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Electroweak Radiative Corrections to b -quark Production

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ABSTRACT

The electroweak radiative corrections to the $Z^0 b\bar{b}$ and $\gamma b\bar{b}$ vertices are calculated in 't Hooft-Feynman gauge in the Standard Model and are given in a compact form in terms of three universal functions. The results are obtained as the equal mass limit of analogous flavour changing neutral current diagrams which avoids the need for fermion mass renormalization. The box diagram containing an internal top quark that occurs in the process $e^+e^- \rightarrow b\bar{b}$ is also calculated.

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1. Introduction

Of all the observable processes at the Z^0 resonance, $e^+e^- \rightarrow b\bar{b}$ is unique in that it receives contributions from one-loop Feynman diagrams that have top quarks attached to external fermion lines. These are namely the box and final-state vertex corrections. The existence of such unique corrections allows one, in principle, to disentangle top quark mass dependences that come essentially from corrections to the ρ parameter and that enter for all final states. The corrections to the ρ parameter [1] are highly model dependent and the presence of new physics [2] could easily mask the effect of the top quark were a second independent handle not available. The same ambiguity can also occur for many possible types of new physics. The independent observable provided by final state b -quarks is an extremely valuable tool to help tie down the type of new physics being observed should significant deviations from Standard Model predictions be observed.

2. Vertex Corrections

In this paper the electroweak (EW) radiative corrections to the process $e^+e^- \rightarrow b\bar{b}$ are studied. Since most of these corrections are available explicitly elsewhere [3–6] we restrict our discussion to those Feynman diagrams that contain internal top quarks. These are theoretically the most significant for the reasons noted above. Henceforth these corrections will be referred to as virtual W corrections. Corrections to the final state vertex $Zb\bar{b}$ are of particular interest because of their known quadratic dependence on the top quark mass, m_t , and because their effects can be well measured by exploiting the high event rates available in e^+e^- machines operating at the Z^0 resonance. These corrections have been calculated previously [7–10] but were either not given in full or not in a particularly transparent form. Here we present results in a form that is a simple generalization of that for massless internal quarks [3, 6]. All corrections are contained in three universal functions that depend only on masses and momenta but not on fermion quantum numbers. The vertex corrections are obtained here by taking the limit of the flavour changing neutral current vertex $Zq\bar{q}$ or $\gamma q\bar{q}$ in the limit $m_q \rightarrow m_b$. It can be demonstrated that this method produces identical results to usual the approach that uses mass counter-terms to produce finite contributions. It should be noted that the expressions for the

vertices are not, by themselves, gauge-invariant. Only when they are combined with self-energy and box diagrams to form complete S -matrix elements are physically meaningful gauge-invariant results obtained. However the leading top quark mass dependence in the vertex is gauge invariant as there is no available corresponding dependence in self-energy diagrams and the box diagram m_t -dependence is weaker. The top mass dependence of the vertex is exactly gauge invariant when the Z^0 is on-shell because it then represents a complete and independent contribution to the physical process $Z^0 \rightarrow b\bar{b}$. The expression for the vertex correction is rendered finite by the inclusion of a vertex counterterm. Throughout this work 't Hooft-Feynman ($R_{\xi=1}$) is assumed. The Euclidean metric, with time-like momenta squared, will be employed throughout. Conversion to the popular Minkowskian is simply a matter of changing the sign of squared momenta in all expressions and adopting consistent definitions for the form factors B_0 , C_0 and D_0 defined later.

For completeness expressions are also given for the box diagram in $e^+e^- \rightarrow b\bar{b}$ with two internal W bosons and an internal top quark. This diagram does not experience a resonant enhancement at $\sqrt{s} = M_Z$ and has asymptotic behaviour $\sim m_t^{-2} \ln m_t^2$. Its contribution is therefore very small near the resonance.

The diagrams contributing to the one-loop virtual W corrections to the vertices $Zb\bar{b}$ and $\gamma b\bar{b}$ are shown in Fig. 1. The particle labelled ϕ_W is the charged Goldstone boson associated with the W . For the case of massless internal quarks the diagrams containing ϕ_W vanish. All the diagrams with the exception of e and f are divergent. At this stage we will not specialize to the case of external b -quarks but rather just assume that the external quark occurs in an $SU(2)_L$ doublet and that it has weak isospin t_3 and electric charge Q . Its isospin partner, denoted i in the diagrams, will be taken to have isospin and charge t_3' and Q' respectively and mass m_i . The notation s_θ and c_θ will be used to denote the sine and cosine of the weak mixing angle θ_W and we define quantities proportional to left-handed coupling of the Z^0 to fermions,

$$\beta_L = \frac{t_3 - s_\theta^2 Q}{s_\theta c_\theta}, \quad \beta_L' = \frac{t_3' - s_\theta^2 Q'}{s_\theta c_\theta}.$$

We adopt a set of Feynman rules for which the left-handed coupling of the Z^0 to fermions is $ie\beta_L\gamma_\mu\gamma_L$, where γ_L is the left-handed helicity projection operator.

To calculate the diagrams in Fig. 1 we make use of previous work by Hou and

Stuart [11]. In Ref. 11 a detailed study was made of flavour changing neutral current (FCNC) decays of the possible 4th generation b' quark. In the course of this study all the diagrams of Fig. 1 were calculated with general couplings and assuming the masses of the incoming and outgoing fermions were different. This set encompasses all possible one-loop vector-boson-fermion vertex diagrams in all possible gauge theories. The results of this calculation were coded in the algebraic manipulation language REDUCE in a form that could be processed by the package, LERG-I [12, 13]. LERG-I reduces the one-loop expressions to a form in terms of a few generic scalar integrals defined by

$$B_0(p^2; m_1^2, m_2^2) = \int \frac{d^n q}{i\pi^2} \frac{1}{[q + m_1^2][q + p]^2 + m_2^2} \quad (2.1a)$$

$$C_0(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) = \int \frac{d^n q}{i\pi^2} \frac{1}{[q + m_1^2][q + p_1]^2 + m_2^2} \frac{1}{[q + p_1 + p_2]^2 + m_3^2} \quad (2.1b)$$

$$D_0(p_1^2, p_2^2, p_3^2, p_4^2, p_5^2, p_6^2; m_1^2, m_2^2, m_3^2, m_4^2) = \int \frac{d^n q}{i\pi^2} \frac{1}{[q + m_1^2][q + p_1]^2 + m_2^2} \frac{1}{[q + p_1 + p_2]^2 + m_3^2} \frac{1}{[q + p_1 + p_2 + p_3]^2 + m_4^2} \quad (2.1c)$$

where $p_5 = p_1 + p_2$ and $p_6 = p_2 + p_3$. Methods for the evaluation of these scalar integrals have been studied by 't Hooft and Veltman [14], Passarino and Veltman [15] and more recently by van Oldenborgh and Vermaseren [16].

With general algebraic expressions of this form available as REDUCE code, it is possible to subject the results to stringent checks that would be difficult or impossible in a hand calculation. The checks that were performed are discussed in detail in [11].

Clearly a computer program of this type provides a very powerful tool since any FCNC vector boson-fermion vertex can be calculated in any gauge theory, (see for example Ref. 17) simply by changing the couplings that are input. Moreover, true vertex diagrams of the form shown in Fig. 1a-f are well behaved in the limit as the masses of the external fermions become equal. Thus expressions obtained for the FCNC vertex can be used directly in the calculation of the flavour-conserving vertex under study. The pairs of diagrams h & i and j & k together have well-defined limits as the external fermion masses become equal. This limit involves

derivatives of the fermion self-energy and therefore needs some modification from the FCNC case. In the general FCNC vertex, UV divergences cancelled in the sum over internal fermions because of the unitarity of the Kobayashi-Maskawa matrix. In the present case we must instead include a vertex counterterm, Fig. 1k. The form of this counterterm depends on the renormalization scheme adopted. It will be assumed that the popular on-shell scheme is used, in which case diagram g has the value $(-2ig^3 c g t_3 / (16\pi^2)) B_0(0; M_W^2, M_W^2)$ for the Z^0 vertex and $(-2ig^3 s g t_3 / (16\pi^2)) B_0(0; M_W^2, M_W^2)$ for the photon vertex. It is not necessary to perform fermion wave function renormalization because the counterterm contributions, thus generated, cancel between diagrams g, l and m.

Consider the generic pair of vertex diagrams Fig. 2 that show corrections to external on-shell fermion legs of mass m . Let the one-loop fermion self-energy be

$$\Sigma(p) = i\not{p} [A_L(p^2)\gamma_L + A_R(p^2)\gamma_R] + mC(p^2) \quad (2.2)$$

where the vertex V_μ is of vector/axial-vector form. The form factor C includes a fermion mass counterterm contribution such that

$$C(-m^2) = \frac{1}{2} [A_L(-m^2) + A_R(-m^2)], \quad (2.3)$$

which comes from requiring that the pole in the fermion propagator occur at the point $p^2 = -m^2$. As usual there is an overall factor of $\frac{1}{2}$ multiplying external leg correction diagrams.

The sum of the diagrams in Fig. 2 is

$$V_\mu \left\{ A_L(-m^2)\gamma_L + A_R(-m^2)\gamma_R + 2m^2 \frac{\partial}{\partial p^2} \left[C(p^2) - \frac{1}{2} (A_L(p^2) + A_R(p^2)) \right] \right\}_{p^2 = -m^2}. \quad (2.4)$$

Here it is assumed that the external fermion cannot decay into its isospin partner and a W boson, so that $A_L(-m^2)$, $A_R(-m^2)$ and $C(-m^2)$ are all real.

It turns out that exactly the same result can be obtained from the analogous FCNC external leg corrections in the limit that the external fermion masses become equal. There is no factor of $\frac{1}{2}$ associated with such diagrams, but instead one finds that pairs of diagrams combine and together produce the derivatives of eq. (2.4).

The result obtained is then identical to eq. (2.4) without the need to impose the fermion mass renormalization condition (2.3).

The final result for the effective $Z^0 b\bar{b}$ takes the compact form

$$\Gamma_\mu^{Z^0 b\bar{b}} = ie \left(\frac{\alpha}{4\pi} \right) \gamma_\mu \gamma_L \left\{ \frac{\beta_L^i}{2s_\theta^2} \rho(q^2, M_W^2, m_i^2) - \frac{t_3 c_\theta}{s_\theta^2} \Lambda(q^2, M_W^2, m_i^2) \right. \\ \left. - \frac{t_3}{c_\theta s_\theta^3} \Xi(q^2, M_W^2, m_i^2) \right\}, \quad (2.5)$$

and for the $\gamma b\bar{b}$ vertex it is

$$\Gamma_\mu^{\gamma b\bar{b}} = ie \left(\frac{\alpha}{4\pi} \right) \gamma_\mu \gamma_L \left\{ \frac{Q^i}{2s_\theta^2} \rho(q^2, M_W^2, m_i^2) - \frac{t_3}{s_\theta^2} \Lambda(q^2, M_W^2, m_i^2) \right\}. \quad (2.6)$$

The functions ρ , Λ and Ξ are given by

$$\rho(q^2, M^2, m^2) = -\frac{1}{q^2 M^2} \left\{ 2q^4 M^2 - 4q^2 M^4 + 2q^2 M^2 m^2 - q^2 m^4 + 2M^6 - 3M^4 m^2 + m^6 \right\} \\ \times C_0(q^2, 0; M^2, m^2) \\ - \frac{1}{2q^2} \frac{1}{(m^2 - M^2)^2} \left\{ 6q^2 M^4 - 5q^2 M^2 m^2 + 2q^2 m^4 - 4M^6 + 6M^4 m^2 - 2m^6 \right\} \\ \times [B_0(q^2; m^2, m^2) - B_0(0; M^2, M^2)] \\ + \frac{1}{2q^2} \left(\frac{m^2}{M^2} \right) \frac{1}{(m^2 - M^2)^2} \left\{ 8q^2 M^4 - 6q^2 M^2 m^2 + q^2 m^4 - 4M^6 + 6M^4 m^2 - 2m^6 \right\} \\ \times [B_0(q^2; m^2, m^2) - B_0(0; m^2, m^2)] \\ - \frac{1}{4q^2 M^2} \frac{1}{(m^2 - M^2)^2} \left\{ 10q^2 M^4 - 9q^2 M^2 m^2 + 5q^2 m^4 - 8M^6 + 12M^4 m^2 - 4m^6 \right\} \\ \times C_0(q^2, 0; M^2, m^2) \\ \Lambda(q^2, M^2, m^2) = \frac{1}{q^2 M^2} \left\{ 4q^4 M^2 - 2q^2 M^2 m^2 + q^2 m^4 - 2M^6 + 3M^4 m^2 - m^6 \right\} \\ \times C_0(q^2, 0; M^2, m^2) \\ + \frac{1}{2q^2} \frac{1}{(m^2 - M^2)^2} \left\{ 2q^2 M^4 + 3q^2 M^2 m^2 - 2q^2 m^4 - 4M^6 + 6M^4 m^2 - 2m^6 \right\} \\ \times [B_0(q^2; M^2, M^2) - B_0(0; M^2, M^2)] \\ - \frac{1}{2q^2} \left(\frac{m^2}{M^2} \right) \frac{1}{(m^2 - M^2)^2} \left\{ 8q^2 M^4 - 6q^2 M^2 m^2 + q^2 m^4 - 4M^6 + 6M^4 m^2 - 2m^6 \right\} \\ \times [B_0(q^2; M^2, M^2) - B_0(0; m^2, m^2)] \\ + \frac{1}{4q^2 M^2} \frac{1}{(m^2 - M^2)^2} \left\{ 18q^2 M^4 - 13q^2 M^2 m^2 + q^2 m^4 - 8M^6 + 12M^4 m^2 - 4m^6 \right\} \\ \times C_0(q^2, 0; M^2, m^2) \quad (2.8)$$

$$\Xi(q^2, M^2, m^2) = \frac{m^2}{2q^2 M^2} \left\{ 4q^2 M^2 - q^2 m^2 + M^4 - 2M^2 m^2 + m^4 \right\} \\ \times C_0(q^2, 0; M^2, M^2, m^2) \\ - \frac{m^2}{2q^2 M^2} \left\{ 2q^2 M^2 - M^4 + 2m^2 M^2 - m^4 \right\} \\ \times C_0(0, q^2, 0; M^2, m^2, m^2) \\ + \frac{m^2}{4q^2 M^2} \left\{ q^2 + 2M^2 - 2m^2 \right\} \\ \times [B_0(q^2; M^2, M^2) - B_0(q^2, m^2, m^2)] \\ + \frac{m}{2M^2}. \quad (2.9)$$

The functions ρ and Λ vanish at $q^2 = 0$. When the mass of the internal quark, m_i , is set to zero, Ξ vanishes and ρ and Λ reduce to the expressions [6],

$$\rho(q^2, M^2, 0) = \frac{5}{2} - \frac{2M^2}{q^2} - \left(3 - \frac{2M^2}{q^2} \right) [B_0(q^2; 0, 0) - B_0(0, M^2, M^2)] \\ - 2q^2 \left(1 - \frac{M^2}{q^2} \right)^2 C_0(0, q^2, 0; M^2, 0, 0) \quad (2.10a) \\ = \frac{1}{2} - \ln(-u - i\epsilon) + 4 \left(1 + \frac{1}{2u} \right) [\ln(-u - i\epsilon) - 1] \\ - 2 \left(1 + \frac{1}{u} \right)^2 \left[\frac{\pi^2}{6} - \text{Sp}(1 + u + i\epsilon) \right] \quad (2.10b) \\ \Lambda(q^2, M^2, 0) = -\frac{9}{2} + \frac{2M^2}{q^2} + \left(\frac{q^2 - 2M^2}{q^2} \right) [B_0(q^2, M^2, M^2) - B_0(0; M^2, M^2)] \\ + 2M^2 \left(\frac{2q^2 - M^2}{q^2} \right) C_0(q^2, 0, 0; M^2, M^2, 0) \quad (2.11a) \\ = -\frac{5}{2} + \frac{2}{u} + \left(1 + \frac{2}{u} \right) l_1 \sqrt{\frac{4}{u}} \sqrt{1 - \frac{4}{u}} + \left(1 + \frac{1}{2u} \right) \frac{4}{u} l^2 \quad (2.11b)$$

for

$$u \equiv -q^2/M^2, \quad l \equiv \ln \frac{\sqrt{1 - 4/u} + 1 + i\epsilon}{\sqrt{1 - 4/u} - 1 + i\epsilon}$$

The coefficient of the function Ξ in eq. (2.5) is just the axial vector coupling of the Z^0 to the external fermion. It is therefore absent in the case of the photon

vertex eq. (2.6). The three functions ρ , Λ and Ξ are all UV finite. The function ρ receives contributions from diagrams a, b and h-k in Fig. 1. Λ receives contributions from diagrams b-k and Ξ from a, b, d-f. The leading quadratic m_i dependence is contained entirely in $\Xi \sim m_i^2/(2M_W^2)$ and arises in diagrams b and d. The remaining vertex corrections that contain virtual Z^0 's are not considered here but can also be expressed just in terms of ρ and Λ with different arguments.

The top mass dependence in this vertex correction has the effect of reducing the partial width $\Gamma(Z^0 \rightarrow b\bar{b})$ as compared to d and s -quarks.

3. Box Diagram

An expression for the charged current box diagram in $e^+e^- \rightarrow \mu^+\mu^-$ is given in Ref. 12. For fermions in doublets the form of the box diagram depends on whether the weak isospin, t_3 , of the in-coming and out-going fermions are the same or different. The box diagram for $e^+e^- \rightarrow b\bar{b}$ is a simple generalization of the expression for final state muons because both are down-type fermions. The box diagram is shown in Fig. 3. If the coupling of the W boson to a left-handed fermion is $ig/\sqrt{2}$ and $s = -(p_1 + p_2)^2$, $u = -(p_1 + p_4)^2$ then the box diagram is given by

$$\frac{g^4}{16\pi^2} I[\gamma\alpha\gamma_L]e[\gamma\alpha\gamma_L]b,$$

where $[e]$, $[b]$ indicate that the enclosed γ matrices act on electron and b -quark wave functions respectively, and

$$\begin{aligned} I = & \frac{1}{2u} \left[B_0(-s; M_W^2, M_W^2) - B_0(-u, m_t^2, 0) \right] \\ & - \frac{(2u^2 + sm_t^2)}{4u^2} C_0(-s, 0, 0; M_W^2, M_W^2, 0) \\ & - \frac{(2u^2 - sm_t^2)}{4u^2} C_0(-s, 0, 0; M_W^2, M_W^2, m_t^2) \\ & + \frac{(2u - m_t^2 + 2M_W^2)}{2u} C_0(0, -u, 0; M_W^2, 0, m_t) \\ & + \frac{1}{4u^2} [m_t^2(-su + sm_t^2 + 2u^2 + 2uM_W^2) - 2u(u + M_W^2)^2] \\ & \times D_0(-s, 0, -u, 0, 0, 0; M_W^2, M_W^2, 0, m_t^2) \end{aligned} \quad (3.1)$$

For $m_t = 0$ this reduces to the result given in Ref. 12 when the differing convention for t and u are taken into account. The evaluation of the D_0 form factor is simplified

by the presence of a massless internal particle. In this case the D_0 can be written as a single C_0 [14, 18]. One obtains

$$\begin{aligned} D_0(-s, 0, -u, 0, 0; M_W^2, M_W^2, 0, m_t^2) \\ = \frac{1}{M_W^4(m_i^2 - u)} C_0\left(k_1^2, k_2^2, k_3^2, \frac{m_i^2}{(m_i^2 - u)^2}, M_W^{-2}, M_W^{-2}\right), \end{aligned} \quad (3.2)$$

in which

$$k_1^2 = k_3^2 = \frac{u(m_i^2 - M_W^2 - u)}{M_W^2(m_i^2 - u)^2}, \quad k_2^2 = -\frac{s}{M_W^4}.$$

This last result is only valid below the thresholds of the D_0 form factor, in particular the $2W$ production threshold, because the transformation used to obtain this relation does not properly preserve the $i\epsilon$ prescription and therefore does not yield the correct imaginary parts. Any imaginary part generated by the right hand side of eq. (3.2) should be discarded.

For a large top quark mass $I \rightarrow -\frac{1}{4}m_t^{-2} \ln m_t^2$, and thus the box contribution is likely to be of limited practical importance. Its presence is required however to yield fully gauge invariant matrix elements.

The vertex corrections presented were recently used to place an upper bound on the top quark mass from LEP data [19].

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FIGURE CAPTIONS

1. Feynman diagrams with internal charged bosons that contribute to the vertices, $Z\bar{b}b$ and $\gamma\bar{b}b$.
2. Generic diagrams representing external fermion leg renormalization.
3. The box diagram in $e^+e^- \rightarrow b\bar{b}$.

Fig. 1

Fig. 3

Fig. 2