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APPLICATIONS OF STATISTICAL NUCLEAR PHYSICS TO NUCLEAR SPECTROSCOPY

P.G. Hansen¹⁾, B. Jonson²⁾ and A. Richter³⁾

- 1) Det fysiske Institut, Aarhus Universitet, DK-8000 Aarhus C
- 2) Fysiska Institutionen, Chalmers Tekniska Högskola,
S-412 96 Göteborg
- 3) Institut für Kernphysik, Technische Hochschule Darmstadt,
D-6100 Darmstadt

Abstract

A number of new results pertaining to properties of complex nuclear spectra are given and illustrated with data from many experiments. We are concerned with the situation in which a group of closely spaced—though, in principle, still resolvable—nuclear levels is observed, and use the statistical model to interpret the properties of the corresponding particle and photon spectra. According to this model, the physics information contained locally in such a spectrum may be expressed in terms of a small number of average quantities: level spacings and reduced partial widths. The local fine structure, on the other hand, remains unpredictable ('chaotic') but is governed by universal statistical laws. Experimental examples are taken from charged-particle and gamma-ray spectra following the beta decay of far-unstable nuclei and from (e,e') reactions studied by high-resolution methods. We discuss i) the determination of level densities, ii) the role of fluctuation corrections in the interpretation of branching ratios and resonance lifetimes, and finally iii) how Monte Carlo simulations of complex spectra may serve to test sensitivity and experimental bias.

**Dedicated to Professor Torleif Ericson
on the occasion of his 60th birthday**

*Ty detta är det härliga hos människan
att hon kan fatta tingens inre väsen,
ej vad de synas, men vad de betyda;
och verkligheten, vårt öga ser,
den är symbolen endast av ett högre. *)*

Esaias Tegnér, Magisterpromotionen i Lund, 1820

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*) Translations of the quotations are given in the Appendix.

1. INTRODUCTION

(Les sciences ne sont) en France qu'une mode qui succède à des autres, et qui passera à son tour. Mais aucun art, aucune science ne doit être de mode. Il faut qu'ils se tiennent tous par la main, il faut qu'on les cultive en tout temps. Je ne veux point payer de tribut à la mode, je veux passer d'une expérience de physique à un opéra ou à une comédie ...

Voltaire, Lettre à Pierre Robert le Cornier de Cideville, 1735

The statistical theory of the nucleus, after having been out of fashion for a considerable time, now shows signs of making a come-back. It has its origin in experimental studies of the interactions of slow neutrons, which provided precise spectroscopic information on a sequence of levels with certain spin and parity quantum numbers, from level number N to level number $(N+n)$. Since, for a heavy nucleus, N is of the order of 10^6 and n of the order of 100, it seemed unlikely that an explanation on the basis of structural quantum numbers could be extended to the millionth level. Instead, an attempt was made to develop a statistical theory in which all quantum numbers except energy, total angular momentum, and parity are washed out. The development of this theory took place between 1950 and 1965 and is well documented in a reprint collection edited and commented by Porter¹⁾. The main ingredient was brought in by Wigner²⁾, who proposed that the energy levels should be represented by the characteristic values of a symmetric matrix with random coefficients. He also suggested an expression for the probability density of finding a given level spacing D between two neighbouring levels

$$P_W(s) = \frac{1}{2} \pi s \exp \left(-\frac{1}{4} \pi s^2 \right), \quad (1)$$

where $s = D/\langle D \rangle$ is the spacing in units of its mean value. This expression is exact for a matrix with two dimensions and normally distributed matrix elements with zero mean, and it is known to be a good approximation for matrices of large dimensions.

The probability density of observing a reduced transition probability γ_f^2 for a single reaction channel involving a final state f was found by Porter and Thomas³⁾:

$$P_{PT}(x) = (2\pi x)^{-1/2} \exp(-x/2), \quad (2)$$

where $x = \gamma_f^2/\langle \gamma_f^2 \rangle$ is the reduced transition probability in units of its mean value. This expression follows immediately from random-matrix theory, since the distribution (2) is that of the square of a Gaussian variable with zero mean and

hence is identical to the χ^2 distribution with one degree of freedom. The theory of intensity fluctuations was subsequently extended into the continuum region by Ericson⁴⁾, who interpreted strongly fluctuating nuclear reaction cross-sections—today referred to as Ericson fluctuations—as being a result of interference in the overlapping reaction amplitudes.

The statistical picture of complex nuclear spectra simplifies their description in a remarkable way. The physics is contained in the values of a few local averages such as $\langle D \rangle$, $\langle \gamma_f^2 \rangle$, or the convenient ratio $\langle \gamma_f^2 \rangle / \langle D \rangle$, called a strength function, which varies slowly with energy. The local fine structure, on the other hand, is taken to be entirely stochastic and to be beyond the reach of theoretical predictions. This agrees well with modern nuclear statistical theory⁵⁾, which sees complex nuclear spectra as an example of chaotic phenomena in quantum physics. It is interesting to note that certain simple quantum-mechanical models⁶⁾ for one particle lead to a distribution of the eigenvalue spacings that is indistinguishable from the distribution of complex nuclear levels. Such a result had—with remarkable foresight—been anticipated by Wigner, who, in a conference contribution in 1957 (ref. 1, p. 225), remarked that the distribution approximated by expression (1) was very likely to be a universal function, so that it would not depend on the details of the underlying model.

The basic elements of statistical nuclear theory are well proven experimentally [see Bohigas and Weidenmüller⁵⁾]. Still, many unanswered questions remain; for example, those concerning correlations between the transition probabilities in different reaction channels—questions that are very difficult and time-consuming to answer experimentally. For this reason, the theory does not possess absolute predictive power.

The present paper is concerned with the application of statistical theory for a practical purpose, namely as a guide to the understanding of complex spectra encountered in experiments. Some of the relevant statistical distributions are derived in Section 2, where it is pointed out that the observed quantities often belong to extremely skew and wide distributions that tend to defy the experimentalist's intuition, rooted in the normal distribution. Some applications of fluctuation analysis for determining experimental quantities from (unresolved) complex spectra form the subject of Section 3, in which the most promising application is for the determination of the density of levels populated in (e,e') reactions. Finally, Section 4 describes how Monte Carlo simulations based on statistical nuclear theory ('pandemonium') can serve to test detection efficiency and experimental bias in measurements of complex decays.

2. PROPERTIES OF COMPLEX DECAYS

*Mephistopheles: Ich wünschte nicht, Euch irrezuführen.
Was diese Wissenschaft betrifft,
Es ist so schwer, den falschen Weg zu meiden;
Es liegt in ihr so viel verborgnes Gift,
Und von der Arzenei ists kaum zu unterscheiden.*

J.W. Goethe, Faust, 1808–32

2.1 The continuum region

Although the present paper is concerned entirely with fluctuations of isolated resonances, it is probably useful at the outset to remind the reader of some properties of fluctuations in the continuum—the Ericson fluctuations. For these, many exit channels are open for particle emission, so that the total width Γ is much larger than the average level spacing $\langle D \rangle$. As a consequence, all the intermediate states in an energy interval Γ around the beam energy will contribute coherently but randomly to the scattering amplitude, leading to statistical fluctuations in the cross-section σ as a function of energy. Ericson derived expressions from which the distributions governing the fluctuations could be obtained; for the autocorrelation function,

$$\psi_E(\tau) \equiv \frac{1}{2T} \int_{-T}^T \sigma(E)\sigma(E+\tau) d\tau = \langle \sigma \rangle^2 \left(1 + \frac{\Gamma^2}{\Gamma^2 + \tau^2} \right) \quad (3)$$

for $T \rightarrow \infty$. It can be seen that provided Γ exceeds the experimental line width (FWHM) $W_{1/2}$, the autocorrelation function will directly determine Γ and hence the level spacing from the usual expression

$$\Gamma = \frac{D}{2\pi} \cdot N, \quad (4)$$

where N is the effective number of open channels. A number of experimental studies of Ericson fluctuations exist^{7,8)}. The theory of Ericson fluctuations has recently been extended by including isospin in compound-nucleus reactions⁹⁾; in these reactions an interesting consequence is that isospin induces correlations between mirror channels¹⁰⁾. A still different kind of correlations is that between different exit channels. In one experiment¹¹⁾, 7.5 MeV protons were scattered elastically and inelastically from an ^{88}Sr target, so that the channels to the 0^+ ground state and the 1.84 MeV 2^+ state could be compared. A strong effect was found, which was attributed to direct reactions involving an isobaric-analogue resonance in ^{89}Y . Similar correlation effects

have been observed¹²⁻¹⁴⁾ for isolated resonances. These are the main subject of the present paper, to which we now turn.

2.2 Properties of groups of isolated resonances

Consider now the case $\Gamma \ll D \ll W_{1/2}$, which we shall refer to as Porter–Thomas fluctuations for reasons that will become clear in a moment. From eq. (4) it is seen that this situation will be encountered if only a single neutron channel is open, so that $D \cong 6\Gamma \gg \Gamma$. This limit was first discussed by Egelstaff¹⁵⁾, who calculated the fluctuations in measured average cross-sections for a neutron beam with large energy spread. For gamma-ray emission and also for the sub-barrier emission of charged particles, the conditions for Porter–Thomas fluctuations can be fulfilled over a wide energy range. We note for later use that the total gamma widths (typically 10–100 meV depending on mass) are constant to a good approximation, owing to the large number of channels that are open.

The fine structure is dominated by intensity fluctuations, for which the normalized variance for a single reaction channel implied by eq. (2), the Porter–Thomas law, is 2. The fluctuations in the nearest-neighbour spacing, also for one spin and parity, can be calculated from eq. (1) to have a normalized variance of 0.27, but the higher-order spacings are known⁵⁾ to have a 'long-range order', so that the normalized variance on the n^{th} order spacing decreases much faster than $0.27/n$. For this reason, it is even permissible^{16,17)}, to a very good approximation, to neglect fluctuations in the level density and to treat the intensity fluctuations alone.

We now derive some intensity and dispersion laws governing complex decays. To this end, it is convenient to introduce Porter–Thomas distributed variables $x, y, z, \dots$ with unity mean value, and to let a, b, c denote constants. The intensity of a beta, gamma, or particle transition involving a single reaction channel is assumed to be a variable of type x .

Consider, as a first example, beta-delayed charged-particle emission. In this process, protons and alpha particles are emitted from levels that lie very much below the Coulomb barrier, and for this reason the partial particle width proportional to y may be much smaller than the (constant) total gamma width. If the feeding of a given resonance by beta decay is proportional to x , the observed intensity is proportional to $v_1 = x \cdot y$, which, according to the usual law for combining probabilities, belongs to the 'product distribution'

$$P_1(v_1) = \frac{K_0 (v_1^{1/2})}{\pi v_1^{1/2}}, \quad (5)$$

where K_0 denotes the modified Bessel function. From the definition, one can see directly that the variable v_1 has a mean value 1, and likewise that $\text{Var } v_1 = \langle x^2 \rangle \langle y^2 \rangle - 1 = 8$.

Take now a decay with two open channels characterized by partial widths $\Gamma_x = x \langle \Gamma_x \rangle$ and similarly for channel y . The branching ratio is $v_2 = x/(x + ay)$, where the constant $a = \langle \Gamma_y \rangle / \langle \Gamma_x \rangle$. The probability density for v_2 is

$$P_2(v_2) = \frac{a^{1/2}}{\pi (1-v_2)^{1/2} v_2^{1/2} [1 + (a-1) v_2]}, \quad (6)$$

which has the mean value $\langle v_2 \rangle = (1 + a^{1/2})^{-1}$ and the normalized variance

$$a = \langle v_2 \rangle^{-2} \text{Var } v_2 = \frac{1}{2} a^{1/2}. \quad (7)$$

It is striking that the weak branch ($a \gg 1$) is proportional to $a^{-1/2}$ and not to a^{-1} , which would seem a reasonable first guess. We suspect that some 'unexpectedly strong' side branches reported in the literature arise from this effect. In general, it is convenient to introduce a fluctuation function¹⁸⁾ $F(a, b)$ defined by $\langle f(x, y, \dots, a, b) \rangle \equiv F(a, b) f(1, 1, \dots, a, b)$, of which examples are given in ref.¹⁷⁾. Here and in the following, the brackets $\langle \rangle$ are used to denote 'true' averages corresponding to ensemble averages, or, for stationary processes, to averages over an infinite interval.

Note that eq. (7) diverges as $a \rightarrow \infty$, so that the distribution approaches that of $v_3 = x/y$, the 'ratio distribution'

$$P_3(v_3) = \frac{1}{\pi v_3^{1/2} (v_3 + 1)}, \quad (8)$$

which is normalized but has no higher moments. Clearly, a search for a weak branch (say, a gamma transition) in competition with a dominating single exit channel (say, neutron emission) is a problem that can be ill-conditioned theoretically.

The effect of correlations can be illustrated by returning to the product distribution eq. (4) and assuming that the x and y channels are fully correlated, $x \equiv y$, so that $v_4 = x^2$. (This is also the function describing elastic scattering of photons or charged particles via resonances with a constant total width.) For the probability density we find

$$P_4(v_4) = \frac{\exp(-v_4^{1/2}/2)}{(8\pi v_4^{3/2})^{1/2}}, \quad (9)$$

Fig. 1 The probability density distributions corresponding to the Porter–Thomas distribution [eq. (2)], the product distribution [eq. (5)], and the ratio distribution [eq. (8)].

which has the mean value 3 and the variance 96. The normalized variance in this case is 10.7, not very much above the value 8 obtained in the uncorrelated case.

Some expressions covering other combinations have been given in the papers quoted above, but in most cases it is necessary to calculate the relevant averages and variances by numerical integration—or, often better, by Monte Carlo techniques. The upshot of this subsection is then that the distributions discussed here and illustrated in fig. 1 are skew, and on sampling will give many small numbers and a few large ones. They are thus ideally suited for providing the 'pronounced peak structure' that makes for the happiness of every dyed-in-the-wool spectroscopist.

3. THE DETERMINATION OF EXPERIMENTAL QUANTITIES VIA FLUCTUATION ANALYSIS

*Jeg kender ikke meget til meget
det jeg kender til er i forvejen meget udbredt
tænk at vide bare een bestemt ting
for eksempel forholdet mellem mariehønsens pletter*

Benny Andersen, Viden, 1965

3.1 Properties of nuclear noise spectra

We give here some expressions that will be used later. Assume for simplicity in the notation that only one intermediate spin and one final state enter, so that summations over these quantities can be neglected. Denote the particle intensity per unit energy I and the detector resolution $W_{1/2}$. (A Gaussian response function is assumed.) We can then write

$$\text{Var } I = \left(\frac{2 \ln 2}{\pi} \right)^{1/2} \frac{I^2 D \alpha}{W_{1/2}}, \quad (10)$$

where α , of the order 2 to 10, is the normalized variance defined by the first part of eq. (7). It is still assumed that $W_{1/2} \gg D$. We see that the fine structure can be used to derive the level spacing D if the overall energy spectrum $I(E)$ and the parameter α are theoretically understood. Since, in the end, I and $\text{Var } I$ must be taken from experiment, it may be important to correct the result [eq. (10)] for finite sample size.

For the useful though entirely theoretical concept of an infinite nuclear spectrum of unit intensity and with constant level spacing D , the autocorrelation function for Porter–Thomas noise can be written

$$\begin{aligned} \psi_{PT}(\tau) &= \langle I(E)I(E+\tau) \rangle \\ &= 1 + \left(\frac{2 \ln 2}{\pi} \right)^{1/2} \frac{\alpha D}{W_{1/2}} \exp [-2 \ln 2 (\tau/W_{1/2})^2]. \end{aligned} \quad (11)$$

In this case, the correlation width depends on the experimental resolution only, and the level density is determined by the correlation amplitude, i.e. the value for $\tau = 0$. This is very different from the autocorrelation function eq. (3), which links the level density to the correlation width. As $I(E)$ and $I(E+\tau)$ are independent stochastic variables for $\tau \neq 0$, the autocorrelation function needs to be corrected for counting statistics for $\tau = 0$ only (see subsection 3.2).

The similarity between nuclear particle spectra and oscillograms of electrical noise suggests that some elements of the theory¹⁹⁾ of the latter could be of use. For example, there is a theorem that links the autocorrelation function to $\nu_{\max}$, the density of maxima for a random function. From this we have

$$\begin{aligned} \nu_{\max} &= \frac{1}{2\pi} \sqrt{\left[-\frac{\Psi(4)}{\Psi(2)} \right]_{\tau=0}} \\ &= \frac{(3 \ln 2)^{1/2}}{\pi W_{1/2}} = \frac{0.456}{W_{1/2}} \end{aligned} \quad (12)$$

for the case of Porter–Thomas noise. Here the symbol $\Psi^{(i)}$ denotes the i^{th} derivative with respect to τ , and the ratio is evaluated at $\tau = 0$. Thus there is one maximum for each 2.2 half widths. It is interesting that the density of true peaks D^{-1} does not enter. Equation (12) may be viewed as a measure of the information content of a nuclear noise spectrum. It is clear that a completely general property of (nuclear) noise spectra is that the density of maxima must scale as b/Γ , where Γ is the correlation width and b is a constant depending on the exact shape of the response function. Expressions of this type have been proposed^{20,21)} for evaluating Γ for Ericson fluctuations. To this end, b has been estimated^{21,22)} numerically to be about 0.55. Applying formula (12) to the autocorrelation function (3) gives $b = \sqrt{3/\pi} = 0.551$.

A group of resonances will not follow an exponential decay law if the total widths differ from resonance to resonance. For example, this effect must be taken into account in experiments^{23–25)} that observe the lifetimes of highly excited states through coincidences between K X-rays and emitted charged particles following beta decay. For the simple case of two competing exit channels [eqs. (6), (7)], and if particle x is observed, the decay law^{17, 24)} is

$$I_x(t) = \frac{\langle \Gamma_x \rangle}{\hbar} \left(\frac{2\langle \Gamma_x \rangle t}{\hbar} + 1 \right)^{-3/2} \left(\frac{2\langle \Gamma_y \rangle t}{\hbar} + 1 \right)^{-1/2} \quad (13)$$

corresponding to a t^{-2} dependence for large values of t . A similar expression was derived by Malaguti et al.²⁶⁾ for use in blocking experiments.

3.2 The determination of level densities

In order to determine the level-spacing parameter D in eq. (9) we must evaluate $\text{Var } I$ from experimental data. We take, as the first example, beta-delayed proton spectra. In this process, nuclei with a large proton excess ('near the proton drip line') will beta-decay to highly excited states, which are unstable with respect to proton (and usually also alpha) emission. Figure 2 shows data for the case of ^{99}Cd . In principle, the fluctuation amplitude should be measured from the 'true' value. We are forced to approximate this by a local average measured in some interval L , which must be greater than the resolution $W_{1/2}$ yet smaller than the energy range over which the average spectral intensity changes considerably. Below we shall outline how this can be done by comparing the spectrum with a smoothed average of itself. This, however, necessitates what is called 'a correction for finite sample size'. An example given below will show how this works.

Fig. 2 Beta-delayed proton spectrum from ^{99}Cd ($T_{1/2} = 16 \pm 3$ s) measured a) with a counter telescope ($W_{1/2} = 41$ keV) and b) with a single Si detector ($W_{1/2} = 17$ keV). Note that the improvement in resolution reveals a much more complicated spectrum, typical of fluctuation phenomena. The inset shows the autocorrelation function, which in this case²⁷⁾ was calculated on the basis of a more complicated smoothing function, a Gaussian plus its second derivative. This function gives a smaller distortion of the gross structure than a simple Gaussian smoothing. The upper point for $\tau = 0$ represents the value before the correction for contributions from counting statistics.

We return to a sequence of lines with unit mean and standardized variance σ . Defining the sample mean by the symbol

$$\bar{I} = \frac{1}{L} \int_0^L I(t) dt \quad (14)$$

and still denoting the global mean by angle brackets $\langle I \rangle = 1$, $\langle I^2 \rangle = \alpha + 1$, we seek the global mean of the sample variance

$$\text{Var}_s I = \langle \bar{I}^2 - \bar{I}^2 \rangle = \alpha + 1 - \langle \bar{I}^2 \rangle \quad (15)$$

$$= \left(1 - \frac{D}{L}\right) \alpha = C \text{Var } I, \quad (16)$$

where the correction factor C is evidently of the form $1-n^{-1}$, where n is the number of lines in the sample. The last term in eq. (15) can quite generally be expressed by means of the autocorrelation function via the transformations

$$\begin{aligned} \langle \bar{I}^2 \rangle &= \left\langle \frac{1}{L^2} \left[\int_0^L I(t) dt \right]^2 \right\rangle \\ &= \left\langle \frac{1}{L^2} \int_0^L dt \int_0^L ds I(t)I(s) \right\rangle \\ &= \left\langle \frac{1}{L^2} \int_0^L ds \int_{-s}^{L-s} dt \psi(\tau) \right\rangle \equiv 1 + \frac{\alpha D}{L}. \end{aligned} \quad (17)$$

Instead of dividing the spectrum into bins of width L , it is possible to perform a smoothing to give a reference spectrum corresponding to the resolution parameter W_γ . Defining $y = W_\gamma/W_{1/2}$, one finds¹⁷⁾ for the correction factor for a Gaussian smoothing,

$$C = 1 + \frac{1}{y} - \sqrt{\frac{8}{1 + y^2}}. \quad (18)$$

In order to take into account the energy dependence of a real spectrum, it is simplest to work with the ratio $I(t)/I_\gamma(t)$; in this case the autocorrelation function takes a more complicated form¹⁷⁾. The results of an autocorrelation analysis of data for ^{99}Cd are shown in the inset of fig. 2, from which one may derive the level density expressed in terms of the parameter a of a Fermi-gas model. The spectrum of ^{115}Xe is shown in fig. 3, together with (inset) a level-density systematics^{17,27,28)}. Note the low level-density in ^{99}Cd ; this is caused by its vicinity to doubly magic ^{100}Sn , until today not observed in any direct experiments.

The determination of level densities from beta-delayed proton spectra is difficult, both with respect to the experiments themselves and to their analysis, and not many cases are to hand. Nevertheless, the method has some merit, as it offers one of the few possibilities for studying level densities in nuclei that lie

Fig. 3 Beta-delayed proton spectrum from ^{115}Xe ($T_{1/2} = 18 \pm 4$ s) measured with a single Si detector, energy resolution 15 keV. The total intensity is $(3.4 \pm 0.6) \times 10^{-3}$ protons per beta decay. The curve shows the spectrum after smoothing with a Gaussian function with a FWHM of approximately 60 keV. The proton decays go, to 42%, to the ground state of ^{114}Te and, to 58%, to its first-excited 2^+ state at 0.709 MeV. The inset shows the level-density systematics expressed in terms of the usual parameter a taken from experiments on neutron capture (open circles) and on beta-delayed protons (filled circles). The line is the estimate $a = A/8$. The experimental data are from refs. 17, 27, and 28.

far from the beta-stability line. A recent application²⁵⁾ determined an average spacing of 3.7 keV for the $1/2^-$ and $3/2^-$ states in ^{69}As at 5.5 MeV excitation energy, populated in the electron-capture and positron decay of ^{69}Se , presumably with spin and parity $3/2^-$. The $5/2^-$ states make a negligible contribution as they can decay to the ^{68}Ge ground state via f-wave protons only. Here we touch on what is probably the basic problem in the application of fluctuation analysis to spectra of beta-delayed particles: namely, that the relative contribution made by states with different intermediate spins is not observed experimentally and enters as a theoretically based assumption. This problem can be largely avoided in the application that we shall discuss next.

In a series of papers^{29–32)}, fluctuation analysis was developed for studying the fine structure in inelastic electron scattering. The Darmstadt high-resolution spectrometer can provide a FWHM resolution of about 30 keV, but at higher excitation energies this is still insufficient for the observation of individual resonances. The great asset of the electron-scattering method is

Fig. 4 Spectrum (a) of electrons scattered inelastically on ^{90}Zr at an incident beam energy of 44 MeV and at 165° scattering angle. After subtraction of the radiative tail from the elastic peak, the autocorrelation function (b) of the stationary cross-section $I(E)/I_0(E)$ was calculated. The solid curve shows a theoretical fit corresponding to $\langle D \rangle = 6.5 \pm 1.6 \text{ keV}$. The statistical uncertainty is negligibly small. Finally, (c) shows an analysis based on the experimental intensity distribution of the stationary cross-section (histogram) sampled in 60 keV bins. The curves are calculated distributions for different values of l , the average number of levels per bin. The best fit, $l = 11$, corresponds to an average level spacing $\langle D \rangle = 5.5 \pm 1.5 \text{ keV}$, also valid for the energy interval 8.1 to 9.7 MeV excitation energy. (Based on ref. 29.)

that it is possible to choose kinematical conditions so that transitions of one-multipole order dominate; in the application^{29,31} to ^{90}Zr (35–66 MeV beam energy, 165° scattering angle), M2 excitations thus lead to 2^- excited levels. The experimental spectrum shown in fig. 4 contains a very large contribution from the radiative tail of the elastic peak. After this contribution was subtracted and the 'stationary cross-section' was calculated, the analysis was performed by two methods: the autocorrelation method outlined above, and one in which the intensity distribution was directly compared with theory. The two methods agreed within the errors, and at $E \approx 8.9 \text{ MeV}$ gave an average spacing of 6 keV. The analysis could be continued up to 12 MeV—where the level spacing has diminished to about 1 keV—and gave an energy dependence that is roughly in agreement with the one expected from theory. This provides a self-consistency argument for the correctness of the estimated radiative tail,

Fig. 5 The spectrum³⁰⁾ of inelastically scattered electrons from ^{44}Ca is the sum of data taken at incident beam energies of 30–50 MeV. The contribution from the radiative tail of the elastic line has been subtracted. The energy resolution is approximately 40 keV. The thick line is the smoothed spectrum $I_s(E)$. The ratio of the two spectra is the 'stationary cross-section', and from this the autocorrelation function can be calculated.

the 'Achilles heel' of this method, and allows an estimate of the total observed magnetic quadrupole strength.

A similar analysis was performed³⁰⁾ for the M1 strength in ^{44}Ca , an isotope which is unique among the even calcium isotopes in having an—at first sight—structureless (e,e') spectrum, but which, on looking closer, reveals a complex fine structure arising from a highly fragmented M1 giant resonance (fig. 5). In this case, also contributions from 2^- states had to be taken into account in order to arrive at an average 1^+ level spacing of 13 ± 3 keV, and at estimates of the M1 strength. For the latter, fluctuation analysis again was important in setting limits on the amount of strength that could have escaped detection. Finally, we mention an application³²⁾ to the isoscalar quadrupole giant resonance in ^{208}Pb , which was complicated by overlapping contributions of E0 and E1 multipole order.

The (e,e') spectra provide a good example of an application of formula (11) for the density of maxima. For the ^{90}Sr data, the statistics in the part of the spectrum shown in fig. 4 are insufficient, but from the summed spectra³¹⁾ 33 maxima are counted in a 4 MeV interval. With an average resolution of 45 keV the calculated number of maxima is 41. For the ^{44}Ca data (fig. 5) there are 39 maxima in a 3.75 MeV interval, as compared with a calculated value of 43 maxima.

Before leaving the subject of fluctuation analysis as a tool for determining level densities in complex spectra, it may be useful to underline that this approach is qualitatively the same as in the analysis of resolved spectra. In both cases the fundamental problem is to provide an estimate of the number of unobserved levels, a task that is always delicate because intensity zero is strongly favoured by the statistical distributions involved (see fig. 1). This problem is a familiar one to practitioners of neutron-resonance spectroscopy, and one that will be illustrated further in our discussion of 'pandemonium' in Section 4.

3.3 Branching ratios and lifetimes

Until now, we have discussed the situation in which some fine structure can be observed. However, even for spectra with no observable fine structure, the statistical behaviour of the individual levels must be kept in mind. The discussion of fluctuation functions in connection with eqs. (6) to (8) shows that the observed overall intensities may be strongly influenced by the underlying microscopic structure. We give two examples of this.

The first example concerns the strength function for the sub-barrier emission of alpha particles from excited states of medium-mass, proton-rich nuclei with $A = 110-120$. This process is observed³³⁻³⁵) in the EC/ β^+ decay of nuclei near the proton drip line. Previous studies (see refs. 16 and 17 and references therein) had shown that the corresponding emission of protons can be well understood in a statistical model, in which the proton strength was taken from an optical-model calculation. The question was now how to understand the magnitude of the corresponding alpha strength function. The answer to this turned out to be simple and intuitively pleasing.

A conventional statistical model without fluctuation corrections works well for the caesium isotopes. This assumes³³) as an 'ansatz' that the alpha strength $S_\alpha = \langle \gamma_\alpha^2 \rangle / D$ is constant in the region of interest, typically 5–12 MeV excitation energy. The observed widths, most often dominated by s-wave emission, are $\Gamma_\alpha = 2P \gamma_\alpha^2$, where P is the barrier penetrability. It is further assumed that S_α arises from cluster levels with reduced width equal to the Wigner limit $\gamma_W^2 = 3\hbar^2 / (2m_\alpha R^2)$, where R is the channel radius, and with a spacing between the cluster levels of $2\hbar\omega \cong 16$ MeV. This leads to the estimate $\langle \Gamma_\alpha \rangle = \gamma_W^2 PD / \hbar\omega$, which turns out to agree extremely well with experiments^{33,35}) on proton/alpha ratios (see table 1). It is important to notice that the calculation of ratios is expected to be especially reliable since errors in the shape of the beta strength function and in the level density will cancel in this case.

Table 1

Beta-delayed emission of protons and alphas
in the decay of caesium and xenon isotopes^{a)}

	$T_{1/2}$ (s)	b_p	b_p/b_α		
			exp	theory	
				without	with ^{b)}
^{114}Cs	0.7 ± 0.2	—	33 ± 12	40	—
	0.57 ± 0.02	$(7 \pm 2) \times 10^{-2}$	$44 \pm 3^c)$		
^{116}Cs	3.5 ± 0.2	$(3.6 \pm 0.8) \times 10^{-3}$	47 ± 2	50	—
$^{116}\text{Cs}^d)$	3.9 ± 0.4	$(6.6 \pm 1.3) \times 10^{-3}$	> 200		
	0.65 ± 0.10	$(2.8 \pm 0.7) \times 10^{-3}$	4.7 ± 1.8		
^{118}Cs	$16.4 \pm 1.2^e)$	$(4.2 \pm 0.6) \times 10^{-4}$	17.2 ± 0.3	18.9	18.7
^{120}Cs	$58.3 \pm 1.8^f)$	$(7 \pm 3) \times 10^{-8}$	0.36 ± 0.10	0.26	—
^{113}Xe	2.8 ± 0.2	est. 0.04	830 ± 50	3500	640
^{115}Xe	18 ± 4	$(3.4 \pm 0.6) \times 10^{-3}$	1100 ± 300	2600	1450

a) Data from refs. 33–35) and newer sources as indicated. See also Nuclear Data Sheets for additional references. The quantities b are branching ratios relative to the number of beta decays.

b) Fluctuation corrections included, see text.

c) R. Kirchner, O. Klepper, G. Nyman, W. Reisdorf, D. Schardt, K. Wien, R. Fass and S. Mattsson, Phys. Lett. **78B** (1978) 393.

d) D.D. Bogdanov, J. Voboril, A.V. Demyanov and L.A. Petrov, Phys. Lett. **71B** (1977) 67. The existence of a short-lived isomer was confirmed by D'Auria et al.³⁴⁾. However, they found no particle decays with this, and could not reproduce the b_p/b_α ratios reported by Bogdanov et al.

e) This decay is a mixture of two isomers, (14 ± 2) s and (17 ± 3) s. See J. Genevey-Rivier, A. Charvet, G. Marguier, C. Richard-Serre, J. D'Auria, A. Huck, G. Klotz, A. Knipper and G. Walter, Nucl. Phys. **A283** (1977) 45.

f) Two isomers, (57 ± 6) s and (64 ± 3) s, see footnote (e).

For the case of the odd-mass xenon isotopes³⁵⁾, the proton decay of the odd-Z iodine daughter is dominated by the ground-state transition, and fluctuation corrections cannot be neglected. (For the caesium decays, the proton spectra emitted from the even Xe intermediate states are more complex; consequently the fluctuation corrections are small.) The corrections calculated for $^{113,115}\text{Xe}$ amount to factors of 5 and 2, respectively³⁵⁾. The authors take this as a reminder that this effect is occasionally important and remark that the effect 'itself is founded on well-established properties of complex nuclear spectra and hardly requires experimental proof'. The underlying assumption for this is, of course, that the alpha, proton and gamma channels are unlikely to be correlated.

The second example concerns measurements of nuclear lifetimes by proton X-ray coincidences. The lifetimes of an atomic K-shell vacancy range from 6×10^{-16} s to 6×10^{-18} s (approximately) for atomic numbers $Z = 20-92$. The corresponding total widths Γ_K of 1–100 eV will, at a certain nuclear excitation energy, be equal to the partial widths for sub-barrier emission of charged particles. This suggests the use of X-ray spectroscopy to determine the order of emission of the X-ray and the charged particle, and hence the lifetime. A review of the history and accomplishments of this method and a comparison with other techniques for measuring short lifetimes can be found in the review by Massa and Vannini³⁶⁾.

The first—and also the most promising—application was developed by Hardy et al.²³⁾, who investigated beta-delayed protons following electron-capture (EC) beta decay. This process is ideally suited because one K vacancy is produced per proton and because the neutron exit channel is closed owing to the high neutron binding energy in proton-rich systems. Following the original application²³⁾ to the decay of ^{69}Se , the method has been applied²⁴⁾ to ^{73}Kr , also an $(N-Z) = 1$ nucleus. Recently, the ^{69}Se experiment has been repeated with improved resolution and statistics by Dessagne et al.²⁵⁾; their results are shown in fig. 6. The large variation found for the X-ray intensity ratio is extremely striking, and the authors suggest that this arises from the presence of two unmixed groups of states with different quadrupole deformations, $\beta_2 = 0.2-0.3$ and $\beta_2 = 0.5$. Such structures are, in fact, expected theoretically in the energy window in question, i.e. at 4.7–6.8 MeV excitation energy in ^{69}As .

Although the non-exponential decay of the proton-emitting resonances [see eq. (12)] has been taken into account, there has so far been no detailed analysis of the fluctuation behaviour of the X-ray ratios. We indicate briefly some of the ingredients that could enter into such an analysis. Let us, for

Fig. 6 Beta-delayed proton spectrum from the decay of ^{69}Se and (inset) the intensity ratio of Ge and As X-rays in coincidence with protons. (Based on ref. 25.) A high value for this ratio implies that the proton was emitted before the X-ray and hence that the levels in this energy bin have short average lifetimes.

simplicity, consider a single proton exit channel with normalized intensity $y = \Gamma_p / \langle \Gamma_p \rangle$ competing with the constant total gamma width Γ_γ . Let the feeding by beta decay be proportional to x , also a normalized Porter–Thomas variable. [We are using the notation defined just before eq. (4).] Introducing the constants $\gamma = \Gamma_\gamma / \langle \Gamma_p \rangle$ and $k = \Gamma_K / \langle \Gamma_p \rangle$ we easily find that the mean X-ray intensities from a single resonance are proportional to

$$\omega(Z-2) = xy / (y + \gamma + k), \quad (19)$$

$$\omega(Z-1) = xyk / [(y + \gamma + k)(y + \gamma)]. \quad (20)$$

The observed X-ray ratios emerge as a ratio of intensity sums for a given energy bin B . We are therefore concerned with averages and variances of a quantity of the type

$$R = [\sum_B \omega(Z-2)] / \sum_B \omega(Z-1). \quad (21)$$

It is simplest to use Monte Carlo techniques to evaluate the statistical properties of R . Assuming a total of 14 levels in an energy bin and studying a

sequence of 20 bins based on one and the same set of pseudorandom numbers, the results may be characterized by the mean and error for a single bin and by $R_{\min}$, $R_{\max}$. For typical values of the constants we find:

γ, k	$\overline{R}$	$R_{\min}$	$R_{\max}$
1.0, 20	0.13 ± 0.05	0.08	0.25
0.5, 20	0.10 ± 0.04	0.05	0.20
1.0, 10	0.25 ± 0.10	0.17	0.45
0.5, 10	0.19 ± 0.08	0.10	0.35

It is seen that the spread, although appreciable, is smaller than that in the inset in fig. 6, and hence provides a tentative support for the interpretation of the data as a persistent selection rule effect. A more detailed analysis would have to take into account the contributions of the various intermediate spins and the (weak) proton branch to the 2^+ state in ^{68}Ge .

Finally, we mention for completeness the pioneering work by Röhl et al.³⁷⁾ and by Chemin et al.³⁸⁾ in which the p-X coincidence method was applied to resonances populated in proton-induced reactions. These experiments are rendered very difficult by the small probability of simultaneous excitation of the nuclear resonance and a K vacancy, and by the high singles count rate for X-rays, which are generated with a cross-section of the order of 60 barn. Some statistical problems that are encountered if this method is applied to Ericson fluctuations have been discussed by von Brentano and Kleber³⁹⁾.

4. TESTS OF EXPERIMENTAL SENSITIVITY IN SPECTROSCOPIC STUDIES OF COMPLEX DECAYS

*Att betrakta världen så är att se
på den med tärningsögon.
Vårt bästa val är att kasta tärningen ofta, ofta.
Att ständigt låta den rulla, att den må falla ut
med en kvot stor nog
för ny förväntan att andas.*

Harry Martinson, Tärningspelet, 1971

In this section we outline an experimental and essentially technical application of statistical nuclear physics, namely the use of Monte Carlo simulated spectra^{40,41)} to test experimental detection efficiency and bias under given circumstances. The basic difficulty, which has already been touched upon in previous sections, is that the intensities of individual lines belong to

extremely skew distributions with a pronounced singularity at the origin, and that consequently the experimentalist must expect to encounter a small number of intense lines in the presence of very many weak—and very likely undetected—ones. Experience shows that nevertheless this comes as a surprise to some, whose intuition, rooted in Gaussian error theory, will tell them that they are in the presence of a 'prominent peak structure' arising from 'selection rule effects', 'intermediate structure', etc. The same problem poses itself, in a slightly different disguise, in attempts to perform 'complete spectroscopy'; that is, experiments in which one aims at statements about the true number and intensities of lines of a given kind.

The first example selected⁴⁰⁾ for the use of simulation techniques was that of the gamma-ray spectroscopy in the complex decay of ^{145}Gd . This isotope had attracted considerable interest because of apparently strong anomalous electron-capture/positron ratios^{42–45)} in the allowed beta decay. The determination of these ratios rested upon the input–output balance of gamma intensities to individual levels and hence upon the completeness of the decay scheme. A quite similar analysis of simulated gamma spectra, for which the 'true' intensities clearly are known, could now be used to estimate the unobserved intensity.

The fictional nucleus created for the simulation of the ^{145}Gd decay was referred to as ^{145}Pn (an isotope of the element 'pandemonium'), and it was assumed to have the same spin and Q-value but to have smooth, structureless level densities and strength functions. The decay scheme was simplified somewhat, and the simulated gamma spectrum was generated, level by level, with a level separation according to the Wigner law and with a beta feeding distributed around the average with a Porter–Thomas distribution. The spectrum was then folded with an experimental response function corresponding to a 3 keV (FWHM) Gaussian resolution, and the energy-dependent efficiency of the detector was incorporated. Finally, the effect of counting statistics was superimposed. Examples of simulated and measured spectra are shown in fig. 7.

The results of the analysis for $E_\gamma > 1.7$ MeV, that is the region taken to be statistical, are summarized in table 2, which also gives a comparison with newer experimental data. The original experimental spectrum Gd-I had somewhat less statistics than Pn-II and observed roughly the same number of peaks. From the comparison with the simulation it was then assumed⁴⁰⁾ that roughly 20% of the intensity was missing in the original experiment. New experiments with better statistics have confirmed this conclusion by recovering an additional 20% (ref. 44), resp. 14% (ref. 45), and the agreement becomes

Fig. 7 Monte Carlo simulated γ -ray spectrum corresponding to the decay of ^{145}Pn , a fictional nucleus created for the simulation of the ^{145}Gd decay. The pseudodata are based upon a different set of random numbers than those of the (low-statistics) spectrum shown previously⁴⁰⁾, and the statistics correspond to the set Pn-III in table 2. The inset shows for comparison part of the γ -ray data from a ^{145}Gd experiment⁴⁵⁾. Note that the real data are on a linear scale.

Table 2

Simulated and measured gamma spectra of ^{145}Gd

Data set	Counts	No of peaks ^{a)}	Intensity observed (%)	Reference
$^{145}\text{Pn-I}$	$\sim 10^6$	8	55	40
$^{145}\text{Pn-II}$	$\sim 10^7$	35	86	40
$^{145}\text{Pn-III}$	$\sim 10^8$	70	95	40
$^{145}\text{Gd-I}$	Slightly below Pn-II	32	x	42, 43
$^{145}\text{Gd-II}$	Similar to Pn-III	280	x + 20	44
$^{145}\text{Gd-III}$	- - -	325	x + 14	45

a) The 'true' number of peaks in the simulated spectra is around 1000. The real spectrum must have additional rays from γ -ray cascades⁴⁵⁾, so the 'true true' number must be several thousand.

Fig. 8 The energy spectrum of beta-delayed neutrons from ^{135}Sb from experiment (a) and from a Monte Carlo simulation of the fictional nucleus ^{135}Pn (b). The simulation used slowly varying strength functions corresponding to the parameters of the real decay. Thus the appreciable fine structure in (b) is essentially a result of Porter–Thomas fluctuations. (From ref. 41.)

better when it is noted that even the improved experiments must be assumed to have missed of the order of 6% of the intensity.

A second application⁴¹⁾ concerned the spectra of beta-delayed neutrons arising from other isotopes of pandemonium. A typical example can be seen in fig. 8, which shows that the peak structure observed experimentally is fully consistent with what could be expected from fluctuations. The paper⁴¹⁾ underlines that the similarity between the measured and simulated spectra does not prove that the real isotope has the same featureless properties as its pandemonium effigy. The point is rather that if local nuclear properties are to be extracted from real data, then the method must be one that would give the correct, structureless picture of pandemonium from its 'data'.

The use of Monte Carlo simulations in nuclear spectroscopy has not met with universal approval (compare the exchange of views in refs. 46 and 47). However, the method is analogous to other applications of Monte Carlo techniques for testing response, efficiency, and background in many areas of physics, and in particular in high-energy physics. If we insist on studying complex spectra, we cannot afford to forget the lesson of slow-neutron physics, learned many years ago.

5. CONCLUDING REMARKS

This article, written for household use by practitioners of nuclear spectroscopy, has been concerned with some applications of statistical nuclear physics. In view of the great current interest in the developing field of quantum chaos, it would clearly also be very valuable to have more experimental contributions going the opposite way. Such experiments are, however, quite difficult. We mention below, as one possible example, a fundamental problem that is, in fact, waiting on the doorstep of nuclear spectroscopy.

In the introductory section of this paper, we took it to be intuitively reasonable (and in any case justified by experience) that level number $N = 10^6$ of a certain energy, spin, and parity could no longer be characterized by other structural quantum numbers, and that its wave function would have to be a complex random mixture of very many basic configurations. But is this also true for level numbers $N = 10^5$, 10^4 , and 10^3 ? Most probably so. Starting from the other end, we would probably agree relatively easily that the 10 lowest eigenstates have much individuality, that they represent rotations, vibrations, two-quasi-particle states, and so on. But we have essentially no knowledge that permits us to chart the transition from order to chaos. Experiments of this kind present an enormous challenge to spectroscopy.

One possible line of attack is suggested by a recent sequence of papers⁴⁸⁾, in which spin-1 states in the strongly deformed nuclei $^{156,158,160}\text{Gd}$ were studied by scattering electrons and photons inelastically. The structural quantum number K , which represents the projection of total angular momentum on the intrinsic nuclear symmetry axis, could be determined from the branching ratios of the gamma rays to the ground state and its first rotational 2^+ state. The values found in the three nuclei, and which concern more than 50 states at excitation energies up to 3.9 MeV, are all consistent with pure assignments, either $K = 0$ or $K = 1$. Thus, well past $N = 10$ there is no sign that the Coriolis force has caused a breakdown of K conservation. Will this happen before $N = 10^2$? We do not know.

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Appendix

Translations of the quotations

This is Man's wonderful ability:
to be able to grasp the inner essence of phenomena,
not what they appear to be, but what they mean,
and the reality that we see with our eyes
is a symbol only of something higher.

In France the sciences are but a fashion that replaces others and will itself be replaced in turn. Yet, no art and no science should be allowed to be a matter of fashion. They all ought to go forward hand in hand; we ought to study them at all times. I do not want to obey fashions, I want to be free to go from a physics experiment to an opera or a play, ...

Mephistopheles: I did not wish to lead you astray.
It is very difficult to avoid the false path
as far as this science is concerned;
in it there is much hidden poison
that one hardly can tell from medicine.

I don't know much about anything;
what I know is already commonplace.
Wouldn't it be nice to know just one fact,
for example the proportions of the spots of the ladybird.

To view the world in this way
is to see it with the eyes of a gambler.
Our best choice is to cast the die again and again.
To let it keep rolling in the hope
that it will give returns high enough
to inspire new expectations.