0$, and that $\eta = 1$ when $Q_2^2 = 0$. The four-momenta of p_1 , p_2 , and p_3 are

$$\begin{aligned} p_1^+ &= x_1 \eta W, & p_1^- &= \frac{\epsilon_1 + \epsilon_3}{x_1 \eta} W, & p_{T1} &= p_T, \\ p_2^+ &= (1-\eta)W, & p_2^- &= (1-\frac{y}{\eta})W, & p_{T2} &= 0, \\ p_3^+ &= (1-x_1)\eta W, & p_3^- &= \frac{\epsilon_3 + \epsilon_1}{(1-x_1)\eta} W, & p_{T3} &= -p_T. \end{aligned} \quad (7.2)$$

For a special case, we take three partons that are massless:

$$\begin{aligned} Q_1^2 &= Q_2^2 = Q_3^2 = 0 & \eta &= 1, \\ p_1^+ &= xW, & p_1^- &= t(1-x)W, & p_{T1} &= p_T, & z_1 &= x + t(1-x), \\ p_2^+ &= 0, & p_2^- &= (1-t)W, & p_{T2} &= 0, & z_2 &= (1-t), \\ p_3^+ &= (1-x)W, & p_3^- &= txW, & p_{T3} &= -p_T, & z_3 &= 1-x+tx, \end{aligned} \quad (7.3)$$

where

$$x = x_1 \quad \text{and} \quad t = \frac{p_T^2}{x(1-x)}. \quad (7.4)$$

For the massless case there are following relations

$$\begin{aligned} z_1 + z_2 + z_3 &= 2, \\ y_{13} &= 1 - z_3 = t, \\ y_{23} &= 1 - z_1 = (1-x)(1-t). \end{aligned} \quad (7.5)$$

There is a delicate problem in connection with the $q\bar{q}g$ primary vertex. To generate this vertex we use the cross-section $d\sigma(z_1, z_2)$, which is calculated taking

q , $\bar{q}$, and g on-mass-shell. Instead of z_1 and z_2 , we can use x and t and determine them from $d\sigma(x, t)$. However, these three quanta have virtuality due to their cascade. In the triplet variables (x, Q^2, p_T) , the three partons have $(x, 0, p_T)$, $(1, 0, 0)$, and $(1 - x, 0, -p_T)$. The antiquark cascades backward to the quark, and the role of p^+ and p^- is reversed for the cascade on the antiquark side. Later, the virtualities have non-zero values. So there is ambiguity when we calculate their four-momenta, although it is of nature beyond the next-to-leading order. There are two ways of doing this.

a) Keep the invariant mass

From x and p_T we calculate y_{13} and y_{23} by using Eq.(7.5). From s , Q_1^2 , Q_2^2 , Q_3^2 , y_{13} , and y_{23} , we calculate four-momenta assuming that y is defined by Eq. (A.9). The transverse momentum of p_2 is 0 and p_T^2 is replaced by

$$\begin{aligned} p_{1x} &= (p_1 p_2 - E_1 E_2)/|p_2|, \\ p_{3x} &= (p_2 p_3 - E_2 E_3)/|p_2|, \\ p_T^2 &= |p_1^2| - p_{1x}^2 = |p_3^2| - p_{3x}^2. \end{aligned} \quad (7.6)$$

b) Keep the angle

From x and p_T we calculate the fraction z_i by using Eq.(7.3). We assume that z_i equals to the momentum fraction. We use the relation in the massless case to determine the angles between the partons, i.e. $\cos \theta_{ij} = 1 - 2(z_i + z_j - 1)/(z_i z_j)$. Next, solving the equation $W = \sum [Q_i^2 + \alpha(z_i W/2)^2]$ on α , the absolute value $|p_i|$ of the space momentum of p_i is given by $\sqrt{\alpha z_i W/2}$. From the $\cos \theta_{ij}$ and $|p_i|$ we can calculate the four-momenta assuming that the transverse momentum of p_2 is 0.

We use method a) in NLLjet as a default. However, method b) and a 'mixed' method of a) and b) are available as options.

Finally, it should be noted that at every step a check must be made to ascertain that the inside of the square root in Eq. (7.1) is positive.

8. ANGULAR ORDERING

Our scheme of angular ordering is fully described in Ref. [10]. It is the higher-order effect, and we can show it explicitly through the study in NLL order. In some codes for the parton shower, the angular ordering is introduced for every branching. As shown in Ref. [10], this is not the correct way. Our angular ordering originates from the analysis of three-body vertices: a) $g(p) \rightarrow g(p_1) + g(p_2) + g(p_3)$, and b) $q(p) \rightarrow q(p_1) + g(p_2) + g(p_3)$. The light-cone fraction is normalized so that $x = 1$ for p .

For quark branching, there is no restriction coming from angular ordering. We consider a branching of a gluon of momentum p_{23} , which corresponds to $p_2 + p_3$. When the mother of the gluon p_{23} is a gluon, the vertex a) is applied, and when it is a quark, the vertex b) is applied. As described in Section 2, the condition is applied after we determine the virtuality of p_{23} , i.e. Q_{23}^2 , and the light-cone fraction of p_2 and p_3 , i.e. x_2 and x_3 , in the conventional way.

a) Mother parton is a gluon:

$$\begin{aligned} x_1 &> x_2, x_3 \\ Q_{23}^2 &< Q^2 \frac{x_1 x_2 x_3}{x_1 x_2 + x_2 x_3 + x_3 x_1} [1 + (3/2)R \log R], \\ R &= \frac{x_2 x_3}{(x_2 + x_3)^2}. \end{aligned} \quad (8.1)$$

It should be noted that when the first inequality does not hold, there is no restriction.

b) Mother parton is a quark:

$$\begin{aligned} Q_{23}^2 &< Q^2 \frac{x_2 x_3}{x_1 x_2 + x_2 x_3 + x_3 x_1} [1 + (3/2)R \log R], \\ R &= \frac{x_2 x_3}{(x_2 + x_3)^2}. \end{aligned} \quad (8.2)$$

9. DOUBLE-CASCADE SCHEME AND THE SEPARATION OF JETS

As described in Ref. [11], we use the 'double-cascade scheme' to improve the symmetry between quark jets and antiquark jets. One can observe that Eqs. (7.3), (7.4) and (7.5) are invariant under the exchange of t and $1-x$ when t is small. Based on this observation, the phase space parametrized by x and t is divided into $1-x > t$ and $1-x < t$. The first part corresponds to jets on the quark side, the second part corresponds to jets on the antiquark side. Using the symmetry, the role of $1-x$ and t is interchanged in antiquark jets. Then

$$t < 1-x \quad (9.1)$$

is required at every branching. This works for separating partons in a quark jet and those in an antiquark jet in an event that originates from primary $q\bar{q}$. For primary $q\bar{q}g$ jets there is no explicit condition. However, constraint on the phase space in $q\bar{q}g$ jets is effective only beyond the next-to-leading order. On the other hand, the introduction of a condition for separating each jet seems to be natural. So we assume that Eq. (9.1) or an equivalent expression should hold at every branching.

The practical implementation of Eq. (9.1) is not simple. We consider partons of momentum k , p_i , p_j , and so on. The parton to be tested by a condition is the mother parton of parton k , i.e. the tested branching is the one that has produced the parton k and its sister partons. We assume that p_i is inside the jet that contains k , and that p_j is outside. There are many equations that are equivalent to Eq. (9.1) in the massless limit:

$$\begin{aligned} (A1) \quad & kp_i < kp_j, \\ (A2) \quad & (k+p_i)^2 < (k+p_j)^2, \\ (B1) \quad & kp_i/E_i < kp_j/E_j, \\ (B2) \quad & 1 - \cos\theta_i < 1 - \cos\theta_j, \end{aligned} \quad (9.2)$$

where (A1) and (A2) are the conditions for separation by invariant mass, and

(B1) and (B2) are for separation by angle. When k , p_i , and p_j are massless, (A1) and (A2) are the same, as are (B1) and (B2). In the massless case, we call them simply (A) and (B).

a) In $q\bar{q}$ primary jets; direct branching.

The word 'direct' means the branching of a parton that belongs to the primary vertex. We consider $e^+e^- \rightarrow q(p_1) + \bar{q}(p_2)$ and $q(p_1) \rightarrow q(k = (x, K^2, p_T)) + g(p_3 = (1-x, Q_3^2, -p_T))$. The branching of p_2 is treated in the same way. The inside parton is p_3 or p_1 , and the outside parton is p_2 . By using the formulae in Section 6, we get

$$\begin{aligned} 2kp_3/s &= \frac{1-x}{x}\epsilon_k + \frac{x}{1-x}\epsilon_3 + \frac{\kappa}{x(1-x)}, \\ 2kp_2/s &= x\eta(1-\frac{\epsilon_1}{\eta}) + (1-\eta)\frac{\epsilon_k+\kappa}{x\eta}, \\ \epsilon_1 &= \frac{\epsilon_k}{x} + \frac{\epsilon_3}{1-x} + \frac{\kappa}{x(1-x)}. \end{aligned} \quad (9.3)$$

The last one comes from Eq. (3.5). The interchange of a quark (k) and a gluon (p_3) gives us an equation similar to that obtained by replacing x with $1-x$.

If we take them as being massless, i.e. $K^2 = Q_3^2 = Q_2^2 = 0$, we have

$$(A), (B) \Rightarrow \frac{\kappa}{x(1-x)} < \frac{x}{1+x}, \frac{1-x}{2-x}. \quad (9.4)$$

For $x \approx 1$, the second relation reduces to Eq. (9.1), which is identical to the condition for separating a gluon from an antiquark in this limit. Since $(1-x)/(2-x) < 1-x$, Eq. (9.4) is stronger condition than Eq. (9.1).

b) In $q\bar{q}$ primary jets; general.

We consider a parton $k = (xx_m, K^2, k_T)$, which belongs to the cascade of the quark (p_1). The k 's sister parton, $k' = ((1-x)x_m, K'^2, k_T')$ is also considered implicitly as we exchange x and $1-x$. Here x_m is the light-cone fraction of

the k 's mother parton, and in the rest of this section z is the relative light-cone fraction. We consider the separation with respect to $p_1(\text{in})$ and $p_2(\text{out})$. Here κ is the normalized k_T^2 in Eq.(A.8).

$$\begin{aligned} 2kp_1/s &= xx_m\epsilon_1 + \frac{\epsilon_k + \kappa}{xx_m}, \\ 2kp_2/s &= xx_m\eta\left(1 - \frac{\epsilon_1}{\eta}\right) + (1 - \eta)\frac{\epsilon_k + \kappa}{xx_m\eta}. \end{aligned} \quad (9.5)$$

In order to compare the results in subsection (a), we take $\epsilon_k = 0$ and $\eta = 1$ for case (A):

$$\epsilon_m < \frac{z}{1-z}x_m^2(1-2\epsilon_1) < \frac{z}{1-z}, \quad (9.6)$$

where $\epsilon_m = x(1-x)\kappa$ is the normalized virtuality of the mother parton. As $z \approx 0$, Eq. (9.6) reduces to Eq. (9.1). We get a similar relation for a parton k that belongs to the cascade of an antiquark after the exchange of $1-z$ and t .

c) In $q\bar{q}g$ primary jets; general.

We consider $e^+e^- \rightarrow q(p_1) + \bar{q}(p_2) + g(p_3)$ and a parton $k = (xx_m, K^2, k_T)$ (and its sister parton). As in the previous subsection, we take $\epsilon_k = 0$. Using the equations in Section 7, we have

$$\begin{aligned} 2kp_1/s &= \eta^2 x_q \frac{1-x}{x_m} \epsilon_m + \frac{\epsilon_1 + \kappa}{x_q} xx_m - 2k_T p_T / s, \\ 2kp_2/s &= (1-\eta)\eta \frac{1-x}{x_m} \epsilon_m + (1-y/\eta)\eta xx_m, \end{aligned} \quad (9.7)$$

where x_q is the light-cone fraction of the quark at the primary vertex. We neglect the last term in the first equation of Eq. (9.7). We consider the separation with respect to p_1 and p_2 , taking k as belonging to the cascade of quarks or antiquarks. We get

$$\begin{aligned} k \text{ in quarks: } \quad \epsilon_m &< \frac{z}{1-z}x_m^2 \frac{1}{x_q} \left[1 - \frac{2\epsilon_1}{x_1} - \frac{(2-x_q)\kappa}{x_q(1-x_q)} \right], \\ k \text{ in antiquarks: } \quad \epsilon_m &> \frac{z}{1-z}x_m^2 \left[1 + \frac{\kappa(2-x_q)}{\eta x_q(1-x_q)} \right]. \end{aligned} \quad (9.8)$$

Using these relations, we define the effective s as shown below.

We summarize the results of this section. The condition of the double-cascade scheme is

$$(Q^2/s) < 1 - z \quad (9.9)$$

for the branching $q(x=1, Q^2) \rightarrow q(x) + \text{anything}$. If the event starts from the $q\bar{q}g$ primary vertex, then replace s in Eq. (9.9) by s' , where s' is given by

$$\begin{aligned} \text{from quarks: } s' &= s \left(x_q + \frac{p_T^2}{x_q s} \right), \\ \text{from antiquarks: } s' &= s \left[1 - \frac{p_T^2}{x_q(1-x_q)s} \right]. \end{aligned} \quad (9.10)$$

The angular separation condition is stronger than the condition imposed by the double-cascade scheme. We do not apply Eq. (9.9) for the branching of gluons since the angular-ordering condition in Section 8 is already imposed for them. For the branching of partons that belong to the primary vertex, we further impose the condition of the angular separation in order to confirm the separation of the jets. This is because the double-cascade scheme gives a weaker condition than that given by the angular separation, and because it does not work very well for massive quarks or for the highly off-mass-shell case. For the $q\bar{q}$ case, we require that

$$|k| < |p_2| \quad (9.11)$$

in the notation of subsection (a), and for the $q\bar{q}g$ case we require (B1) in Eq. (9.2).

10. DETERMINATION OF MOMENTA

As described in Section 2, we generate four-momenta from their triplet variables (z, Q^2, p_T) . The direction of p^+ defines the IMF and is related to the selection of the gauge vector. In our formulation, the vector of the light-cone gauge is parallel to the momentum of the primary antiquark, and the antiquark

does not radiate a gluon. The introduction of the double-cascade scheme and the existence of hard three-jets modify this situation. The relations in Sections 3 and 4 hold for a specific direction of p^+ . Using these relations, we determine p^+ and p_T , and then determine p^- ; E and p_z are determined by p^+ and p^- . We have determined four-momenta of partons as if p^+ points to the positive z -axis. After that, we rotate the partons according to the direction of the primary parton from which it originates. The polar angle of a primary parton is defined as follows:

$$\begin{aligned} q & \quad \theta = 0 \quad , \\ \bar{q} & \quad \theta = \pi \quad ; \end{aligned} \quad (10.1)$$

b) $q\bar{q}$

$$\begin{aligned} q & \quad \theta = 0 \quad , \\ \bar{q} & \quad \theta = \pi \quad , \\ g & \quad 0 \text{ for } (p_1 + p_3)^2 < (p_2 + p_3)^2, \pi \text{ for } (p_1 + p_3)^2 > (p_2 + p_3)^2 \end{aligned} \quad (10.2)$$

where p_1 , p_2 , and p_3 are the momenta of a quark, of an antiquark, and of a gluon, respectively.

For the $q\bar{q}$ case, a) is the only possible choice. However, for the $q\bar{q}g$ case, we can define a different scheme b').

$$\text{b') } q\bar{q}g \quad \theta_i = \text{polar angle of } p_i \quad . \quad (10.3)$$

In NLLjet the default method is b'), but b) is also available as an option. We would like to stress that the difference between b) and b') is beyond NLL, because a part of the kinematics in the last section is also beyond it.

11. MASS OF THE QUARK

Proper treatment of the quark mass m is beyond the validity of the present formulation since it is constructed assuming that the quark is massless. However, we must somehow take it into account. As is suggested in the string picture, the probability of the pair creation of quarks from a vacuum is suppressed as $\sim \exp(-cm^2)$. If we neglect the mass of the quark in the generation of partons, we have trouble with their hadronization when heavy flavour is included. To incorporate these observations we have introduced m into NLLjet. The uses of m are listed below. Some of them are very natural conditions, some are essential in order not to break the kinematics, but some are only experimental relations and need further elaboration.

a) Primary vertex

We reject any configuration that is not allowed kinematically when we take the quark mass into account.

b) Modification of Q_0^2 .

$$Q_0^2 = Q_0^2 + m^2 \quad . \quad (11.1)$$

Since the minimum of Q^2 is changed, it affects the evolution of partons in many steps.

c) Modification of δ .

$$\delta = \max \text{ of } (\delta, 2\sqrt{Q_0^2/W}) \quad . \quad (11.2)$$

When a radiative correction is made, this relation has to be tested for each event since W is not constant.

d) Double-cascade

We modify the condition Eq. (9.9) as

$$(Q^2/s) - (m/W) < 1 - x \quad , \quad (11.3)$$

where $W = \sqrt{s}$ for $q\bar{q}$ and $W = \sqrt{s}$ for $q\bar{q}g$.

12. e^+e^- PHYSICS

In order to study experimental data in e^+e^- reactions and to determine QCD Λ , a hadronization model must be linked with NLLjet. Analyses of MARK II data at $W = 29$ GeV and other sets of data have been done by Kamae and Shirahashi using the LUND string fragmentation [16]. They get $\Lambda_{\overline{MS}} = 235 \pm 52$ MeV. Analysis of VENUS data at TRISTAN has also been made using EPOCS hadronization [17], and a value of $\Lambda_{\overline{MS}} = 254^{+55}_{-47} \pm 56$ (sys.) MeV was obtained [18].

The radiative correction for e^+e^- initial-state radiation is important. NLLjet includes a function of the correction based on a formula similar to that of Berends, Kleiss and Jadach [19]. However, modules for the hard-photon emission are designed on the assumption that the energy dependence of the hadronic cross-section is moderate. This assumption is allowed in the PEP/PETRA/TRISTAN region and below the Z^0 pole. In the SLC/LEP region the generation of the hard photon is not efficient so that it is recommended to switch off the flag for the radiative correction.

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Appendix A. BASIC NOTATION

We use two coordinate frames. The first one is the infinite momentum frame (IMF) and the second one is the centre-of-mass frame (CMF). The evolution of the parton shower is done in the IMF. The direction of boost is the direction of the positive z -axis, and the transverse plane is the x - y plane:

$$s = q^2 = W^2 = (\text{squared c.m. energy}) , \quad (A.1)$$

$$p^+ = E + p_z ,$$

$$p^- = E - p_z ,$$

$$E = (p^+ + p^-)/2 , \quad (A.2)$$

$$p_z = (p^+ - p^-)/2 ,$$

$$Q^2 = (\text{virtuality}) ,$$

$$Q_0^2 = (\text{minimum virtuality or minimum virtuality of the gluon}) , \quad (A.3)$$

$$Q_q^2 = (\text{minimum virtuality of the quark}) ,$$

$$\mathbf{k}_T, \mathbf{p}_T = (\text{transverse momentum}) , \quad (A.4)$$

$$\bar{p}^- = \frac{Q^2 + \mathbf{p}_T^2}{p^+} ,$$

$$p^2 = p^+ p^- - \mathbf{p}_T^2 , \quad (A.5)$$

$$p \cdot p' = (1/2)(p^+ p'^- + p^- p'^+) - \mathbf{p}_T \mathbf{p}'_T ,$$

$$z = \frac{p^+}{P} = (\text{light-cone fraction in IMF}) , \quad (A.6)$$

$$P = (\text{infinite momentum}) .$$

Now we define some dimensionless quantities:

$$\epsilon = \frac{Q^2}{s} , \quad (A.7)$$

which are used as ϵ_0 , ϵ_q , and ϵ_k for Q_0^2 , Q_q^2 and K^2 , respectively, and as ϵ_i for

a momentum p_i :

$$\kappa = \frac{p_T^2}{s} \text{ or } \frac{k_T^2}{s}, \quad (\text{A.8})$$

which may have subscripts similar to that of ϵ . Also $\kappa_{ij} = p_{iT} \cdot p_{jT}/s$, and similar notation is used:

$$y_{ij} = \frac{(p_i + p_j)^2}{s}, \quad (\text{A.9})$$

$$z_i = \frac{2E_i}{W} = (\text{energy fraction in CMF}) \quad (\text{A.10})$$

Finally, we define colour factors for $N_{colour} = 3$.

$$\begin{aligned} C_F &= 4/3 & T_a T_a &= C_F \cdot I, \\ C_A &= 3 & f_{abcd} &= C_A \delta_{cd}, \\ T_R &= 1/2 & T_r(T_a T_b) &= T_R \delta_{ab}. \end{aligned} \quad (\text{A.11})$$

Appendix B. LIST OF SUBROUTINES

In this Appendix, we present the modules in NLLjet, with a short explanation of each function. The first line shows the SUBROUTINE statement or the FUNCTION statement, which appears just as the first (non-comment) statement of the module in the NLLjet program. For an argument, we show its name in typewriter fonts and sometimes add a tag (*in*) and/or (*out*) to specify that the argument is referred and/or defined in the module. Modules and variables used in the program are also shown in typewriter fonts.

Before the generation of events, two routines, QCINPT and QCPREP, are to be called for the input of parameters and the initialization. After that, QCGEN is called repeatedly to generate events. Samples of main program are found in Appendix E.

SUBROUTINE QCINPT(WBEAM,NEVEN,NRADC,NPRNT)

reads the control parameters for NLLjet from the file assigned to unit FT05F001. WBEAM(*out*) is the centre-of-mass energy in GeV, NEVEN(*out*) is the number of events to be generated, NRADC(*out*) is a flag for radiative correction (NRADC=IRC in input parameters), and NPRNT(*out*) is the interval to print the list of generated events.

BLOCK DATA QCDATA

sets default values of parameters that are to be read in QCINPT.

SUBROUTINE QCPREP

controls the prologue procedure for NLLjet. It calculates the variables in common areas and calls the modules for calculating the table of the Sudakov factor and those for calculating the cross-section for the primary vertex. In order to save computer time, the table and the cross-section may be read from a file if they have already been calculated and saved in the file.

SUBROUTINE QCOOT (NEVE, NCHK, WCNS)

prints the four-vectors of partons (PARTN) and the status of partons (LISTP) in the generated event. If NCHK(IN)=1, the momentum conservation is examined, assuming that the event is generated in the centre-of-mass system with the total energy of WCNS(IN), NEVE(IN) is the sequential number of the event in the present generation.

SUBROUTINE QCROT

The parton shower evolves taking the z-axis as the direction of the IMF. In this routine, four-momenta of generated partons are rotated using the distribution calculated by the coupling of the quark to the electroweak current.

SUBROUTINE TBFND (RS, QN, GN, NWEAKX)

calculates the parameters for the angular distribution of two-jets. These parameters are used in QCROT.

REAL FUNCTION FJET2 (COST, NF, ITOT)

gives the differential or the total cross-section of two-jets; NF specifies the flavour. When ITOT=0, FJET2 = $(1/\sigma_{\mu\mu})d\sigma/d\Omega$ for $\cos\theta = \text{COST}$, and when ITOT=1, FJET2 = $(1/\sigma_{\mu\mu})\sigma$.

REAL FUNCTION FJET3 (X, NF, NOK)

gives the differential cross-section of three-jets; NF specifies the flavour; FJET3 = $(1 - z_1)(1 - z_2)d\sigma/dz_1 dz_2 d\Omega_1 d\Omega_2$ in unit of $\sigma_{\mu\mu}(\alpha_s/\pi)(3/8\pi^2)$.

SUBROUTINE TBPRV

calculates the fraction of primary two-jets, S2MOD in /PRVTX/.

SUBROUTINE JR3IST (DELT, G0)

calculates the fraction of primary three-jets of order α_s by integrating the differential cross-section in leading order.

SUBROUTINE JR3ND (DELTA, WL, G1, GR1)

calculates the fraction of primary three-jets of order α_s^2 and the average of $y_1 y_2$ by interpolating the numbers that have been calculated numerically for some $\delta = \text{DELTA}(\text{IN})$.

SUBROUTINE INITA

is responsible for the following initialization. It prepares coefficients for the calculation of the strong coupling constant $\alpha_s(Q^2)$. It prepares the array for numerical integration by the Simpson method, which is used in PNEXX to calculate the Sudakov-factor table. Also it checks the mass of the quarks in order to save CPU time.

SUBROUTINE TBOUT

is a service routine to print the Sudakov-factor table.

SUBROUTINE TBPNE

The table of non-branching probability, i.e. the Sudakov factor, is calculated in this subroutine using subroutines PNEXX, which are explained in the following. The tables are stored in /TPNEXX/.

SUBROUTINE PNE1A (QQ0, QQ2, NF, PNE)

calculates the leading part of the Sudakov factor for $q \rightarrow q + X$; Q^2 integration from $Q_0^2(=QQ0)$ to $Q^2(=QQ2)$ is done analytically.

SUBROUTINE PNE1B (QQ1, QQ2, NF, PNE)

calculates the sub-leading part of the Sudakov factor for $q \rightarrow q + X$ by numerical integration from $Q_1^2(=QQ1)$ to $Q_2^2(=QQ2)$.

SUBROUTINE PNE2A (QQ0, QQ2, PNE)

calculates the leading part of the Sudakov factor for $g \rightarrow g + X$; Q^2 integration from $Q_0^2(=QQ0)$ to $Q^2(=QQ2)$ is done analytically.

SUBROUTINE PNE2B(QQ1,QQ2,PNE)

calculates the sub-leading part of the Sudakov factor for $g \rightarrow g + X$ by numerical integration from $Q_1^2(=QQ1)$ to $Q_2^2(=QQ2)$.

SUBROUTINE PNE3C(QQ1,QQ2,NFLAV,PNE)

calculates the Sudakov factor for $g \rightarrow g + \bar{q}$ by numerical integration from $Q_1^2(=QQ1)$ to $Q_2^2(=QQ2)$.

SUBROUTINE PHELL

The above subroutines **PNE1X** use the fact that the argument of α_s is $p_T^2 = x(1-x)Q^2$. This calculates the Sudakov factor in the leading order for $\alpha_s(Q^2)$.

SUBROUTINE QCGEN(IFLAV)

controls the generation of one event. The meaning of **IFLAV(in)** is explained in **GENPRV**. First, it resets the common area /QCEVNT/ that records the generated event, and calls **GENPRV** to create primary partons. It iteratively calls **GENQBR** and **GENGBR**, which process the branching of the quark and of the gluon, respectively. After the evolution ends, the four-momenta of the partons are determined: this subroutine calls **CMSPRI** in order to register ancestors and calculates the four-momenta of partons at the primary vertex by **CMSPRV**. Here the direction of the primary partons is recorded in /CMSANG/, which forces them to forward the positive z-axis; **CMS2BD** and **CMS3BD** are called iteratively to determine the four-momenta of partons at each two- and three-body branching. The direction of the partons is recovered by **CMSROT**. Finally, the event is rotated by **QCROT** according to the coupling of the quark to the electroweak current.

SUBROUTINE GENPRV(IFLAV)

determines the type of primary vertex and generates either the $q\bar{q}$ state or the $q\bar{q}g$ state. If **IFLAV(in)** is zero, the quark flavour is selected according to the R-ratio. Otherwise, events with flavour=**IFLAV** are always generated.

SUBROUTINE GENQBR(NQUA)

NQUA(in) is the address of the quark to be processed in this subroutine. The quark is tested for branching or for no branching. If it branches, then its virtuality, the mode of branching, and the light-cone fractions (x) of the daughter partons are determined.

SUBROUTINE GENGBR(NGLU)

NGLU(in) is the address of the gluon to be processed in this subroutine. The gluon is tested for branching or for no branching. If it branches, then its virtuality, the mode of branching, and the light-cone fractions (x) of the daughter partons are determined.

SUBROUTINE PRVQQG(X,PT,NF,NCND)

determines the $x(=X(out))$ and $|p_T|(=PT(out))$ for primary three-jets of flavour=**NF(in)**. When **NCND=1**, this means that the trial in this routine has failed.

SUBROUTINE CMSPRV(WERROR)

determines the four-vectors of the primary partons from a given virtual mass squared and the fractions z of light-cone variables. It returns with **WERROR=1** if the kinematics for three-partons is broken. After recording the direction of the partons (Section 10) in /CMSANG/, all the primary partons are rotated to evolve along the positive z-axis. See Sections 6 and 7.

SUBROUTINE CMS2BD(WADR)

determines the four-vectors of the daughter partons in two-body branching from the four-vectors of their mother parton, and the virtualities and z of the daughter partons; **WADR(in)** is the address of the mother parton. Variables that cannot be fixed—namely, azimuthal angles—are chosen at random. See Section 3.

SUBROUTINE CMS3BD(NADR)

determines the four-vectors of daughter partons in three-body branching from the four-vectors of their mother parton, and the virtualities and z of the daughter partons; $NADR(in)$ is the address of the mother parton. Variables that cannot be fixed are chosen at random. See Section 4.

SUBROUTINE CMSPRI

By using of LISTP(5,), this subroutine registers the ancestor of each parton. The ancestor is a parton that is produced at the primary vertex. Thus LISTP(5,) = 2 or 3 (q or $\bar{q}$) for an event which starts from a two-jet primary vertex, and LISTP(5,) = 2, 3, or 4 (q , $\bar{q}$, or g) for one that starts from a three-jet primary vertex.

SUBROUTINE CMSROT

According to /CMSANG/, the direction of the partons is restored in this subroutine.

SUBROUTINE LIMAO(X3,RR,Z,MOTHER,NOK)

checks whether the branching is valid with respect to the condition of angular ordering; $NOK(out)=1(0)$ means for that it is accepted (not accepted). See Section 8.

SUBROUTINE LIMDC(NQUA,NTD,K2,Z,NBR)

checks whether the branching is valid with respect to the condition for the double-cascade scheme; $NBR(out)=1(0)$ stands for that it is accepted (not accepted). See Section 9.

SUBROUTINE LIM3(KCASE,NADD,NBR,Q2,X)

checks whether the generated primary three-jet system is consistent with the kinematical condition; $NBR(out)=1(0)$ means that it is accepted (not accepted). When $KCASE(in)=0$, the test is done for the three partons that belong to the primary vertex. When $KCASE=1$, the test is done for the branching of one of the three partons in the primary vertex. See Section 9.

SUBROUTINE LIM2(NADD,Q2,Z,NBR)

checks whether the generated primary two-jet system is consistent with the kinematical condition. The test is done for the branching of a parton at the two-jet primary vertex. See Section 9.

SUBROUTINE LIMBR(NADD,Q2MAX)

determines the maximum virtuality $Q2MAX(out)$ in the branching of a parton at address $NADD(in)$. See Eqs. (3.6), (4.3), and (6.3).

SUBROUTINE CHKN1(S,Y12,Y23,Q12,Q22,Q33,NCND)

tests whether the three-parton state does not break the kinematics; $NCND(out)=0$ means that the kinematics is satisfied. See item (a) in Section 7.

REAL FUNCTION EFF3JT(YA,YB,YCUT,ALPHA,NL,NSL)

gives the differential cross-section of three-jets up to the order α_s^2 .

REAL FUNCTION EF3JTF(YA,YB,YCUT0,ALPHA)

gives the non-singular contribution of the order α_s^2 differential cross-section of three-jets.

FUNCTION EF3JCT, EF3JAO, EF3JH, EF3JK, EF3JQ0, EF3JFC, EF3JFW

These seven functions are parts of EF3JTF.

REAL FUNCTION APQX(Z,K2)

is the Altarelli-Parisi function for the inclusive decay of a quark of a light-cone fraction $z(=Z(in))$ and a virtuality $Q^2(=K2(in))$.

REAL FUNCTION APQX1(Z)

gives the value of the second-order Altarelli-Parisi function for a quark.

REAL FUNCTION APQX1A(Z)

gives the leading term of APQX1 around $z = 1$.

REAL FUNCTION APMX(Z,K2)

is the Altarelli-Parisi function for the inclusive decay of a gluon of a light-cone fraction $z(=Z(in))$ and a virtuality $Q^2(=K2(in))$.

REAL FUNCTION APMQ(Z,K2)

is the Altarelli-Parisi function for $g \rightarrow q\bar{q}$ for a gluon of a light-cone fraction $z(=Z(in))$ and a virtuality $Q^2(=K2(in))$.

SUBROUTINE HAPQX(K2,ZMIN,ZMAX,ALPH,Z)

determines a light-cone fraction $x(=Z(out))$ for $ZMIN(in) < x < ZMAX(in)$ at the branching of a quark with a virtual mass squared $K2(in)$; $ALPH(in)$ is the strong coupling constant.

SUBROUTINE HAPGX(K2,EPSL,ALPH,Z)

determines a light-cone fraction $x(=Z(out))$ for $EPSL(in) < x < 1-EPSL$ at the branching of a gluon with a virtual mass squared $K2(in)$; $ALPH(in)$ is the strong coupling constant.

SUBROUTINE HAPQQ(K2,EPSL,ALPH,Z)

determines a light-cone fraction $x(=Z(out))$ for $EPSL(in) < x < 1-EPSL$ at the branching of a gluon to $q\bar{q}$ with a virtual mass squared $K2(in)$. $ALPH(in)$ is the strong coupling constant.

REAL FUNCTION V3QGG(Z,Y)

gives the probability distribution of three-body branching $g \rightarrow q(x_1) + g(x_2) + g(1-x_1-x_2)$; $Z(in)=x_1$ and $Y(in)=x_2$.

REAL FUNCTION V3QQQ(Z,Y,LF)

gives the probability distribution of three-body branching $g \rightarrow q(x_1) + q'(x_2) + \bar{q}'(1-x_1-x_2)$; $Z(in)=x_1$ and $Y(in)=x_2$. When $LF(in)=1(2)$, $q \neq q'$ ($q = q'$).

REAL FUNCTION V3GGG(Z,Y)

gives the probability distribution of three-body branching $g \rightarrow g(x_1) + g(x_2) + g(1-x_1-x_2)$; $Z(in)=x_1$ and $Y(in)=x_2$.

REAL FUNCTION V3GQQ(Z,Y)

gives the probability distribution of three-body branching $g \rightarrow g(x_1) + q(x_2) + \bar{q}(1-x_1-x_2)$; $Z(in)=x_1$ and $Y(in)=x_2$.

REAL FUNCTION SV3QGG(Z,EPSL,ALPH)

gives the value of the integral of the three-body branching function V3QGG by x_2 fixing $x_1(=Z)$.

REAL FUNCTION SV3QQQ(Z,EPSL,ALPH,LF)

gives the value of the integral of the three-body branching function V3QQQ by x_2 fixing $x_1(=Z)$. The meaning of $LF(in)$ is the same as that in V3QQQ.

REAL FUNCTION SV3GGG(Z,EP,SL,ALPH)

gives the value of the integral of the three-body branching function V3GGG by z_2 fixing $z_1(=Z)$.

REAL FUNCTION SV3GQQ(Z,EP,SL,ALPH)

gives the value of the integral of the three-body branching function V3GQQ by z_2 fixing $z_1(=Z)$.

SUBROUTINE SV3GW, FJW, FJQ, FJY, FDF, FMF

These six functions are the parts of SV3GGG.

SUBROUTINE HV3QGG(Z,YMIN,Y)

determines the light-cone fraction $z_2(=Y(out))$ of a gluon for a fixed fraction $z_1(=Z(in))$ of a quark at the branching $q \rightarrow q(z_1) + g(z_2) + g(1 - z_1 - z_2)$.

SUBROUTINE HV3QQQ(Z,YMIN,Y,LF)

determines the light-cone fraction $z_2(=Y(out))$ of a quark for a fixed fraction $z_1(=Z(in))$ of a quark at the branching $q \rightarrow q(z_1) + q'(z_2) + \bar{q}'(1 - z_1 - z_2)$.

SUBROUTINE HV3GGG(Z,YMIN,Y)

determines the light-cone fraction $z_2(=Y(out))$ of a gluon for a fixed fraction $z_1(=Z(in))$ of a gluon at the branching $g \rightarrow g(z_1) + g(z_2) + g(1 - z_1 - z_2)$.

SUBROUTINE HV3GQQ(Z,YMIN,Y)

determines the light-cone fraction $z_2(=Y(out))$ of a quark for a fixed fraction $z_1(=Z(in))$ of a gluon at the branching $g \rightarrow g(z_1) + q(z_2) + \bar{q}(1 - z_1 - z_2)$.

REAL FUNCTION ALPS(QQ,MODE)

calculates the strong coupling constant $\alpha_s(Q^2)$ by iteration; $QQ(in)=Q^2$ and it gives α_s of leading (next-to-leading) order for $MODE(in)=0(1)$. Parameters in /COMAL1/ are set by INITA.

REAL FUNCTION ALPY(QQIN), ALPY(QQIN,Z), ALPD(QQIN)

These functions give the strong coupling constant at $Q^2=QQIN(in)$; ALPY is used as $\alpha_s(Q^2)$ in many routines; ALPY is the $\alpha_s(z(1-z)Q^2)$ for $z=Z(in)$; ALPD is the non-leading contribution of $\alpha_s(Q^2)$.

SUBROUTINE HBR1DM(PNE,TP,TMIN,TMAX,DELT,NTMX,T1,NBR)

determines virtuality at a branching according to the table of probabilities PNE(in). The virtuality is given by $\Lambda^2 \exp(T1(out))$.

SUBROUTINE INTRPL(PNE,T,TMIN,DELT,NTMX,P)

gives the interpolated value P(out) for T(in) using the table PNE(in).

REAL FUNCTION SPENCE(X), SPO(X), COMPLEX FUNCTION CSP(Z)

These functions give the Spence function, $sp(x) = -\int_0^x dt \log(1-t)/t$, for $z=X(in)$; SPO is used by SPENCE, and CSP is complex-valued for complex argument Z(in).

REAL FUNCTION P2BODY(AM00,AM01,AM02)

gives the momentum of two-body decay, in the centre-of-mass system, for a particle with a given mass AM00(in) to particles with masses AM01(in) and AM02(in).

SUBROUTINE BOOST\$(PPPP,PREP,NNN1,NNW2)

executes a Lorentz boost of a set of four-vectors, PPPP(4,I)(in & out). For a given four-vector PREP(4)(in), the system is transformed into another system in which the space component of four-vector PREP(4) is 0. Here, I runs from NNN1(in) to NNW2(in).

SUBROUTINE ERRPRI

controls the printing of error messages. It has a number of entry points as ERRPxx.

SUBROUTINE ROTAT\$(PPPP,PHI1,COST,PHI2,NUMP,NDIM)

executes the Euler rotation of a set of vectors.

SUBROUTINE SWAPD(X,Y), SWAPI(X,Y)

interchange two arguments $X(in \& out)$ and $Y(in \& out)$; **SWAPD** is for real variables and **SWAPI** is for integer variables.

REAL FUNCTION RANDOM(IDUM)

generates a uniform random number between 0 and 1; **IDUM(in)** is a dummy argument. This module is dependent on the system.

SUBROUTINE XDATE(CDATE)

To record the run, this subroutine provides some information, e.g. the date of the run, as **CDATE(out)**, which is a string of 8 bytes. This module is dependent on the system.

SUBROUTINE QRCGEN, QRCBES, RCGEN\$, RCGIN\$, RCGCN\$, RCGRS\$, RCGAN\$, RCICN\$, RCIRS\$, RCGOE\$, RRATF\$ RCICN\$, RCIRS\$, RCGOE\$, RRATF\$

These subroutines are responsible for the radiative correction for the initial-state radiation. The method is similar to that used in Ref. [19]. Detailed explanation is skipped here. The usage is shown in Appendix E.

Appendix C. COMMON AREAS

In this Appendix we present all the common areas used in **NLLjet**, with a short explanation. The name of the common area and that of the variable are shown in typewriter font.

/QCEVNT/ PARTN(10,1000), LISTP(10,1000), NATOP, NSIZE

Common area for recording the generated event in the form of a list of particles:

PARTN : **PARTN(J,I)** records the momentum of the parton at **address=I**. The meaning of first index, **J**, is as follows:

PARTN(1,I) z , the fraction of the light-cone variable

PARTN(2,I) Q^2 , the squared virtual mass, in GeV^2

PARTN(3,I) $|\mathbf{p}_T|$, the magnitude of the transverse momentum, in GeV

PARTN(4,I) ϕ , azimuthal angle of $\mathbf{p}_T$

PARTN(5,I) p_x , the x -component of the momentum, in GeV

PARTN(6,I) p_y , the y -component of the momentum, in GeV

PARTN(7,I) p_z , the z -component of the momentum, in GeV

PARTN(8,I) E , energy, in GeV

PARTN(9,I) p^+ -component of the momentum, in GeV

PARTN(10,I) p^- -component of the momentum, in GeV

LISTP : **LISTP(J,I)** records the history and status of the parton at **address=I**. The meaning of the first index, **J**, is as follows:

LISTP(1,IX) Code number of the parton:

0 is a gluon, and 1, 2, 3, 4, 5 and 6

are u -, d -, s -, c -, b -, and t -quarks, respectively.

For antiquarks, the code number has a negative sign.

The primary vertex whose address is 1

has a special code number: 99.

When the radiative correction is included,

the radiated photon may be added at the

bottom of the list with the code number=100.

LISTP(2,I)Relative address of the mother parton.

The address of the mother parton is I-LISTP(2,I).

LISTP(3,I)Number of the daughter partons. If it is 0, the parton is

one of those that are found after the evolution ends.

LISTP(4,I)Relative address of the eldest daughter parton.

The addresses of the daughters are from

I+LISTP(4,I) to I+LISTP(4,I)+LISTP(3,I)-1.

LISTP(5,I)Status of the parton. This is used in two ways.

In the phase of parton evolution, it is used as follows:

0 means that z and Q^2 are undetermined.

1 means that z is fixed but Q^2 is undetermined.

2 means that z and Q^2 are fixed.

After the evolution, four-momenta are determined

by z and Q^2 . It is used to record its ancestor parton.

NATOP The top address in the event.

NSIZE Size of PARTN and LISTP. At present, it is 1000.

/NLLPAR/ ROOTS, Q2MAX, Q02, FLAMD, FLMD2, DELTA(10),

QMASS(10), SCHEME, NFLAV

Important physical parameters for NLLjet.

ROOTS0 : Input value of centre-of-mass energy, in GeV.

ROOTS : Centre-of-mass energy, in GeV of the generated event. It may differ from ROOTS0 owing to radiative correction. In physical routines of NLLjet, ROOTS is always referred.

Q2MAX : Square of ROOTS, in GeV².

Q02 : Minimum virtuality, Q_0^2 , in GeV².

FLAMD : QCD Λ parameter, in GeV.

FLMD2 : Square of FLAMD, in GeV².

DELTA : Parameter δ which separates the hard scattering and the shower in the first branching. It may depend on the quark flavour.

QMASS : Quark masses, in GeV.

SCHEME : Scheme parameter of the P-function for quark branching.

NFLAV : The number of flavours N_f .

/QCMLL/ NLAP, NLAL, KZAP, KZQQG, NSHUF1, NSHUF2, NLMODE, NAO

Control parameters for NLLjet. These are flags for selecting the various modes.

NLAP : Selection of the Altarelli-Parisi P-function; 1(0) stands for NLL(LL).

NLAL : Selection of the form of the strong coupling constant α_s . When it is 1(0), α_s is calculated in the form of NLL (LL).

KZAP : Selection of the argument of the strong coupling constant in the evolution of partons. It is Q^2 for KZAP=0, $x(1-x)Q^2$ for KZAP=1, and $(1-x)Q^2$ for KZAP=2. The last two cases correspond to the transverse momentum squared.

KZQQG : Selection of the argument of the strong coupling constant at the primary $q\bar{q}g$ vertex. The meaning is the same as for KZAP.

NSHUF1, NSHUF2 : (Now unused.)

NLMODE : When it is 1(0), the evolution is done in the NLL (LL) mode.

NAO : When it is 1(0), the condition of angular ordering is imposed (not imposed).

/TPNEGG/ PNEGG(0:NIDIM), DELTG, NTMAG, NINTG

Table of non-branching probability, Π_{NE} for gluons.

PNEGG : PNEGG(K) is $\log \Pi_{NE}(Q_{max}^2, Q_k^2)$ for gluons. Q_{max}^2 is the squared centre-of-mass energy and $\log Q_k^2 = \log Q_{mn}^2 + K \cdot DELTG$. The value of this table is neg-

ative, and $PNEGG(NTWIG)=0$ by definition. It should be noted that $\log(Q_{max}^2/\Lambda^2)$ and $\log(Q_{min}^2/\Lambda^2)$ are found in $/LINKIN/$ as QML and $QOQGL$.

$DELIG : \log(Q_{max}^2/Q_{min}^2)$, where Q_{min}^2 is the available minimum virtuality.

$NTWIG : \text{The size of the table.}$

$NINTG : \text{(Now unused.)}$

$/TPNEQQ/ PNEQQ(0:NIDIM,10), DELTQ(10), NTWIG, NINTQ$

The same tables as in $/TPNEGG/$, but for quarks. The second index of $PNEQQ$ and the index of $DELTQ$ specify the quark flavour.

$/TPNEGG/ PNEGG(0:NIDIM), DELGQ, NTWIG, NINGQ$

The same tables as in $/TPNEGG/$, but for the branching of gluon into $q\bar{q}$. In order to be consistent, the size of the table and the maximum and the minimum of Q^2 are identical to those in $/TPNEGG/$.

$/PRVTX/ S2MOD(10), FMODE(10)$

Fractions for selecting the quark flavour and the primary vertex.

$S2MOD : \text{The fraction of the two-jet, which depends on the quark flavour.}$

$FMODE : \text{The flavour fraction.}$

$/COLOR/ CF, CA, TR, PI, BETA0, BETA1$

Values for colour factors and π . See Eq. (A.11).

$CF : C_F$

$CA : C_A$

$TR : T_R$

$PI : \pi = 3.14159\dots$

$BETA0 : \text{The lowest-order coefficient of the beta function, } \beta_0 = (11C_A - 4T_R N_f)/3, \text{ where } N_f \text{ is the number of flavours.}$

$BETA1 : \text{The next-to-leading-order coefficient of the beta function, } \beta_1 = (34C_A^2 - (20C_A + 12C_F)T_R N_f)/3.$

$/LINKIN/ QM(0:10), QOM(0:10), QO2M(0:10), QOQG2(0:10),$

$QOQG2(10), QOQGL(0:10), QML$

Values for the minimum virtual mass and related quantities. The index of arrays denotes a gluon as 0, and u, d, s, c, b , and t -quark as 1, 2, 3, 4, 5, and 6.

$QM : \text{Quark masses, in GeV } QM(0)=0.$

$QOM : \text{The lowest virtual mass, } \sqrt{Q_0^2 + QM^2}, \text{ in GeV.}$

$QOM2 : \text{Square of } QOM, \text{ in GeV}^2.$

$QOQG2 : \text{The minimum virtual mass squared for a system of a quark and a gluon, in GeV}^2.$

$QOQQ2 : \text{The minimum virtual mass squared for a system of a quark and an antiquark, in GeV}^2.$

$QOQGL : \log QOQG2/\Lambda^2.$

$QML : \log s/\Lambda^2$, where s is the centre-of-mass energy squared.

$/CWEAK0/ NWEAK$

$NWEAK : \text{determines the quark-current coupling; 1 (0) stands for the electroweak theory (QED).}$

$/CWEAK2/ ZMASS, WMASS, HMASS, SIN2W, ZGANN, WGANN, WKCP2,$

$COEF1X(2), COEF2X(2), COEF3X(2),$

$COEF4X(2), COEF5X(2), COEF6X(2)$

Physical parameters for the electroweak theory. Some of these are set in $QCINPT$ and $QCADATA$ as input values, and the remainder are set in $TBFMD$.

$ZMASS, WMASS, HMASS : \text{Masses of the } Z^0, \text{ the } W^\pm, \text{ and Higgs particles.}$

SIN2W : $\sin^2 \theta_w$.
ZGAMM, WGAMM : Width of the Z^0 and the $W^\pm$.
WCPC2 : $1/16 \sin^2 \theta_w \cos^2 \theta_w$.
COEFFXI(J) : represents coefficients for two-jet distribution; J=1 represents the up-state of weak isospin and J=2 the down-state. These are used in the package of radiative correction.
/DSTRJT/ RQMS(10), RGMS(10), COEF(6,10), F2J(4,6), PI
Coefficients for the angular distributions of three-jets and two-jets. The index of the above arrays represents the quark flavour.
RQMS : $2m_s/\sqrt{s}$ for the quarks.
RGMS : $2Q_0/\sqrt{s}$ for all quarks.
COEF : The differential cross-section of two-jets for the quark is $d\sigma/d\Omega = (9/16\pi) v (C_{VV} f_{VV} + C_{AA} f_{VV} + C_{VA} f_{VA})$ in unit of $\sigma_{\mu\mu}$, where $C_{VV} = \text{COEF}(1,1)$; $C_{AA} = \text{COEF}(2,1)$; $C_{VA} = \text{COEF}(3,1) + \text{COEF}(4,1)$; $f_{VV} = 2 - v^2 + v^2 \cos^2 \theta$; $f_{AA} = v^2 + v^2 \cos^2 \theta$; $f_{VA} = 2v \cos \theta$, and $v = \sqrt{1 - 4m_q^2/s}$.
F2J : The differential cross-section of two-jets for the quark is given by the function FJET2.
FJET2 = FJ2(1,1) + FJ2(2,1) $\cos \theta$ + FJ2(3,1) $\cos^2 \theta$, and the maximum of FJET2 is FJ2(4,1).
PI : $\pi = 3.14159 \dots$
/QCRECV/ NRECOV
NRECOV : In QCPREP, it controls the save/load action of variables in /TPNEGG/, /TPNEQG/, /TPNEGG/, and /PRVTX/. When it is 0, a data file is not used. When it is negative, the tables are written on a file of unit number =-NRECOV. When it is positive, the tables are read from a file of unit number =NRECOV.

/QCRND/ NSEED
NSPEED : Value for the seed of the random number.
/COMAL1/ C0, C1, FL2, FL2M, QCUT
C0 : $\beta_0/4\pi$
C1 : $\beta_1/4\pi\beta_0$
FL2 : Λ^2
FL2M : $\tilde{\Lambda}^2$ which is defined as $\tilde{\Lambda}^2 = Q_0^2 \exp(-4\pi/\beta_0\alpha_s(Q_0^2))$. Here $\alpha_s(Q_0^2)$ is the second-order coupling for Λ^2 .
QCUT : Q_0^2
/COMAL2/ B1, B2, B3, B4, BS
These values are the coefficients of the asymptotic form of the next-to-leading P -function of the quark. At present, $b_1(=B1)$ is used as $P^{(1)}(x) \approx b_1/(1-x)$ as $x \rightarrow 1$.
/COMPES/ NSAME(10)
NSAME : If the mass of the I-th quark and that of the J-th quark (J>I) are same, we set NSAME(J)=I to skip calculation of the Sudakov factor for the J-th quark. Otherwise, NSAME(J)=0.
/COMPEW/ WG1(0:N1), WG2(0:N2), WG3(0:N3)
Weights for numerical integration by the Simpson method.
/ALPHAL/ QCUT, ALPHMX
To set the upper limit of the strong coupling constant.
qCUT : The minimum Q^2 for the argument of the strong coupling constant.
ALPHMX : The maximum value of the strong coupling constant.

/CHSANG/ C0(3), SI(3)

These keep the direction of primary partons. They are set in CHSPRV and used in CHSROT to rotate the partons four-momenta.

/CKIN1/ P1P2, P1P3, P2P3, E1, E2, E3, P1, P2, P3, C12, C13, C23

These values give $p_i p_j$, E_i , $|p_i|$ and $\cos\theta_{ij}$ in the $q\bar{q}g$ primary vertex. They are set in CHKKW1.

/RCCOM1/, /RCCOM2/, /RCCOM3/, /RCCOM4/, /RCCOM5/

These five common areas are used by the package for radiative correction.

/SV3GWZ/ Z, Z1

This common area is used by SV3GW to transfer variables to some low-level routines.

/QCWARN/ NWARN

NWARN : (Now unused.)

Appendix D. INPUT DATA

NLLjet is controlled by a set of input parameters. In this Appendix we describe how to specify them. Input data are read from unit FT05F001 and consist of a number of lines. Each line consists of a keyword and of a numerical parameter or a string parameter. For some keywords, there is no parameter. The number of parameters is one, except for the QMASS line. The keyword and parameters are to be separated by space character(s). The location of a keyword and of parameters in a line is arbitrary. The ordering of keywords is also arbitrary, except for the QMASS line. Since each parameter has its default value, only values that the user wants to change are to be given. Below, we briefly present keywords and their meaning. The keywords must be typed in upper case as they are shown. After a keyword, we specify the type of parameter by [I], [R], [C], or [X] to denote that the type is integer, floating, or string, or that there is no parameter for the keyword. The value quoted in parenthesis is the default value.

TITLE	[C]	The title string. (String to specify date)
ROOTS	[R]	Center-of-mass energy, in GeV. (50)
Q02	[R]	Minimum virtuality of partons, in GeV^2 . (1.0)
LAMBDA	[R]	$\Lambda_{\overline{MS}}$, in GeV. (0.2)
NFLAV	[I]	Number of quark flavours. (5)
QMASS	[R]	Quark masses, in GeV. This keyword is exceptional for two reasons. First, it must appear after the NFLAV line, if this line exists. Second, the number of parameters is not 1 but NFLAV.
		Ordering of the quarks is u, d, s, c, b, and t. (0.0, 0.0, 0.0, 1.5, 5.0, 30.0)
NRECOV	[I]	Specify the unit number of the file for storing the Sudakov-factor table and the fraction of jets; this is to save computer time during the initialization stage. If it is 0, then no Read/Write action is done for the file. If it is negative, then calculated values are written on the file of unit number=-NRECOV.

NEVEN	[I]	If it is positive, the step for calculating tables is skipped and the values are read from the file of unit number=NRECOV. (0) Number of events to generate. (10) Please note that NEVEN and the following NPRINT are used in the standard main program that is distributed with NLLjet . When the user employs his own main program, these parameters become meaningless. Interval for printing the list of generated events. Events are printed once in NPRINT events. (10) Parameter for separating primary two- and three-jets. (0.1) To order that the quark current be coupled only to a photon and not to a Z_0 . This is not the default status. Mass of the Z_0 in GeV. (91.1) Width of the Z_0 in GeV. (2.5) $\sin^2 \theta_w$. (0.223) Seed number for random number generation. When the user modifies the random-number routine to suit the host machine, this value becomes be meaningless. (1234567879) To order that the leading-logarithmic mode be used. This is not the default status. To order that the application of the angular ordering condition be suppressed. This is not the default status.
LL	[X]	
NGAO	[X]	
The following parameters are for the control of radiative correction.		
IRC	[I]	Flag to switch radiative correction. When IRC=0, radiative correction is not taken into account. When IRC=1 or 2, correction for initial radiation is included. The difference between IRC=1 and IRC=2 is that the former is based on the one-photon approximation, while the latter uses the formula in which soft photons are exponentiated. (0)
EPMIN	[R]	Minimum of photon energy, in GeV. No meaning for IRC=0

EPHAX	[R]	Maximum of photon energy, in GeV. No meaning for IRC=2. (1.0)
BSPREAD	[I]	IRC=0. (ROOTS*0.95/2.0) Flag for the width of beam energy, in GeV. BSPREAD=0/1 is off/on. (0)
SIGMO	[R]	Width of beam energy, in GeV. (0.0)

Appendix E. EXAMPLE OF MAIN PROGRAM AND JCL FOR FACOM AT KEK

The following are two examples of main program for generating events by NLLjet. After calling QCINPT and QCPREP, QCGEN is called repeatedly.

```

C-----
C Sample main program for NLLjet (1)
C Generate events NEVEN times
C and print them once for NPRNT events.
C (default: NEVEN=10, NPRNT=10)
C No radiative correction, No hadronization.
C-----
C
C      IMPLICIT REAL*8(A-H,O-Z)
C
C ...  Read parameters
C      CALL QCINPT( WBEAM, NEVEN , MRADC , NPRNT )
C
C ...  Prologue for NLLjet
C      CALL QCPREP
C
C* If you want to do some initialization, do it here.
C
C ...  Event Loop
C      DO 100 I = 1, NEVEN
C
C ...  Generate one event
C      IFLAV=0
C      CALL QCGEN ( IFLAV )
C
C* If you want to hadronize partons, call hadronization
C routines here.
C
C* Now one event generated. If you want to analyse it, do it here.
C
C* If you do not need to print, delete following lines.
C ...  Print out parton list
C      NEVE=I

```

```

NCHK=1
IF (MOD(1,NPRNT).EQ.0) CALL QCOUT ( NEVE , NCHK , WBEAM )
C
100 CONTINUE
C
C* NEVEN events generated. If you want to do summary, do it here.
C
STOP
END

```

When the user wants to use the radiative correction package in NLLjet, the main program is as follows. We need the common area /NLLPAR/ and PHOT(4) for the photon's momentum. QCRGCGN generates the photon for a radiative event and also determines the flavour IFLAV. Since the development of parton shower is done in the centre-of-mass system of reduced energy, the system of partons is transformed into the laboratory system by QRCES. We also shift δ for a massive quark since it is impossible to generate primary three-jets of massive quarks when the radiated photon takes away a large amount of energy.

```

C-----
C Sample main program for NLLjet (2)
C Generate events NEVEN times
C and print them once for NPRNT events.
C (default: NEVEN=10, NPRNT=10)
C With/without radiative correction.
C-----
C
C      IMPLICIT REAL*8(A-H,O-Z)
C
C PHOT(4)= 4-momentum of radiative photon
C IPHOT = 0 .. Non-radiative event
C      = 1 .. Event with a radiative photon
C      DIMENSION PHOT(4)
C      COMMON /NLLPAR/ ROOTS0 , ROOTS , Q2MAX , Q02 , FLAND , FLMD2
C      * , DELTA(10) , QMASS(10) , SCHEME , NFLAV
C
C ...  Read parameters

```

```

C      CALL QCINPT( WBEAM, NEVEN, NCHK, NCHK, WBEAM )
C      CALL QCOUT ( NEVE, NCHK, WBEAM )
C      END IF
C
C      100 CONTINUE
C
C* NEVEN events generated. If you want to do summary, do it here.
C
C      STOP
C      END

```

Below we show an example of job-control statements to run NLLjet on FA-COM at KEK. Here XXXX is a user-id and XXXX.NLLJET.FORT is a partitioned dataset that stores the FORTRAN source of NLLjet. The member NLLJET consists of 99 modules, described in Appendix B, and MAIN0 is the main program given as the first example. As a stream file, FT05F001 is defined to specify control parameters, which are explained in Appendix D. Since MRECOV is defined as a control parameter, the file XXXX.0F29.DATA is declared in order to save the Sudakov-factor table and that of the fraction of the primary vertex.

```

//XXXX JOB CLASS=M
//*-----
//* NLLjet : JOB0 : Generate Parton Shower
//*-----
// EXEC FORT7CLG,
// PARM.FORT='OPT(3),SOURCE',
// PARM.LKED='NOMAP,NOLIST'
//FORT.SYSIN DD DSN=XXXX.NLLJET.FORT(MAIN0),DISP=SHR
// DD DSN=XXXX.NLLJET.FORT(NLLJET),DISP=SHR
//GO.FT05F001 DD *
TITLE TEST RUN
ROOTS 29.0
LAMBDA 0.20
DELTA 0.1
Q02 1.0
NFLAV 5
QMASS 0.325 0.325 0.5 1.6 5.0

```

```

C      CALL QCINPT( WBEAM, NEVEN, NCHK, NCHK, WBEAM )
C      CALL QCOUT ( NEVE, NCHK, WBEAM )
C      END IF
C
C      100 CONTINUE
C
C* If you want to do some initialization, do it here.
C
C      STOP
C      END
C
C ... Radiative correction, generate photon
C      IF( NRADCO .NE. 0 ) THEN
C      CALL QRCOR( ROOTSX, ROOTS, PHOT, IPHOT, IFLAV )
C
C      IF( IPHOT .NE. 0 ) THEN
C      DELSAV = DELTA(IFLAV)
C      DELTA(IFLAV)
C      = MAX( DELTA(IFLAV), 2*(QMASS(IFLAV)+SQRT(Q02))/ROOTS )
C      END IF
C
C      END IF
C
C ... generate partons
C      CALL QCGEN( IFLAV )
C
C ... boost system if initial state photon is radiated.
C      IF( NRADCO .NE. 0 ) THEN
C      CALL QRCOR( ROOTSX, PHOT, IPHOT )
C      IF( IPHOT.NE.0 ) DELTA(IFLAV) = DELSAV
C      END IF
C
C* If you want to hadronize partons, call hadronization
C* routines here.
C
C* Now one event generated. If you want to analyse it, do it here.
C
C* If you do not need to print, delete following lines.
C ... Print out parton list and hadron list
C      IF(MOD(I,NPRINT).EQ.0) THEN
C      NEVE=I

```

```

NEVEN 10000
NPRINT 1000
NRECOV -30
IRC 0
/*
//GO.FT30F001 DD DSN=XXXI.6F29.DAT, DISP=(NEW,CATLG),
// SPACE=(TRK,(2,2),RLSE),DCB=(RECFM=VBS, BLKSIZE=23689)
//

```

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