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THE POSSIBILITY OF A LIQUID CRYSTAL PHASE IN NUCLEAR MATTER

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ABSTRACT

We show on the basis of a flux tube model that nucleons are likely to have large shape correlations in nuclear matter at high density. The possibility that this could lead to a new phase of nuclear matter is discussed.

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It is generally accepted that the nucleons in nuclei are in a gaseous phase, and that at high density or temperatures the nucleons will dissociate to a quark-gluon plasma. There have been many suggestions of more exotic phases: for example, the ground state of nuclear matter at medium densities may consist of strange matter¹, and various forms of boson condensates have been suggested². In this note we consider the possibility that there may be an intermediate phase when the nucleons behave as a liquid crystal.

There are a wide variety of models where the quarks interact via either strings or flux tubes^{3,4}. One of us⁵ has suggested that an appropriate description would be massless quarks interacting via a flux tube, which effectively provides the mass of the hadron. The vibrational modes of the string correspond to non-zero separation between the quarks: in other words, the meson has an instantaneously string-like shape. A free hadron (we consider only $l=0$ states and ignore spin) will necessarily be spherically symmetric, but the properties of a hadron buried in a nuclear medium may differ considerably. It is, for example, widely believed that at least part of the explanation of the EMC effect is due to the swelling of a nucleon in nuclear matter⁶, although alternative explanations are not ruled out⁷. Hence in a string model one should not be surprised if sizeable short-range shape correlations exist between neighbouring hadrons. In view of the elongated nature of the individual hadrons, it is plausible that this could give rise to a phase where the deformations were strongly correlated: instead of the nucleons forming a gas of hard spheres, they would form a liquid of rods at high density. It is appropriate to refer to this as a liquid crystal phase.

It is possible to provide a partial answer via numerical techniques. One of us has developed a Monte-Carlo method based on the flux tube model, which provides a plausible description of nuclear matter, without any explicit reference to nucleons. An early version was described in Ref. 8; a much more sophisticated version has now been developed and will be described in detail elsewhere⁹. Currently the model is restricted to SU(2) colour ($n_c = 2$), which means that the nucleons are made of a pair of quarks with total spin 0, but in general we can consider a model in which the N quarks can have n_s spin degrees of freedom, n_c colour s and n_f flavours. In the present work we take $n_s=n_f=2$, and consider up to N = 16 quarks. Although the model cannot be taken seriously as a quantitative model for nuclear matter (because of its restrictions to SU(2) colour), it shows nuclear binding, saturation and the swelling of nucleons which can be used to explain the EMC effect. It is therefore natural to use it to look for other short-range correlation phenomena.

It is assumed that each pair is instantaneously a flavour and colour singlet, and then the inclusion of a conventional spin-spin interaction effectively forces each nucleon to be a spin singlet. There are no flavour changing interactions in this model, and since the nucleons are ud combinations, there is no need to perform any explicit antisymmetrisation for the quarks in a given nucleon. If we consider two pairs of quarks, assumed to be distinguishable, then the wave function would normally be written as a product of the form

$$\psi(\vec{r}) = \phi(\vec{r}_{12}) \phi(\vec{r}_{34}) \quad (1)$$

where the r_{ij} are chosen so as to minimise the total length of the flux tubes: we refer to this as the "optimum set" of linkages. The two-body wave function is chosen to be

$$\phi(\vec{r}_{1j}) = \exp\left\{-\beta\left|\vec{r}_1 - \vec{r}_j\right|\right\}^\alpha \quad (2)$$

where α and β are variational parameters. It has been argued by Isgur and Paton¹⁰ that this extreme model is unlikely to be correct, because one expects that where two adiabatic surfaces are close, the region where they intersect will be softened. For example, if we consider the arrangement in fig 1a, then as the quarks d_1 and d_2 are displaced along the axis x, then the extreme flip-flop model would give an interaction like V_H in fig 1c. The lower curve is probably closer to reality, giving rise to a long range attraction between nucleons. There is no convenient parametrisation of this effect that can be applied to the many-body situation under consideration, but one can include the effect of near optimum arrangements by writing

$$\psi(\vec{r}) = \phi(\vec{r}_{12}) \phi(\vec{r}_{34}) + \phi(\vec{r}_{14}) \phi(\vec{r}_{32}) \quad (3)$$

If the surfaces are well separated, then only the optimum set contributes, while if two surfaces are close, then the second term is important and the wave function will be enhanced: in a sense, it contains the ghosts of other adiabatic surfaces in its structure. The actual wave function in the many body case can be written

$$\psi(\vec{r}) = \sum_{\text{perms.}} \left[\prod \phi(\vec{r}_{ij}) \right] \quad (4)$$

In practice, the number of permutations is very large, but the only ones which contribute significantly are those which permute two quarks from the original optimum set, and these are the ones which are included in the calculation.

A useful test problem, which mimics some of the features of the present calculation, is the double oscillator potential

$$V(\vec{r}) = \sigma \left(x^2 + y^2 + z_{\min}^2 \right) \quad (5)$$

$$z_{\min} = \min(|z-z_0|, |z+z_0|)$$

whose solution is analytically known¹¹. The wave functions which are the analogues of (1) and (3) are

$$\psi(\vec{r}) = \exp\left\{-\beta(x^2 + y^2) - (\gamma z_{\min})^\alpha\right\} \quad (6)$$

and

$$\psi(\vec{r}) = \exp\left\{-\beta(x^2 + y^2)\right\} \left[\exp\left\{-(\gamma(z-z_0))^\alpha\right\} + \exp\left\{-(\gamma(z+z_0))^\alpha\right\} \right] \quad (7)$$

These differ little except close to $z = 0$, but (7) lies lower in energy, and provides a better approximation to the exact wave function, because the (unphysical) cusp in (6) gives rise to an increase in the kinetic energy. In the same way, (3) has an energy lying lower than that of (1).

The actual solution of the many-body Schrödinger equation is found by using the standard Metropolis algorithm. The double oscillator provides a useful quantitative check of the algorithm, and agreement is very good. In principle the best values of the variational parameters α and β in (2) can be found by choosing them to minimise the energy $E(\alpha, \beta)$. In practice the minimum is very flat, so it is adequate to set α to the value which optimises the solution for an isolated hadron, which turns out to be $\alpha = 1.747$ for a purely linear interaction, and use β as the sole variational parameter.

Following the arguments of Greenberg and Hietarinen and Robson¹², quarks connected by different arrangements of flux tubes are taken to be orthogonal: in other words, there is no need for explicit antisymmetrisation of the wave function between quarks in separate nucleons. Note that the two terms in (3) are orthogonal in colour space, so the terms need to be added in quadrature. It is to be emphasised that colour is used for labelling quarks, rather than as a genuine dynamical variable. The wave function is an instantaneous one: as the Monte-Carlo proceeds pairings, spins and colours will change. One can obviously interpret this as the exchange of a gluon along with the tube flip. We have, in fact, made the calculation with the quarks antisymmetrised via a Slater determinant of plane waves, one for each degree of freedom. Although this makes a considerable difference to the overall energy, it turns out to make almost no difference to the effect under consideration.

Normal liquid crystals have a permanent axis associated with the molecule, which for simplicity can be taken to be cylindrical. It is traditional to use the order parameter¹³

$$s = \frac{1}{2} \langle 3 \cos^2(\theta) - 1 \rangle \quad (8)$$

where θ is the angle between the molecular axis and an external preferred axis. Obviously $s=1$ corresponds to perfect alignment and $s=0$ to a purely random arrangement. This criterion cannot be used here, since the nucleons do not have anything that could be regarded as a permanent axis, so it is necessary to consider instantaneous correlations. Using the vectors $\vec{r}_1, \vec{r}_2$, and $\vec{R}$ as shown in fig (1d), one can define the correlates

$$\begin{aligned} f_1(\vec{r}_1, \vec{r}_2, \vec{R}) &= \widehat{r}_1 \cdot \widehat{r}_2 \\ f_2(\vec{r}_1, \vec{r}_2, \vec{R}) &= \frac{1}{2} (\widehat{r}_1 + \widehat{r}_2) \cdot \widehat{R} \\ f_3(\vec{r}_1, \vec{r}_2, \vec{R}) &= \widehat{r}_1 \times \widehat{r}_2 \cdot \widehat{R} \end{aligned} \quad (9)$$

The absolute values are used since there is no meaning to the sense of the vectors. These obviously satisfy

$$0 < f_i < 1 \quad (10)$$

It is not a priori clear whether one expects any correlation at all, and if so what vectors are likely to be correlated. For example, the liquid crystal type correlation of fig 1a depends on the closeness of the

adiabatic surfaces of fig 1c. This would give rise to $f_1 \sim 1$, f_2 , $f_3 \sim 0$. However there are several arrangements where there will be nearby adiabatic surfaces: arrangements of the type shown in fig 1e would correspond to $f_1 \sim 1$, $f_2, f_3 \sim 0$, fig 1f would correspond to $f_1 \sim 0$, $f_2 \sim 1/2$ and $f_3 \sim 1$ and fig 1g would correspond to $f_1 \sim -1/2$, $f_2 \sim 0$ and $f_3 \sim 0$. In a string model, where only continuous changes are allowed, (fig 1h), one would only expect the first type of correlations. For randomly oriented pairs the probabilities for the various f_i are

$$\begin{aligned} P(f_1) df_1 &\propto df_1 \\ P(f_2) df_2 &\propto (1-f_2) df_2 \\ P(f_3) df_3 &\propto (\pi/2 - \sin^{-1} f_3) df_3 \end{aligned} \quad (11)$$

so that

$$\langle f_1 \rangle = 0.5, \langle f_2 \rangle = 1/3 \text{ and } \langle f_3 \rangle = \pi/8 \quad (12)$$

The results of the Monte-Carlo are shown for large R ($= 1.25$ fm) are shown in fig (2). It is clear that the agreement with (10) is excellent.

If we consider correlations between nearest neighbours only, it is very clear that there is a very pronounced parallel alignment at short distances. In fig (3) we show plots of $P(f_1)$ for three small values of R , and $\rho = 0.5 \text{ fm}^{-3}$. In fig (4) we show the average value $\langle f_1(R) \rangle$ as a function of R . The correlation dies away to zero very rapidly beyond 0.5 fm. There are small effects for f_2 and f_3 . It is interesting that even at normal nuclear densities the effect remains: in fact the value of $\langle f_1(R) \rangle$ is almost independent of density, but the probability $\tilde{P}(\rho, R)$ of having two hadrons at a small separation R decreases at low density. This can be seen from the global correlation

$$\overline{f_1(\rho)} = \int \tilde{P}(\rho, R) \langle f_1(\rho, R) \rangle dR \quad (13)$$

which slowly increases as a function of density. It tends to be swamped by rather distant, randomly-oriented nearest neighbours even at a density of $\rho = 1.0 \text{ fm}^{-3}$. For separations of less than 0.5 fm the average value of f_1 is 0.57 , roughly independent of density.

Although not dramatic, the effect is very persistent. The values of f_1 remain much the same whether or not the wave function is antisymmetrised, or whether colour Coulomb effects are included. It is only slightly affected by varying the mass or the string tension, by changing the degrees of freedom or in going from local colour to global colour, where colour is permanently attached to a given quark. It is somewhat reduced if a colour magnetic interaction is included. It seems reasonable to conclude that there is a strong short-range shape correlation between neighbouring nucleons. It is surprising that the evidence is so clear, since the original choice of wave function (2) is spherically symmetric. It also seems likely that there will be similar spin correlations, and this is under investigation. However from this study, we are unable to conclude that this actually implies a new phase: although nuclear matter should become ordered, it would seem likely that the transition to a (totally disordered) quark-gluon plasma would come first. The errors on the Monte Carlo tend to become very large at high densities, which indicates that the choice of wave function is no longer a

good one. To answer the question of whether a new phase exists definitively will require the use of a correlated asymmetric wave function, as well as (ultimately) SU(3) colour.

It is worth emphasising that this effect of shape correlation persists even at normal nuclear density, for distances of less than 1/2 fm. There is, unfortunately, no very obvious experimental signature: for example, large momentum transfer due to multiple scattering off nearby quarks in neighbouring nucleons would be indistinguishable from scattering of quarks in the same nucleon. If this could be seen, it would provide dramatic evidence for the importance of quarks in ordinary nuclei: not only do conventional nucleons have no shape, but the idea of separate nucleons being separated by much less than their radius is quite outside conventional nuclear theory.

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Figure Captions

Fig 1a) Short-range alignment of quark pairs.

1b) Displacement of quarks leading to intersecting adiabatic surfaces,

1c) Schematic sketch of interaction energy for fig (1b)

1d) Definitions of the vectors used in defining the correlation functions.

1e), 1f), 1g) Three possible alternative arrangements of flux tubes, leading to nearby adiabatic surfaces.

1g) String deformation, leading to preferred parallel alignment.

Fig 2) Plot of correlation functions at large inter-nucleon separation.

Fig 3) Plot of f_1 at small separations.

Fig 4) Plot of $\langle f_1(R) \rangle$ for $r = 0.5 \text{ fm}^{-3}$ is shown.

Fig 2

Fig. 3

Fig 4