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Supersymmetry of the three-dimensional Einstein-Hilbert
gravity in the Landau gauge.

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Abstract

In analogy with the Chern-Simons action in the Landau gauge, we find for the 3D Einstein-Hilbert gravity an extra symmetry whose Ward operator, together with the linearized Slavnov operator, gives rise to a $N=1$ supersymmetric algebra. We discuss furthermore the inclusion in the action either of a cosmological constant or of a topological term which permits the exploitation of the tight analogy of 3D gravity with the Chern-Simons models built on complex gauge groups.

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1. Introduction

Three-dimensional Einstein-Hilbert gravity, due to its peculiar properties, has been the object of numerous investigations [1,2]. Recent analyses [3,4,5] have shown that the model shares various features with the three-dimensional Chern-Simons gauge theory; indeed, after an appropriate redefinition of the spin connection, the action of the three-dimensional gravity looks like a Chern-Simons action.

The aim of this paper is to show that, as in the Chern-Simons case [6,7], once the Landau gauge has been chosen, the three-dimensional gravity also possesses an extra symmetry whose generators carry a Lorentz index. These generators connect commuting with anticommuting fields and, together with the *BRS* operator associated with local Lorentz transformations and diffeomorphisms, form an algebra which, after the introduction of suitable external fields, closes off-shell on the translations and then allows for a supersymmetric interpretation of the model.

The existence of a supersymmetric structure in the Landau gauge is not a property of a few peculiar models, but involves a large class of systems; for instance, such a supersymmetry is also present [8] in the three- and four-dimensional *BF* systems considered in [9,10]. Quite in general one could expect that such a supersymmetry is a common feature of all the models which do not have local degrees of freedom in the ordinary sense of quantum field theory.

The work is organized as follows: in Sec.2 we discuss the pure Einstein-Hilbert gravity; Sec.3 is devoted to the case of a non-vanishing cosmological constant and, finally, we discuss the relationship with the Chern-Simons gauge model built with a complex gauge group.

2. Pure Einstein-Hilbert gravity

Let us start by considering the three-dimensional Einstein-Hilbert action without cosmological term:

$$S_{EH} = \frac{1}{4} \int_M d^3x \, \epsilon^{\mu\nu\rho} \epsilon_{abc} e_\mu^a \left(\partial_\nu \omega_\rho^{bc} - \partial_\rho \omega_\nu^{bc} + \omega_\nu^b \omega_\rho^{dc} - \omega_\rho^b \omega_\nu^{dc} \right), \quad (2.1)$$

where M is the three-dimensional space-time and e_μ^a , $\omega_\mu^{ab} = -\omega_\mu^{ba}$ are the vierbein and the spin connection that are to be considered as independent fields. Lorentz indices $a, b, \dots$ are contracted by means of the Minkowski metric $\eta_{ab} = \text{diag}(1, -1, -1)$.

As pointed out by Witten [4], the action (2.1) admits a Chern-Simons interpretation; indeed, if one redefines the spin connection

$$\omega_{\mu a} = -\frac{1}{2} \epsilon_{abc} \omega_\mu^{bc}, \quad (2.2)$$

the Einstein-Hilbert action becomes the Chern-Simons action $S_{CS}(e, \omega)$:

$$S_{CS} = -\frac{1}{2} \int_M d^3x \, \epsilon^{\mu\nu\rho} e_\mu^a \left(\partial_\nu \omega_\rho^a - \partial_\rho \omega_\nu^a + \epsilon_{abc} \omega_\nu^b \omega_\rho^c \right). \quad (2.3)$$

The action (2.3) is invariant under the transformations:

$$\begin{aligned} \delta e_\mu^a &= -(D_\mu \rho)^a - \epsilon_{bc}^a e_\mu^b \tau^c, \\ \delta \omega_\mu^a &= -(D_\mu \tau)^a, \end{aligned} \quad (2.4)$$

where $(D_\mu \rho)^a$ is the covariant derivative with respect to the spin connection ω_μ^a , i.e. :

$$(D_\mu \rho)^a = \partial_\mu \rho^a + \epsilon_{bc}^a \omega_\mu^b \rho^c. \quad (2.5)$$

The transformations (2.4) represent both local Lorentz transformations and diffeomorphisms [4]. To quantize the action (2.3) we interpret the parameters τ^a and ρ^a as ghost fields, we introduce two antighost fields ξ^a and λ^a , two Lagrangian multipliers d^a and b^a , and an operation s

$$\begin{aligned} s \omega_\mu^a &= -(D_\mu \tau)^a \\ s \tau^a &= \frac{1}{2} \epsilon_{bc}^a \tau^b \tau^c \\ s e_\mu^a &= -(D_\mu \rho)^a - \epsilon_{bc}^a e_\mu^b \tau^c \\ s \rho^a &= \epsilon_{bc}^a \tau^b \rho^c \\ s \xi^a &= d^a, \quad s d^a = 0 \\ s \lambda^a &= b^a, \quad s b^a = 0, \end{aligned} \quad (2.6)$$

which is easily seen to be nilpotent. The second step in the quantization of (2.3) consists of introducing a gauge-fixing term. As is well known, the gauge-fixing procedure requires the introduction of an arbitrary reference metric $g_{\mu\nu}^{(0)}(x)$ which is a priori unrelated to e_μ^a and ω_μ^a .

In the present work, the choice that we perform corresponds to a Landau gauge [4,11,12] with flat constant diagonal reference metric

$$g_{\mu\nu}^{(0)}(x) = h_{\mu\nu}. \quad (2.7)$$

With this choice, the gauge-fixing action reads:

$$\begin{aligned} S_{gf} &= \int d^3x s(\xi^a \partial^\mu \omega_{\mu a} + \lambda^a \partial^\mu e_{\mu a}) \\ &= \int d^3x (d^\mu \partial^\mu \omega_{\mu a} + \xi^a \partial^\mu (D_\mu \tau)_a + b^a \partial^\mu e_{\mu a} + \lambda^a \partial^\mu (D_\mu \rho)_a - e_{abc} (\partial^\mu \lambda^a) e_\mu^b \tau^c), \end{aligned} \quad (2.8)$$

where

$$\partial^\mu = h^{\mu\nu} \partial_\nu. \quad (2.9)$$

It should be stressed that, with the introduction of the gauge-fixing metric $g_{\mu\nu}^{(0)} = h_{\mu\nu}$, the correlation functions of the model unavoidably depend on the "unphysical" reference metric $h_{\mu\nu}$. The genuine physical metric $g_{\mu\nu}(x)$, which has a dynamical interpretation, is given instead by

$$g_{\mu\nu}(x) = \eta_{ab} e_\mu^a(x) e_\nu^b(x). \quad (2.10)$$

Note that the introduction of the physical metric $g_{\mu\nu}$ in the gauge-fixing term would imply the appearance in the action of the inverse of the vierbein which would destroy the polynomial character of the model. The properties we are really describing here are therefore those of the model with respect to the reference metric $h_{\mu\nu}$, the hope being to probe the physical sector a little bit in this way. In fact, the arguments used in ordinary gauge theories suggest that, in the vacuum expectation values of $ISO(2,1)$ invariant quantities, the dependence on the gauge-fixing metric $h_{\mu\nu}$ should disappear.

Similarly to what happens in ordinary Chern-Simons gauge theories [7], the gauge-fixed action

$$S = S_{CS} + S_{gf}, \quad (2.11)$$

also possesses in our case an extra symmetry whose generators carry a vector index. Indeed, one easily verifies that the expression (2.11) is invariant under the following transforma-

tions:

$$\begin{aligned} \delta_\rho \omega_\mu^a &= \epsilon_{\mu\alpha\tau} \partial^\tau \lambda^a \\ \delta_\rho e_\mu^a &= \epsilon_{\mu\alpha\tau} \partial^\tau \xi^a \\ \delta_\rho \tau^a &= -\omega_\rho^a, & \delta_\rho \rho^a &= -e_\rho^a \\ \delta_\rho d^a &= \partial_\rho \xi^a, & \delta_\rho \xi^a &= 0 \\ \delta_\rho b^a &= \partial_\rho \lambda^a, & \delta_\rho \lambda^a &= 0, \end{aligned} \quad (2.12)$$

where the generators δ_ρ have the properties:

$$\begin{aligned} \{ \delta_\rho, \delta_\sigma \} &= 0 \\ \{ s, \delta_\rho \} &= \partial_\rho + \text{eq. of motion}, \end{aligned} \quad (2.13)$$

which allow for an $N = 1$ on-shell supersymmetry. Moreover, as in the pure gauge case [7], the on-shell algebra (2.13) can be promoted to an $N = 1$ off-shell SUSY by introducing suitable auxiliary fields. To do this, let us first translate the BRS invariance of the action (2.11) into a Slavnov identity. This is easily achieved by introducing a set of external sources coupled to the non-linear BRS variations (2.6)

$$S_s = \int d^3x \left(-\Omega^{\alpha\mu} (D_\mu \tau)^\alpha + \frac{1}{2} L^\alpha \epsilon_{abc} \tau^b \tau^c - \gamma^{\alpha\mu} ((D_\mu \rho)_a + \epsilon_{abc} e_\mu^b \tau^c) + N^\alpha \epsilon_{abc} \tau^b \rho^c \right), \quad (2.14)$$

where

$$\Omega^{\alpha\mu} = h^{\mu\nu} \Omega_\nu^\alpha, \quad \gamma^{\alpha\mu} = h^{\mu\nu} \gamma_\nu^\alpha. \quad (2.15)$$

The classical Slavnov identity reads:

$$\int d^3x \left(\frac{\delta \Sigma}{\delta \Omega^{\alpha\mu}} \frac{\delta \Sigma}{\delta \omega_{\mu a}} + \frac{\delta \Sigma}{\delta \gamma^{\alpha\mu}} \frac{\delta \Sigma}{\delta e_{\mu a}} + \frac{\delta \Sigma}{\delta L^\alpha} \frac{\delta \Sigma}{\delta \tau_a} + \frac{\delta \Sigma}{\delta N^\alpha} \frac{\delta \Sigma}{\delta \rho_a} + b^\alpha \frac{\delta \Sigma}{\delta \lambda^a} + d^\alpha \frac{\delta \Sigma}{\delta \xi^a} \right) = 0, \quad (2.16)$$

where

$$\Sigma = S_{CS} + S_{gf} + S_s. \quad (2.17)$$

As in standard gauge theories [13], one also imposes the equations of motion of the Lagrangian multipliers

$$\frac{\delta \Sigma}{\delta d^a} = \partial^\mu \omega_{\mu a}, \quad \frac{\delta \Sigma}{\delta b^a} = \partial^\mu e_{\mu a}, \quad (2.18)$$

and of the antighosts:

$$\partial^\mu \frac{\delta \Sigma}{\delta \Omega^{\alpha\mu}} + \frac{\delta \Sigma}{\delta \xi^a} = 0, \quad \partial^\mu \frac{\delta \Sigma}{\delta \gamma^{\alpha\mu}} + \frac{\delta \Sigma}{\delta \lambda^a} = 0. \quad (2.19)$$

Let us now try to translate the δ_ρ invariance into a Ward identity for the full action Σ .

Due to the introduction of the external sources (2.14) the operator δ_ρ has to be modified; a simple algebraic manipulation shows that the δ_ρ invariance is replaced by the Ward identity:

$$\mathcal{W}_\sigma \Sigma = \Delta_\sigma, \quad (2.20)$$

where

$$\begin{aligned} \mathcal{W}_\sigma = \int d^3x \left(\varepsilon_{\mu\sigma\tau} (\gamma^{\sigma\tau} + \partial^\tau \lambda^a) \frac{\delta}{\delta \omega_\mu^a} - \omega_\sigma^a \frac{\delta}{\delta \tau^a} - N^a \frac{\delta}{\delta \sigma^a} \right. \\ \left. + \varepsilon_{\mu\sigma\tau} (\Omega^{\sigma\tau} + \partial^\tau \xi^a) \frac{\delta}{\delta e_\mu^a} - e_\sigma^a \frac{\delta}{\delta \rho^a} - L^a \frac{\delta}{\delta \sigma^a} \right), \end{aligned} \quad (2.21)$$

and

$$\begin{aligned} \Delta_\sigma = \int d^3x \left(\varepsilon_{\mu\sigma\tau} \partial^\mu \partial^\nu \gamma_\sigma^a + \varepsilon_{\mu\sigma\tau} \partial^\mu \partial^\nu \Omega_\sigma^a + L^a \partial_\sigma \tau_a \right. \\ \left. - \Omega^{\mu\nu} \partial_\sigma \omega_{\mu\nu} - \gamma^{\mu\nu} \partial_\sigma e_{\mu\nu} + N^a \partial_\sigma \rho_a \right). \end{aligned} \quad (2.22)$$

The term Δ_σ , being linear in the quantum fields, is a classical breaking. Such a feature is once again in complete analogy with the Chern-Simons gauge theory [7]. By introducing the linearized Slavnov operator

$$\begin{aligned} B_\Sigma = \int d^3x \left(\frac{\delta \Sigma}{\delta \Omega^{\mu\nu}} \frac{\delta}{\delta \omega_{\mu\nu}^a} + \frac{\delta \Sigma}{\delta \omega_{\mu\nu}^a} \frac{\delta}{\delta \Omega^{\mu\nu}} + \frac{\delta \Sigma}{\delta \gamma^{\mu\nu}} \frac{\delta}{\delta e_{\mu\nu}^a} + \frac{\delta \Sigma}{\delta e_{\mu\nu}^a} \frac{\delta}{\delta \gamma^{\mu\nu}} \right. \\ \left. + \frac{\delta \Sigma}{\delta L^a} \frac{\delta}{\delta \sigma^a} + \frac{\delta \Sigma}{\delta \sigma^a} \frac{\delta}{\delta L^a} + \frac{\delta \Sigma}{\delta \rho^a} \frac{\delta}{\delta N^a} + \frac{\delta \Sigma}{\delta N^a} \frac{\delta}{\delta \rho^a} \right. \\ \left. + \partial^\mu \frac{\delta}{\delta \xi^a} + \partial^a \frac{\delta}{\delta \lambda^a} \right), \end{aligned} \quad (2.23)$$

one verifies the algebra:

$$\begin{aligned} \{ B_\Sigma, B_\Sigma \} &= 0 \\ \{ \mathcal{W}_\sigma, \mathcal{W}_\rho \} &= 0 \\ \{ B_\Sigma, \mathcal{W}_\sigma \} &= \mathcal{P}_\sigma, \end{aligned} \quad (2.24)$$

where $\mathcal{P}_\sigma$ is the translation Ward operator

$$\mathcal{P}_\sigma = \sum_{(\text{all the fields } \phi)} \int d^3x (\partial_\sigma \phi) \frac{\delta}{\delta \phi}. \quad (2.25)$$

One sees, then, that the introduction of the external sources (Ω, γ, L, N) closes the SUSY algebra (2.13) and, in this sense, the external sources play the role of the auxiliary fields of the standard linearly-realized supersymmetry.

3. Inclusion of a cosmological constant

Introducing into the action a cosmological constant α , which has the physical effect of considering a space-time with a constant curvature instead of a flat one, the action (2.1) is generalized to:

$$S_{EH}^{\alpha} = \frac{1}{4} \int_M d^3x \varepsilon^{\mu\nu\rho} \varepsilon_{abc} \left(e_\mu^a (\partial_\nu \omega_\rho^{bc} - \partial_\rho \omega_\nu^{bc} + \omega_\nu^b \omega_\rho^c - \omega_\rho^b \omega_\nu^c) - \frac{2}{3} \alpha e_\mu^a e_\nu^b e_\rho^c \right), \quad (3.1)$$

and the corresponding Chern-Simons action (2.3) reads [4]:

$$S_{CS}^{\alpha} = -\frac{1}{2} \int_M d^3x \varepsilon^{\mu\nu\rho} \left(e_\mu^a (\partial_\nu \omega_\rho^a - \partial_\rho \omega_\nu^a + \varepsilon_{abc} \omega_\nu^b \omega_\rho^c) + \frac{\alpha}{3} \varepsilon_{abc} e_\mu^a e_\nu^b e_\rho^c \right). \quad (3.2)$$

The action (3.2) is now invariant under the transformations:

$$\begin{aligned} \delta e_\mu^a &= -(D_\mu \rho)^a - \varepsilon_{bc}^a e_\mu^b \tau^c, \\ \delta \omega_\mu^a &= -(D_\mu \tau)^a - \alpha \varepsilon_{bc}^a e_\mu^b \rho^c. \end{aligned} \quad (3.3)$$

In order to recover the nilpotency of the operation s , it is necessary to add an α -dependent term to the variation of the ghost τ^a given in (2.6). The resulting action of the BRS operator on the fields is

$$\begin{aligned} \hat{s} \omega_\mu^a &= -(D_\mu \tau)^a - \alpha \varepsilon_{bc}^a e_\mu^b \rho^c \\ \hat{s} \tau^a &= \frac{1}{2} \varepsilon_{bc}^a (\tau^b \tau^c + \alpha \rho^b \rho^c) \\ \hat{s} e_\mu^a &= -(D_\mu \rho)^a - \varepsilon_{bc}^a e_\mu^b \tau^c \\ \hat{s} \rho^a &= \varepsilon_{bc}^a \tau^b \rho^c \\ \hat{s} \xi^a &= \partial^a \rho, \quad \hat{s} \partial^a = 0 \\ \hat{s} \lambda^a &= \partial^a \rho, \quad \hat{s} \partial^a = 0, \end{aligned} \quad (3.4)$$

and

$$\hat{s}^2 = 0. \quad (3.5)$$

The gauge-fixing term, with the choice of the reference metric made in the previous section, now takes the form

$$\begin{aligned} S_{gf}^{\alpha} = \int d^3x \left(\partial^\mu \partial^\nu \omega_{\mu\nu}^a + \xi^a \partial^\mu (D_\mu \tau)_a + \partial^\mu \partial^\nu e_{\mu\nu}^a \right. \\ \left. + \lambda^a \partial^\mu (D_\mu \rho)_a - \varepsilon_{abc} (\partial^\mu \lambda^a) e_\mu^b \tau^c - \alpha \varepsilon_{abc} (\partial^\mu \xi^a) e_\mu^b \rho^c \right). \end{aligned} \quad (3.6)$$

Note that the cosmological constant appears in the gauge-fixed action only in terms which

are cubic in the fields. This fact, together with the linearity of the symmetry transformations δ_σ (2.12) implies that the equations

$$\delta_\sigma (S_{CS}^a + S_{gf}^a) = 0, \quad (3.7)$$

and

$$\begin{aligned} \{\delta_\sigma, \delta_\rho\} &= 0 \\ \{s, \delta_\rho\} &= \partial_\rho + \text{eq. of motion}, \end{aligned} \quad (3.8)$$

still remain valid even in the presence of a cosmological constant with unchanged δ_σ . The associated Slavnov identity reads:

$$\int d^3x \left(\frac{\delta \Sigma^a}{\delta \Omega^{a\mu}} \frac{\delta \Sigma^a}{\delta \omega_{\mu\alpha}} + \frac{\delta \Sigma^a}{\delta \gamma^{a\mu}} \frac{\delta \Sigma^a}{\delta \epsilon_{\mu\alpha}} + \frac{\delta \Sigma^a}{\delta L^a} \frac{\delta \Sigma^a}{\delta \tau_a} + \frac{\delta \Sigma^a}{\delta N^a} \frac{\delta \Sigma^a}{\delta \rho_a} + b^a \frac{\delta \Sigma^a}{\delta \lambda^a} + d^a \frac{\delta \Sigma^a}{\delta \xi^a} \right) = 0, \quad (3.9)$$

with

$$\Sigma^a = S_{CS}^a + S_{gf}^a + S_s^a, \quad (3.10)$$

and

$$\begin{aligned} S_s^a &= \int d^3x \left(-\Omega^{a\mu} ((D_\mu \tau)^a + \alpha \epsilon_{abc} \dot{e}_\mu^b \rho^c) + N^a \epsilon_{abc} \tau^b \rho^c \right. \\ &\quad \left. + \frac{1}{2} L^a \epsilon_{abc} (\tau^b \tau^c + \alpha \rho^b \rho^c) - \gamma^{a\mu} ((D_\mu \rho)_a + \epsilon_{abc} \dot{e}_\mu^b \tau^c) \right). \end{aligned} \quad (3.11)$$

With respect to the case discussed in Sec.2 nothing changes in the equations of motion of the Lagrangian multipliers and of the antighosts, as well as in the Ward identity of the δ_σ symmetry.

4. Complex gauge group

In the presence of a non-vanishing cosmological constant, the action (3.2) can be generalized [4] by introducing a topological term which does not modify the classical equations of motion. The resulting action shares a common structure with the Chern-Simons models built with a complex gauge group [14]. So we consider now the general case in which the gauge group G_C is defined by the complex extension of the Lie algebra of a compact simple group G .

Let $\{T^a\}$ be the Hermitian generators of G in the defining representation with

$$[T^a, T^b] = i f^{abc} T^c, \quad (4.1)$$

and

$$\text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}. \quad (4.2)$$

In terms of the complex fields

$$\begin{aligned} A_{+\mu} &= (A_\mu^a + i B_\mu^a) T^a \\ A_{-\mu} &= (A_\mu^a - i B_\mu^a) T^a, \end{aligned} \quad (4.3)$$

the action of the Chern-Simons model with gauge group G_C takes the simple form [14]:

$$\begin{aligned} S &= \frac{t_+}{8\pi} \int d^3x \epsilon^{\mu\nu\rho} \text{Tr} (A_{+\mu} \partial_\nu A_{+\rho} + i \frac{2}{3} A_{+\mu} A_{+\nu} A_{+\rho}) \\ &\quad + \frac{t_-}{8\pi} \int d^3x \epsilon^{\mu\nu\rho} \text{Tr} (A_{-\mu} \partial_\nu A_{-\rho} + i \frac{2}{3} A_{-\mu} A_{-\nu} A_{-\rho}). \end{aligned} \quad (4.4)$$

The two complex coupling constants t_+, t_- appearing in (4.4) are related by conjugation $t_+ = k + is$, $t_- = k - is$ and, for topological reasons, k is quantized [14]. By construction, the action (4.4) is real and, in terms of the real vector components A_μ^a and B_μ^a , the expression (4.4) reads:

$$\begin{aligned} S &= \frac{k}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} \left(\frac{1}{2} A_\mu^a \partial_\nu A_\rho^a - \frac{1}{2} B_\mu^a \partial_\nu B_\rho^a - \frac{1}{6} f^{abc} A_\mu^a A_\nu^b A_\rho^c + \frac{1}{2} f^{abc} A_\mu^a B_\nu^b B_\rho^c \right) \\ &\quad + \frac{s}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} \left(-A_\mu^a \partial_\nu B_\rho^a + \frac{1}{2} f^{abc} A_\mu^a A_\nu^b B_\rho^c - \frac{1}{6} f^{abc} B_\mu^a B_\nu^b B_\rho^c \right). \end{aligned} \quad (4.5)$$

Eq.(4.5) displays an important property of the theory described by the action S ; namely, the tensorial group structure is determined by the maximal compact subgroup of G_C , i.e.

precisely by G . The action S is invariant under G_C gauge transformations defined by

$$\begin{aligned}\delta A_{+\mu} &= D_\mu(\Lambda_+) \Lambda_+ = \partial_\mu \Lambda_+ + i[\Lambda_{+\mu}, \Lambda_+] \\ \delta A_{-\mu} &= D_\mu(\Lambda_-) \Lambda_- = \partial_\mu \Lambda_- + i[\Lambda_{-\mu}, \Lambda_-],\end{aligned}\quad (4.6)$$

where

$$\begin{aligned}\Lambda_+ &= (\Lambda_1^a + i\Lambda_2^a) T^a \\ \Lambda_- &= (\Lambda_1^a - i\Lambda_2^a) T^a.\end{aligned}\quad (4.7)$$

Eq.(4.6) is equivalent to

$$\delta A_\mu^a = \partial_\mu \Lambda_1^a - f^{abc} A_\mu^b \Lambda_1^c + f^{abc} B_\mu^b \Lambda_2^c, \quad (4.8)$$

and

$$\delta B_\mu^a = \partial_\mu \Lambda_2^a - f^{abc} A_\mu^b \Lambda_2^c - f^{abc} B_\mu^b \Lambda_1^c. \quad (4.9)$$

By introducing the auxiliary real fields (L^a, M^a) and the anticommuting ghost fields ($c^a, \bar{c}^a, \bar{d}^a, d^a$), the combination of the gauge-fixing and the ghost Lagrangians in the Landau gauge takes the form

$$\begin{aligned}S_{gf} &= \int d^3x (L^a \partial^\mu A_\mu^a + M^a \partial^\mu B_\mu^a) \\ &+ \int d^3x (\bar{c}^a \partial^\mu (\partial_\mu c^a - f^{abc} A_\mu^b c^c + f^{abc} B_\mu^b \bar{c}^c)) \\ &+ \int d^3x (\bar{d}^a \partial^\mu (\partial_\mu d^a - f^{abc} A_\mu^b \bar{d}^c - f^{abc} B_\mu^b c^c)).\end{aligned}\quad (4.10)$$

In terms of the real components ($A_\mu^a, B_\mu^a, L^a, M^a$) and of the ghost fields ($c^a, \bar{c}^a, d^a, \bar{d}^a$), the existence of SUSY transformations under which the total action $S + S_{gf}$ is invariant is not obvious. In addition, the particular structure of the physical content of the theory is not immediately recognized. It is convenient then to perform a linear change of variables. By

defining the fields

$$\begin{aligned}B_+^a &= \frac{1}{2}(L^a - iM^a) \\ B_-^a &= \frac{1}{2}(L^a + iM^a) \\ \psi_+^a &= \frac{1}{\sqrt{2}}(c^a + i\bar{d}^a) \\ \psi_-^a &= \frac{1}{\sqrt{2}}(c^a - i\bar{d}^a) \\ \bar{\psi}_+^a &= \frac{1}{\sqrt{2}}(\bar{c}^a - i\bar{d}^a) \\ \bar{\psi}_-^a &= \frac{1}{\sqrt{2}}(\bar{c}^a + i\bar{d}^a),\end{aligned}\quad (4.11)$$

the expression (4.10) can be written as

$$\begin{aligned}S_{gf} &= \int d^3x (B_+^a \partial^\mu A_{+\mu}^a + \bar{\psi}_+^a \partial^\mu (\partial_\mu \psi_+^a - f^{abc} A_{+\mu}^b \psi_+^c)) \\ &+ \int d^3x (B_-^a \partial^\mu A_{-\mu}^a + \bar{\psi}_-^a \partial^\mu (\partial_\mu \psi_-^a - f^{abc} A_{-\mu}^b \psi_-^c)).\end{aligned}\quad (4.12)$$

The structure of the total action, obtained by combining (4.4) with (4.12), shows the most striking feature of the Chern-Simons theory with complex gauge group G_C : the model contains two sets of decoupled degrees of freedom ($A_{+\mu}^a, B_{+\mu}^a, \psi_+^a, \bar{\psi}_+^a$) and ($A_{-\mu}^a, B_{-\mu}^a, \psi_-^a, \bar{\psi}_-^a$). In other words, with respect to the case of the compact group G , there is a "doubling". Each of the two sub-systems coincides with the standard Chern-Simons model with group structure constants inherited by G and coupling constants t_+ and t_- respectively. In particular it is easily recognized that the combination of the actions (4.4) and (4.12) possesses a SUSY invariance which is a straightforward extension of the transformations defined in [7].

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