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STRONG – WEAK COUPLING DUALITY AND NON-PERTURBATIVE EFFECTS IN STRING THEORY

A. Font

Depto. de Fisica, Univ. Central de Venezuela,
Aptdo. 20513, Caracas 1020-A, Venezuela

L.E. Ibáñez

CERN, 1211 Geneva 23, Switzerland

D. Lüst

CERN, 1211 Geneva 23, Switzerland

and

F. Quevedo

Theoretical Division LANL
Los Alamos, NM 87545, USA

ABSTRACT

We conjecture the existence of a new discrete symmetry of the modular type relating weak and strong coupling in string theory. The existence of this symmetry would strongly constrain the non-perturbative behaviour in string partition functions and introduces the notion of a maximal (minimal) coupling constant. An effective Lagrangian analysis suggests that the dilaton vacuum expectation value is dynamically fixed to be of order one. In supersymmetric heterotic strings, supersymmetry (as well as this modular symmetry itself) is generically spontaneously broken.

Modular invariance appears in a variety of physical problems [1]. These symmetries involve an invariance under the inversion of coupling constants along with the discrete translations of a “theta term”. The first example of this type of symmetry in field theory was discovered by Cardy [2] who showed that the phase structure of the Abelian Higgs model on the lattice exhibits such a type of invariance under inversion of couplings and shift of the θ -parameter. These transformations generate an infinite discrete group $SL(2, \mathbb{Z})$. In the context of string theory such a sort of symmetry seems also ubiquitous. The one-loop partition functions in terms of the world sheet modular parameter τ must be explicitly modular invariant. More recently [3] it has been realized that the spectrum and interactions of compactified strings are invariant under the inversion of the compactification length $R \rightarrow \alpha'/R$ (α' denotes the string tension) if one simultaneously replaces quantized momenta by winding modes (target space duality). This symmetry survives order by order in perturbation theory [4]. Therefore all inequivalent compactifications are unambiguously characterized by radii R larger (smaller) than the minimal (maximal) length $\sqrt{\alpha'}$, the scale set by the extension of the string itself. In the case of a two-dimensional compactification the duality symmetry is extended to the full modular invariance due to the existence of the B_{mn} antisymmetric tensor which acts as a θ -parameter. In more realistic six-dimensional compactifications (like e.g. orbifolds) the same structure (conveniently generalized) is also found. This target-space modular invariance strongly constrains the form of the low-energy effective action as a function of the compactification moduli [5]. It can also give interesting information about the possible form of non-perturbative string corrections (like e.g. supersymmetry breaking [6], [7]) if duality were an exact symmetry of string theory (possibly broken spontaneously but not explicitly) [6].

In the present letter we conjecture the existence of a further modular invariance symmetry in string theory. This includes a duality invariance under which the string dilaton (whose vev. yields the coupling constant in heterotic strings) gets inverted. In four dimensions the dilaton comes along [8] with a pseudoscalar (“axion”) field θ . Both degrees of freedom form a complex scalar

$$S = \frac{1}{g^2} + i\theta \quad (1)$$

which is the lowest component of a chiral superfield in supersymmetric 4-D strings. Thus we conjecture an invariance under the modular symmetry

$$S \rightarrow \frac{aS - ib}{icS + d} \quad (2)$$

where $a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1$. Such a symmetry strongly constrains the theory since it relates the weak and strong coupling regimes as well as the different “ θ -sectors” of the theory. We do not have a proof that this is actually a symmetry of string theory, but we will try to convince the reader that there are a number of arguments which make our conjecture plausible. Then we will show that, if this symmetry is actually there, there are several interesting consequences. We do most of our analysis in terms of the effective ($N=1$ supergravity) low energy field theory. One then finds that the S -modular invariance property makes the string tree-level results (sphere) for the effective action badly unstable and that the vacuum expectation value of the string dilaton is dynamically fixed to be of order one. We also find that supersymmetry is generically broken in heterotic strings.

In order to motivate our discussion we will first describe some facts about a better known type of modular invariance, the one associated to the target space duality. This we do because of the close parallelism existing between both types of duality. We will then briefly review some facts which lead us to believe that the new modular invariance of eq. (2) is actually a symmetry of string theory. We finish by presenting some consequences of the existence of this symmetry.

Consider any heterotic string compactification from ten to four dimensions. There is always one massless complex scalar, usually denoted T , whose vev is associated to the compactification size R and the antisymmetric background B_{mn} as follows

$$T = \frac{R^2}{\alpha'} + iB = \frac{1}{g_\sigma^2} + iB \quad (3)$$

where $g_\sigma^2 = \alpha'/R^2$ is the coupling constant of the 2-d non-linear σ -model. The presence of this mode is related to the existence of a marginal internal operator $\partial X_L(\bar{z}) \times \partial X_R(z)$. Duality in g_σ occurs if there are non-contractible loops around which the string can wind. Then the spectrum and interactions are invariant

under $g_\sigma^2 \rightarrow 1/g_\sigma^2$. For example, in the case of compactification of the closed bosonic or heterotic string on a one-dimensional circle, one gets a mass formula

$$m^2 = \frac{1}{\alpha'}(n^2 g_\sigma^2 + \frac{l^2}{g_\sigma^2}) = \frac{1}{2}(P_L^2 + P_R^2) \quad (4)$$

which is explicitly invariant under that symmetry if one simultaneously changes quantized momenta n by winding number l . In fact this duality admits an interpretation in terms of world-sheet “electric-magnetic” charge duality in the following sense. For this kind of toroidal compactification the theory has a gauge symmetry $U(1)_L \times U(1)_R$. Then $e = \sqrt{\alpha'} n/R$ is the (world-sheet) “electric” charge and $g = lR/\sqrt{\alpha'}$ is the “magnetic” charge of a (world-sheet) “dyon”. The electric $U(1)_e = U(1)_{L+R}$ current $\oint d\bar{z} \partial X_L(\bar{z}) + \oint dz \partial X_R(z)$ measures precisely $e = \frac{1}{2}(P_L + P_R)$ whereas its dual $U(1)_m = U(1)_{L-R}$ measures $g = \frac{1}{2}(P_L - P_R)$. The theory is selfdual (it is invariant under the exchange $e \longleftrightarrow g$) and is characterized by a maximal (minimal) coupling constant g_σ . The duality transformation acts like $\partial X_L \rightarrow \partial X_L, \partial X_R \rightarrow -\partial X_R$, i.e., it exchanges $U(1)_e \leftrightarrow U(1)_m$. Considering the heterotic string theory, toroidal compactification of some or all six internal dimensions leads to enhanced $N = 2, 4$ space-time supersymmetry. Then $\oint dz \partial X_R(z)$ appears as a central extension in the $N = 2, 4$ supersymmetry algebra [9] since this operator is contained in the operator product expansion of two right-moving supercharge operators. Thus the eigenvalues $e - g$ are the non-vanishing central charges of the (massive) “electrically” and “magnetically” charged states in a sense similar to Witten and Olive [10].

More generally $R \rightarrow \alpha'/R$ duality can be interpreted in terms of field theoretical “duality” rotations between field equations and Bianchi identities on the world-sheet [11], [12]. The compactified torus dimensions only appear in the world-sheet sigma model through $\partial_\sigma X$. We may then define a “dual field strength”

$$\epsilon_{\alpha\beta} \partial^\beta X = \partial_\alpha \tilde{X}, \quad (5)$$

where $\alpha, \beta = 1, 2$. One can then show that the partition functions corresponding to the two world-sheet actions

$$G_{mn} \partial X^m \partial X^n \longleftrightarrow G^{-1mn} \partial \tilde{X}_m \partial \tilde{X}_n \quad (6)$$

are equivalent so that there is $G \rightarrow G^{-1}$ duality ($R \rightarrow 1/R$ duality in the simplest toroidal case). The indices m, n run over the compactified dimensions.

In a two-dimensional complex compactification there is a new periodic variable B associated to the background of the antisymmetric field and the duality symmetry is promoted to a complete invariance under $PSL(2, \mathbb{Z})$

$$T \rightarrow \frac{aT - ib}{icT + d}. \quad (7)$$

In more realistic six-dimensional compactifications the situation is rather more involved, but a subgroup $PSL(2, \mathbb{Z})$ associated to the field describing the volume of the 6-manifold is always expected. Of special interest (from the phenomenological point of view) is the case in which there is (at least on the sphere) an unbroken space-time supersymmetry in four dimensions. In this case the effective low-energy field theory is a $N = 1$ supergravity Lagrangian [13] which is completely determined (up to two derivatives) by the Kähler potential G and the gauge kinetic function f . The function G may be split into a part which cannot be written as a sum of holomorphic and antiholomorphic fields (the scalar kinetic function K) and the superpotential term so that

$$G(\phi, \phi^*) = K(\phi, \phi^*) + \log |W(\phi)|^2. \quad (8)$$

At the tree level the Kähler function $K(T, T^*)$ is given by the Zamolodchikov metric [14] which leads to

$$K(T, T^*) = -3 \log(T + T^*) \quad (9)$$

i.e. the well-known field theoretical truncation limit [8]. In fact that field theoretical limit possesses a global non-compact symmetry $SU(1, 1)$ and the T -field lives in a σ -model which parametrizes the coset $SU(1, 1)/U(1)$. This symmetry contains an invariance under dilatations as well as a continuous Peccei-Quinn symmetry under which $T \rightarrow T + i\alpha$, $\alpha \in \mathbb{R}$.

The field-theoretical truncation results get explicitly broken by σ -model (i.e. world-sheet) instantons [15] which contribute to the superpotential W with terms

$\sim e^{-T}$. However the modular subgroup $SL(2, \mathbb{Z}) \in SU(1, 1; \mathbb{R})$ remains unbroken at this level since target-space duality is an exact symmetry (at least in string perturbation theory). Thus, e.g. the continuous Peccei-Quinn symmetry is broken to a discrete shift subgroup. Still, the kinetic function associated to the T -field is given by the Kähler metric of the $SU(1, 1)/U(1)$ σ -model, i.e. the world-sheet instanton corrections appear only through the superpotential. That has been explicitly checked in orbifold compactifications [16], [17] for the case of the sphere. The Yukawa couplings between twisted fields at different fixed points vanish in the field theory limit. World-sheet non-perturbative effects generate a superpotential of the form

$$W \sim F(T) C_i^3 \quad (10)$$

where the twisted matter fields C_i and $F(T)$ transform like

$$C_i \rightarrow \frac{C_i}{(icT + d)^2} ; \quad F(T) \rightarrow (icT + d)^3 F(T) \quad (11)$$

under modular transformations. Here the function $F(T)$ is related to characters $\chi(T)$ of some Kac-Moody algebra, e.g.

$$F(T) \sim \eta(T)^6 \chi(T) \sim \sum_n a_n e^{-b_n T}, \quad (12)$$

where $\eta(T)$ is the well-known Dedekind function. Thus the overall Kähler potential G is modular invariant since the variation of K is cancelled by that of $|W|$. We recognize in eq. (12) that $F(T)$ contains the expected world-sheet instanton behaviour e^{-T} in a duality invariant way.

Since T corresponds to a modulus, we know that the purely T -dependent superpotential has to vanish order by order in perturbation theory so that the vev of T remains undetermined. However one expects that non-perturbative (infinite genus) effects will generate a superpotential for T . It was argued in ref. [5] that the simplest expression for $W(T)$ compatible with modular invariance and absence of singularities in the fundamental domain is*

* More general possibilities can be obtained by using eq.(24) in the variable T with $r = -3$.

$$W(T) \sim \eta(T)^{-6}. \quad (13)$$

In this case the complete T -dependent Kähler potential can be written as

$$G(T, T^*) = -3 \log(T_R |\eta(T)|^4) \quad (14)$$

A Kähler potential as in eq.(14) gives rise to a scalar potential for the modulus

$$V(T) \sim \frac{1}{(T + T^*)^3} \frac{1}{|\eta(T)|^{12}}. \quad (15)$$

This potential looks like a bosonic partition function whose modular parameter is T . Indeed it can be argued [6] that a superpotential like that in eq. (13) is in fact generated in the presence of gaugino condensation in some four-dimensional heterotic strings. The potential in eq. (15) has the property that it diverges in the $Re\ T \rightarrow \infty$ and the $Re\ T \rightarrow 0$ limits since one has

$$\eta(T)^{-n} = e^{\frac{n\pi^2}{12}} (1 + ne^{-\pi n T} + \frac{n}{2}(3+n)e^{-2\pi n T} + \dots). \quad (16)$$

Thus e.g. the ("weak coupling") decompactification limit $Re\ T \rightarrow \infty$ is singular and hence cannot formally be reached. This behaviour does not occur at the tree-level (sphere) nor at any finite order in perturbation theory since we know that the non-renormalization theorems guarantee the absence of any superpotential for T . However there are hints that string perturbation theory breaks down for large (or small) T . For example, the one-loop contribution to the supergravity gauge kinetic function f as computed in ref. [18] yields for some compactifications a result $f \sim \log(Re T |\eta(T)|^4)$ which also diverges in the decompactification limit. Thus it is not surprising that an "infinite genus" result like the non-perturbative induced superpotential for T also diverges. This has the important physical consequence [6] that the theory is energetically forced to be compactified since a potential of the form in eq.(15) has its minimum at a finite $Re\ T \sim \alpha'$ **

** One can write down superpotentials which are not singular in the decompactification limit by multiplying the $\eta(T)^{-6}$ function by appropriate inverse powers of the absolutely modular invariant function $j(T)$. However [19] one then necessarily introduces spurious singularities with difficult physical interpretation at finite values of T .

Let us now discuss the case of the complex scalar field S of eq.(1). S is associated to the existence of a marginal operator (in the light-cone gauge) $\partial X_L^\pm(z) \otimes \partial X^-(z)_R$, where $\partial X^\pm = \partial X^3 \pm i\partial X^4$, $X^{3,4}$ being the uncompactified transverse coordinates. This is in complete analogy with the marginal operator associated to the modulus T . Again the associated Zamolodchikov metric dictates the form of the Kähler potential on the sphere

$$G(S, S^*) = K(S, S^*) = -\log(S + S^*). \quad (17)$$

The complete tree-level supergravity action contains the bosonic terms

$$e^{-1}\mathbf{L} = \frac{1}{S_R^2}|\partial_\mu S|^2 + R + S_R F_{\mu\nu} F^{\mu\nu} + \dots \quad (18)$$

where $S_R = S + S^*$. One can perform a Weyl rescaling of the metric (and also of the fermionic fields) of the form $g_{\mu\nu} \rightarrow S g_{\mu\nu}$ and then one gets

$$e^{-1}\mathbf{L} = S_R \left(\frac{1}{S_R^2} |\partial_\mu S|^2 + R + F_{\mu\nu} F^{\mu\nu} + \dots \right) \quad (19)$$

As it is well known, this classical action possesses [8] two kind of symmetries.

- 1) A continuous Peccei-Quinn symmetry under $S \rightarrow S + ia$, $a = \text{real constant}$. This forbids a possible S -dependence of the superpotential in perturbation theory (non-renormalization theorem [20]).
- 2) A classical invariance under which $e^{-1}\mathbf{L} \rightarrow \lambda e^{-1}\mathbf{L}$ if one scales $S \rightarrow \lambda S$. In fact the S field lives in a σ -model which parametrizes the coset $SU(1,1)/U(1)$ and the symmetry $SU(1,1)$ contains the above two symmetries. Of course, this full symmetry cannot survive at the quantum level. For example, it is obvious that the dilatation invariance will be broken by loop effects and only the scaling parameter $\lambda = 1$ will be allowed. The conjecture we want to make here is that the continuous non-compact symmetry $SU(1,1)$ associated to the dilaton complex scalar S σ -model is not completely broken by perturbative and non-perturbative effects but a subgroup $SL(2, \mathbf{Z})$, a modular discrete group, survives unbroken. As we advertised above, we are far from having a proof of this conjecture. However we would like now to discuss some plausibility arguments in its favour.

From the conformal field theory point of view both modes S and T are very similar, both corresponding to $\partial X_L \otimes \partial X_R$ marginal deformations involving different coordinates X (with periodic boundary conditions for the case of T). In fact we further compactified the two transverse space coordinates the parallelism between S and T would be strict but, of course, that is not the case.

One of the most obvious differences between S and T is that ReT is the breathing mode associated to the volume of the six-dimensional compact manifold whereas S does not have a breathing mode interpretation of this sort. However, from the (point-like) field theory point of view S may admit such an interpretation: if we obtain our ten-dimensional supergravity Lagrangian by dimensional reduction (circle compactification) from the eleven-dimensional supergravity Lagrangian (as was in fact first obtained [21]), the ten-dimensional dilaton (and hence S) is a breathing mode of the compactifying circle. Moreover, a proof of duality on the lines of equations (5) and (6), can be done for the case of extended objects (p-branes) [12], in particular for the $11 - D$ supermembrane. Since the type II A superstring can be obtained from those membranes from a double dimensional reduction mechanism, the duality transformations for the membrane include those of the string together with a duality transformation for the dilaton field. (Type II A strings, however, are not selfdual but "dual" to type II B strings [22], [23]). Of course, there is at the moment no idea about how a ten-dimensional heterotic string could be obtained from any eleven-dimensional extended structure, but that is certainly an open possibility. If this was the case, duality in both T and S would be expected.

The S -duality we are discussing includes an invariance under the transformation of the string coupling constant $g \rightarrow 1/g$. Montonen and Olive [24] conjectured some time ago that this type of duality invariance does in fact occur in field theory models of the Georgi-Glashow type (and for any other gauge group with adjoint scalars). They argued that both the spectrum and (classical) interactions of the elementary fields in these theories are identical to the spectrum and interactions of the monopole sector of the theory (after doing the replacement $g \rightarrow 1/g$). They also conjectured that this duality was also true quantum-mechanically. The Montonen and Olive conjecture is particularly compelling in the $N = 4$ extended supersymmetry case [25],[10]. In $N = 4$ the monopole sector of the theory nec-

essarily fits [25] into the same type of supermultiplet as the elementary fields since there is only one type of relevant supermultiplet available, and for the same reason the interactions are universal (up to the replacement $g \rightarrow 1/g$). There is a universal formula for the mass of any particle with electric and magnetic charge (dyon)

$$m^2 = v^2(Q_e^2 g^2 + \frac{(4\pi\hbar)^2 Q_m^2}{g^2}) \quad (20)$$

where v is the vev of the symmetry breaking scalar field and Q_e and Q_m are respectively the electric and magnetic charges. Notice that the dyon formula is the analogue of the string mass formula (4) where the role of quantized momenta and winding is played by the electric and magnetic charges. Now the electric and magnetic charges $Q_e g$, Q_m/g respectively appear as central charges in the extended supersymmetry algebra and the mass formula (20) therefore becomes exact in perturbation theory [10].

Since $N = 4$ super-Yang-Mills may be obtained as the massless limit of a toroidal compactification of the heterotic string, it seems reasonable to expect a similar phenomenon to occur in compactified heterotic strings. In this case there should be extended objects whose massless gauge sector would be dual (in the Montonen-Olive sense) to the $N = 4$ gauge sector of the compactified heterotic string. The existence of this type of classical solutions has been recently pointed out [26] by Strominger. He found a solution to the low-energy heterotic string field equations corresponding to a 10-dimensional “5-brane” which is dual to the 10-dimensional heterotic string. The existence of this “heterotic” 5-brane was conjectured by Duff [27] who also suggested that its low energy limit should correspond to the “dual” version of $D = 10$ supergravity which contains a 7-form field strength [21], [28] (instead of the standard version with a 3-form which appears in the low energy limit of the heterotic string). These two $D = 10$ theories are “dual” in a sense quite analogous to the way in which the two world-sheet actions in eq.(6) are dual.

In the case of four dimensions, four of the five spatial dimensions of the 5-brane may be compactified on the internal variety and thus these classical configurations would look like strings from the four-dimensional point of view. These strings would be dual to the standard elementary strings [26]. From the

low energy effective Lagrangian point of view, these two dual theories would look identical (up to the replacement $g \rightarrow 1/g$).

At this point one realizes the similarity of these two dual theories with the dual theories considered in ref.[1] and [2]. In fact, since the 5-brane is supposed to describe the physics of the heterotic string at large coupling, one can really talk of two different phases of a single theory instead of talking about two theories dual to each other. Then it is natural to conjecture that the complete theory, the one which describes simultaneously elementary and solitonic states, should present a modular symmetry of the type described in refs.[2],[1]. Thus we conjecture that the Montonen-Olive type duality of the heterotic strings will lead to a modular invariance symmetry as in eq.(2).

The effective Lagrangian should be explicitly duality invariant, as happens with the $R \rightarrow 1/R$ duality. The analogy is quite complete. For small g (big R) the “elementary” strings dominate and the “dual” particles (winding modes in T -duality) are very massive. For big g the opposite occurs. Thus a duality $g \rightarrow 1/g$ in the effective four-dimensional field theory should exist if the above arguments are correct. It follows that inequivalent theories are characterized by coupling constants g smaller (larger) than some critical value. (The notion of a maximal (minimal) coupling could possibly be understood in the sense that the coupling constant determines the “size” of the internal gauge group manifold which should not be “smaller” than the typical scale in string theory.) In analogy with T -duality, one also expects the continuous Peccei-Quinn symmetry $S \rightarrow S + ia$, $a \in \mathbf{R}$ not to be completely broken but a discrete subgroup ($a \in \mathbf{Z}$) to survive. In this way the effective field theory should be invariant under a full modular group $SL(2, \mathbf{Z})$ generated by the transformations in eq.(2).

Something similar should be true for the complete (non-perturbative) partition function Z_∞ of the theory. This partition function should contain not only the usual string excitations but also all the solitonic sectors (monopoles and multimonopole solutions and their partners), very much in the same way that the usual perturbative partition function contains momenta and windings in toroidal compactification. The existence of S -modular invariance will then

mean the invariance

$$Z_\infty(S) = Z_\infty(S'), \quad S' = \frac{aS - ib}{icS + d} \quad (21)$$

for the complete partition function in four dimensions. Such a symmetry would imply strong constraints on the possible form of such partition functions. It would also tell us that the non-perturbative partition function shares the same symmetry properties as the lattice field theory models described by Cardy [2]. In principle one can think of many possible partition functions $Z_\infty(S)$ consistent with the symmetry in eq. (21) like e.g.

$$Z_\infty \sim \frac{1}{(S + S^*)|\eta(S)|^4} \quad (22)$$

which looks like a bosonic partition function, or many others.

The invariance property in eq. (21) is not sufficient to fix uniquely the form of Z_∞ since, of course, there are many modular invariant possibilities. On the other hand one can try to extract some physical information by using an effective Lagrangian approach as in [5]. We would like now to explore some of the consequences of the existence of this new modular invariance by using such a kind of four-dimensional effective Lagrangian approach. Notice first that since the axion mode θ is related to the antisymmetric tensor through a duality transformation $\partial^\mu \theta \sim \epsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma}$, the transformation $\theta \rightarrow \theta + a$ is only consistent with Lorentz invariance in four (or less) dimensions. This does not mean that such symmetries are not present in higher dimensions, only that they would manifest themselves in a different way. We will thus focus here on the constraints that S -duality imposes on the form of the low-energy effective four-dimensional $N = 1$ supergravity action. To start with, the Kähler potential in eq. (17) is not modular invariant by itself. To make the full function G modular invariant the superpotential must transform as

$$W \rightarrow \frac{e^{i\alpha}}{icS + d} W \quad (23)$$

under modular transformations. Here α is some phase (sometimes called in the mathematical literature the multiplier of W). Let us forget for the moment the

rest of the scalar fields. In general one expects that non-perturbative effects may generate a purely S -dependent superpotential $W(S)$ with the appropriate modular transformation properties. Holomorphicity and absence of unphysical singularities at finite values of S severely constrains the possible forms of W . In particular, the most general regular holomorphic modular function of weight r can be written [29]

$$\Delta(S) = (G_6(S))^p (G_4(S))^t (\eta(S))^{2r-12p-8t} P(j(S)) \quad (24)$$

(with $p, t \geq 0$) where G_6 and G_4 are the weight 6 and 4 Eisenstein functions and $P(j)$ is an arbitrary polynomial on the absolutely modular invariant function $j(S)$. The superpotential must have weight $r = -1$ and thus one has for the most general regular superpotential

$$W(S) = \frac{c}{\eta(S)^2} \left(\frac{G_6(S)}{\eta(S)^{12p}} \right) \left(\frac{G_4(S)}{\eta(S)^{8t}} \right) P(j(S)). \quad (25)$$

whose multiplier is $\alpha = \epsilon^{-2-12p-8t}$, ϵ being the multiplier of $\eta(S)$. This class of superpotentials all diverge as $S \rightarrow \infty, 0$ and are finite for finite values of S . They also might have zeros at some points. That is the case for p or $t > 0$ in which case $W(S)$ would have zeros at $S = 1, e^{i\pi/3}$. The simplest superpotential in this class is given by

$$W(S) = \frac{c}{\eta(S)^2} \quad (26)$$

where c is a constant. This is analogous to the result we discussed for T . Thus a non-perturbative purely S -dependent superpotential is constrained to have the form in eq. (25). The superpotentials involving matter fields should also transform as in eq.(23). Since (unlike what happened for the T -duality) the matter fields C do not seem to transform under this symmetry, this suggests that matter superpotentials should be of the form $W = W(C)W(S)$ where $W(S)$ here is a function of the type shown in eq.(25). Then the Yukawa resp. scalar self-coupling constants are given by $\lambda \sim \frac{|W(S)|^2}{s_R}$ which is the modular invariant extension of the sphere result $\lambda \sim 1/s_R$.

As a hint about the physics obeyed by the dilaton one could consider a situation neglecting the contribution of any other matter fields. One can then compute the scalar potential of the S field induced by the simple potential in eq. (26). In this case one has

$$G(S, S^*) = -\log(S_R |\eta(S)|^4) = \frac{1}{2\pi} \int^S dS' (G_2(S') - \frac{\pi}{S_R}) + h.c. \quad (27)$$

The scalar potential obtained from eq. (27) is

$$V(S) = e^G (G_S G_{\bar{S}})^{-1} G_{\bar{S}} - 3 = \frac{1}{S_R |\eta(S)|^4} \left(\frac{S_R^2}{4\pi} |\hat{G}_2(S)|^2 - 3 \right). \quad (28)$$

where $\hat{G}_2 = G_2 - 2\pi/S_R$ is the weight two (non-holomorphic) Eisenstein function. This potential diverges (just as happened with T) in the limits $S \rightarrow \infty, 0$ and has minima at finite values of S , close to the critical value $S = 1$. Thus the dilaton vev (and hence the value of the coupling constant) is determined to be $g \sim 1$. This is true for all the generic superpotentials of eq. (25). This is to be compared with the sphere result which yields $V(S) \sim 1/S_R$ implying [30] a run-away classical solution towards a non-interacting $g = 0$ theory. In our case this run-away solution is impossible because the duality property stabilizes the potential. This is just as well, since we know that in a duality invariant theory there is never a consistent perturbative regime since either the “electric” or “magnetic” charge are necessarily large.

Let us make some comments about the physical implications of the S -duality hypothesis. One of them is the immediate consequence that the sphere results, i.e. the perturbative vacuum must be badly unstable. This seems to be consistent with the results [31] found for the perturbative expansion of the bosonic string which turns out to be divergent and non-Borel summable. This behaviour at $Re S \rightarrow \infty$ points to the possibility of a completely different analysis of the dilaton potential in general. Previously, it was believed that this limit represents just a free field theory and therefore the dilaton potential should vanish there. This is the basis of the arguments of reference [30], where the impossibility of a weak coupling physical vacuum in string theory is argued. If $Re S \rightarrow \infty$ actually

does not represent a free theory then those arguments will have to be reconsidered. Another important consequence is that supersymmetry is generically broken in four-dimensional heterotic strings. One has a dilaton dependence for the gravitino mass of the type

$$m_{3/2} = e^G \sim \frac{1}{S_R |\eta(S)|^4} \quad (29)$$

which is nowhere vanishing. Any other superpotential of the more general class in eq.(25) yields the same qualitative result. Thus supersymmetry breaking seems difficult to avoid (since G_S is generically non-vanishing), at least to the extent to which one can trust the effective Lagrangian approach. The natural scale for this breaking seems to be the string scale, and hence it is not obvious to us how a small susy-breaking scale could be obtained. Let us finally remark that the S -field modular invariance will itself also be generically spontaneously broken. A scalar potential for the S field (as in eq.(28)) will be minimized for some specific S -value (and its modular transformed counterparts) and once this happens this modular invariance will be spontaneously broken, unless the minimum lies at a selfdual point. From then on the vev's of all S -dependent coupling constants like λ or the gauge coupling constant will just give us the boundary condition at the string scale and further evolve according to the renormalization group equations. Of course these latter conclusions depend somewhat on an analysis based on the effective four-dimensional supergravity Lagrangian. One should try to derive more stringy consequences from the general invariance of partition functions as in eq. (21). For example, combining this modular invariance with general factorization arguments for the perturbative partition functions, it may be possible to find a modular invariant “string equation” in the spirit of reference [32], that could determine the non-perturbative partition function. Nevertheless we think that the effective Lagrangian analysis reveals some general properties which could be expected to be present in a more stringy analysis.

The previous considerations could in principle be generalized if the full modular group does not necessarily factorize but involve the transformations of T and S together with other charged and neutral moduli. If this is the case, model dependence for S could be introduced in this scheme and mechanisms to select

between the different vacua may eventually be discovered. Although all the above discussion is very speculative, we think that the existence of this possible symmetry in string theory would be very important, especially in connexion with the problem of the undetermined value of the string dilaton at the level of the sphere. The existence or not of this duality should thus be addressed in order to make any further progress.

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