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ETERNAL EXTENDED INFLATION AND GRACEFUL EXIT FROM OLD INFLATION WITHOUT JORDAN-BRANS-DICKE

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ABSTRACT

Recently a possible solution to the graceful exit problem of the old inflation was proposed in the context of the Jordan-Brans-Dicke theory (extended inflation). In this paper we will argue that inflation in this theory occurs in a most natural way if it starts near the Planck density, as in the standard version of chaotic inflation. With most natural initial conditions, the inflationary universe in the JBD theory enters the stage of permanent reproduction of new inflationary domains (eternal extended inflation). In order to realize the extended inflation scenario at least two classical scalar fields driving inflation are necessary, as distinct from the simplest versions of new and chaotic inflation. It is shown that in the theory of two scalar fields one can solve the graceful exit problem even without modifying the Einstein gravity theory, due to the possibility that the decay rate of the false vacuum in old inflation depends on the value of the second scalar field and hence on time.

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1 Introduction

One of the most dramatic stages of the development of inflationary cosmology was related to the solution of the graceful exit problem in the old inflationary universe scenario of Guth [1]. This scenario was based on three fundamental propositions:

1. The universe initially expands in a state with a very high temperature, which leads to the symmetry restoration in the early universe, $\varphi(T) = 0$, where φ is some scalar field driving inflation (the inflaton field).
2. The effective potential $V(\varphi, T)$ of the scalar field φ has a deep local minimum at $\varphi = 0$ even at a very low temperature T . As a result, the universe remains in a supercooled vacuum state $\varphi = 0$ (false vacuum) for a long time. The energy-momentum tensor of such a state rapidly becomes equal to $T_{\mu\nu} = g_{\mu\nu} V(0)$, and the universe expands exponentially (inflates) until the false vacuum decays.
3. The decay of the false vacuum proceeds by forming bubbles containing the field φ , corresponding to the minimum of the effective potential $V(\varphi)$. Reheating of the universe occurs due to the bubble-wall collisions.

Unfortunately, as was pointed out by Guth in [1] and confirmed later in [2,3], this scenario has a major defect. If the rate of the bubble formation is bigger than the speed of the universe expansion, then the phase transition occurs very rapidly and inflation does not take place. On the other hand, if the vacuum decay rate is small, then the universe after the phase transition becomes extremely inhomogeneous, and in a considerable part of the physical volume of the universe the phase transition to the minimum of $V(\varphi)$ never completes. This is called the graceful exit problem.

The main idea of the old inflationary universe scenario was very simple and attractive, and the role of the old inflationary scenario in the development of modern cosmology was extremely important. However, all attempts to solve or avoid the graceful exit problem and to suggest a successful inflationary universe scenario failed until cosmologists managed to surmount a certain psychological barrier and renounce all three of the aforementioned assumptions, while retaining the main idea of ref. [1] that the universe underwent inflation during the early stages of its evolution. The invention of the new inflationary universe scenario [4] marked the departure from the assumptions (2), (3). Later it was shown that the assumption (1) also does not hold in all realistic models of inflation, for two main reasons. First of all, the time which is necessary for the field φ to roll down to the minimum of $V(\varphi, T)$ is typically too large, so that either inflation occurs before the field rolls to the minimum of $V(\varphi, T)$, or it does not occur at all. On the other hand, even if the field φ is occasionally near the minimum of $V(\varphi, T)$ from the very beginning, inflation typically starts very late, when thermal energy drops from M_p^4 down to $V(0, T)$. In all realistic models of inflation $V(0, T) < 10^{-10} M_p^4$, hence inflation may start in a state with $\varphi = 0$ not earlier than at $t \sim 10^4 M_p^{-1}$. During such a time a typical closed universe would collapse before the conditions necessary for inflation could be

realized [5].

The assumption (1) was finally given up with the invention of the chaotic inflation scenario [6]. The main idea of this scenario was that instead of making the *assumption* that the scalar field should be in a state corresponding to a minimum of $V(\varphi, T)$ from the very beginning, one should study various initial distributions of the scalar field φ and investigate in which case the inflationary regime may occur. The result of this investigation is that there exists a wide class of theories where inflation occurs under quite natural initial conditions (for a review see [5]). This class includes in particular all theories of a scalar field φ minimally coupled to gravity, with a polynomial effective potential $V(\varphi)$, or even with an exponential effective potential $V \sim \exp \frac{c\varphi}{M_p}$ with $c \lesssim 5$.

No satisfactory realization of the old and new inflationary scenarios based on the assumption (1) is known to us at present. In this sense all realistic inflationary models suggested so far can be considered as different realizations of the idea of chaotic inflation. However, to distinguish between various realizations of this idea we will still speak about the old inflationary scenario if inflation occurs in a state corresponding to a deep local minimum of $V(\varphi)$ and reheating of the universe occurs due to the bubble wall collisions. On the other hand, we will speak about the chaotic inflation scenario if inflation occurs during a slow rolling of some scalar fields from their non-equilibrium initial positions.

There was no way to suggest a realistic version of old inflation in the context of theories with one-scalar field. Recently, however, this problem was solved in the context of the so-called extended inflation scenario [7] based on the investigation of inflation in the Jordan-Brans-Dicke theory [8] with the action of the following type:

$$S = \int d^4x \sqrt{-g} \left[\frac{f(\phi)R}{16\pi} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + L(\varphi) \right]. \quad (1)$$

Here $\phi^2 = \frac{b}{2\pi} \Phi$, where Φ is the Jordan-Brans-Dicke field; φ is the inflaton scalar field. In the first papers on the extended inflation scenario it was assumed that the effective gravitational constant $G = M_p^{-2} = f^{-1}(\phi)$ is equal to $\frac{b}{2\pi g^2}$, with $b \gg 1$. It was assumed also that $V(\phi) = 0$ and that the effective potential of the field φ has a deep minimum at $\varphi = 0$, as in the old inflationary universe scenario [7]. Then it was understood that a successful realization of this scenario is possible only for $1 \ll b < 25$, which is incompatible with the experimental constraints on this parameter in the JBD theory, $b > 500$ [9,10], and to cure this problem it was suggested to add to the theory a non-vanishing effective potential of the field ϕ [10]. Later it was argued that a much better scenario (hyperextended inflation) can be suggested if one takes $f(\phi) = M^2 + \xi \phi^2 - \alpha \phi^4 / M^2$ [11] (see also [12]).

The reason why this theory may resolve the graceful exit problem is very simple. The moment when the phase transition from the false vacuum completes is determined by the condition $\epsilon(t) \equiv \Gamma / H^4(t) \geq \epsilon_\tau$ where Γ is the number of bubbles nucleated per unit four-volume, $\epsilon_\tau \approx \frac{3}{4} [3,13]$. At the first stages of inflation $H(t)$ is big, $\Gamma \ll H^4(t)$ and $\epsilon(t) \ll \epsilon_\tau$. In the ordinary Einstein theory this relation holds forever, which is the origin of the graceful

exit problem. But in the theory (1) inflation is not exponential. The Hubble constant $H(t)$ decreases in time due to the change of the field ϕ . At some moment $t = t_c$ the value of $\epsilon(t)$ becomes smaller than ϵ_c and inflation completes.

The discovery of the new class of inflationary models is certainly very important. Of course, those who wish to have just one preferable scenario may be disappointed by the appearance of new candidates, but in my opinion it is better to know all existing possibilities, which should give us a better chance to suggest a complete cosmological theory which would be consistent with the future theory of all fundamental interactions. In any case, since the chaotic inflation scenario is at present rather well understood [5], it is quite reasonable to study other, more complicated versions of inflationary cosmology.

In order to continue investigation of the new class of models one should first understand their relation to other models and theories suggested so far. Many relevant comments on this question are contained in different papers on extended inflation. However, some of these comments were strongly biased by the understandable desire to show the advantages of the new models. For example, it was claimed that chaotic inflation requires fine tuning of parameters and exotic quantum effects at the Planck density. It was claimed also that the modifications of the Einstein theory, which are necessary for the realization of the extended inflation scenario, naturally appear in virtually every realistic theory of elementary particles, including the superstring theory. Some authors believe that extended inflation provides the only way to produce exponentially big domain walls and strings, and practically everybody believes that this scenario provides the only way to solve the graceful exit problem of the old inflation.

In this paper we will try to examine the validity of these statements. We will also discuss the problem of initial conditions for extended inflation and suggest a further extension of this scenario, which we call eternal extended inflation. Finally, we will suggest a possible solution of the graceful exit problem of old inflation which does not require any modifications of the Einstein gravity theory.

2 Chaotic Inflation and Extended Inflation

Let us first remember some facts about chaotic inflation (for a detailed discussion and a comparison of various versions of the inflationary universe scenario see [5]).

- 1) In the simplest models of the chaotic inflation scenario with the power-law effective potential $V(\phi) \sim \frac{\lambda}{n} \phi^{2n}$ inflation occurs at $\phi \gtrsim n M_p / 12$. This means that for small λ inflation may occur well outside the dangerous region of Planck densities, at $V(\phi) \ll M_p^4$.
- 2) In the models with the effective potential which looks like $V_0 \exp(\frac{c\phi}{M_p})$ at $\phi > \phi_0$, $c \lesssim 5$, $\phi_0 < M_p$, inflation occurs for all $\phi > \phi_0$. In this case one can have both $V(\phi) \ll M_p^4$ and $\phi \ll M_p$ during inflation (though the last condition is not necessary at all for the validity of

the chaotic inflation scenario).

- 3) In our opinion, the most natural initial conditions for chaotic inflation should be imposed as close as possible to the hypersurface of the Planck density, for the simple reason that this is the *first* time when the *initial* conditions can be imposed on *classical* fields in a *classical* space-time. However, if one wishes, one can impose initial conditions at a density much smaller than M_p^4 . It is most important that for small λ the crucial process of the self-reproduction of the inflationary universe [14,15] may occur at densities much smaller than the Planck density.

Thus, we believe that no exotic processes at the Planck density are necessary for the realization of the chaotic inflation scenario.

Now let us consider the issue of fine tuning. In order to do it we will give a simple example of a semi-realistic theory where chaotic inflation is possible [5]:

$$L = \frac{RM_p^2}{16\pi} + L_{GUT} - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 - \beta \varphi^2 \phi^2. \quad (2)$$

Here L_{GUT} is a Lagrangian of some basic grand unified theory, φ is some gauge singlet field with a mass $m \leq 10^{12} - 10^{13} \text{ GeV}$ interacting with some scalar field (or fields) ϕ from the basic theory with a coupling constant $\beta \sim 10^{-6}$. For example, in the $SU(5)$ theory the corresponding interaction can be either $-\beta \varphi^2 H_2^2$ or $-\beta \varphi^2 T \cdot \Phi^2$. Due to radiative corrections the effective potential $V(\varphi)$ at $\varphi^2 > m^2/\beta$ acquires a contribution $C \frac{\beta}{16\pi^2} \varphi^4 (\ln \frac{\beta \varphi^2}{m^2} + O(1))$, where $C = O(1)$ is some constant which depends on the properties of the field ϕ in the $SU(5)$ theory (whether it stands for H_2 or Φ). This theory leads to inflation with all desirable properties including density perturbations $\frac{\delta \rho}{\rho} \sim 10^{-5}$. This model is similar to the Shafi-Vilenkin model [16], but is simpler and leads to a more efficient reheating of the universe [5]. This model is also much simpler than any version of extended inflation suggested so far.

Is there any fine tuning of parameters in this model? In order to understand it let us represent the last two terms in (2) in the form

$$\left(\frac{1}{2} (c\beta M_p)^2 - \beta \phi^2 \right) \varphi^2, \quad (3)$$

where $c \leq O(1)$. The only feature of this model which can be considered unnatural is the presence of a small coupling constant $\beta \sim 10^{-6}$. The need for such a small parameter in the theory is related to the experimental fact that the anisotropy of the microwave background radiation $\frac{\delta T}{T}$ is "unnaturally" small, $\frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} \sim 10^{-5}$. We are not sure that the existence of a small parameter in the theory which explains the existence of some other experimentally measured unnaturally small quantity should be considered as a fine tuning. For example, nobody considers as a fine tuning the presence of a small coupling constant $2 \cdot 10^{-6}$ of the interaction between the electron and the Higgs boson in the standard theory of weak and electromagnetic interactions, since this constant is responsible for the smallness of the electron mass. In any case, a similar small parameter $M_F/M_p < 10^{-5}$ is present for a similar reason in all models of extended inflation [10,17], and the authors of these models do not consider

it as a fine tuning. On the other hand, if one is allowed to modify the Einstein theory and consider the Jordan-Brans-Dicke theory with the parameter $b \sim 10^3$ [7], one may consider as well the induced gravity theory with the Lagrangian $\xi \phi^2 R$ with $\xi \sim 10^3$ (or one may just add such a term to the Lagrangian (2)). In such a theory the coupling constant β should be of the order $50 \cdot 10^{-6} \sim 10^{-1}$, which does not look like a fine tuning at all [18]. In the chaotic inflation scenario based on supergravity one can get $\frac{gT}{\mu} \sim \frac{g}{\mu} \sim 10^{-5}$ without introducing any dimensionless parameters smaller than 10^{-2} [19], and in the models based on the Kaluza-Klein theory one may avoid introducing any small parameters whatsoever [20].

Therefore the only real problem of the chaotic inflation scenario is not the fine tuning problem or the problem of exotic processes at the Planck density, but the question whether it is possible to realize it in the context of a completely realistic theory of elementary particles. Since the notion of a realistic theory of elementary particles nowadays changes every 5 years (GUTs, supergravity, superstrings...), one should know all possible versions of the inflationary universe scenario in order to be better prepared for the future changes of the particle theory. This, rather than the "defects" of chaotic inflation, should serve as a justification of the investigation of such relatively complicated theories as the extended inflation scenario.

The possibility to justify the modifications of the Einstein theory proposed in [7,10,11] in the context of realistic theories of elementary particles does not seem very easy. Whereas radiative corrections of the type of $\xi \phi^2 R \ln \phi$ with some small coefficients proportional to coupling constants do naturally appear in many theories, neither the JBD theory with $b \gg 1$ nor the hyperextended inflation with $f(\phi) = M^2 + \xi \phi^2 - \alpha \phi^4 / M^2$ arise as a natural consequence of "virtually every" realistic theory of elementary particles. In particular, the low energy limit of the superstring theory is not the JBD theory but the ordinary Einstein gravity. In principle, one can represent the superstring theory in a form similar to the JBD theory, but the corresponding coefficient b in this theory proves to be a small (and not adjustable) parameter. As a result, instead of inflation one typically has a regime in which the scale factor $a(t)$ linearly grows in time [21]. (For a review of problems with inflation in the context of the superstring theory see [22] and references therein.)

Now let us investigate the problem of initial conditions in the extended inflation scenario. In our opinion, the original idea that in this scenario inflation starts after the high temperature symmetry restoration, does not work in a nice way for the same reason as in the original old inflation scenario and in the new inflation scenario; see the beginning of this paper and also refs. [5,23]. The best way to investigate this scenario which we see at present is to consider whether it is likely or not that at the very early stages of the evolution of the universe (i.e. near the Planck time) the inflaton field φ was occasionally in a state near the local minimum of $V(\varphi)$ in some part of the universe bigger than the horizon H^{-1} . This means that one should proceed exactly as in the chaotic inflation scenario.

A similar approach to the old and new inflation would not give us very encouraging results. The reason is that in all realistic models of new inflation, as well as in the old inflation scenario, the false vacuum energy is smaller than $O(10^{-10})M_P^4$. Therefore the field φ can be sufficiently

homogeneous at a scale of the horizon at the moment when inflation may start, but only if at the Planck time it is homogeneous on a scale many orders of magnitude bigger than the Planck length, which seems unnatural [5].

There is no such problem in the chaotic inflation scenario since inflation may start directly at the Planck time. If one wishes to avoid uncertainties related to the processes at the Planck density, one may consider inflation starting with a density two, ten or one hundred times smaller than M_P^4 , but there are no problems related to the absence of the possibility of starting inflation at the energy density bigger than $10^{-10}M_P^4$. Therefore in the chaotic inflation scenario inflation may start if the scalar field was originally sufficiently homogeneous on a scale just a few times bigger than the Planck length [5], [24].

Fortunately, several versions of the extended inflation scenario have a similar advantage! Indeed, in this scenario the notion of the Planck density depends on the value of the field ϕ :

$$M_P^4(\phi) = f^2(\phi). \quad (4)$$

Therefore inflation may start near the Planck density with the fields ϕ and φ related to each other as follows [25]:

$$V(\varphi) \sim f^2(\phi). \quad (5)$$

In this sense extended inflation, just like chaotic inflation, is free from the difficulties with the initial conditions which hampered a successful realization of the old and new inflation (see, however, below).

One can get an independent (though speculative) confirmation of this conclusion by an investigation of the quantum creation of the universe from "nothing" in the extended inflation scenario. According to [26], the probability of quantum creation of the inflationary universe filled with the field φ with $V(\varphi) \lesssim M_P^4$ is suppressed by the factor

$$P \sim \exp\left(-\frac{3M_P^4}{8V(\varphi)}\right). \quad (6)$$

In the extended inflation scenario this expression should be replaced by

$$P \sim \exp\left(-\frac{3f^2(\phi)}{8V(\varphi)}\right), \quad (7)$$

which implies that the process of quantum creation of the inflationary universe is not exponentially suppressed only if the relation (5) holds.

One should note, however, that in some versions of the extended inflation scenario inflation may occur only at $V(\varphi) \ll f^2(\phi)$, as in the standard old inflation scenario. For example, it can be easily shown that the condition $M_P^4 \sim f^2(\phi)$ is incompatible with inflation in the hyperextended inflation model suggested in [11] ($f(\phi) = M^2 + \xi \phi^2 - \alpha \phi^4 / M^2$ with $M \gg M_P$,

²According to (7), quantum creation is not suppressed for $V(\varphi) > f^2(\phi)$ as well. However, in this case one cannot speak about creation of a classical space-time.

$\xi, \alpha = O(1)$. (Other difficulties with the hyperextended inflation with "natural" parameters ξ and α are discussed in [17].)

One more problem related to the initial conditions for extended inflation is connected with the volume of the phase space of all possible values of the fields ϕ and φ satisfying the condition (5). Eq. (5) typically has an infinitely big set of solutions and only a finite part of it leads to extended inflation. For example, if $V(\varphi)$ is a polynomial function of φ with a local minimum at $\varphi = 0$, then the extended inflation scenario is realized only for the initial values of φ corresponding to a finite vicinity of $\varphi = 0$, whereas for all other initial values of φ from $-\infty$ to $+\infty$ either there is no inflation or the chaotic inflation scenario is realized along the lines of ref. [25]. It seems therefore that even though there is no *exponential* suppression of the probability of initial conditions satisfying eq. (5), the set of those initial conditions which lead to a realization of extended inflation is vanishingly small as compared with the set of those initial conditions for which extended inflation does not occur.

A possible way to resolve this problem is to say that the probability of initial conditions with big $f^2(\phi)$ and $V(\varphi)$ for some reasons is extremely small. At the moment we do not see why this conjecture could be true in the simple models of extended inflation considered so far.

Another possible resolution is related to the process of permanent self-reproduction of inflationary domains which occurs in this scenario at a sufficiently large energy density $V(\varphi)$ [25].

3 Eternal Extended Inflation

Let us consider the original extended inflation model (1) with $b \gg 1$ and $V(\phi) = 0$ and let us study the stage of inflation with the field $\varphi = 0$ corresponding to the local minimum of $V(\varphi)$. As in ref. [7], we will denote $V(\varphi = 0) = M_F^4$.

During inflation one can neglect $\frac{d}{dt}$ as compared with $H \equiv \frac{a}{a} \dot{a}$ and $\ddot{\phi}$ as compared with $3H\dot{\phi}$ and $\dot{\phi}^2$. In such a case equations for a and ϕ in the theory (1) with $V(\phi) = 0$ and $b \gg 1$ take a very simple form:

$$H = \frac{2}{\phi} \sqrt{\frac{b}{3}} M_F^2, \quad (8)$$

$$3H\dot{\phi} = \frac{4}{\phi} M_F^4. \quad (9)$$

From these equations one can easily see that during the typical time $\Delta t = H^{-1}$ the field ϕ grows by $\Delta\phi = \phi/b$, which leads to a gradual decrease of H . According to [13], inflation completes in the whole universe at the moment when $\frac{3H^4}{4\pi}$ becomes smaller than Γ , the number of bubbles nucleated per unit four-volume.

However, taking into account quantum fluctuations of the field ϕ , this conclusion is not entirely correct. According to [27], quantum fluctuations during inflation produce perturbations $\delta\phi$ of the classical scalar field ϕ with a wavelength bigger than H^{-1} . The average amplitude of perturbations generated during the time $\Delta t = H^{-1}$ is given by $|\delta\phi| = H/2\pi$. This is bigger than the classical growth $\Delta\phi = \phi/b$ of this field for

$$\phi < M_F \left(\frac{b^3}{3\pi^2} \right)^{1/4}. \quad (10)$$

This result has important consequences.

It is well known that the process of inflation in any spatial domain of radius H^{-1} occurs independently of any events outside it ("no-hair theorem" for de Sitter space). In this sense any inflationary domain of initial size exceeding $2H^{-1}$ can be considered as a separate mini-universe, expanding independently of what occurs outside it.

To investigate the behaviour of such a mini-universe, with quantum fluctuations taken into account, let us consider an inflationary domain of initial size $O(H^{-1})$ containing a sufficiently homogeneous field ϕ . During a typical time interval $\Delta t = H^{-1}$ the mean value of the field ϕ inside this domain will grow by $\Delta\phi = \phi/b$. However, if $\Delta\phi \ll |\delta\phi|$, the average growth of the field ϕ is not important. During the time $\Delta t = H^{-1}$ the domain grows e times and its volume grows e^3 times. Because the typical wavelength of perturbations $\delta\phi(x)$ generated during this time is H^{-1} , the whole domain effectively becomes divided into e^3 separate domains (mini-universes) of size $O(H^{-1})$. In almost half of these domains the field ϕ does not grow, but rather decreases by $|\delta\phi(x)| - \Delta\phi \sim H/2\pi$. This means that during this time the total volume of the universe occupied by the decreasing field ϕ grows approximately by $e^3/2 \sim 10$ times. A similar statement remains true for the next time interval $\Delta t = H^{-1}$, etc. This means that any inflationary domain of initial size bigger than $O(H^{-1})$, filled with the field $\phi < M_F \left(\frac{b^3}{3\pi^2} \right)^{1/4}$, permanently reproduces new and new inflationary domains of a similar type, and this process never completes (eternal extended inflation). The theory of this effect is completely analogous to the theory of a similar effect in the eternal chaotic inflation scenario [14,15].

It is important that the process of permanent reproduction of inflationary domains occurs in a wide interval of values of the scalar field ϕ , corresponding to

$$\frac{3}{4b} M_F^4(\phi) < M_F^4 < M_F^4. \quad (11)$$

This means that in order to prove the existence of the regime of self-reproduction of the universe in the extended inflationary universe scenario with $b \gg 1$, just as in the chaotic inflation scenario, there is no need to study exotic quantum effects at the Planck density. The results obtained above mean that even though the fraction of initial conditions which lead to extended inflation seems to be vanishingly small, the total volume of the parts of the universe which experience extended inflation may be exponentially large.

Note, that the effects discussed above do not return us to the problem with the impossibility of completing the phase transition in the old inflation scenario. Indeed, in the eternal

extended inflation scenario many domains with $\phi > M_F \left(\frac{\phi^3}{3\pi^2}\right)^{1/4}$ are produced as well. In such domains fluctuations of the field ϕ are small and the phase transition completes. This just means that when computing the size of the homogeneous part of the universe in the extended inflation scenario [7] one should use $\phi \sim M_F \left(\frac{\phi^3}{3\pi^2}\right)^{1/4}$ instead of the initial value $\phi \sim M_F \left(\frac{\phi}{3\pi}\right)^{1/2}$ corresponding to the Planck energy density. A similar result is valid for the chaotic inflation as well [5].

4 Solution of the Graceful Exit Problem Without Modifying the Einstein Gravity

As we have seen in the previous sections, the most interesting realizations of chaotic and extended inflation have many common features. One such feature is the possibility to produce exponentially big bubbles, strings and domain walls due to phase transitions which may occur during inflation. In the context of chaotic inflation this possibility was first suggested three years ago [28], and has now been revived again in the extended inflation scenario.

But is it possible to solve the graceful exit problem of the old inflation scenario with the help of chaotic inflation without considerable modifications of the Einstein gravity? Or, to be more precise, is it possible to avoid the graceful exit problem if the inflaton field which drives the last stages of inflation in the Einstein gravity theory has a potential with a deep local minimum? As we will show now, the answer to this question is positive.

The most straightforward way to do so³ is just to make a conformal transformation and to recast the JBD theory in the Einstein form with a complicated effective potential being a product of $V(\phi)$ and $\exp C\phi$ where C is some constant [29,17,30]. In such a formulation the extended inflation scenario is equivalent to a peculiar version of the chaotic inflation scenario where inflation is driven by two scalar fields simultaneously. (This is the deep reason why many results obtained in the context of the chaotic inflation scenario remain valid for extended inflation as well.) However, in the chaotic inflation scenario with two scalar fields a much more natural possibility does exist, related to the theory of phase transitions during inflation mentioned above [28].

Indeed, let us consider again the simplest version of the chaotic inflation scenario (2) with the Lagrangian $\mathcal{L}_{GV}$ containing the effective potential $V(\phi)$ with a deep local minimum at $\phi = 0$, as in the old inflationary universe scenario,

$$V(\phi) = M_F^4 + \frac{\alpha}{2} M_F^2 \phi^2 - \frac{\gamma}{3} M_F \phi^3 + \frac{\lambda}{4} \phi^4. \quad (12)$$

Then the total effective potential of the fields ϕ and φ in the theory (2) takes the following

³This way was suggested to me by Keith Olive

form:

$$V(\phi, \varphi) = M_F^4 + \frac{\alpha}{2} M_F^2 \phi^2 - \frac{\gamma}{3} M_F \phi^3 + \frac{\lambda}{4} \phi^4 - \left(\frac{1}{2} (c\beta M_F)^2 - \beta\phi^2 \right) \varphi^2 + C \frac{\beta^2}{64\pi^2} \varphi^4 (\ln \frac{\beta\varphi^2}{m^2} + O(1)), \quad (13)$$

where $C, c = O(1)$.

As we have already mentioned, the theory (2) with $\beta \sim 10^{-6}$ leads to a natural realization of the chaotic inflation scenario even without taking into account $V(\phi)$ (12). Now let us consider the behaviour of the fields ϕ and φ in the theory (2), (13).

According to our previous arguments, the most natural initial values of the fields ϕ and φ should satisfy the constraint $V(\phi, \varphi) \sim M_P^4$ (though, of course, one can easily take $V(\phi, \varphi)$ several times smaller). Then the field ϕ will roll down to the minimum of $V(\phi, \varphi)$ more rapidly than the field φ since the effective mass of the field ϕ is much bigger. Soon after that inflation will be driven mainly by the field φ . At this stage the effective mass squared of the field ϕ acquires a very big positive contribution,

$$M_{eff}^2(\phi) = \alpha M_F^2 + 2\beta\varphi^2. \quad (14)$$

Therefore at this stage of the evolution of the universe (at $\varphi \gg M_P$) the effective potential of the field ϕ has the only minimum at $\phi = 0$, the field ϕ rapidly rolls down to this minimum and stays there.

With a decrease of the field φ , its contribution to the vacuum energy density becomes smaller than M_P^4 , and after that inflation is driven by the field ϕ , as in the old inflation scenario.

Let us take for definiteness $M_F \sim 10^{15}$ GeV, which is a natural GUT scale. Then the old inflation takes over at $\varphi < M_P$, i.e. at the very end of the preceding stage of chaotic inflation. For smaller values of β old inflation starts earlier, i.e. at bigger φ . At this stage the field φ continues its slow rolling down to the minimum of its effective potential. (Recall that the massive scalar field in de Sitter space decreases as $\exp(-\frac{m^2}{3H})$ for $m < H$.) Therefore the effective mass squared $M_{eff}^2(\phi)$ of the field ϕ will change rather slowly, see eq. (14).

The rest of the story is quite obvious. The probability of bubble production Γ is extremely sensitive to the value of the effective mass of the field: it rapidly increases with the decrease of $M_{eff}^2(\phi)$. Thus, the parameter $\epsilon(t) \equiv \Gamma/H^4$ depends on time, though for a quite different reason than in the extended inflation scenario. In the extended inflation scenario the Hubble parameter H decreases in time. Here the decay rate Γ increases in time. In both cases the phase transition completes when/if $\epsilon(t)$ becomes sufficiently big. The theory of the phase transition crucially depends on the values and signs of the coupling constants. If the effective value of α ($\alpha_{eff} = M_{eff}^2/M_P^2$) in the process of decrease of the field φ always remains big and positive, then the action for tunneling is very big and the phase transition never completes. If the constant γ is big and α_{eff} gradually becomes very small, then the phase transition completes. If the constants γ and λ are small, then the phase transition completes only when α_{eff} becomes negative. In that case no big bubbles appear after the phase transition. One

can easily check by the standard methods that with the values of parameters similar to those discussed above one can easily get density perturbations $\frac{\delta \rho}{\rho} \sim 10^{-5}$.

This means that one can actually suggest models in which the universe at the last stages of inflation is described by the old inflationary scenario, and that the graceful exit problem of this scenario can be solved without any modification of the Einstein gravity.

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