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On the Behavior of the Adjoint Wilson Line and Susceptibility around Deconfinement in SU(2) Lattice Gauge Theory

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ABSTRACT

The Wilson line $\bar{L}_a$ and the susceptibility χ_a in the adjoint representation are investigated in SU(2) lattice gauge theory around deconfinement. Our simulations are performed on $N_\sigma^3 \times 6$ lattices with $N_\sigma = 8, 12, 18, 24$. In the region close to the deconfinement temperature for $T \rightarrow T_c^+$ we find that the behavior of the adjoint Wilson line is consistent with the predictions of $(\phi^4)_3$ field theory. The adjoint susceptibility is shown to be a smoothly increasing function around the deconfinement temperature. Thus χ_a does not show the critical fluctuations.

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1. Introduction

Pure SU(2) lattice gauge theory shows the essential feature of QCD in particular the finite temperature deconfinement phase transition. Both theoretical arguments¹⁻⁻² as well as numerical results³⁻⁻⁵ indicate that the transition from the Z_2 - symmetric confined phase to the deconfined phase with spontaneously broken Z_2 symmetry is second order. The most relevant degrees of freedom for the deconfinement of a source in the fundamental representation, $J = 1/2$, are those connected with the thermal Wilson line L_f . According to universality conjecture of Svetitsky and Yaffe² the Wilson line defined in (3+1)-dimensional SU(2) lattice gauge theory is analogous to the magnetization in the 3-dimensional Ising model. Thus just above the critical temperature $\bar{L}_f$ should show non-analytic power-like behavior with the same critical exponents as found for the magnetization in the Ising model. The most recent finite-size scaling analyses⁶ of SU(2) lattice gauge theory and Monte Carlo renormalization group studies⁶ have given numerical evidence in favor of the universality conjecture predicting correctly the values of the critical exponents. Nevertheless, one could expect that finite temperature (d+1)-dimensional SU(2) lattice gauge theory contains far more nontrivial operators than can be constructed from the equivalence of this theory with d-dimensional Z_2 spin model. In particular the properties of the Wilson lines which transform as a higher irreducible representation of the gauge group have been considered in the literature⁷⁻⁻¹² as the possible candidates for observing critical behavior in the lattice gauge theory. It is immediately obvious that all representations which transform nontrivially under the center Z_2 of the color gauge group SU(2) are just as good order parameters for the deconfinement phase transition as is the fundamental representation. The Wilson line in representations which are invariant under Z_2 transformations are not an order parameter for spontaneously breaking of the global center symmetry. In particular the adjoint source can not be confined below T_c because of the screening by the glueballs just as the fundamental source will be screened by $\bar{q}q$ mesons when the dynamical quarks are incorporated in a system. However, no symmetry argument by itself forces the Wilson line of the representations which are not changed under the center transformations to show any nontrivial behaviour around the phase transition. The Monte Carlo results^{8,10} obtained in SU(2) and SU(3) lattice gauge

theory have shown that all higher representation Wilson lines exhibit an abrupt change at the same temperature T_c as the fundamental Wilson line. For a gauge theory which exhibits a continuous phase transition it is natural to ask the question what are a critical exponents related with higher representation Wilson lines; those which transform nontrivially under the center of the gauge group. The mean field results obtained by Damgaard⁹ in a strong coupling SU(2) effective gauge theory suggest that the expectation value of the Wilson line in J representations are simply related with the fundamental Wilson line, $\bar{L}_J \sim \bar{L}_f^{2J}$ for $\beta \rightarrow \beta_c^+$. The Monte Carlo data^{9,11} from the $8^3 \times 2$ and $18^3 \times 4$ lattices have shown remarkably good agreement with the mean field predictions. It was, however, not clear to what extend the Monte Carlo data of Ref. 9 and Ref. 11 represent the results of the infinite volume limit. Since the gluon screening is a large distance effect, it could well be that these results will be modified when going to larger lattices. These Monte Carlo simulations were also performed in the range of the coupling constants which were at some distance from the critical region, $(\beta - \beta_c) > 0.015$. If one considers larger lattices and/or performs the Monte Carlo simulations closer to the critical coupling, one can not exclude that the adjoint source will no longer show critical behavior. In this case the gluon screening could also modify the indices of odd representations of the SU(2) gauge group bringing them down to the fundamental critical exponents. Just recently Kiskis¹² has presented theoretical predictions for the behavior of higher representation Wilson lines based on the flux tube model¹³ for the deconfined phase and $(\phi^4)_3$ field theory.¹⁴⁻⁻¹⁵ In the critical region for $\beta \rightarrow \beta_c^+$ the odd representation Wilson lines were found to scale as in the fundamental representation, $\bar{L}_{J=odd} \sim \bar{L}_{J=1/2}$. In the adjoint representation, ($J = 1$) the above theoretical models predict that $\bar{L}_a = a + b(\beta - \beta_c)^{0.88}$. The Monte Carlo data of Ref. 12 obtained on $15^3 \times 2$ and $19^3 \times 2$ lattices favor the prediction of the $(\phi^4)_3$ model over the mean field result in the region sufficiently close to the critical temperature.

In this letter we shall present Monte Carlo data for the adjoint Wilson line around deconfinement phase transition. In an extension of the discussion presented in Ref. 12 our Monte Carlo computations were performed on larger lattices in both the space-like and the time-like directions. We shall also discuss the influence of the lattice size on the behavior of the adjoint Wilson line. With the lattice size used in Ref. 12, ($N_\tau = 2$) we are rather

far away from the asymptotic scaling regime. In order to be closer to the continuum limit, we have performed the simulation on lattices with $N_\tau = 6$.

The discussion of the behavior of the adjoint source around deconfinement in finite temperature $SU(2)$ lattice gauge theory has been performed up to now in the literature on the level of measurements of thermal expectation value of the adjoint Wilson line. It is a further challenge to investigate whether any apparently nontrivial behavior in the critical region can be observed in terms of other physical operators which are related with a higher representation adjoint source. In this letter we shall also discuss the properties of the susceptibility for the adjoint Wilson line.

2. Lattice calculations

The partition function for pure QCD can be written in terms of gluonic link matrices U and the standard Wilson action $S_G(U)$ as follows:

$$Z(N_\sigma, N_\tau) = \int \prod dU e^{-S_G(U)} \quad (2.1)$$

with

$$S_G = \frac{4}{g^2} \sum_P \text{Tr} U_P. \quad (2.2)$$

The order parameter for deconfinement in pure $SU(N)$ lattice gauge theory is the fundamental Wilson line which is defined as a product of link variables which are $SU(N)$ matrices in the fundamental representation

$$L_f(\vec{x}) = \frac{1}{N} \text{Tr} \prod_{\tau=1}^{N_\tau} U(\vec{x}, \tau), \quad (2.3)$$

and

$$L_f = \frac{1}{N_\sigma^3} \sum_{\vec{x}} L_f(\vec{x}). \quad (2.4)$$

The higher representation Wilson lines due to the recursion relation for $SU(N)$ characters can be expressed as a polynomial of the fundamental Wilson line. In particular the adjoint Wilson line L_a reads

$$L_a(\vec{x}) = (4L_f^2(\vec{x}) - 1)/3. \quad (2.5)$$

With the partition function given by Eq. (2.1) the thermal average of any physical operators A_P is defined as follows:

$$\bar{A}_P = \frac{1}{Z} \int \prod dU A_P e^{-S_G}. \quad (2.6)$$

The adjoint susceptibility, χ_a reads

$$\chi_a = N_\sigma^3 (\bar{L}_a^2 - \bar{L}_a)^2. \quad (2.7)$$

We have simulated the above theory for an $SU(2)$ gauge group on a $N_\sigma^3 \times 6$ lattice with $N_\sigma = 8, 12, 18, 24$ using the full $SU(2)$ heatbath program. In particular we compute the expectation value of the adjoint Wilson line and the adjoint susceptibility from Eq. (2.5)-Eq. (2.7). Our data were obtained in the vicinity of the critical coupling constant β_c for $2.40 < \beta < 2.46$. The value of β_c varies with the lattice size used in actual Monte Carlo simulations from 2.427 on the $24^3 \times 6$ to 2.437 on the $12^3 \times 6$ lattice. The finite-size scaling analysis¹⁶ of $SU(2)$ lattice gauge theory gives, $\beta_c = 2.4265 \pm 0.003$. For a given value of the coupling constant we have run between 320 up to 550 thousand sweeps. The first 2000 sweeps were dropped for thermalization.

In Fig. 1 we show the numerical results for the adjoint Wilson line on an $N_\sigma^3 \times 6$ lattice with $N_\sigma = 8, 12, 18, 24$. It is immediately clear from Fig. 1 that $\bar{L}_a$ shows a very strong dependence on N_σ . This is especially true for $\beta \leq 2.43$ which corresponds to the confinement region of the fundamental source. When going sufficiently far above the phase transition for $\beta > 2.44$ the finite-size effect is less pronounced. On the $8^3 \times 6$ lattice the adjoint Wilson line is a rather smooth function of β . However, the data obtained on $24^3 \times 6$ lattice already show a steep increase of $\bar{L}_a$ in the range of the coupling constant where one expects deconfinement. The behavior of $\bar{L}_a$ presented in Fig. 1 is qualitatively similar to that observed for the fundamental Wilson line^{2,4}. Nevertheless, the values of $\bar{L}_a$ are significantly lower than those obtained for $\bar{L}_f$ when compared at the same value of β . This is apparently in good agreement with the results of the weak coupling expansion¹⁷ which indicate that by increasing the representation J the value of $\bar{L}_J$ decreases.

In order to deduce the behavior of the adjoint Wilson line just above critical temperature assigned to the deconfinement of fundamental source, we compare our data with the predictions of both $(\phi^4)_3$ and mean field approach. In Fig. 2 we plot the difference

$\bar{L}_a(\beta) - \bar{L}_a(\beta_c)$ normalized to $\Delta\beta^{0.88}$ with $\Delta\beta = \beta - \beta_c$. The result¹² of $(\phi^4)_3$ theory suggests that this quantity should be independent of $\Delta\beta$. As seen in Fig.2 our data are compatible within error bars with the prediction of $(\phi^4)_3$ field theory. However, the value of b in the parameterization of $\bar{L}_a = a + b\Delta\beta^{0.88}$ depends on the lattice size. In the derivation of these results the critical temperature was taken to be 2.430 independent of the lattice size. Within the error bar for the estimation of β_c this approximation for $\beta_c^{(N_c)}$ is acceptable. The typical errors for the determination of the constant b is also indicated in Fig.2. This error is mainly related with the uncertainty in the determination of the critical coupling and the value of $\bar{L}_a$ at $\beta = \beta_c$.

Having in mind the mean field prediction⁹ and the previous Monte Carlo high statistical results¹¹ obtained on an $18^3 \times 4$ lattice, we show in Fig.3 the adjoint Wilson line in normalized form, $\bar{L}_a^{3/2}/\Delta\beta$. As seen in Fig.3 for $0.01 < \Delta\beta < 0.03$, $\bar{L}_a^{3/2}/\Delta\beta$ is independent of $\Delta\beta$ within the errors. This result coincides with our previous Monte Carlo results¹¹ on an $18^3 \times 4$ lattice which in the range $0.014 < \Delta\beta < 0.05$ also shows good agreement with the prediction of the mean field calculations of Damgaard.⁹ However, when going closer to the critical temperature for $0.0 < \Delta\beta < 0.01$ a strong deviation from the mean field result is observed as indicated in Fig.3. Thus, in the whole parameter range studied in our Monte Carlo computations we see that the prediction of the $(\phi^4)_3$ model for $\bar{L}_a$ is favored over the mean field result. These properties agree with the results presented in Ref. 12.

Due to a gluon screening mechanism, mentioned in the introduction, the free energy of a static adjoint source should be finite at $T = T_c$. Thus, on the infinite lattice at the critical temperature the adjoint Wilson line should have a finite nonzero value, contrary to $\bar{L}_f$ which is exactly zero at $T = T_c$. In Fig.4 we show the behavior of $\bar{L}_f^{(N_c)}$ at $\beta = \beta_c$ normalized to $\bar{L}_f^{(N_c=8)}$ ($\beta = \beta_c$) as a function of N_c/N_f for the fundamental and the adjoint representation Wilson line. From Fig.4 we see that finite-size effects influence both representation of the Wilson lines in the same way. They both decrease with increasing N_c/N_f . Even on a $24^3 \times 6$ lattice they are still far away from the result expected in the infinite volume limit. Thus, in order to find a value of $\bar{L}_a$ at $T = T_c$, a much larger lattice would be required. Nevertheless, since the gluon free energy is large at $T = T_c$, the value of $\bar{L}_a(\beta = \beta_c)$ can be expected to be very small. Due to typical errors for the Monte Carlo computations it will be certainly very difficult to conclude that the value of $\bar{L}_c$ is different

from zero. Detailed finite-size scaling analyses could give a final result for $\bar{L}_a(T_c)$. These, however, would require high statistical computations on different large lattices.

The fundamental Wilson line is not the only parameter which can give insight into the critical behavior in a pure gluonic medium. The fundamental susceptibility χ_f , which measures the fluctuations in the system will also show deconfinement. For a second order transition χ_f diverges at $T = T_c$. On a finite lattice, however, one can only observe a peak structure for χ_f at the critical temperature which is more strongly pronounced when going to larger N_c lattices⁵. In Fig.5 we present our Monte Carlo result for the adjoint susceptibility. We have not included the error bars for χ_a in Fig.5. The rough estimates suggest that these errors can be large.

Contrary to the behavior obtained for the fundamental susceptibility⁵, χ_a does not show a peak structure at the point where χ_f does. This property is valid independently of the lattice size used in our actual Monte Carlo computations as seen in Fig.5. Thus χ_a seems to be not effected by the deconfinement transition and consequently our data indicates that the adjoint Wilson line is not a true order parameter related with spontaneously breaking of the Z_2 global symmetry. The abrupt change of $\bar{L}_a$ at T_c could be related to the appearance of the mass gap¹⁸ in a system when crossing the transition point. Below T_c the thermal expectation value of the adjoint Wilson line is determined by the free energy of a color singlet bound state of the static adjoint source with gluons. Whereas above T_c it relates to the effective gluon mass.¹⁹

3. Conclusions

We have measured the adjoint Wilson line and the susceptibility in $SU(2)$ gauge theory on $N_c^3 \times 6$ lattices with $N_c = 8, 12, 18, 24$. We show that just above the deconfinement transition, in the region $0 < (\beta - \beta_c) < 0.03$, the behaviour of the adjoint Wilson line is compatible with the $(\phi^4)_3$ field theory as given in Ref. 12, which suggests that $\bar{L}_a = c + b(\beta - \beta_c)^{0.88}$. However, at some distance above T_c , $(\beta - \beta_c) > 0.01$, the mean field parameterization⁹ $\bar{L}_a \sim (\beta - \beta_c)^{2/3}$ for the adjoint Wilson line is also not excluded by our Monte Carlo data. The adjoint susceptibility was found to be an increasing function of the coupling constant in the region where the fundamental source shows the deconfinement transition which appears as a pronounced peak in χ_f at the critical temperature.

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6. Figure Caption

- Figure 1: The adjoint Wilson line, $\bar{L}_a$ as a function of the coupling constant obtained on $N_\sigma^3 \times 4$ lattices with, $N_\sigma = 8, 12, 18, 24$.
- Figure 2: $(\bar{L}_a(\beta) - \bar{L}_a(\beta_c)) / (\beta - \beta_c)^{0.88}$ versus $\beta - \beta_c$. The lattice sizes as in Fig.1
- Figure 3: $\bar{L}_a^{3/2} / (\beta - \beta_c)$ versus $\beta - \beta_c$. The lattice sizes as in Fig.1
- Figure 4: $\bar{L}_j^{(N_\sigma)} (\beta = \beta_c) / \bar{L}_j^{(N_\sigma=8)} (\beta = \beta_c)$ versus N_σ / N_r for the adjoint (circle) and the fundamental (crosses) representation Wilson line.
- Figure 5: The adjoint susceptibility, χ_a versus β . The lattice sizes as in Fig.1.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5