Vol. 29 - Pinning Mechanisms in YBCO Tapes

Spera, Marcello (Politecnico di Milano)

01 May 2015
Pinning Mechanisms in YBCO tapes

Marcello Spera
780662

Relatore: Ezio Puppin
Correlatore: Amalia Ballarino

A.A. 2013-2014
A Nonna Elena, la persona più forte che io conosca.
Abstract

In this thesis work, a study on flux pinning mechanisms of commercial YBCO tapes is presented. This study has been performed via critical current characterization using transport (via direct I-V curves) and magnetization (via a Vibrating Sample Magnetometer) measurements. The latter ones turned out to be better concerning the comprehension of the pinning landscape of the provided samples, as a wider range of magnetic fields and temperatures is available for those measurements in the setup I used. The comparison of the experimental data with existing theoretical models allowed me to draw a picture of the pinning mechanisms underlying in each sample, and they turned out to be quite different one another. Moreover, for high-performance research tapes, another interesting feature has been found: the counterplay between the self-field critical current and the in-field one. Very well engineered artificial pinning structures limit the self-field critical current density due to the high density of defects, while samples with high degree of crystallinity have their in-field performances limited due to the poor pinning properties.

The work is organized as follows: in chapter 1, a general introduction on superconductivity is given, in order to introduce all the concepts and definitions necessary to understand the forthcoming discussions. Also a brief outline of the BCS theory, even if it does not suit for high-temperature superconductors, is presented, in order to give the reader an idea of how superconductivity is treated microscopically.

In chapter 2, the properties of the material I investigated, YBCO, and other basic concepts concerning pinning are presented, focusing more on the main topic of this thesis rather than general superconductivity.

Chapter 3 is dedicated to a bibliographic research made in order to have a general idea about the current performances of state-of-the-art commercial and research tapes, and the pinning mechanisms employed up to now to increase the critical current performances of the tapes. It is followed, in chapter 4, by a brief summary of the pinning mechanisms in the low-temperature superconductors mainly used for applications, such as Nb-Ti, Nb$_3$Sn and MgB$_2$, in order to have a wider idea of the similarities and differences of how the pinning is achieved in different materials.

A pinned type-II superconductor features magnetic hysteresis: in chapter 5 I focused the attention on the existing theoretical models accounting for the magnetization behaviour of high-temperature superconductors and its link to critical currents. Besides, this chapter deals with the legitimacy of the extrapolation of the critical current density from the magnetization hysteresis loop, which has been used for the data analysis.
In chapter 6 the experimental setup used during my measurements is described, while in chapter 7 the achieved results are presented and widely discussed.

The pinning picture inferred from the experimental data for the measured samples is summarized in chapter 8; two appendixes have been added for the sake of completeness, containing a brief outline of the theoretical models used during the data analysis.
Abstract

In questo lavoro di tesi viene presentato uno studio sui meccanismi di pinning dei flussoni su nastri commerciali di YBCO. Lo studio è stato effettuato tramite caratterizzazione in corrente critica dei campioni a mia disposizione, utilizzando misure di trasporto (misure dirette della caratteristica I-V) e magnetizzazione (con un sistema VSM). Queste ultime si sono rivelate essere migliori per la comprensione dei meccanismi di pinning, dato che nel setup a mia disposizione un ampio range di campi magnetici e temperature era disponibile durante le misure magnetiche. Il confronto dei dati sperimentali con modelli teorici già esistenti mi ha permesso di comprendere le proprietà di pinning di ogni campione, che si sono rivelate essere abbastanza differenti.

Inoltre, per quanto riguarda i nastri in ricerca e sviluppo ad alte performance, è stata investigata un’altra importante caratteristica: il controbilanciamento delle performance in autocampo e di quelle in campo. Strutture di pinning artificiale altamente ingegnerizzate limitano la densità di corrente critica in auto-campo, vista l’alta densità di difetti presente nel volume per avere un efficace pinning, mentre i campioni con un alto grado di cristallinità sono limitati in campo a causa delle loro scarse proprietà di pinning.

Il testo è organizzato come segue: nel capitolo 1, viene presentata una introduzione generale sul fenomeno della superconduttività, al fine di introdurre tutti i concetti e le quantità di base necessarie per comprendere i capitoli successivi. Viene inoltre presentata brevemente parte della teoria BCS, nonostante non sia adatta per studiare i superconduttori ad alta temperatura, per dare un’idea al lettore di come la superconduttività viene trattata tramite teorie microscopiche.

Nel capitolo 2 vengono presentate le proprietà del materiale che ho studiato, l’YBCO, e altri concetti di base riguardanti il pinning, andando a focalizzare l’attenzione sul principe di pinning utilizzato per aumentare la corrente critica. È seguita, nel capitolo 4, da un breve riassunto sui meccanismi di pinning presenti nei superconduttori a bassa temperatura che vengono principalmente utilizzati per applicazioni, come il Nb-Ti, il Nb₃Sn e il MgB₂, per avere un’idea più generale di come il pinning viene ottenuto in diversi materiali.

I superconduttori di tipo II con pinning dei flussoni presentano isteresi magnetica: nel capitolo 5 ho concentrato l’attenzione sui modelli teorici
esistenti che spiegano la magnetizzazione dei superconduttori ad alta temperatura e il suo legame con la corrente critica. Inoltre, questo capitolo discute la legittimità dell’estrapolazione della corrente critica dal ciclo di isteresi, concetto che è stato poi utilizzato durante l’analisi dei dati.

Nel capitolo 6 viene descritto l’apparato sperimentale utilizzato durante le misure, mentre nel capitolo successivo sono presentati i risultati e la loro discussione.

I meccanismi di pinning dedotti dai dati sperimentali sono riassunti nel capitolo 8; sono state poi aggiunte due appendici per completezza, contenenti un breve riepilogo dei modelli teorici utilizzati per l’analisi.
Li miei compagi fec’io si aguti,
con questa orazio piccia, al cammino,
che a pena poscia li avrei ritenuti;

e volta nostra poppa nel mattino,
de’ remi facemmo ali al folle volo,
sempre acquistando dal lato mancino.
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<td>( 2p )</td>
<td>Perimeter of the sample</td>
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<tr>
<td>( ab )</td>
<td>Plane in the crystallographic structure of YBCO where the copper planes lie</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Coefficient for the second order term in the Ginzburg-Landau free energy expansion</td>
</tr>
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<td>( \beta )</td>
<td>Coefficient for the fourth order term in the Ginzburg-Landau free energy expansion</td>
</tr>
<tr>
<td>( c )</td>
<td>In the text, direction in the crystallographic structure of YBCO perpendicular to the copper planes</td>
</tr>
<tr>
<td>( c )</td>
<td>In formulae, speed of light</td>
</tr>
<tr>
<td>( A )</td>
<td>Area of the sample</td>
</tr>
<tr>
<td>( A_{\text{coil}} )</td>
<td>Area of the coil</td>
</tr>
<tr>
<td>( \mathbf{A} )</td>
<td>Magnetic vector potential</td>
</tr>
<tr>
<td>( \mathbf{B} )</td>
<td>Magnetic induction</td>
</tr>
<tr>
<td>( \mathbf{B}_a )</td>
<td>Applied magnetic induction</td>
</tr>
<tr>
<td>( \mathbf{B}_c )</td>
<td>Critical magnetic induction</td>
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<td>( \mathbf{B}_{c1} )</td>
<td>Lower critical magnetic induction</td>
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<tr>
<td>( \mathbf{B}_m )</td>
<td>Maximum magnetic induction applied in a magnetization measurement</td>
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\( B_p \)  Full penetration magnetic induction
\( B_s \)  Full penetration magnetic induction for the reverse currents
\( d \)  Penetration depth in Bean's model
\( d_{att} \)  Flux line lattice spacing
\( \delta \)  Oxygen vacancy in YBCO
\( e \)  Electron charge
\( E \)  Energy
\( E \)  Electric field
\( E_{cr} \)  Critical electric field
\( \Phi_0 \)  Flux quantum
\( \Phi_B \)  Magnetic induction flux
\( G \)  Gibb’s free energy
\( h \)  Planck’s constant
\( \hbar \)  Reduced Planck’s constant
\( H \)  Magnetic field
\( H_a \)  Applied magnetic field
\( H_c \)  Critical magnetic field
\( H_{c1} \)  Lower critical magnetic field
\( H_{c2} \)  Upper critical magnetic field
\( H_m \)  Maximum magnetic field applied in a magnetization measurement
\( H_p \)  Full penetration magnetic field
\( H_s \)  Full penetration magnetic field for the reverse currents
\( i \)  Imaginary unit
\( I \)  Current
\( I_c \)  Critical current
\( J \)  Current density
\( J_c \)  Critical current density
<table>
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<tr>
<td>$J_{\text{engineering}}$</td>
<td>Engineering current density</td>
</tr>
<tr>
<td>$l$</td>
<td>Length of the sample</td>
</tr>
<tr>
<td>$L$</td>
<td>Thickness of the sample for the study made in chapter 5</td>
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<tr>
<td>$\lambda_l$</td>
<td>London penetration depth</td>
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<td>Mass</td>
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<td>$\mu$</td>
<td>Power decay exponent</td>
</tr>
<tr>
<td>$\vec{\mu}$</td>
<td>Magnetic moment</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Vacuum permeability</td>
</tr>
<tr>
<td>$M$</td>
<td>Magnetization</td>
</tr>
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<td>$M^+$</td>
<td>Upper branch of the magnetization loop</td>
</tr>
<tr>
<td>$M^-$</td>
<td>Lower branch of the magnetization loop</td>
</tr>
<tr>
<td>$\Delta M$</td>
<td>Width of the magnetization loop</td>
</tr>
<tr>
<td>$n$</td>
<td>Exponent for the I-V superconductor’s Power Law ($n - \text{value}$)</td>
</tr>
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<td>$n_s$</td>
<td>Supercarrier density</td>
</tr>
<tr>
<td>$p$</td>
<td>Parameter indicating the strength of the critical current dependence from the magnetic field in chapter 5</td>
</tr>
<tr>
<td>$q$</td>
<td>Electric charge</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of the sample</td>
</tr>
<tr>
<td>$RR$</td>
<td>Ramp rate of the applied magnetic field</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Resistivity</td>
</tr>
<tr>
<td>$s_p$</td>
<td>Effective size of the sample</td>
</tr>
<tr>
<td>$S$</td>
<td>Entropy</td>
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<tr>
<td>$t$</td>
<td>Thickness of the sample</td>
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<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Critical temperature</td>
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<tr>
<td>$T_0$</td>
<td>Weak pinning characteristic temperature</td>
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<tr>
<td>$T^*$</td>
<td>Strong pinning characteristic temperature</td>
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<tr>
<td>$U$</td>
<td>Internal energy</td>
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\( v \) \hspace{1cm} \text{Velocity}

\( w \) \hspace{1cm} \text{Width of the sample}

Note: the symbols used in the appendices are not reported here
Chapter 1

Introduction

1.1 Overview

The phenomenon of superconductivity was discovered in 1911 by Heike Kamerlingh Onnes while he was performing experiments on cryogenics and resistivity of metals. He found that below a certain temperature, the resistivity of the mercury sample he was measuring dropped abruptly to an infinitesimal value. The resistivity of a normal metal is known to increase linearly with temperature, due to the increasing amplitude of the lattice vibrations that can scatter electrons, as the vibrating crystal differs from the perfect one, in which the Bloch electrons are eigenstates of the Hamiltonian. Moreover, at very low temperatures the resistivity can reach a constant value, due to the scattering by defects, whose presence does not depend on temperature, or even slightly increase, due to the Kondo effect.

The abrupt drop found by Onnes was really unexpected, and no one could find a good explanation at that time. After some years, in 1933, Walther Meissner and Robert Ochsenfeld discovered another peculiar characteristic of superconductors, related to their magnetic properties. Indeed they found that when superconductors are immersed in a magnetic field they behave as perfect diamagnets, completely expelling the applied field. Besides this effect is not due to the absence of resistance, as I will briefly explain: imagine a superconductor and a perfect conductor. The first one has the characteristic of zero resistance and perfect diamagnetism, while the second one only features infinite conductivity. Let us start the analysis at a temperature higher than the critical one (defined as the temperature below which the superconductivity takes place): the magnetic field will penetrate the superconductor and the perfect conductor, as their peculiar properties are not active in this temperature range. If we cool down the superconductor, it will become a perfect diamagnet and therefore completely expel the flux, leaving its interior field-free. If the field is then removed, the supercon-
ductor will demagnetize in order to keep its internal field zero. The same will happen to the superconductor if it is first cooled and then a field is applied. For the perfect conductor, instead, if the sample is first cooled and then a field is applied the material will behave as a superconductor, as the screening currents due to the Faraday - Neumann - Lenz law will be able to flow without resistance, completely screening the field. But if the field is applied first and the sample is field cooled, nothing will happen, as there is no change in the magnetic flux. Moreover, if one then removes the applied field, the sample will get magnetized, as it wants to keep the same field configuration as before, due to the induction law. From this simple example one can easily understand that the magnetic properties of superconductor are not trivially related to the absence of resistivity.

The superconducting properties can be destroyed not only by raising the temperature, but also in two other ways: with a magnetic field and with an injected current. The details and the explanation will be given later on, for the time being I just want to introduce some nomenclature: the value of injected current at which the superconductor starts to develop a finite resistance is called critical current $I_c$, and the value of applied field at which the superconductivity is lost is called critical field, $H_c$. It was found that there exist some superconductors in which flux penetration is somehow allowed, these are called type II superconductors, in contrast to the ones with full screening that are called type I. In these superconductors there are actually two critical fields: the lower critical field, $H_{c1}$, at which the flux starts to penetrate the superconductors volume, and the upper critical field, $H_{c2}$, at which the superconducting properties are lost. The way how the field penetrates inside the type II superconductors is peculiar, as the flux is quantized in vortex lines, also called fluxons or fluxoids, that arrange themselves in a triangular 2D lattice. Up to the late 60s, all the superconductors that had been discovered had a very low critical temperature, limiting their applications as they could only be cooled with liquid helium. In 1987, Paul Chu and his co-workers found a compound of Yttrium, Barium, Copper and Oxygen that featured a critical temperature of about 90 K. This was a breakthrough for both applications and theory, as this critical temperature is higher than the boiling temperature of Nitrogen, which is way cheaper and easier to produce than liquid Helium, and also because this high temperature superconductivity is impossible to explain with the theories that had developed in the meantime. Actually, the most complete theory available (I will talk about it later on) had put an upper limit on the critical temperature that was passed over by the discovery of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$, YBCO in an abbreviated way.
1.2 Macroscopic Theories: London Theory

The first persons who tried to give an explanation to the phenomenon of superconductivity were Fritz and Heinz London in 1935, with a macroscopic theory that could explain the Meissner effect, but did not explain the actual mechanism that brought to the absence of resistance. The starting point of their theory is the simple Drude model of metals, without the frictional term, to take infinite conductivity into account. The original equation is:

$$m\ddot{\mathbf{v}} = q\mathbf{E} - \gamma \mathbf{v} \quad (1.1)$$

Removing the frictional term and assuming a negative charged carrier one gets:

$$m\ddot{\mathbf{v}} = -|q|\mathbf{E} \quad (1.2)$$

From now on I shall denote by $q$ the absolute value of the charge of the carrier. The current density can be written as:

$$\mathbf{J} = -qn_s \mathbf{v} \quad (1.3)$$

The quantity $n_s$ is the density of the supercarriers, this yields:

$$\frac{d\mathbf{J}}{dt} = \frac{q^2 n_s}{m} \mathbf{E} \quad (1.4)$$

This is known as the first London equation. Now with the help of Maxwell’s equations, remembering that $\nabla \times \mathbf{E} = -\frac{dB}{dt}$, taking the curl of both sides of the previous equation and exchanging the time derivative with the curl, as they act on different variables, one gets:

$$\frac{d}{dt} \left[ \nabla \times \mathbf{J} + \frac{q^2 n_s}{m} \mathbf{B} \right] = 0 \quad (1.5)$$

Finally, with the help of the fourth Maxwell equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ we have:

$$\frac{d}{dt} \left[ \nabla \times \nabla \times \mathbf{B} + \frac{q^2 n_s \mu_0}{m} \mathbf{B} \right] = 0 \quad (1.6)$$

Now a second assumption is made in order to take the Meissner effect into account: the internal field must be zero, so not only the derivative of the quantity in brackets but also that quantity should be zero. This, introducing the London penetration depth $\lambda_L = \sqrt{\frac{m}{q^2 n_s \mu_0}}$, yields the second London equation:

$$\nabla \times \nabla \times \mathbf{B} + \frac{1}{\lambda_L^2} \mathbf{B} = 0 \quad (1.7)$$
Remembering that the divergence of $B$ is zero, one obtains:

$$\nabla^2 B - \frac{1}{\lambda_L^2} B = 0 \quad (1.8)$$

This equation can be easily solved for a given geometry, giving an exponential decay of the magnetic field inside the superconductor over a length scale of about $\lambda_L$. This result just reproduces the Meissner effect. One thing that should be pointed out is that two different assumptions had to be made, one concerning the zero resistance and one concerning the absence of flux inside the volume of the superconductor, underlining again that the Meissner effect is not a direct consequence of the absence of resistivity. If one takes the curl of equation (1.4), upon exchanging again the time derivative with the spatial operator and using the last Maxwell equation, an analogue result for the current can be found:

$$\nabla \times \nabla \times J + \frac{1}{\lambda_L^2} J = 0 \quad (1.9)$$

Meaning that the current flows on a superficial thin layer, in type I superconductors. This brings us to the conclusion that type I superconductors are not the best materials for current carrying applications.

### 1.3 Macroscopic Theories: Ginzburg - Landau theory

Another macroscopic but more refined theory was proposed by Vitaly Lazarevich Ginzburg and Lev Landau, who applied their theory on the second order phase transitions to the superconducting state. The starting point of the general theory is defining the order parameter. For example, in a ferromagnetic system, the order parameter can be taken as the normalized magnetization: indeed, when the ferromagnet is at zero temperature its order parameter is 1, and it slowly decrease to 0 at its Curie temperature due to thermal fluctuations that disorder the spin alignment. The same can be said for a superconductor, considering as the order parameter the fraction of supercarriers. The second assumption is that one can expand the free energy in terms of this parameter, which is to say, we study its properties around the transition temperature. Moreover, as there is no difference between a ferromagnet with spins oriented “downwards” (along a certain defined axis) and one with spins oriented “upwards”, the energy expansion can feature only even powers of the order parameter. The expression of the free energy density for a ferromagnet thus becomes:

$$F(r, T) = F_0(r, T) + \alpha m^2 + \frac{1}{2} \beta m^4 + \frac{1}{4} \gamma |\nabla m|^2 + \ldots \quad (1.10)$$
Considering the order parameter constant in the selected volume, one can easily minimize the free energy with respect to \( m \), obtaining two possible equilibrium values for the order parameter:

\[
\frac{\partial F}{\partial m} = 2\alpha m + 2\beta m^3 = 0 \quad (1.11)
\]

\[
m = 0 \quad m = -\sqrt[3]{\frac{\alpha}{\beta}} \quad (1.12)
\]

The coefficients \( \alpha \) and \( \beta \) can both depend on temperature. A more detailed analysis shows that \( \alpha \) has a zero’th order term equal to zero, so one must take the first order correction, while for \( \beta \) the zero’th order term is finite and the expansion can be stopped at that order. So we take:

\[
\alpha(T) \simeq \alpha_1(T - T_c) \quad \beta(T) \simeq -\beta
\]

(1.13)

So now, if the temperature is higher than the critical one, \( \alpha \) is positive and the value under square root in equation (1.12) is negative, so the only possible solution is \( m = 0 \). If the temperature is below the transition one, the term under square root becomes positive, and an equilibrium position with order parameter different from 0 is possible. This is a very simple and phenomenological explanation for all the possible phase transition that can happen in nature, let us apply it for the case of the superconductor. In a superconductor, the order parameter is chosen to be something that resembles the collective wavefunction of the supercarriers. Indeed, to explain the zero resistance behaviour of superconductors one could assume that the carriers somehow behave as a boson condensate, so the scattering of one particle is heavily suppressed as they all tend to remain in the same ground state, and a change of momentum by a scattered carrier would bring a change of momentum of the whole condensate, which is very unlikely to happen. This will be discussed more deeply in the following sections. The square modulus of the order parameter should represent the “strength” of the superconductor, and it is then chosen to be equal to the supercarrier density. As this only requirement only concerns the squared modulus of the order parameter, an arbitrary phase should also be added, giving:

\[
\Phi(r) = \sqrt{n_s}e^{i\theta(r)} \quad (1.14)
\]

Following the same reasoning made for the ferromagnet, the free energy can be written as:

\[
F(T) = \int_V \left[ F_n(r, T) + \alpha |\Phi(r)|^2 + \frac{1}{2}\beta|\Phi(r)|^4 - \frac{1}{2m}((-i\hbar \nabla + qA(r))\Phi(r))^2 + \frac{B^2}{2\mu_0} d^3r \right] \quad (1.15)
\]

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The fourth term under the integral represent the gradient term of equation (1.10) written in an appropriate quantum mechanical manner, i.e. the kinetic energy operator with the canonical momentum, \(A(r)\) being the vector potential of an hypothetically applied magnetic field; while the fifth term is just the magnetic field energy density. Now for the energy minimization, I make the order parameter slightly vary, I write down the difference in free energy due to this variation and I impose it equal to zero. In formulae:

\[
\Phi(r) \rightarrow \Phi(r) + \delta\Phi
\]

\[
\Phi^*(r) \rightarrow \Phi^*(r) + \delta\Phi^*
\]

\[
\delta F(T) = \int_V \left[ \alpha \Phi(r) + \beta |\Phi(r)|^2 \Phi(r) - \frac{1}{2m} (-i\hbar \nabla + qA(r))^2 \Phi(r) \right] \delta\Phi^* d^3r = 0 \quad (1.16)
\]

As the variation of the order parameter is arbitrary, the only way the integral can be zero is by putting equal to zero the term in the squared brackets, this leads to the first Ginzburg Landau equation:

\[
-\frac{1}{2m} (-i\hbar \nabla + qA(r))^2 \Phi(r) + \beta |\Phi(r)|^2 \Phi(r) = -\alpha \Phi(r) \quad (1.17)
\]

This equation has the form of a nonlinear Schrödingers equation with total energy \(\alpha\). For a homogeneous superconductor the gradient term vanishes and one finds the equilibrium solutions already discussed for the ferromagnet. Repeating the free energy minimization procedure illustrated before varying the vector potential instead of the order parameter leads to the second Ginzburg Landau equation, which defines the supercurrent:

\[
J = -\frac{q}{m} [\Phi^*(-i\hbar \nabla + qA)\Phi + c.c.] \quad (1.18)
\]

Which is the analogue of the probability current density one can find with basic quantum mechanics.

An expression of the critical current can also be found within this simple theory. Let me consider a sample with uniform current distribution in its volume, and also with a very small thickness, for which the self-field contribution can be considered negligible. With these assumptions, and remembering the expression of the order parameter, one has for the free energy density:

\[
f = f_n + \alpha n_s + \frac{1}{2} \beta n_s^2 + \frac{n_s v_s^2}{2m} \quad (1.19)
\]

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Taking the first derivative of this expression with respect to \( n_s \), the supercarrier density, one can find the value of the density at equilibrium, and therefore the current with the well-known expressions. In formulae:

\[
\frac{\partial f}{\partial n_s} = \alpha + \beta n_s + \frac{v_s^2}{2m} \quad \rightarrow \quad n_s = -\frac{1}{\beta} \left[ \alpha + \frac{v_s^2}{2m} \right]
\]

\[
J_s = qn_s v_s = -\frac{q v_s}{\beta} \left[ \alpha + \frac{v_s^2}{2m} \right]
\]

Maximizing this expression with respect to \( v_s \) one obtains the maximum current sustainable by the superconductor, which is the critical current. Remembering the expression for the coefficient \( \alpha \), one finally obtains:

\[
J_c = -\frac{4q}{3\beta} \sqrt{\frac{2m}{3}} \alpha_1(0)^\frac{3}{2} (T_c - T)^\frac{3}{2}
\]

(1.20)

Giving a power law behaviour for the critical current with temperature. One must remember, anyway, that one of the assumptions of GL theory is that we are expanding the energy next to the critical temperature. So this behaviour is justified only for a small range of temperatures.

1.4 Flux Quantization

Let me now discuss briefly the importance of the phase factor in the order parameter. Let us take a closed path inside the superconductor’s volume: the order parameter at the beginning and at the end of the loop will have to be the same. As in basic quantum mechanics, the phase factor gained by a quantity is due to its propagation in time and space, i.e. \( \theta = p \cdot x \) where \( p \) and \( x \) are respectively the 4-momentum and 4-position of the quantity under consideration. Leaving apart the time-energy component, as I am working on a just position dependent order parameter, and substituting the canonical momentum one gets:

\[
\theta = \frac{1}{\hbar} (-i\hbar \nabla + qA(\mathbf{r})) \cdot \mathbf{r}
\]

(1.22)

Which can be transformed into:

\[
\nabla \theta = \frac{1}{\hbar} (-i\hbar \nabla + qA(\mathbf{r}))
\]

(1.23)

The condition on the phase reads:

\[
\oint \nabla \theta \, dl = 2n\pi
\]

(1.24)
That becomes:

$$\oint \nabla(-i\hbar \nabla + qA(r)) \, dl = 2n\pi \hbar$$  \hspace{1cm} (1.25)

The first term is closely related with the supercurrent, as it corresponds to the carrier momentum. So if one takes the integration path far enough from the surface so that the current is zero, the first term vanishes and one has:

$$\oint qA(r) \, dl = nh$$  \hspace{1cm} (1.26)

Remembering that $B = \nabla \times A$ and using Stokes theorem, I finally get:

$$\Phi_B = n\frac{\hbar}{q} = n\Phi_0$$  \hspace{1cm} (1.27)

Now, if the integration path is taken in a bulk superconductor, without any topological constraint, the value of $n$ can be zero, as I can ultimately shrink the loop to a point. On the other side, if the domain is not simply connected, I cannot take the $n = 0$ value, meaning that the flux that can penetrate the superconductor is quantized, and its quantization depends on the supercarriers’ charge.

### 1.5 Type I and Type II superconductors

With some thermodynamic considerations, one can rather easily find a remarkable result about an important distinction between two kinds of superconductors. Let me consider the Gibbs free energy of the superconductor, defined as:

$$G = U - TS$$  \hspace{1cm} (1.28)

In presence of an external magnetic field, the total energy of the superconductor will be the sum of its free energy and the energy necessary to expel the magnetic field, equal to the field self-energy, namely:

$$G_{\text{super}}(B) = G_{\text{super}}(0) + \frac{B^2}{2\mu_0}V$$  \hspace{1cm} (1.29)

From this equation one can find an expression for the upper critical field: indeed, the breakdown of superconductivity will take place when $G_{\text{super}}(B)$ will become equal to the normal state free energy, thus leading to:

$$B_c = \sqrt{2\mu_0 V [G_{\text{normal}} - G_{\text{super}}(0)]}$$  \hspace{1cm} (1.30)

The difference in free energy between the normal and superconducting state can be seen as a condensation energy (the term will be clearer in the following
sections) for the carriers, which is to say, the energy needed for a carrier to become a supercarrier, and it is equal to:

\[ G_{\text{normal}} - G_{\text{super}}(0) = \Delta = \frac{B_c^2}{2\mu_0}V \]  

(1.31)

Now consider a normal-superconductor interface: the field will be able to penetrate in the normal material, and it will decay exponentially on a length \( \lambda_L \) in the superconductor. The order parameter, on the other hand, will initially be zero at the interface, to grow to its saturation value (i.e. the supercarriers density) in the bulk. I define the characteristic length to reach the saturation value for the supercarriers density the coherence length \( \xi \) of the superconductor. Now, what is the energy cost (or gain) for creating such an interface? Within \( \lambda_L \) there will be a gain of energy due to the fact that the magnetic field does not need to be expelled, giving an energy gain per unit area equal to

\[ \frac{\Delta E_{\text{magn}}}{A} = \frac{B_c^2}{2\mu_0} \lambda_L \]  

(1.32)

On the other hand, there will be a loss of condensation energy within \( \xi \), due to the fact that a fraction of carriers is not superconducting yet, giving:

\[ \frac{\Delta E_{\text{magn}}}{A} = \frac{B_c^2}{2\mu_0} \xi \]  

(1.33)

So I will have an overall energy gain if \( \lambda_L > \xi \): this means that if this condition is satisfied, in the presence of a magnetic field it will be more energetically favourable for the superconductor to split in different normal-superconducting regions and let pass part of the applied field. This estimation is quite rough, with more detailed calculation, Ginzburg-Landau theory reaches a similar result, defining the Ginzburg-Landau parameter as \( \kappa = \lambda_L/\xi \):

- Type I superconductor: \( \kappa < \frac{1}{\sqrt{2}} \)
- Type II superconductor: \( \kappa > \frac{1}{\sqrt{3}} \)

So, a type II superconductor will feature an alternation of different normal-superconducting regions, and the magnetic field will partially penetrate the superconductor in the normal regions: from the flux quantization condition shown in the previous paragraph, this means that the magnetic flux penetrating a type-II superconductor will be quantized in single flux lines carrying one flux quantum. Abrikosov \[1\] studied theoretically the properties of a flux line lattice and he found that the most favourable arrangement is a triangular 2D lattice, with lattice parameter equal to:

\[ d = \sqrt{\frac{2\Phi_0}{\sqrt{3}B}} \]  

(1.34)
Moreover, the area of the tubes will be, approximately:

\[ A_{\text{flux line}} = \pi \xi^2 \]  

(1.35)

With these quantities, one can estimate also the upper critical field, \( B_{c2} \): at this value of the field the superconductor is fully penetrated, meaning that the lattice spacing is almost equal to the tube diameter, \( d \simeq 2\xi \). This yields:

\[ B_{c2} = \frac{\Phi_0}{2\pi \xi^2} \]  

(1.36)

Finally, in a type-II superconductor the critical field used at the beginning of this paragraph (called the thermodynamical critical field) is defined as a geometric mean of the lower and upper critical field

\[ B_{c}^{th} = \sqrt{B_{c1}B_{c2}} \]  

(1.37)

while for type-I superconductor the thermodynamical critical field coincides with the critical field.

1.6 Microscopic Theory: Introduction

In 1957, John Bardeen, Leon Neil Cooper, and John Robert Schrieffer, presented a microscopic theory of superconductivity [3, 5], named BCS in their honour, and nowadays one of the greatest achievements of condensed matter physics that granted them the Nobel Prize in Physics in 1972. The main clue to their theory was given by the isotope effect, discovered in 1950 by two independent groups, one led by Emanuel Maxwell and the other formed by C. A. Reynolds, B. Serin, W. H. Wright, and L. B. Nesbitt. They had found that the critical temperature of samples made with the same material but with different isotopes was inversely proportional to the mass of the isotope used: usually the electronic properties are not strongly affected by the lattice ions’ mass, as in a mean field theory the ions contribute only in creating the crystal field, then screened by the mean charge density of the other electrons. This brought to think that there could be a contribution due to lattice vibrations that could actually change the way how the electrons mutually interact. Moreover, one year before the theory was presented, Cooper had demonstrated that, if there is a net attractive interaction between two electrons, they will always form a bound state, no matter how weak the interaction is. The main concept of the BCS theory is to think that the electrons interact attractively via interactions with the crystal lattice, forming quasi particles called Cooper pairs, that have integer spin and then almost behave as bosons (they are not really bosons, as the total wavefunction is symmetric with exchange of Cooper pairs, but anti-symmetric for exchange of single electrons in the same pair). Then, it seems
that in order to understand the interaction that couples electrons, one must take into consideration lattice excitations in the electronic problem: in this way, one is not making use of the Born–Oppenheimer approximation, that is the main assumption of all the molecular and Solid State Physics. The lattice contribution is explained as follows: when an electron travels through the lattice, the ions will feel its negative charge and be attracted via the Coulomb interaction. This will produce a local lattice distortion, and then a local accumulation of positive charge. This local charge can be felt by another electron, that will then be coupled to the initial one. In other words, the interaction between electrons is mediated by phonons. But these two electrons will also interact via Coulomb repulsion: in order to minimize this interaction the best configuration is the one in which they are Bloch States with momenta pointing in opposite directions, so that they go far away from each other in a short time. In this picture, the supercurrent is then carried by Cooper pairs that have double the mass of the electron and double its negative charge. The Cooper pairs behave like a Bose liquid, all occupying the same ground state: in this way, one can identify the Ginzburg–Landau order parameter as the collective wavefunction of the pairs condensate. In this picture the flux quantum derived in the previous section takes the value:

$$\Phi_0 = \frac{\hbar}{2e} \simeq 2 \cdot 10^{-15} \text{ Wb} \quad (1.38)$$

In order to understand first the Cooper problem and then the basics of BCS theory, it is better to introduce the second quantization formalism for particles in a solid.

### 1.7 Second Quantization

In a many electron problem, it is well known that, if the total Hamiltonian is the sum of single particle Hamiltonians, the exact solution is written in the form of a Slater determinant. However, if one has 2 particles operators inside the Hamiltonian, the slater determinant is just an approximation and the real wavefunction is a weighted sum of Slater determinants arising from all the possible excitations. In a crystal, the electronic Hamiltonian contains various contributions:

$$H = T_e + V_{e-n} + V_{e-e} \quad (1.39)$$

In this expression, the first two operators are single particle operators, while the third one is a two-particle operator. Calling $h_1$ the single particle operator and $h_2$ the two particle one, $\psi_0$ the slater determinant wavefunction and $\psi_i$ the single particles wavefunctions (including the spin part, also called spin-orbitals) used to build the determinant, their matrix elements are:

$$\langle \psi_0 | h_1 | \psi_0 \rangle = \sum_i \langle \psi_i | h_1 | \psi_i \rangle \quad (1.40)$$
\[
\langle \psi_0 | h_2 | \psi_0 \rangle = \frac{1}{2} \sum_{ij} \langle \psi_i \psi_j | h_2 | \psi_i \psi_j \rangle - \frac{1}{2} \sum_{ij} \langle \psi_i \psi_j | h_2 | \psi_j \psi_i \rangle \tag{1.41}
\]

Now, let me define the Slater operator \( A \) as follows:

\[
| \psi_0 (N) \rangle = A \{ \psi_1, \psi_2, \ldots, \psi_{1N} \} \tag{1.42}
\]

To build a state with one more electron, I introduce the creation operator \( c^\dagger_m \), that creates an electron in the spin orbital \( \psi_m \). Then I get:

\[
| \psi_0 (N+1) \rangle = c^\dagger_m | \psi_0 (N) \rangle = A \{ \psi_m, \psi_{11}, \psi_{12}, \ldots, \psi_{1N} \} \tag{1.43}
\]

Note that if \( \psi_m \) is already an occupied spin-orbital, the determinant vanishes, as there are two equal columns. If I want to add two electrons, I will use the creation operator twice, yielding:

\[
| \psi_0 (N+2) \rangle = c^\dagger_n c^\dagger_m | \psi_0 (N) \rangle = A \{ \psi_n, \psi_m, \psi_{11}, \psi_{12}, \ldots, \psi_{1N} \} \tag{1.44}
\]

Note that if \( m = n \), the determinant vanishes, giving a first relation for the creation operators:

\[
c^\dagger_n c^\dagger_m = c^\dagger_m c^\dagger_n = 0 \tag{1.45}
\]

Moreover, the well-known properties of determinants also yield:

\[
c^\dagger_n c^\dagger_m | \psi_0 (N) \rangle = -c^\dagger_m c^\dagger_n | \psi_0 (N) \rangle \tag{1.46}
\]

Which leads to the fermion anticommutation relation:

\[
\{ c^\dagger_n, c^\dagger_m \} = 0 \tag{1.47}
\]

With this operator, any known state can be built starting from the ground state an applying the creation operators for each occupied state, namely:

\[
| \psi_0 (N+2) \rangle = c^\dagger_{i1} c^\dagger_{i2} \ldots c^\dagger_{in} | 0 \rangle \tag{1.48}
\]

In the same way, one can define an annihilation operator, that removes an electron from a given state:

\[
c_m A \{ \psi_m, \psi_{11}, \psi_{12}, \ldots, \psi_{1N} \} = A \{ \psi_{11}, \psi_{12}, \ldots, \psi_{1N} \} \tag{1.49}
\]

If the state to be annihilated is not in the front row, permutations with their corresponding sign change must be performed before annihilating; this reads:

\[
c_m A \{ \psi_1, \psi_m, \psi_{12}, \ldots, \psi_{1N} \} =
- c_m A \{ \psi_m, \psi_{11}, \psi_{12}, \ldots, \psi_{1N} \} = -A \{ \psi_{11}, \psi_{12}, \ldots, \psi_{1N} \} \tag{1.50}
\]
Using these properties one can find the remaining anticommutation relations for the creation and annihilation operators, giving the known fermion relations:

\[ \{c_n, c_m\} = 0 \quad \text{and} \quad \{c_n, c_m^\dagger\} = \delta_{nm} \quad (1.51) \]

With these new tools one can derive the matrix elements introduced before, finding:

\[ \langle \psi_0 | h_1 | \psi_0 \rangle = \sum_{m,n} \langle \psi_m | h_1 | \psi_n \rangle c_m^\dagger c_n \quad (1.52) \]

\[ \langle \psi_0 | h_2 | \psi_0 \rangle = \frac{1}{2} \sum_{k,l,n,m} \langle \psi_k \psi_l | h_2 | \psi_m \psi_n \rangle c_k^\dagger c_l^\dagger c_n c_m \quad (1.53) \]

Finally, the number operator for a chosen state is defined as:

\[ n_i = c_i^\dagger c_i \quad (1.54) \]

This operator has the property of counting the number of particles present in the selected state.

### 1.8 Non interacting electron gas

In order to establish a basis for understanding of the BCS mechanism, I will discuss the simple case of a free electron gas. The allowed energy values are given by the dispersion relation, and depend on the wavevector \( \mathbf{k} \). Moreover, I denote the spins with the letter \( \sigma \). The free electron Hamiltonian in the second quantization formalism becomes:

\[ H = \sum_{\mathbf{k},\sigma} \epsilon_\mathbf{k} c_{\mathbf{k},\sigma}^\dagger c_{\mathbf{k},\sigma} \quad (1.55) \]

Only the kinetic energy term is present, as I am considering a non-interacting picture. In a crystal, the eigenvalues of the kinetic energy term \( \epsilon_\mathbf{k} \) will be renormalized with the effective mass to take into account the crystal field, and the creation and annihilation operators will act on Bloch States rather than free particle states, but the overall formalism does not change. The Fermi sphere is filled up with electrons with opposite spins up to the Fermi level, so that the number (operator) of electrons in the ground state satisfies:

\[ n_{\mathbf{k},\sigma} |\psi_0\rangle = \Theta(\epsilon_F - \epsilon_\mathbf{k}) |\psi_0\rangle \quad (1.56) \]

\( \Theta(x) \) is the Heaviside step function, with the clear meaning that every energy state below the Fermi level is filled, while each state above is empty. The density of states can be rigorously calculated with the aid of the single particle Green’s function of the system, but can also be inferred just counting
the number of states at a given energy $\epsilon_k$ for every possible value of $k$. The result is:

$$D(E) = \sum_k \delta(E - \epsilon_k)$$ (1.57)

To take into account the Coulomb interactions, one should add the repulsive term to the Hamiltonian, which definitely complicates the picture. To make use of the formalism introduced previously, it is useful to write down the Fourier transform of the Coulomb potential, the integral can be solved easily and gives:

$$V_{k,k',q} = \frac{1}{4\pi\epsilon_0^2} \frac{2\pi e^2}{q^2}$$ (1.58)

This will be useful in the forthcoming paragraphs. Note that this potential only depends on the transferred momentum, and not on the initial and final scattering states. Instead of going deep inside the theory of Fermi liquids, let me now talk about what happens if an attractive overall interaction between electrons is present, in addressing the so called Cooper problem [4].

### 1.9 The Cooper Problem

The situation is the following: two electrons above the Fermi energy interact attractively only if their momenta point in opposite directions and if their energy lies within a small interval between $\epsilon_F$ and $\epsilon_F + E_0$, otherwise they do not interact at all; all the other electrons are inert in the ground state below the Fermi energy. I denote the plane wave state of the two electrons without any interaction with $|k, -k\rangle$ and the state with interaction by $|1, 2\rangle$, the Hamiltonian of the system is given by the sum of the kinetic energy term and the effective potential. Therefore:

$$H_0 |k, -k\rangle = 2\epsilon_k |k, -k\rangle$$ (1.59)

$$(H_0 + V_{\text{eff}}) |1, 2\rangle = E |1, 2\rangle$$ (1.60)

With the restriction that $\epsilon_k > \epsilon_F$. I then write the interacting state as a weighted sum of the non-interacting basis functions, assuming that they form a complete set.

$$|1, 2\rangle = \sum_k a_k |k, -k\rangle$$ (1.61)

I put this in equation (1.60), multiply by the bra $\langle k', -k'|$, and exploit the orthogonality relations between basis functions and equation (1.59) to get:

$$a_k [2\epsilon_k - E] = -\sum_{k'} a_{k'} \langle k', -k'| V_{\text{eff}} |k, -k\rangle$$ (1.62)

Now I take into account the assumption of the problem to have a suitable expression for the scattering matrix element on the right side of equation...
(1.62): this quantity will be zero if the energy of the particles is above \( \epsilon_F + E_0 \) and will take a constant negative value in the region \( \epsilon_F < \epsilon_k < \epsilon_F + E_0 \), namely:

\[
\langle k', -k' | V_{\text{eff}} | k, -k \rangle = \begin{cases} 
0 & \text{if } \epsilon_k > \epsilon_F + E_0 \\
-V & \text{if } \epsilon_F < \epsilon_k < \epsilon_F + E_0 
\end{cases}
\]  

(1.63)

With this assumption, equation (1.62) simplifies, yielding:

\[
a_k[2\epsilon_k - E] = V \sum_{k'} a_{k'} \Theta(\epsilon_k - \epsilon_F) \Theta(E_0 - |\epsilon_k - \epsilon_F|)
\]  

(1.64)

Now I transform this sum on the \( k \) space in an integral over the first Brillouin zone, and then I make use of the density of states defined previously to pass to an integral in energy.

\[
a_k[2\epsilon_k - E] = V \int_{-\infty}^{+\infty} \sum_{k'} a(\epsilon) \delta(\epsilon - \epsilon_k \Theta(\epsilon - \epsilon_F) \Theta(E_0 - |\epsilon - \epsilon_F|) d\epsilon =
\]

\[
V \int_{-\infty}^{+\infty} a(\epsilon) D(\epsilon) \Theta(\epsilon - \epsilon_F) \Theta(E_0 - |\epsilon - \epsilon_F|) d\epsilon
\]  

(1.65)

Renaming the variables I get:

\[
a(\epsilon)[2\epsilon - E] = V \int_{\epsilon_F}^{\epsilon_F + E_0} a(\epsilon') D(\epsilon') d\epsilon'
\]  

(1.66)

The right side of this equation does not depend on \( \epsilon \), this means that also the left side should not depend on \( \epsilon \), so the general form of \( a(\epsilon) \) will be:

\[
a(\epsilon) = \frac{C}{2\epsilon - E}
\]  

(1.67)

\( C \) is a renormalization constant that is not important and can be taken to be equal to 1. This transforms equation (1.66) in its new self-consistent form:

\[
1 = V \int_{\epsilon_F}^{\epsilon_F + E_0} D(\epsilon') d\epsilon'
\]  

(1.68)

Moreover, I assume that the density of states slowly varies with energy, taking its value at the Fermi energy, cast the coupling constant \( \lambda \) defined as \( \lambda = V D(\epsilon_F)/2 \), perform the simple integration, and define \( \Delta = 2\epsilon_F - E \) obtaining:

\[
\frac{1}{\lambda} = \ln \left[ 1 + \frac{2E_0}{\Delta} \right]
\]  

(1.69)

The sign convention on \( V \) was taken so that \( V \) must be a positive value in order to have an attractive interaction, so also \( \lambda \) is a positive number. So
also the left side of equation (1.69) must be positive, which means $\Delta > 0$ that yields $E < 2\epsilon_F$. This is a stunning result, as the initial assumption was that the added electrons were above the Fermi energy. This means that with an attractive interaction, something happens to the Fermi sphere and the occupied states below the Fermi energy become accessible, making us think that this new pair does not obey to the Pauli Exclusion Principle. Solving equation (1.69) for $\Delta$ yields:

$$\Delta \simeq 2E_0 e^{-\frac{1}{\lambda}}$$

(1.70)

So one can see that $\Delta$ has an essential singularity for $\lambda = 0$, meaning that this result cannot be obtained at any order of perturbation theory in $\lambda$. Moreover, $\lambda$ also depends on the density of states, that in a free electron gas goes as $\sqrt{\epsilon}$. So if the density of states is small, i.e. the Fermi sea disappears, also $\Delta$ goes to zero, meaning that in order to have the bound state, one must also have the other electrons present. In vacuum one would not obtain this bound state from the same attractive interaction.

### 1.10 Phonon mediated interactions

I will discuss now the interacting electron gas, considering phonon mediated attractive interactions, trying to find if there is a way in which the weak attractive interaction can become dominant over the repulsion. The Hamiltonian is:

$$H = \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_{k,k',q,\sigma,\sigma'} \frac{1}{4\pi\epsilon_0} \frac{2\pi\epsilon^2}{q^2} c_{k+q,\sigma}^\dagger c_{k',q,\sigma'}^\dagger c_{k,\sigma} c_{k',\sigma'} + V_{e\text{-phonon}}$$

(1.71)

With:

$$V_{e\text{-phonon}} = \sum_{k,q,\sigma} M_q (a_{-q}^\dagger + a_q) c_{k+q,\sigma}^\dagger c_{k,\sigma}$$

(1.72)

$M_q$ is the matrix element for electron phonon coupling, the $a$ operators are the creation and annihilation operators for phonons, that obey to the Bose commutation relations:

$$[a_{q'}^\dagger, a_q] = \delta_{q,q'}$$

(1.73)

Now one can turn this electron-phonon term into an electron-electron effective coupling term by considering the phonon as a interaction mediator and calculating its propagator. Moreover, the matrix element for the electron phonon coupling is difficult to calculate, and I can take it as a free parameter, that I assume to be small. A study on the Green’s function for the Hamiltonian of free phonons allows finding the free phonon propagator:

$$D_0(q,\hbar\omega) = \frac{2\hbar\omega_q}{(\hbar\omega)^2 - (\hbar\omega_q)^2 + \eta}$$

(1.74)
From which one can obtain the phonon mediated part of the Hamiltonian:

\[ V_{\text{eff}}(\mathbf{q}, \hbar \omega) = \frac{2|M_q|^2}{(\hbar \omega)^2 - (\hbar \omega_q)^2} \]  

(1.75)

In this way, the whole Hamiltonian can be written by means of an effective potential, taking into account both attraction and repulsion:

\[ H = \sum \epsilon_{k,\sigma} c_{k,\sigma}^\dagger c_{k,\sigma} + \sum \tilde{V}_{\text{eff}}(\mathbf{q}, \hbar \omega) c_{k+\mathbf{q},\sigma}^\dagger c_{-\mathbf{q},\sigma'}^\dagger c_{k,\sigma} c_{k',\sigma'} \]  

(1.76)

With:

\[ \tilde{V}_{\text{eff}}(\mathbf{q}, \hbar \omega) = \frac{1}{4\pi\epsilon_0} \frac{2\pi e^2}{q^2} + \frac{2|M_q|^2}{(\hbar \omega)^2 - (\hbar \omega_q)^2} \]  

(1.77)

Here one can see that approaching \( \hbar \omega_q \) from below, the phonon part is negative, i.e. attractive, and goes to infinite, overcoming the Coulomb repulsion, no matter how small the phonon matrix element is. This happens when the frequency is of the order of the lattice vibration characteristic frequencies, namely when the electron has waited a minimum time for the lattice vibration to propagate and interact. This attractive energy range is very small compared to the Fermi energy, and this can lead to further conclusions. Indeed, if one wants to remain within the same small energy range after an interaction, the situation that maximizes the phase space is the one in which the interacting electrons have opposite momenta. Moreover, the attractively interacting electrons must have the possibility to be close in space (but at a different time), so for the Pauli Exclusion Principle the most favourable configuration is the singlet one. This brings me to further simplifications of the Hamiltonian, leading to:

\[ H = \sum \epsilon_{k,\sigma} c_{k,\sigma}^\dagger c_{k,\sigma} + \sum \tilde{V}_{k,k'} c_{k,\uparrow}^\dagger c_{-k,\downarrow}^\dagger c_{k',\downarrow} c_{k',\uparrow} \]  

(1.78)

A remarkable fact to be pointed out is that superconductivity within this interpretation requires a good electron-phonon coupling. This means that above the transition temperature, a material that is more likely to be a good superconductor will be a poor normal conductor, as the interaction with phonons will increase the resistivity. This is the reason why the best normal conductor known nowadays, Copper, cannot reach the superconducting phase.

### 1.11 The Gap equation

Starting from the previous Hamiltonian one can find an explanation to every experimental fact observed in superconductors; I will just focus, for completeness, on the derivation of the Gap equation and the critical temperature equation.
The gap equation is found by writing down the free energy of the superconductor and minimizing it with respect to the gap parameter. A mean field picture is considered, and I define the expectation values of the creation and annihilation operators as:

\[ b_k = \langle c_{-k,\downarrow} c_{k,\uparrow} \rangle \]
\[ b_k^\dagger = \langle c_{k,\uparrow}^\dagger c_{-k,\downarrow} \rangle \]

Then I have:

\[ c_{-k,\downarrow} c_{k,\uparrow} = b_k + c_{-k,\downarrow} c_{k,\uparrow} - b_k = b_k + \delta b_k \]

Putting this in equation (1.78) and neglecting terms in \( \delta b_k^2 \), one finds:

\[ H = \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^\dagger c_{k,\sigma} - \sum_{k,k'} V_{k,k'} \left[ b_k^\dagger c_{-k',\downarrow} c_{k',\uparrow} + b_k c_{k,\uparrow}^\dagger c_{-k',\downarrow} - b_k^\dagger b_k \right] \]

(1.79)

And then, defining the gap parameter as:

\[ \Delta_{k'}^\dagger \equiv -\sum_k V_{k,k'} b_k^\dagger \]
\[ \Delta_k \equiv -\sum_{k'} V_{k,k'} b_{k'} \]

(1.80)

The Hamiltonian takes the form:

\[ H = \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^\dagger c_{k,\sigma} - \sum_k \left[ \Delta_{k'}^\dagger c_{-k,\downarrow} c_{k,\uparrow} + \Delta_k c_{k,\uparrow}^\dagger c_{-k,\downarrow} - b_k^\dagger \Delta_k \right] \]

(1.81)

Note how the terms \( c_{k,\uparrow}^\dagger c_{-k,\downarrow}^\dagger \) and \( c_{-k,\downarrow} c_{k,\uparrow} \) represent pair creation and annihilation, respectively. Because of these terms, the Hamiltonian is no longer diagonalized, and one must perform a base change under a unitary rotation to re-diagonalize it. Two new fermion operators are introduced, with the relations:

\[ c_{k,\uparrow} = \cos \theta \eta_k - \sin \theta \gamma_k \]
\[ c_{-k,\downarrow}^\dagger = \sin \theta \eta_k + \cos \theta \gamma_k \]

Now the procedure is to insert these new operators in the Hamiltonian and choose the angle \( \theta \) that diagonalizes it, i.e. that leaves only terms in \( \eta_k^\dagger \eta_k \) and \( \gamma_k^\dagger \gamma_k \). The calculation is rather simple and, considering \( \Delta_{k'}^\dagger = \Delta_k \), gives:

\[ \tan 2\theta = -\frac{\Delta_k}{\epsilon_k} \]

(1.82)
And finally the Hamiltonian can be written as:

$$H = \sum_{k,\sigma} \left( \epsilon_k \Delta_k b_k^\dagger \right) + \sum_k E_k \left[ \eta_k^\dagger \eta_k - \gamma_k^\dagger \gamma_k \right] \quad (1.83)$$

Where:

$$E_k = \sqrt{\epsilon_k^2 + |\Delta_k|^2} \quad (1.84)$$

This Hamiltonian contains a constant term and a part describing a spinless fermion system with excitation energies equal to $E_k$ and $-E_k$. Before I had one kind of particles with two possible spin states, now I have two kinds of (quasi) particles with no spin: the number of degrees of freedom has not changed. Finally, $\Delta_k$ represents a gap for the particles. In this form, it is easy to write down the partition function of the system (with $\beta$ the Boltzmann factor), and hence the free energy:

$$Z = e^{-\beta H_0} \prod_k \left( 1 + e^{-\beta E_k} \right) \left( 1 + e^{\beta E_k} \right) \quad (1.85)$$

$$F = \frac{1}{\beta} \ln Z = H_0 - \frac{1}{\beta} \sum_k \left[ \ln \left( 1 + e^{-\beta E_k} \right) + \ln \left( 1 + e^{\beta E_k} \right) \right] \quad (1.86)$$

Minimizing this quantity with respect to the gap parameter gives an expression for $b_k^\dagger$, which can be put inside equation (1.80) to find a self-consistent equation for the gap, the BCS gap equation:

$$\Delta_k = -\sum_{k'} V_{k,k'} \Delta_{k'} \frac{\tanh \left( \frac{\beta E_{k'}}{2} \right)}{2E_{k'}} \quad (1.87)$$

From this general mean field equation, in which there is no specification about the exact pairing mechanism, it is possible to derive a general expression for the transition temperature. One could actually expand the gap parameter and the pairing potential with known basis functions reflecting the symmetry of the particular crystal considered, but let me just assume that the coupling matrix element takes a constant value $-V$ for electrons lying in a small region around the Fermi energy and zero elsewhere. For phonon assisted superconductivity, the upper limit will correspond to the energy cut-off of the mediators, which corresponds to the Debye frequency $\omega_D$. With this approximation, the matrix element can be taken outside the sum in equation (1.87), and inspecting its two sides one deduces that the gap parameter is $k$-independent (the right side does not contain any term depending on $k$, so also the left one must not depend on $k$). This leads to:

$$1 = V \sum_{k'} \frac{\tanh \left( \frac{\beta E_{k'}}{2} \right)}{2E_{k'}} \quad (1.88)$$

This can be easily solved in two special cases: $T = 0$ and $T = T_c$. 40
1.11.1 Gap equation for $T \approx T_c$

In this case, the gap vanishes so $E = \epsilon$, one can replace the sum with an integral, limited up to the Debye energy, using the density of states. Moreover, assuming that the Fermi level is far away from Van Hove singularities, one can assume a constant density of states around the small energy interval $\omega_D$. Finally, I introduce the dimensionless coupling constant as done in the Cooper problem, $\lambda = V D(\epsilon_F)/2$, finding:

$$1 = \lambda \int_0^{\hbar \omega_D} \frac{\tanh \left( \frac{\beta \epsilon}{2} \right)}{\epsilon} d\epsilon = \lambda \left[ \tanh \left( \frac{\beta \hbar \omega_D}{2} \right) \ln \left( \frac{\beta \hbar \omega_D}{2} \right) - \int_0^{\beta \hbar \omega_D} \frac{\ln x}{\cosh^2 x} dx \right] (1.89)$$

In the weak coupling limit, $\lambda \ll 1$, meaning that $\beta \hbar \omega_D/2$ must be much bigger than one. This simplifies the second integral, as the upper limit can be taken as infinite. Making use of the Euler-Mascheroni constant, defined as:

$$\gamma = \lim_{m \to \infty} \left( \sum_{l=1}^{m} \frac{1}{l} - \ln m \right) \simeq 0.5772156649 \quad (1.90)$$

The solution of the above equation for the critical temperature is:

$$k_B T_c = \frac{2 e^\gamma}{\pi} \hbar \omega_D e^{-\frac{1}{\lambda}} \simeq 1.13 \hbar \omega_D e^{-\frac{1}{\lambda}} \quad (1.91)$$

1.11.2 Gap equation for $T=0$

In the case of $T=0$, the argument of the hyperbolic tangent goes to infinite, and the function tends to 1, giving:

$$1 = V \sum_{k'} \frac{1}{2 E_{k'}} = \lambda \int_0^{\hbar \omega_D} \frac{d\epsilon}{\sqrt{\epsilon^2 + \Delta^2}} = \lambda \sinh^{-1} \left( \frac{\hbar \omega_D}{\Delta} \right) \quad (1.92)$$

In the weak coupling limit, the argument of the hyperbolic sinus must be big, so that I can write $\sinh^{-1} (h \omega_D/\Delta) \simeq \ln (2 h \omega_D/\Delta)$, finally obtaining:

$$\Delta(T=0) = 2 \hbar \omega_D e^{-\frac{1}{\lambda}} \quad (1.93)$$

Again, as in the Cooper problem, both results have essential singularities for $\lambda = 0$. The values of the transition temperature and the energy gap depend on the coupling constant, which is in general difficult to get. But if one takes the ratio of these two quantities, all the material-dependent parameters cancel out, giving:

$$\frac{2 \Delta(T=0)}{k_B T_c} = \frac{2 \pi}{e^\gamma} \simeq 3.52 \quad (1.94)$$
This universal ratio gives a direct correspondence between the energy gap and the transition temperature. Many other quantities can be calculated with the aid of BCS theory, and one important feature is the upper limit for the critical temperature, given by the cutoff of phonon energies and presented in equation 1.91. According to BCS theory, indeed, there cannot be a superconductor with critical temperature above 40 K, approximately: this calculation can be refined by taking into account the possible phonon spectra and using the equations derived previously. The presence of high temperature superconductors in nature cannot then be explained with this theory, meaning that the coupling mechanism in these new materials must be different. Nevertheless, BCS theory works extremely well with low temperature superconductors.

1.12 References

Chapter 2

YBCO and pinning

In this chapter, I will discuss about the superconductor that I have been using during my experiments, focusing the attention on its characteristics and other important concepts concerning pinning, which is the main topic of my thesis.

2.1 Overview of high-temperature superconductors

The first high-temperature superconductor was discovered in 1986 by Bednorz and Muller, two IBM scientists who received the Nobel Prize in Physics is 1987. This material, which belongs to the Ba-La-Cu-O systems, was never developed for applications. The first high-temperature superconductor material with $T_c$ above 77 K was the Yttrium-Barium-Copper Oxide (YBCO), which was discovered in 1987 by Paul Chu and his colleagues from the University of Alabama [1]. It featured a critical temperature of 92 K, and opened whole new set of possibilities for superconductors applications and mysteries about the theory, as its critical temperature is far above the limit imposed by the BCS theory. In the following years, many other high-temperature superconductors have been discovered, but the only having a quite important role in applications is the BSCCO, Bismuth Strontium Calcium Copper Oxide, discovered in 1988 by H. Maeda, Y. Tanaka, M. Fukutumi, and T. Asano [2]. There are the BSCCO 2212 and the BSCCO 223 and they feature critical temperatures of 95 K and 110 K, respectively. These high-temperature superconductors belong to the family of the cuprates. Cuprate is a general term to refer to compounds containing copper anions, in different possible ways. In YBCO, the structure is organized in stacked copper planes and columnar copper ribbons, as it can be seen from figure 2.1. BSCCO is the only high-$T_c$ cuprate not containing Rare Earths (Strontium is an alkaline metal), and it is also characterized by a planar structure made by stacked copper planes.
Another cuprate worth to be named is HBCCO (Mercury Barium Calcium Copper Oxide), the superconductor with the highest critical temperature so far, 135 K at atmospheric pressure, with peaks of critical temperature higher than 150 K under extremely high pressure [4] [5].

2.2 YBCO structure and characteristics

There can be different kinds of YBCO in dependence of the oxygen content: the exact chemical formula for YBCO is actually YBa$_2$Cu$_3$O$_{7-\delta}$, where $\delta$ indicates the oxygen vacancy. Its properties change as follows with respect to this parameter:

- for $0.65 < \delta \leq 1$ the material is not superconducting
- for $0 \leq \delta \leq 0.65$ the material is superconducting below $T_c$
- at $\delta \simeq 0.07$ the material features the highest $T_c$ (95 K) and the highest $B_{c2}$ (120 T for B perpendicular to the CuO$_2$ planes and 250 T for B parallel to them, at 0 K) [6].

This oxygen vacancy provides hole doping for YBCO, in which the charge carriers (coupled somehow) are holes. In figure 2.2 one can see the phase diagram of YBCO with respect to the hole doping and temperature, and it can be seen that it features many different strange behaviours.

The YBCO (and some other cuprates as well) unit cell has a perovskite structure, essentially made by layers. As it can be seen from figure 2.1, the unit cell features planar CuO$_2$ units sharing 4 vertices, that seen from the top form a checkboard structure. Perpendicular to these planes there are CuO$_4$ ribbons: the Yttrium is placed between the CuO$_2$ planes, while the Barium is between the CuO$_2$ planes and the CuO$_4$ ribbons. The role of Barium is to stabilize the structure and eventually dope it. The conduction occurs preferentially in the CuO$_2$ planes, while the CuO$_4$ ribbons act as a charge reservoir: for these reasons this systems actually behave as 2-dimensional, and this is believed to be one of the characteristics that enables superconductivity at high temperature. The direction perpendicular to the CuO$_2$ planes is named the $c$ axis, whereas the other two directions (actually not exactly equivalent) are named $ab$ and define the $ab$ plane.

This layered and almost 2-D structure implies a large anisotropy in the characteristic properties of YBCO. For example, the normal state conductivity for transport along the $c$ direction is about 0.1 times the normal state conductivity along $ab$, namely $\sigma_c \sim 0.1 \sigma_{ab}$. Also the superconducting properties feature high anisotropy: as an example the characteristic lengths introduced in the previous chapter, at 86.5 K are [7]:

$$\lambda_{||}(86.5\text{K}) = 800 \text{nm}$$
Figure 2.1: Atomic structure of YBCO. Source [3]
This anisotropy in the crystal structure has consequences also on the macroscopic structure: indeed, it is very unlikely to produce wires of high-$T_c$ superconductors, while it is much easier to produce tapes. The growth on tapes obviously has a preferential direction, that coincides with the $c$ axis, and nevertheless presents some particular issues. First, a perfectly textured growth of a superconducting layer is very difficult to achieve, so a discrete number of grain boundaries are present in YBCO films; the best properties are achieved when the grain boundaries are aligned. Another source of disorder derives from the fact, anticipated in the previous paragraph, that the $a$ and $b$ directions are not completely equivalent, but very similar. This implies that during crystal growth it is very easy energetically to switch from the propagation in the $a$ direction to the propagation in the $b$ direction. This phenomenon is called twinning, and creates some irregularities in the crystal structure of the superconductor, which are called twin boundaries. Grain boundaries and twin boundaries have a very peculiar role for the next concept that will be introduced, flux pinning.
2.3 Flux Pinning: introductory concepts

I have shown in the introduction the difference between Type-I and Type-II superconductors: the first ones completely expel any applied magnetic flux, and carry currents only on their surface, while the latter ones are penetrated by quantized flux lines and have currents flowing in their entire volume. For these reasons, Type-II superconductors are the ones used for applications. But now let us imagine a Type-II superconductor (from now on I will generally refer to it with the word ‘superconductor’) immersed in a magnetic induction field \(B\), in which I want to inject a current. The magnetic field will penetrate the superconductor in form of quantized fluxons, arranged in the already discussed Abrikosov lattice. If a current is injected, on the single flux line will act a Lorentz force \(F_{\text{Lorentz}}\) due to the motion of the charges in presence of a magnetic flux density (a more detailed derivation, leading to the same results, can be done calculating the energy of 2 interacting flux lines in the framework of Ginzburg-Landau theory), that can be expressed as:

\[
F_{\text{Lorentz}} = J_{\text{ext}} \times B \tag{2.1}
\]

This force will move the flux line, creating a source of dissipation for the superconductor, which is actually no longer superconducting, as there is some kind of resistance opposing to the current flow. What is necessary, then, is to pin the flux lines somehow, so that there is a force opposing to the Lorentz one preventing the movement of the flux lines. The name for this force is *pinning force* \(F_{\text{pin}}\).

In order to better understand this concept, let us consider a picture like the following: keeping the applied field fixed, we ramp up the current and see what happens to the flux line system. At the beginning, the current is low and so is the Lorentz force, the pinning force is much greater and the fluxons remain pinned. By increasing the current, we will eventually reach the critical one: at the critical current the superconducting properties disappear and dissipation is detected. This means that the flux lines have started to move and dissipate power, indicating that the Lorentz force has overcome the pinning one. According to this picture, a practical definition of the pinning force is introduced, which is the Lorentz force for an injected current equaling the critical one:

\[
F_{\text{pin}} = J_c \times B \tag{2.2}
\]

In most of the cases, the current flows perpendicular to the direction of the applied field, so one can take the standard product of the magnitudes of these quantities.

How can one pin a flux line? Basically, one needs to introduce a potential well for the flux lines, with a local energy minimum. In order to do so, let us recall from the introduction what happens in a Type-II superconductor: the field penetrates in a region, and there the order parameter is
as low as possible, and then it increases in the not penetrated region up to its saturation value over a length $\xi$. This means that the field is more energetically favoured to penetrate in regions which are not superconducting. So all the possible defects, such as precipitates, oxygen vacancies, grain boundaries and twin boundaries, where the superconducting properties are poor, are effective pinning sites. The flux line lattice actually distorts itself to accomodate as many lines as possible in the pinning sites, balancing the energy gained by pinning the flux lines with the strain energy needed for the distortion (an example of theory of pinning on low angle boundaries is given in appendix A). The characteristics of these potential wells will clearly depend upon the applied field and the temperature, and the pinning energy is closely related to the pinning force. So one can write, with the assumption of a current flowing perpendicular to the external field:

$$\begin{align*}
F_{\text{pin}}(B,T) &= J_c(B,T) \cdot B \\
J_c(B,T) &= \frac{F_{\text{pin}}(B,T)}{B}
\end{align*}$$

(2.3)

(2.4)

From this equation one can see that the easiest way to get information on the pinning force and mechanisms happening in a superconductor is the measurement of the critical current behaviour at different temperatures and fields. So this parameter is not only a standard way to test the performances of a superconductor, but also a useful tool to understand the physics of the pinning mechanisms from the experimental point of view.

The pinning of the flux lines also has many other implications, like the amazingly stable magnetic levitation for the Meissner effect found in YBCO bulk samples and the magnetization curve for pinned Type-II superconductors. Indeed, these materials feature hysteresis in the magnetization loop, mainly due to the fact that once a flux line is pinned, and the field is decreased to zero, it will eventually remain trapped inside the superconductor. The magnetization behaviour can be linked to the critical current, so a magnetization measurement can give information about the critical current. This will be discussed deeply in the next chapters.

When the vortices start to move, resistive behaviour is detected. There are at least three different possible and different resistive behaviours found in high-temperature superconductors: TAFF, Flux Creep and Flux Flow.

2.3.1 TAFF

TAFF is an acronym for Thermally Assisted Flux Flow, and actually refers to the depinning of flux lines due to temperature, for currents lower than the critical one. The probability for a flux line to go beyond the pinning barrier is governed by the Boltzmann’s factor (with $U(B,T)$ the height of
the pinning potential well):

\[ P_{\text{depinning}} \sim e^{-\frac{U(B,T)}{k_B T}} \]  

(2.5)

Even for small temperatures, there is some probability that a flux line could jump out of its well and wander freely, creating dissipation. The term TAFF has been introduced specifically for high temperature superconductors in order to explain the current-voltage characteristic in the low current region, which often exhibits some resistive behaviour due to thermally assisted flux flow, as it can be seen in figure 2.3.

### 2.3.2 Flux Creep

The flux creep refers to the beginning of vortex motion mainly caused by the current reaching the critical value, as it can be seen in figure 2.3. It can be modelled stating that as the current increases, the pinning energy \( U(J) \) decreases somehow. In order to actually take into account the experimental data, where a power law of the kind \( E \sim J^n \) is usually used in the flux creep regime, the suitable functional form for \( U(J) \) is:

\[ U(J) = U(0) \ln \left( \frac{J_c}{J} \right) \]  

(2.6)
The electric field caused by vortex motion is proportional to the total Lorentz force acting on the free fluxons, which is proportional to the number of unpinned vortex lines. This number is closely related to the Boltzmann factor introduced in the previous section, so one has

\[ E \sim e^{-\frac{U(J)}{K_bT}} = e^{-\frac{U(0)}{K_bT} \ln \left( \frac{J}{J_c} \right)} \]  

(2.7)

Using the well-known properties of logarithms and exponentials, one gets:

\[ E \sim \left( \frac{J}{J_c} \right)^\frac{U(0)}{K_bT} \]  

(2.8)

Assuming \( n = U(0)/K_bT \), equation 2.8 reproduces exactly the experimental behaviour.

The power law index \( n \) is called the \emph{n-value} of the superconductor. It is a very important parameter which indicates the performance of the superconductor. Indeed, the higher it is, the steeper is the transition and the higher is the pinning energy. Usually, low-temperature superconductors have very high values of \( n \), and their transition is very steep, while for high-temperature superconductors \( n \) is usually lower and the transition is wider.

\section*{2.3.3 Flux Flow}

This is the regime where all the vortices are no longer pinned, and they can move freely in the superconductor causing dissipation. The Lorentz force will be at a certain point balanced by some viscous force exerted by the material, in formulae (using force densities):

\[ \Phi_0 J = \eta v \]  

(2.9)

Where \( \Phi_0 \) is the flux quantum, \( \eta \) is the dynamic viscosity of the medium and \( v \) is the velocity. The electric field associated with the Lorentz force will then be:

\[ E = Bv = \frac{B\Phi_0 J}{\eta} \]  

(2.10)

From which one can identify a flux flow resistance \( \rho_{ff} \) defined as:

\[ \rho_{ff} = \frac{1}{\eta} B\Phi_0 \]  

(2.11)

Bardeen and Stephen \[10\] worked out a theory for the flux flow regime, and they found:

\[ \rho_{ff} = \rho_n \frac{B}{B_{c2}} \]  

(2.12)

Where \( \rho_n \) is the normal-state resistivity. It should be pointed out that \( B/B_{c2} \) is the fraction of the volume occupied by vortex cores: when all the volume is penetrated by the field, the flux flow resistance equals the normal-state one.
2.4 Definitions of critical current

If the I-V curve for a superconductor is complicated, how can one define the critical current in a practical way? There are actually three different standard criteria used to identify the critical current from a transport measurement. These three criteria are the electric field criterion, the resistivity criterion and the offset criterion, some visual examples are provided in figure 2.4. It should be noted that differently from other superconductors, the critical current of YBCO tapes is often reported as value per centimetre width of the supercondcutor. The critical current can also be inferred from magnetic measurements, and I will fully discuss about this in chapter 5.

2.4.1 The electric field criterion

This criterion is the simplest and most used standard: the critical current is defined as the current at which the superconductor voltage drop across a given length reaches a critical value. In order to make this definition independent from the length over which the voltage is measured, a critical electric field is defined. The value is usually 0.1 µV/cm for low-temperature superconductors and 1 µV/cm for high temperature superconductor, due to the less steep transition (i.e. lower $n$ – value) of the latter. This criterion features some problems if there is a resistive contribution due to current sharing in the I-V curve, or if the experimental noise is higher than the measured signal.

2.4.2 The resistivity criterion

In the resistivity criterion, a certain resistivity is chosen and the critical current is defined as the current at which the superconductor I-V characteristic intersects a line with the defined slope. In other words, once defined a certain resistivity limit, the critical current is the one at which the resistivity developed in the superconductor equals the resistivity set as a limit. Typical resistivity values of $10^{-14}$ Ω · m and $10^{-14}$ Ω · m are used for low-temperature superconductors and high-temperature superconductors, respectively. This criterion is not suitable for critical current measurements made on a wide range of currents, as for low critical currents the determination can be difficult due to noise, while for high values of critical currents the actual voltage that needs to be reached can be very high.

2.4.3 The offset criterion

The offset criterion tries to deal with the issues of both the previous criteria. A critical field is defined, as in the electric field one, but the critical current is defined taking a line from the intersection of the superconductor’s I-V curve with the critical field line, tangent to the I-V curve and taking its
Figure 2.4: Visual examples of the criteria described in section 2.4 for the definition of critical current. From top to bottom: electric field criterion, resistivity criterion, offset criterion. Source [11].
intersection with the I axis. For superconductors with high values of $n$, this criterion is essentially the same as the electric field one.
2.5 References


Chapter 3

State of the art superconducting REBCO tapes: critical current and pinning mechanisms

In the past years, most of the research has been focused on increasing the superconductors’ critical current density $J_c$, in order to achieve better performance of the material in applications, like superconducting magnets, rotating electric machines or cables. In order to do this, a strong vortex pinning is necessary. In this chapter, a general overview of the current state of the art high-temperature superconducting tapes is given, in order to have a more precise idea on what has been already done and what can still be improved in order to enhance the performance of superconducting REBCO tapes. First, the attention will be focused on the most common structures and tapes, the ones provided by the companies producing the material, like SuperPower Inc., Fujikura Ltd., American Superconductors, Furukawa electric co. Ltd. and Bruker Corporation; secondly some more exotic structures, growth methods and pinning mechanisms will be considered.

3.1 SuperPower

The tapes produced by SuperPower have the following structure (from bottom to top, in figure 3.1) [1]:

- Cu stabilizer
- Ag Overlayer
- Substrate (Hastelloy)
- Buffer stack, which provides a diffusion barrier to the metal substrate and improves the lattice matching with the upper layer
- ReBCO [RE=Rare Earth] layer
- Ag overlayer
- Cu stabilizer

Figure 3.1: Structure of SuperPower REBCO tape. Source [1]

The width of the tape is usually 4 or 12 mm. The growth of the buffers is achieved through Ion Beam Assisted Deposition (IBAD), while for the superconducting layer Metal-Organic Chemical Vapour Deposition (MOCVD) is used. The tape is then covered with a thin protective silver layer and electroplated on both surfaces with copper.

One of the key points in trying to raise the $J_c$ is to insert defects in the REBCO layer, in order to provide pinning centres for the fluxoids. In 2013, Yimin Chen and Changhui Lei from SuperPower studied [2] how the addition of Zr and Rare Earth composition affected the performance of the tape. It is to say that YBCO (and other compounds of the REBCO family as well) has a strongly anisotropic behaviour due to its complex structure made of CuO$_2$ planes in the $ab$ direction and copper ribbons in the $c$ direction. The aim of the study by Chen and Lei was to insert pinning centres in both directions and study the interaction between the two at a temperature around 30 - 40 K and in an external field of 3 T. They found out that the optimal content of Zr (which is added to REBCO by MOCVD) is 15 wt%. The Zr interacts with the other precursors creating BaZrO$_3$ nanorods, preferentially oriented along $c$. A high $c$ peak is indeed found in angle resolved critical current measurements at 77 K and 1 T. The $c$ peak raises and broadens lowering the temperature and increasing the field. Having fixed the Zr content to
15 wt %, the Rare Earth composition had been studied: the Re also reacts with the precursors forming Re$_2$O$_3$ nano-dots organized in layers. At least three interesting facts have shown up:

- $I_c$ is more sensitive to Gd and Y concentration at low temperature than at high temperature;
- At 40 K and 3 T, the optimal value for the Gd and Y concentration is 0.6 for both;
- The enhancement in critical current is in the ab plane.

Other compositions have been studied with TEM and X-ray diffraction. It has been seen that bigger quantities of RE inhibit the creation of long nanorods along the c axis, degrading the pinning along that direction. The optimal structure remains the one with an addition of Zr of the 15 wt% and a stoichiometry of the Rare Earths of 0.6 for both Gd and Y, which allows the growth of ordered nano-wires, and at the same time nano-dots planes. The Cu composition has been varied as well and it has been found that also in this case the critical current is more sensitive to composition at low temperatures rather than at high ones. With these optimal values, at 40 K and 3 T, the highest critical current values found were around:

$$I_c/\text{unit width} = 700 \text{A/cm} - \text{width} \quad B||c,$$

$$I_c/\text{unit width} = 1400 \text{A/cm} - \text{width} \quad B||ab.$$

More data for the behaviour at 4.2 K can be found in [11]. The critical current values per unit width are:

$$I_c/\text{unit width} \approx 500 \text{A/cm} \quad \text{at 4.2 K, 15 T, B } || c$$

$$I_c/\text{unit width} \approx 2400 \text{A/cm} \quad \text{at 4.2 K, 15 T, B } || ab$$

An interesting study has been carried out by Selvamanickam et al. [3] on the anisotropy of the critical current. They measured the $I_c$ at various temperatures as a function of the external magnetic field applied along the ab plane (the percentage of Zr was 7.5%). The critical current at 77 K and self-field was 475 A per 12 mm width; at 30 K and 2.5 T they found a value of 2700 A per 12 mm width and at 20 K and 9 T they found a $I_c$ of 2650 A per 12 mm width. Unfortunately, when performing measurements with $B||c$ the improvement was not as high. At 30 K and 2.5 T the current was 1250 A per 12 mm width, but anyway better than the values one would have had without Zr addition. Better angular resolved measurements have been carried out: at 77 K and 1 T, the critical current exhibits two minima at about 80° and −60° (angle between the applied field and the c axis), a high peak along c, and two relative maxima for $B||ab$. A reference is given in figure 3.2.
Figure 3.2: Angular resolved critical current plot for a YBCO SuperPower tape. Source [3]
temperature, it has been seen that the c peak substantially decreases (see figure 3.3), so “future efforts should be more directed towards improving the critical current value in the vicinity of $B \parallel c$ since that is the limiting factor at lower temperatures” [3]. Another interesting feature that was pointed out concerns the Full Width at Half Maximum of the ab peaks: researchers noticed that the FWHM increases when lowering the temperature (this is probably due to additional pinning mechanisms that come into play in that temperature range), which can be useful for applications where a substantial component of the magnetic field is around the ab direction. On the other hand, increasing the applied field makes the peaks sharper. In the latter part of the paper, measurements with a different percentage of Zr (15%, the optimal pointed out in [2]) are shown. The interesting feature is that at high temperature the performance of the 7.5% sample is better, but when reducing the temperature to 40 K the 15% sample takes the high-ground for every orientation. The 15% concentration is substantially better for every low temperature application. A different kind of structure was proposed by Khatri, Majikic, Shi, Chen and Selvamanickam [4]: they grew Platinum nanorods on LaMnO$_3$ in a triangular lattice, coated them with a layer of BaZrO$_3$, and deposited the REBCO by MOCVD later. After the BZO coating, SEM images showed that the rods remained in their previous place, but they acquired some curvature due to the sputtering process.

![Figure 3.3: Angular resolved critical current data from [3]; in the left curve the temperature is fixed at 30 K and the applied field is varied, while in the right curve the field is fixed at 3 T and the temperature is varied. The shrink of the c peak ($\theta = 0$) can be easily noticed.](image)

The critical current densities do not increase at 77 K and in self-field:

$$J_c = 4.93 \cdot 10^6 \text{A/cm}^2$$ for the sample without nanorods,

$$J_c = 1.63 \cdot 10^6 \text{A/cm}^2$$ for the sample with nanorods.
This tendency is also found for higher values of the applied magnetic field at 77 K. The interesting fact, however, is the $dJ_c/dB$ behaviour: it has been found that the critical current is less sensitive to the magnetic field, for the sample with nanorods than for the sample without them. This applies both in the c direction and in the ab direction. Fitting the experimental curves with a power law decay,

$$J = kB^{-\alpha}$$

one finds that for $B \parallel c$, $\alpha$ goes from 0.85 to 0.66 adding the Pt rods, and for $B \parallel ab$ the value drops from 0.62 to 0.36. The tendency is interesting but limited at a temperature of about 77 K, which is not of main interest for the most advanced applications.

3.2 Fujikura

Some interesting values have been given by S. Fujita et al. from Fujikura Ltd. [12]. To begin with, the structure of the tape is slightly different from the one from SuperPower, and presents (from bottom to top, see figure 3.4):

- Hastelloy Substrate
- Buffer layer: MgO + Al$_2$O$_3$ + Y$_2$O$_3$ + CeO$_2$
- REBCO
- Ag layer, for protection
- Cu stabilizer
- Insulating tape (if required)

The growth of the buffer layer is made via Ion Beam Assisted Deposition (IBAD), and for the superconducting layer a hot wall heating Pulsed Laser Deposition (PLD) is performed, which leads to a better $I_c$ performance of the tape with respect to the ones grew with simple PLD. The edges of the wires (10 mm wide) are cut with a laser and then coated with Ag also on the sides to prevent deterioration. The data reported showed a current per tape length record in 2011, 572A x 816m, with a good uniformity along all length. More interesting data concern the angle dependence and the behaviour at 4.2 K. On the first issue, a big anisotropy and the absence of the $c$ peak is found also in these tapes, the R&D section of the company is working on introducing artificial pinning sites along $c$. While the tapes had a critical current per unit width equal to 498 A/cm at 77 K and self-field, at 4.2 K the results are the following [12]:

$$I_c/width = 518 \text{ A/cm} \quad \text{at 4.2 K, 17 T, B } \parallel c$$
$I_c/width \simeq 4000 \text{ A/cm}$ at 4.2 K, 15 T, B || ab

These values are even better than those from SuperPower, making us think that this technology is very promising.

**Structure**

- Insulating Tape [Polyimide]
- Stabilizer [Cu]
- Protection Layer [Ag]
- Superconducting Layer [GdBa$_2$Cu$_3$O$_y$]
- Buffer Layer [MgO, etc.]
- Substrate [Hastelloy]

Figure 3.4: *Structure of REBCO tape of Fujikura*

### 3.3 American Superconductors

The data from American Superconductors are difficult to find, as they do not provide the original papers on their website. Some data can be found for the so called Copper Laminated Amperium Wire [13], which can carry a critical current equal to 250 A (for a 12 mm wide tape at 77 K and self-field), which corresponds to a $J_{\text{engineering}}$ (the $J_c$ referred to the whole cross section of the tape, including all the non-superconducting layers) equal to 10300 A/cm$^2$. The growth of the superconducting layer is made via Metal Organic Deposition (MOD). Scaling the $I_c$ value with respect to the width of the tape tapes, around 1 cm, we find a $I_c$/unit length around 250 A/cm, half of the value found by Fujikura.

### 3.4 Furukawa

Also in this case the data cannot be found so easily. The layer structure is similar to the one from SuperPower, but the width is usually 5 mm or 3 mm after laser cutting. In a recent paper by M. Yagi et al. [14] about the development of a superconducting tape, some values are given. They state a critical current around 100 A, which gives a $I_c$/unit width, using the average width of 4 mm, of approximately 250 A/cm at 77 K and self-field, which is almost the same found for American Superconductors.
3.5 Bruker

The REBCO tapes produced by Bruker have a structure very similar to the one of SuperPower:

- Substrate (steel type)
- Buffer Layers
- YBCO layer
- Au or Ag protection layer
- Copper all around the structure

For the growth, many techniques are used: first, the substrate is cleaned, then a buffer of yttria-stabilized zirconia oxide is grown by Alternating Beam Assisted Deposition (ABAD). The CeO$_2$ buffer and the YBCO layer are added via high-resolution Pulsed Laser Deposition, while the protection layer is grown by physical vapour deposition. The copper encasement is achieved by electroplating.

The commercial tapes provided by Bruker can have widths of either 4 mm or 12 mm, with respective critical currents (77 K, self field) of:

\[ I_c(77\text{K}, \text{sf}, 4\text{ mm width}) = 135 \text{ A} \]
\[ I_c(77\text{K}, \text{sf}, 12\text{ mm width}) = 420 \text{ A} \]

giving a critical current per centimeter width between 337.5 and 350 A/cm-width. Concerning the in-field and low-temperature performances, from the data sheet on the Bruker website [19] one can see that a critical current greater than 4800 A/cm-width has been achieved in a magnetic field applied parallel to the \( \text{ab} \) direction of 18 T.

The reported results from all the companies are summarized in table 3.1.

3.6 Manufacturing processes

In this section I will briefly summarize all the manufacturing processes used by the different companies to grow the layers of their tapes. While the buffer layers are grown via Ion Beam Assisted Deposition (IBAD), a technique which combines sputtering and ion implantation, by all the companies, there are different techniques concerning the superconducting layer and the substrate.

The superconducting layers of the SuperPower tapes are grown via Metal-Organic Chemical Vapour Deposition (MOCVD), through which the growth
of the crystal is obtained by chemical reactions and not by physical deposition. The precursors are very pure gases which are dosed to the exact stoichiometry in a reactor, and then deposited on the substrate where a surface reaction of the metalorganic compounds creates the condition for an epitaxial growth. The substrate is not textured, so the buffer layers are many in order to provide a good initial structure for the superconductor’s growth. The copper is electroplated on all the sides of the tape.

The growth by Fujikura is instead obtained via (hot wall) Pulsed Laser Deposition. In this technique, a plasma of the precursors is created via laser heating and ablation. The components of the plasma then deposit on the hot substrate via physical reaction. Also in this case the substrate is not textured, so the structure requires many buffer layers. The protective Cu layer is soldered on the silver one.

American Superconductors uses substrates grown by RABiTS™, a technique which allows the substrate to be textured. In this case, the buffer layers are less and just provide a good lattice matching. Moreover, the growth of the superconducting layer is obtained via MOCVD as SuperPower. The Cu protection layer is soldered onto the tape.

For what concerns Bruker, they do not use a textured substrate, thus more buffer layers are needed. The YBCO layer is added via PLD, like for Fujikura, and the Cu is electroplated like for the tapes by SuperPower.

### 3.7 Other techniques

The first topic I would like to discuss refers to studies made mainly by H. W. Weber and his co-workers [15, 16, 17]. They try to enhance pinning in YBCO samples and REBCO commercial tapes by neutron irradiation. If the neutron energy is high enough (∼ 1 KeV) at the collision with an atom, an avalanche-like process takes place, where a large number of atoms displace from their original position. The recoil energy is transferred to the

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<tr>
<th></th>
<th>SuperPower</th>
<th>Fujikura</th>
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<tr>
<td>77 K, Self-Field</td>
<td>~ 300 A/cm</td>
<td>498 A/cm</td>
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<tr>
<td>4.2 K, ~ 15 T, B</td>
<td></td>
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<td>4.2 K, 15 T, B</td>
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<td>ab</td>
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<td>American Superconductors</td>
<td>250 A/cm</td>
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<td>Bruker</td>
<td>77 K, Self-Field</td>
<td>~ 340 A/cm</td>
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<td></td>
<td>4.2 K, 18 T, B</td>
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Table 3.1: Summary of the performance data reported in the literature.
crystal lattice in a very small region, causing local melting and amorphous re-solidification: these defects are called defect cascade. First of all, it needs to be pointed out that in self-field configuration there is no enhancement of the critical current, as the defects created by neutrons actually degrade the tape. Nevertheless, the critical temperature does not show any relevant decrease after irradiation. When an external field is applied, these defects act as effective pinning centres, randomly distributed inside the sample’s volume, changing the shape of the anisotropy curve of the sample. At 64 K, the \(ab\) peak remains almost unchanged, as it is due to intrinsic pinning of the \(ab\) Cu planes, while in the region around \(c\) the angular resolved critical current exhibits different behaviours at different fields and different angles. This feature would either contradict the theory of random pinning or mean that the pinning due to neutron irradiation is not really random, which is actually found in many papers (see references inside [15]). An explanation for this feature can be proposed, considering that the random pinning centres actually interact with the already existing pinning landscape, creating an interacting pinning structure which behaves differently (different interactions) at different fields, angles and temperatures. Following this line of research, Topal et al. [18], compared the effects of neutron irradiation on pinning of YBCO and SmBCO samples. They managed to achieve a maximum improvement of a factor 7 in critical current at 50 K and self-field for the SmBCO sample, while only a factor 4 was found for the YBCO. This is explained by the different absorption cross-section of Sm and Y atoms.

An interesting and different pinning method has been proposed ten years ago by D. B. Yan et al. [5]. The idea was to grow an uniaxial anisotropic ferromagnetic layer of CoPt on the top of YBCO and use its domain walls as pinning centres. The anisotropy ensures that when an external field is applied, the spins substantially remain in their initial orientation, and so the domain walls do not change conspicuously, too. Unfortunately, the coercive field was measured to be 600 G \(\simeq 60\) mT, so this kind of pinning can work only at very low fields. The temperature range of the measurements, as well, is not one of the best for applications, as the measurements were performed at 86 K and 75 K. The results showed an improvement of \(J_c\) of about 2-3 times its value without the ferromagnetic layer, the order of magnitude is about \(10^6\) A/cm\(^2\). The pinning mechanism is very interesting, but for actual applications one should find ferromagnetic materials with stronger anisotropy and test them at lower temperatures.

Remaining on magnetic-based pinning methods, a study by Un-Cheong Sou, H. C. Yang et al. [6], showed an improvement of critical current near \(T_c\) and at low applied fields. The pinning has been engineered growing a \(\text{La}_{0.7}\text{Sr}_{0.3}\text{MnO}_3\) magnetic dot array. However, the application of this technique is very limited by the high temperature and low field regime.

I. Brilik et al. [7] used a very different kind of approach: the YBCO was grown by chemical solution deposition, and doped with Zr, so that
nanoparticles of BaZrO$_3$ were included in the film and acted as pinning centres. This type of growth does not need vacuum, but a complex heat treatment is necessary after spin coating of precursors. The measurements were performed at 77 K, with fields up to 6 T and with different dopant concentrations. The experiment showed that the best Zr concentration is 6 mol%, in this case the $J_c$ is $2.95 \cdot 10^6$ A/cm$^2$ in self-field, about 7% less than in the undoped case. Increasing the field, however, the 6 mol% doped sample has the best performance, with a final $J_c$ value around $5 \cdot 10^3$ A/cm$^2$ for a field of 6 T.

In 2012, M. Diirrschnabel et al. [8], managed to reach a critical current of 1018 A/cm, at 77 K and in self-field, which is one of the best performance achieved so far. They used DyBCO, with Dysprosium instead of Yttrium.

A. Patel, S. C. Hopkins and B. A. Glowacki, in their recent paper about field trapping in stacked superconductors [9], gave us some numbers about the best performing tapes: “The best critical current performance values (per tape width) for existing ReBCO tapes include 600 A/cm for long 600 m lengths (Fujikura) […]”. Diirrschnabel [8] as well gave us some orders of magnitude about current devices: “The critical current density, $J_c$, of coated conductors is in the range of $10^5 - 10^7$ A/cm$^2$. Coated conductors are being commercialized; however, their maximum critical currents are still limited to about 300 - 400 A/cm at 77K and zero magnetic field”.

To finish with, a pinning mechanism that was very popular during the nineties consisted in creating columnar defects in the material by ion irradiation [10]. The damage tracks left by the ions probably consist of amorphous regions where the superconductivity is suppressed. Precise data about critical currents could not be found.

It can be said that up to now the best engineered and tested pinning methods remain the ones provided by SuperPower, which work well also at high fields and low temperatures, although the performances of the tapes from Fujikura are better even without any artificial pinning structure (see table 3.1). Studies on the anisotropy to avoid the shrinking of the B||c peak with decreasing temperature are necessary, and the cited papers can give us some clues on new types of pinning, though the ones experimented up to now have just been tested in a low-field and high-temperature region.

### 3.8 References

field critical current characteristics of Zr-Added (Gd,Y)BCO superconducting tapes, Supercond. Sci. Technol. 25 (2012) 125013
[18] U. Topal, L. Dorosinskii, H. Sozeri, Effect of neutron irradiation on pin-

Chapter 4

Pinning in low-\(T_c\) superconductors

For the sake of completeness and in order to have a more general overview on the superconductors today used for application, I would like to analyse how pinning is achieved in low-temperature superconducting materials. Nb-Ti, Nb\(_3\)Sn, MgB\(_2\) and some thin films are discussed in this chapter, with more focus on the physics of the mechanisms than on the critical current values.

4.1 Nb-Ti alloy

NbTi is a low \(T_c\) Type-II superconductor, with a critical temperature of 9.2 K. It is the only used superconductor for magnet applications concerning particles accelerators; just in the Large Hadron Collider at CERN about 1200 tons of Nb-Ti cables are used.

In Nb-Ti the pinning is mostly natural, due to the \(\alpha\)-Ti phase in the alloy, but some artificial pinning mechanisms have been proposed during the past years. Before starting the explanation, I would like to cite a very important result, which should be taken into account also in the case of YBCO and together with the anisotropy of the coherence length could give a first and qualitative explanation of the shrinking of the \(c\) peak at low temperatures. "Peak current densities are achieved when the pinning centre spacing are matched to the spacing of the flux line lattice (FLL) at field of operation". [1] Some early theoretical results give a simple formula for the triangular fluxoid lattice spacing [2]:

\[
d_{\text{latt}} = \sqrt[3]{\frac{h}{3eB}}
\]

where \(h\) is the Plank’s constant and \(e\) the magnitude of the electron charge. So the pinning should be engineered choosing the applied field at which
the device will be working and then putting pinning centres at the right spacing to achieve the best performance. L. R. Motowildo, B. A. Zeitlin et al. [1] proposed two types of artificial pinning, one made introducing Nb interfaces inside the material (barrier pinner geometry), and one made by growing Nb islands on the material, always put in a triangular lattice as needed. They found that the barrier type is better in the mid-field region, while the island one is better for low fields, i.e. lower than 3 T. At high fields they both exhibit similar $J_c$ and $H_{c2}$ characteristics. The barrier approach has been followed also by T. Kitai et al. [3] and they found that the artificial pinning dominates with respect to the natural one when the period of the superconducting plus normal layer thickness is less than 100 nm. Other types of artificial pinning have been briefly described by T. Wong et al. [4], they include Nb, Ti, ferromagnetic materials, combined with the superconducting phase in various ways, such as stacked sheets, rods-tubes, rods-rods and gun-drilled billets-rods. To conclude, another artificial pinning mechanism has been described by C. Bormio-Nunes et al. [14], which consisted in gun barrel drilling a NbTi rod, and then introducing a normal metal as pure Cu, Nb, Ta.

4.2 Nb$_3$Sn

The critical temperature of Nb$_3$Sn is 18.3 K and, as all the superconductors used to carry current, it is a Type-II. It is more expensive than Nb-Ti, but can withstand magnetic fields up to 30 T, whereas Nb-Ti can withstand fields only up to roughly 10 T [5]. The pinning is natural in this material and R. M. Scanalan, W. A. Fietz and E. F. Koch [6] studied what was the most consistent mechanism between grain boundaries and second phase particles. The results of their paper showed that grain boundaries were the only defect present in sufficient density to account for flux pinning in the multifilamentary Nb$_3$Sn material.

4.3 MgB$_2$

This material has the highest $T_c$ among all the low-temperature superconductors, 39 K. The lattice structure and the elements that form the compound allow various types of artificial pinning. The first one I want to illustrate is based on SiC doping: the Silicon and Carbon atoms substitute Boron, creating nano particle inclusions which act as pinning centres. In a study made with TEM images and XRD spectra, S. X. Dou et al. [7] showed that the nanoparticles were mostly made of Mg$_2$Si or unreacted SiC. Nano-Y$_2$O$_3$ doping is also performed, but the $J_c$-field behaviour is better for nano-SiC doping. [8] Other techniques are oxygen alloying of MgB$_2$ films and proton irradiation of MgB$_2$ powders, but these processes are quite dif-
ficult to perform. Due to the hexagonal lattice of MgB\textsubscript{2}, another way to dope the material has been experimented by K. S. B. De Silva et al. [9] The doping was done with Graphene and the mechanism is explained by the tensile strain induced by Graphene, which leads to spatial fluctuations of the transition temperature ($\delta T_c$ pinning). One of the most interesting pinning mechanisms I will talk about has been proposed by W. K. Yeoh, J. Horvat, S. X. Dou and V. Keast, and implies the doping of the superconductor with multi walls carbon nanotubes (CNT) by solid state reaction [10]. They found a significant enhancement in critical current when the whole CNT could be incorporated, which is to say, when the sintering process was kept at temperatures below 900\textdegree C. The pinning was efficient because the carbon nanotubes had approximately the same of the fluxons, whose dimensions are of the order of magnitude of the coherence length $\xi$ of MgB\textsubscript{2}.

4.4 Thin Films

Pinning in multilayers of PbGe and WGe [11], [12] has been achieved in the second half of the nineties by creating regular arrays of submicron holes via lithographic techniques [13]. The mechanisms were substantially two: strong pinning at antidots (the name given to the array of holes) and weak pinning at interstices. Obviously samples with the holes had a higher critical current, more than 1.5 times than in the samples without holes, but the interesting thing is the peculiar shape of the magnetization hysteresis loop. Indeed, it presents various steps, due to the matching of the applied external flux density with the density of the pinning sites.

4.5 References

In order to obtain information on the critical current of a superconductor, various types of measurements can be performed. One is the simple transport measurement, in which a current is let to flow inside the superconductor and the critical current value is defined with one of the criteria presented in section 2.4. In my measurements I always used the electric field criterion with a critical field of 1 µV/cm. Another kind of measurement is indirect and involves the use of the magnetization hysteresis loop of the superconductor. In this chapter a wide analysis of the magnetization models is given, with emphasis on the limitations and the advantages that each of them offers.

5.1 The Bean’s model

The Bean’s model was first proposed in 1962 for the reversible magnetization and then completed in 1964 with the whole hysteretic behaviour [1] [2]. It was first introduced for an infinite slab and infinite cylinder geometries, with the field applied along the “infinite” direction. The model is macroscopic and does not take into account the Meissner state, i.e. $B_{c1} \ll B_a$, where $B_a$ is the externally applied magnetic field, and states that when a magnetic field is applied to the superconductor, currents of the magnitude of the critical current will flow inside it up to a penetration depth at which the internal local field becomes zero. The vortex structure is ignored, in the sense that macroscopically the vortices penetrating from the surface and the circulating supercurrents are seen as a total averaged internal flux and supercurrent. Moreover, it is assumed that the critical current does not depend on the applied field, i.e. $J_c(B) = \text{const}$. In the infinite slab geometry the equations governing the phenomenon are the simple Maxwell’s Equations with the assumption that the current can only take the critical value. So we have, in
Figure 5.1: Reference for the slab geometry (left) and field profile (right) before the full penetration in the superconducting slab with Bean’s assumptions. The full penetration field is reached when $d = w/2$.

SI units:

$$\nabla \times \mathbf{H} = \mathbf{J}_c$$

(5.1)

which, in the infinite (along $z$ and $y$) slab geometry becomes (see figure 5.1 as a reference):

$$\frac{dH}{dx} = J_c$$

(5.2)

An important parameter of the theory is the full penetration field, defined as the field at which the penetrating flux reaches the centre of the superconductor. From the equations is easy to show that:

$$H_p = \frac{J_c w}{2}$$

(5.3)

The magnetic induction $\mathbf{B}$ is defined as the average value of $\mathbf{H}$ inside the volume times the $\mu_0$ constant, and it can be easily calculated for the cases $H_a < H_p$ and $H_a > H_p$, where $H_a$ is the externally applied magnetic field. Then, the magnetization can be obtained as well, with the known relation $\mathbf{M} = \mathbf{B}/\mu_0 - \mathbf{H}$. The results are the following:

$$M(H_a) = -H_a + \frac{H_a^2}{J_c w} \quad \text{for} \quad H_a < H_p$$

(5.4)

$$M(H_a) = -\frac{J_c w}{4} \quad \text{for} \quad H_a > H_p$$

(5.5)
Figure 5.2: Calculated Magnetization loop with Bean’s model. The width used is 20 µm, the \( J_c \) used is the one measured for a YBCO tape at 10 T, around \( 5.7675 \times 10^{10} \text{A/m}^2 \), the maximum applied field is 10 T.

With the same calculation, the entire hysteresis loop can be calculated, starting from a maximum value of the field \( H_m \), giving:

\[
M(H_a) = H_m - H_a + \frac{(H_m - H_a)^2}{2 J_c w} - \frac{J_c w}{4}
\]

for \( H_m - 2 H_p < H_a < H_m \) (5.6)

\[
M(H_a) = \frac{J_c w}{4}
\]

for \( H_a < H_m - 2 H_p \) (5.7)

For the part from \(-H_m\) to \(+H_m\), the expressions can be found by replacing \( H_a \) with \(-H_a\) and \( M \) with \(-M\); the hysteresis loop found is shown in figure 5.2.

With the given equations, one can easily see that the width of the magnetization loop can give direct information on the critical current (\( M^+ \) and \( M^- \) indicate the upper and lower branches of the magnetization loop, respectively):

\[
M^+ - M^- = \frac{J_c w}{4} - \left( -\frac{J_c w}{4} \right) = \frac{J_c w}{2}
\]

and defining \( \Delta M = |M^+| + |M^-| \) one finds:

\[
J_c = \frac{2 \Delta M}{w}
\]

This formula can be extended also to the case of finite length \( l \) along the
y direction (see figure 5.1), yielding [4], [11]:

\[ J_c = \frac{2\Delta M}{w \left(1 - \frac{w}{l}\right)} \]  

(5.10)

The \( w \) and \( l \) dimensions are those perpendicular to the applied field direction with \( w < l \). The real problem about this model is the assumption of constant \( J_c \). Indeed, experimental data show that there is often a field dependence of the critical current, and most of the times it is also very strong. The \( J_c(B) \) dependence in the magnetization models was first taken into account by Y. B. Kim [3], who developed a theory considering a dependence of the type:

\[ J_c(B) = \frac{k}{(B_0 + |B|)} \]  

(5.11)

with \( k \) and \( B_0 \) free parameters. Other dependencies have been hypothesized, with the physical constraint that \( J_c(B) \) should be a decreasing function of \( B \), here as an example [4]:

\[ J_c(B) = A - C|B| \]  \hspace{1cm} (Watson)  

(5.12)

\[ J_c(B) = k_1|B|^{-q} \]  \hspace{1cm} (Yamafuji)  

(5.13)

\[ J_c(B) = A_1e^{-\frac{|B|}{C_1}} \]  \hspace{1cm} (Fietz)  

(5.14)

These dependencies modify the magnetization loop, in which a peak at \( B < 0 \) appears (see figure 7.7).

What is usually done to evaluate the \( J_c \) from the magnetization loop is to use the equation derived from the Bean’s model and to apply it locally at a given applied magnetic field, as in the surroundings of that field there is a quasi-equilibrium state with \( J_c(B) = \text{const} \). I will now discuss the legitimacy of this assumption.

The formula derived from the Bean’s model should be applicable only to samples infinitely long in the \( z \) direction, as the assumption of the theory states. As soon as the sample thickness, \( t \), becomes finite, demagnetizing effects due to the bending of the force lines start to take place. Moreover, the \( J_c(B) \) behaviours summarized before concern the internal field, which can become very different from the external applied field, leading to misinterpretation of the experimental data. In order to quantify these differences, Alvaro Sanchez and Carles Navau have performed a study [5] on the internal field, magnetization loops and critical currents of superconducting cylinders, with the field applied parallel to the cylinder’s axis. Their model relies on magnetic energy minimization under changes in the critical current distribution. The superconductor’s volume is divided into a regular grid of coaxial rings in which currents can flow. Chosen an applied field \( B_a \), the circuit that leads to the greatest decrease of energy is found by calculating the mutual
inductances between circuits and therefore the overall energy. Then the new currents are set obeying to the particular material law \( J(B_i) \), where \( B_i \) is the internal field, and when there is no more decrease in energy the internal magnetic field and the magnetization can be calculated. The process is then repeated for every value of the applied magnetic field. They chose an exponential \( J_c(B_i) \) dependence of the kind:

\[
J_c = J_{c0} e^{-\frac{|B_i|}{B_1}}
\]  

(5.15)

with \( J_{c0} \) and \( B_1 \) free parameters, and calculated the magnetization loops and field profiles for different values of the \( L/R \) (disc thickness over disc radius) ratio and \( p \). The value \( p \) is defined as \( p = J_{c0} R \mu_0 / B_0 \) and is related to the strength of the \( J_c(B) \) dependence; \( p = 0 \) reduces the analysis to the Bean constant \( J_c \) case, while high values of \( p \) characterize a strong dependence of the critical current on the internal field. For each value of \( p \), as the ratio \( L/R \) becomes finite, the slopes of the virgin and reverse branches become bigger; this is due to the demagnetizing field that enhances the total internal field. As soon as \( p \) becomes different from zero, the magnetization loop presents a peak and a star-like shape. The peak can be interpreted as the point at which the averaged absolute value of the internal field reaches a minimum, so that the currents are the highest and therefore also the magnetization. Another feature that can be seen is that by decreasing the \( L/R \) ratio, the

Figure 5.3: Example of hysteresis loops obtained with Kim model compared with experimental data (Source: [3])
magnetization peak shifts towards $B = 0$. I will discuss about this feature later on. The model also allows to calculate the internal field profiles and it has been found that for a strong $J_c(B)$ dependence ($p = 10$) the internal field is much more uniform for thin samples than for thick ones (see figure 5.4).

These results lead to the following conclusions:

- For thin film geometry, which is the case of YBCO tapes, the magnetization peak should lie close to $B_a = 0$.

- For thin film geometry, the internal field is practically uniform inside the superconductor’s volume for values of the applied field above $H_p$, and it does not change too much below the full penetration field.

- The value of the internal field for thin samples is basically equal to the external field except within a narrow window close to $B_a = 0$. This is due to the fact that currents are usually low, except when $B_a$ is small, as the demagnetizing field contributes in the $J_c(B_i)$ behaviour, leading then to a small contribution to the total field. Moreover, if the sample is thin, the total current is obviously less than the one of a thicker sample.

This allows to state that the extraction of critical current values from magnetization loops width is most accurate for the thin film geometry, which is
the case for YBCO tapes when the field is applied along the c axis.

Now I will briefly discuss about the peak position of the magnetization loop. The previous study showed the shift of the peak towards $B_a = 0$ for thin films, that can be easily understood from the conclusions stated above: as the peak is supposed to appear where the internal field is at its minimum absolute value, as far as in thin films the internal field is almost equal to the applied one the minimum value will be for $B_i \simeq B_a = 0$. If one is not satisfied by this phenomenological explanation, a work by D. V. Shantsev, M. R. Koblischka, et al. [6] presents an analytical demonstration of the fact that for thin film geometry and for critical currents depending only on the magnitude of the internal field, the maximum of the magnetization curve lies at $B_a = 0$. Experimental data are also provided in the paper to corroborate this thesis. Besides, if the superconductor presents granularity, trapped currents between grains that modify the internal field bring to a shift of the peak towards positive values of $B$. With these results, one can easily achieve information about the microstructure of the superconductor just by looking at the peak position of the magnetization loop. One further question would be how to model the flux penetration and the magnetization for a thin film geometry. E. H. Brandt has done many works on flux profiles and current distributions in superconducting strips of rectangular cross section in perpendicular field [7], [8]. The results show a weak logarithmic singularity at the edges for the local field, as shown in figure 5.5. Solving the equation for that geometry is however quite challenging and time consuming, especially for the derivation of the magnetization loop. An approximation can be done which leads to good analytical results. It considers the cross-section of the YBCO tape being of elliptical shape, which is actually legit if one considers that for YBCO tapes the thickness and the width differ by about 3 orders of magnitude (µm vs. mm respectively). This approximation will be discussed in detail in the next paragraph.

5.2 Type-II superconducting strip with elliptical cross section

This model has been presented by F. Gömöry et al. [9]. In the paper, a numerical model based on magnetic energy minimization is first used to calculate the current profiles inside the ellipse. The results showed that the currents profiles can be well approximated with an ellipse, especially for thin samples. Starting from this hypothesis, they developed the analytical model. The geometry considered is the one illustrated in figure 5.6: the semi-axes of the ellipse have length $a$ and $b$ (major and minor, respectively), and the assumption on the flux fronts is that they are ellipses with semi-axes $a\epsilon$ and $b\epsilon^{1/n}$. The variable $\epsilon$ is the core of the model, as it indicates how much the flux has penetrated into the superconductor; $\epsilon = 0$ indicates full
Figure 5.5: Perpendicular local magnetic field profiles for a superconducting strip of width $2a$, centered at $y = 0$ and in perpendicular field, as calculated by Brandt (Source: [8])

Figure 5.6: Reference for the considered geometry of the strip with elliptical cross section, infinitely long on the $z$ direction. Source: [9]
penetration. The aspect ratio $b/a$ is named $\beta$, the critical current is assumed to be constant with the applied field and it flows in the grey area of figure 5.6 according to the equation:

$$J(\epsilon, r, \theta) = \begin{cases} 
J_c & \text{for } r_\epsilon < r < r_1 \cap -\pi/2 < \theta < \pi/2 \\
0 & \text{for } r < r_\epsilon \cup r > r_1 \\
-J_c & \text{for } r_\epsilon < r < r_1 \cap \pi/2 < \theta < 3\pi/2 
\end{cases}$$ (5.16)

The outer border of the strip and the flux front shape are given by:

$$r_1(\theta) = \frac{b}{\sqrt{\beta^2 \cos^2 \theta + \sin^2 \theta}}$$ (5.17)

$$r_\epsilon(\theta) = \frac{eb}{\sqrt{\beta^2 \cos^2 \theta + e^{2-2/n} \sin^2 \theta}}$$ (5.18)

Applying the integral Biot-Savart law, the $y$ component of the field in the centre of the ellipse is:

$$H_y(\epsilon) = -\frac{2J_c}{\pi} \int_0^{\pi/2} \cos \theta (r_1(\theta) - r_\epsilon(\theta)) d\theta$$ (5.19)

Which can be solved, with $H_p = 2J_c b/\pi$ the penetration field of a round wire of radius $b$ and $g(x)$ the auxiliary function defined as:

$$g(x) = \begin{cases} 
x^{1/\pi} \arctan \sqrt{\frac{x^2}{x^2 - 2/\pi} - 1} & \text{if } \beta > x^{1-1/n} \\
x^{1/\pi} \text{arctanh} \sqrt{\frac{1 - x^2}{1 - x^2 - 2/\pi}} & \text{if } \beta < x^{1-1/n} 
\end{cases}$$ (5.20)

giving:

$$H_y(\epsilon) = -H_p(g(1) - g(\epsilon))$$ (5.21)

This allows to get the full penetration field, $H_{\delta}$, as the value of $|H_y|$ when $\epsilon = 0$. Using the generalized formula for the magnetic moment, $\mu = \frac{1}{2} \int_V r \times J dV$, the magnetic moment per unit length along $z$ (assumed to be infinite) becomes:

$$\frac{\mu(\epsilon)}{l} = -4J_c \int_0^a xb \sqrt{1 - \frac{x^2}{a^2}} dx - 4J_c \int_0^{\epsilon a} x^{1/n} b \sqrt{1 - \frac{x^2}{\epsilon^{2+1/n} a^2}} dx$$ (5.22)

That can be easily solved by a change of variables, i.e. for the first one $\tau = x/a$ and $u = 1 - \tau^2$, yielding:

$$\frac{\mu(\epsilon)}{l} = -\frac{4J_c ba^3}{3} (1 - \epsilon^{2+1/n})$$ (5.23)
Dividing by the cross section of the tape, the magnetization is obtained:

\[ M(\epsilon) = -\frac{2H_p}{3\beta}(1 - \epsilon^{2+1/n}) \] (5.24)

In the virgin state, the currents will penetrate the ellipse up to a value \( \epsilon_0 \) so that the applied field will be completely shielded. Using the previous equations the condition reads:

\[ H_a = H_p(g(1) - g(\epsilon_0)) \quad \text{if} \quad H_a < H_s \]
\[ \epsilon_0 = 0 \quad \text{if} \quad H_a > H_s \] (5.25)

This implicit relation linking the external field to the value of \( \epsilon_0 \) allows us to find the first part of the magnetization curve, by substituting \( \epsilon \) with \( \epsilon_0 \).

With this result it is also possible to find the amplitude susceptibility for an applied AC field in the form \( H_a = H_m \cos(\omega t) \). The relation linking \( H_m \) to the maximum penetration \( \epsilon_m \) is the usual: \( H_m = H_p(g(1) - g(\epsilon_m)) \) if \( H_m < H_s \) and \( \epsilon_m = 0 \) if \( H_a > H_s \). The amplitude susceptibility is then equal to:

\[ \chi_a = \frac{M_0(\epsilon_m)}{H_m} = -\frac{2}{3\beta} \frac{1 - \epsilon_m^{2+1/n}}{g(1) - g(\epsilon_m)} \] (5.26)

For very small amplitudes, the penetration is very small and one can take the limit \( \epsilon_m \approx 0 \), obtaining (absolute value):

\[ \chi_0 = \frac{2}{3\beta} \frac{(2n + 1)(\beta^2 - 1)}{(n\beta^2 - 1)g(1) - n + 1} \] (5.27)

The analytic expression for \( \chi_0 \) has been calculated by J. A. Osborn [10] with the result \( \chi_0^{\text{analytic}} = 1 + \frac{1}{\beta^2} \). Comparing the two equation one could find the optimal value of \( n \) for each \( \beta \), reducing then the number of free parameters of the model and yielding:

\[ n_{\text{opt}} = \frac{2\beta + 3g(1) - 5}{3\beta^3} \] (5.28)

The hysteretic part of the loop can be described as follows: keeping the existing currents at \( \epsilon_m \), new reverse currents will enter in the volume, as the field is decreased from \( H_m \) to \(-H_m\), to shield a field equal to \((H_m - H_a)/2\). The implicit relations are the usual ones, but with different conditions:

\[ (H_m - H_a)/2 = H_p(g(1) - g(\epsilon_1)) \quad \text{if} \quad H_a > H_m - 2H_s \]
\[ \epsilon_1 = 0 \quad \text{if} \quad H_a < H_m - 2H_s \] (5.29)

\( H_m - 2H_s \) is the penetration field of the reverse supercurrents introduced by Bean. The magnetization is then given by the superposition of the
magnetization given by the frozen currents and the one due to the reverse ones:

$$M_1(\epsilon_1) = M_0(\epsilon_m) - 2M_0(\epsilon_1) = \frac{2HP}{3\beta} \left( \epsilon_m^{2+1/n} + 1 - 2\epsilon_1^{2+1/n} \right)$$ (5.30)

For the other part of the cycle, it is sufficient to recall the symmetry with respect to the origin of the magnetization loop. Making use of this model, one can find the magnetization loops as the one presented in figure 5.7, calculated for $a = 4$ mm, $b = 1$ μm, the optimized value of $n$ for the given aspect ratio $n = 21.984038$ and the critical current density taken from transport measurements at the maximum field reached, $5$ T, and at a temperature of $4.2$ K; $J_c = 9.5675 \cdot 10^{10}$ A/m$^2$.

There are many important facts to be underlined about the plot shown in figure 5.7: to begin with, one can see that for this geometry the full penetration field is very small (it does not go beyond 200 mT), this allows us to state that, for the results presented in the previous paragraph, the internal field is uniform for almost all the cycle. Reducing the aspect ratio of the sample, the penetration field increases, making the internal field less uniform and supporting the results reported in the previous paragraph. Secondly, one can see that the shape of the loop is quite similar to the one obtained with the Bean’s model, but there are some important aspects to be discussed:

- This model is developed for a sample confined in two dimensions, while the Bean’s one takes into account an infinite slab geometry.
• This model is developed for perpendicular fields, while in the Bean’s model (even extended to two confined dimensions) the field is assumed to be parallel to the infinite size of the sample.

• The absence of a $J_c(B)$ dependence can be noted by the fact that there is no peak in the loop, the next step will be to include a $J_c(B)$ dependence in this analytical model, that seems to suit very well the experimental conditions in which the YBCO magnetization measurements are usually performed.

• Once fixed the aspect ratio, the model does not depend on any other parameter.

This chapter introduced the basic concepts for linking the magnetic measurements to the critical current analysis: I discussed the simplest models and their extensions, and provided some theoretical and experimental basis for the interpretation of the magnetization data. I will show later (see chapter 6) the details of the experimental setup used for my measurements, but I can anticipate that the best model for analysing those measurements is the extension of the Bean’s model for a sample with rectangular cross section using the local width of the magnetization loop. The model presented in the last section would be even better (once introduced a $J_c(B)$ dependence), if it did not consider an infinitely long superconducting strip. In the setup that I used the shape of the sample is almost squared, with the width very similar to the length, so the infinite length approximation is not suitable. The investigated samples were commercial YBCO tapes produced according to the techniques presented in chapter 3, so they can be treated as thin films. The results found in literature showed that the application of Bean’s critical state formula in the perpendicular field orientation for thin films is legitimate and gives correct results.

5.3 References

Pust, M. Jirsa, Central peak position in Magnetization Loops of High-Tc superconductors, Physical Review Letters, 82, 14, 1999
Chapter 6

Experimental setup

In this chapter I describe the experimental details of the measurements that I have performed, with particular attention to the sample preparation and the experimental procedure.

6.1 Transport Measurements

The principle of these measurements is simple: a current is injected in the superconductor via a power converter, and the voltage drop between two chosen points on the superconductor is measured. A simple scheme of the measurement is given in figure 6.1. When the pinning is effective, the superconductor does not develop any form of resistance, and the measured voltage is zero. The only source of resistance in this state is the one due to the Thermally Assisted Flux Flow, which consists in the depinning of vortices due to thermal activation. This contribution is very small and negligible, at low temperatures. As the current is increased, the Lorentz force acting on the fluxoids increases as well, and when it becomes higher than the pinning force the vortices will start to move, developing a voltage inside the superconductor. This will cause a voltage drop due to the finite resistance in the superconductor, which is measured. As the transition is not sharp, it is necessary to choose a criterion among the ones presented in section 2.4 to associate the critical current with the measured voltage drop. In YBCO thin films and, in general, high temperatures superconductors, it is common to use the electric field criterion with a critical field of 1 µV/cm. The $I_c$ vs. $V$ behaviour near the transition can be well fitted with a power law:

$$\frac{E}{E_c} = \left( \frac{I}{I_c} \right)^n$$

The exponent is defined as the $n - value$ of the superconductor, and it is an indicator of the pinning potential, as shown in chapter 2:

$$n = \frac{U_{pin}(B, T)}{k_b T}$$
The higher the $n$-value, the higher is the pinning potential and then the better will be the in-field current performance of the superconductor.

### 6.1.1 Experimental setup for liquid nitrogen tests

In order to perform preliminary measurements at 77 K and self-field, the samples were clamped onto a straight steel sample holder with copper parts for current transfer, as shown in figure 6.2. Voltage taps are soldered on the superconductor, usually at a distance between 2 and 3 cm, with particular attention paid to the soldering temperature, which must not exceed the limits imposed by the producer (the maximum soldering temperature is usually below 220$^\circ$) to prevent sample degradation. The sample is then put in a liquid nitrogen bath, a current is injected into it via a current supply, and the voltage drop in the voltage taps region is measured and recorded. The data are acquired via a LabView® software, which allows to reconstruct the $I-V$ curve and then the critical current and $n$-value with the criteria introduced previously are calculated. With this setup, only in self-field measurements are performed.
6.1.2 Experimental setup for liquid helium tests

To perform measurements in liquid helium, with the magnetic field applied perpendicular to the wide face of the sample (i.e. along the c axis), the tape must be put onto an insert, as shown in figure 6.3, that can be put in the cryostat with all the necessary links to inject the current and measure the voltage drop. In order to do this, a small U-shaped sample holder made of stainless steel has been designed by J. Fleiter [3], as shown in figure 6.4. The YBCO tape is soldered along all the U-shape with its superconducting side on the external part of the sample holder; a soldering flux (Kester 135) is used to improve quality of the soldering.

The tape is clamped onto the holder with several aluminium pieces, in which the cylindrical heaters for the soldering are inserted. The soldering is done with eutectic Sn-Pb alloy at a temperature around $200 \div 210^\circ C$. Once the first soldering is made, the current leads are added with a similar soldering procedure, then after another cooldown to room temperature, the voltage taps can be added manually with a soldering iron. The voltage taps are usually put at a distance between 2 and 3 cm, the current leads for liquid helium tests are made of Nb-Ti superconductor and they overlap with the YBCO over a distance of approximately 4 cm. The most critical part of this procedure is ensuring that the YBCO tape is soldered completely parallel to the sample holder surface, which is controlled manually by pulling the tape during the first soldering, while keeping it fixed on the other side with suitable clamps. Copper wires have been soldered as voltage taps, long enough to be wired in the room temperature connections.

The bus-bars are then soldered on the support, bending them a little
Figure 6.3: Insert which allows the sample to be put in the cryostat for measurements in liquid helium.

A detail to be taken into consideration is the correct placement of the + and − voltage taps and the direction of the injected current. In fact, it was decided to choose a current direction so that the Lorentz force developed on the tape during the measurements is directed on the internal part of the sample holder, in order to avoid mechanical damage of the tape or delamination. Moreover, the voltage taps must be put in the right order, so that the voltage signal is positive and the built-in data analysis software can actually work. The measurements have been performed in the Ic2 station of the Superconducting Laboratory in building 163 at CERN, at temperatures of 4.2 K and 1.9 K and in perpendicular magnetic fields ranging from $\sim 2 \div 2.5$ T to 11 T. The actual minimum field that one can employ, though, is linked to the maximum current of the power converter, around 850 A, as the current carrying capability of the sample increases with decreasing applied fields.

The data is gathered with an automatic data acquisition system and elaborated with a LabView program designed for this purpose. The software also offers a data analysis tool that can, once specified the electric field criterion, calculate the $I_c$ values and the $n$ values, that were chosen to be calculated between 0.1 $\mu$V/cm to 1 $\mu$V/cm.

For these types of measurements, one needs rather long samples (more than 10 cm). A delicate soldering procedure is necessary, the self-field effects
Figure 6.4: *U-Shape sample holder used for the transport measurements.*
Source [3]
have to be considered, and only two different temperatures are available in the Ic2 test station at CERN: 1.9 K and 4.2 K.

### 6.2 Magnetization Measurements

Magnetization measurements are an indirect way to derive $I_c$, and they exploit the relationship lying between the width of the magnetization loop and the critical current density derived from the Bean’s model [1], [2] and presented in detail in chapter 5. The core of the experiment is then to measure a magnetization hysteresis loop at various temperatures, with fixed values of the maximum applied fields. The magnetic moment is measured with a Vibrating Sample Magnetometer (VSM): this instrument was invented by Simon Foner at the Lincoln Laboratory, MIT, in 1955 and published in [4]. The basic principle of the VSM described in the original paper is the following: a DC magnetic field is applied to the sample via a permanent or electro-magnet. The sample is made vibrate at a chosen frequency via a loudspeaker, keeping the applied field fixed. The vibration of the magnetized sample creates a change of magnetic flux that is detected by two coils via the Faraday-Neumann-Lenz law as an induced voltage, while the applied field contribution is known as the field ramp rate is fixed. A reference vibrating magnet and its own detection coils are placed for calibration. A schematic idea of the experiment is provided in figure 6.5. In a more mathematical way (SI units):

\[
B = \mu_0 (H_a + M) \quad (6.1)
\]

\[
V = -\frac{d\Phi_B}{dt} = -A_{\text{coil}} \frac{dB}{dt} = -A_{\text{coil}} \mu_0 \left( \frac{dH_a}{dt} + \frac{dM}{dt} \right) \quad (6.2)
\]

where $A_{\text{coil}}$ is the area of the detection coils and $\Phi_B$ is the magnetic flux in the coils. The time derivative of the applied field is known, so the induced voltage measured by the coils gives direct access to the time derivative of the magnetization. Since the vibration frequency is fixed and known, one can measure the value of the magnetization of the sample. Indeed, assuming a sine-wave vibration with constant phase, one gets:

\[
M = M_0 \sin (\omega t + \phi) \quad (6.3)
\]

\[
\frac{dM}{dt} = \omega M_0 \cos (\omega t + \phi) \quad (6.4)
\]

yielding:

\[
V = -A_{\text{coil}} RR - A_{\text{coil}} \mu_0 \omega M_0 \cos (\omega t + \phi) \quad (6.5)
\]

from which the value of the magnetization $M_0$ can be found. The parameter $RR$ is the ramp rate of the applied field. If phase information is required, one can use the information available through the reference vibrating magnet.
The VSM present in the Superconducting Laboratories at CERN clearly features some differences from the original design. The possible vibrational frequencies span between 10 and 100 Hz, and usually a frequency around 20.4 Hz is used. Moreover, a Lock-in Amplifier is present for the signal detection, and it is tuned to the sample’s vibrational frequency through the reference signal. With this tool, there is no restriction on the direction of the applied field as in the original design, as the Lock-in Amplifier will just see the frequency component due to the magnetized sample. Instead of having a loudspeaker, the vibration is provided by a vibrator controlled by a digital function generator, which also provides a rectangular frequency-matched signal to the Lock-in Amplifier. The field direction is *vertical* and the pickup coils are designed to measure the *vertical* component of the magnetic moment of the sample. A schematic diagram of the magnetometer setup is provided in figure 6.6.

For magnetization measurements, in our setup we can perform tests in a wide range of temperatures (from 1.9 K to room temperature), the sample preparation is much easier than for the transport ones, the samples are actually shorter (around 6 mm long) and one can get also other useful information, such as the AC losses, etc. On the other hand, also in this setup the self-field effect must be considered, and the applicability of Bean’s model (although there are positive result in literature, as stated in chapter
5) needs to be verified before drawing any particular conclusion.

6.3 Evaluations on the contribution of an angle mismatch in a VSM measurement

Before proceeding with the experimental results and analysis, it is worth to discuss the possible errors arising from a sample not completely aligned with the field direction in a measurement made with a Vibrating Sample Magnetometer. In chapter 5, I already discussed the legitimacy of the application of Bean’s critical state model to infer critical current densities from magnetization measurements from the theoretical point of view. Now, let me discuss it from the experimental side, taking into account the two main contributions arising from an angular mismatch: the anisotropy of the critical current and the variation of the current path due to misalignment.

6.3.1 Anisotropy of the critical current

The first thing to consider in connection with an angular mismatch of the applied field is the fact that in YBCO the critical current is affected by anisotropy with respect to the direction of the applied field [5]. Due to its complex crystal structure, indeed, YBCO features different pinning mechanisms at different field orientations and at different temperatures: as an example, when $B||ab$, the CuO$_2$ planes act as intrinsic pinning centres, thus increasing the critical current along that direction. At 77 K, the angular dependence of the critical current is rather complex [5], as it features two relative maxima, one along the $c$ direction and one along the $ab$ direction, as it can be seen in figure 3.2. Lowering the temperature, though, the $c$ peak
substantially disappears and one finds a more or less wide plateau, its width depending on the field, around the $c$ axis. Studies by A. Xu et al. [6] showed that at a field of 1 T and at 4.2 K, the plateau extends until 5°, as shown in figure 6.7. V. Selvamanickam et al. [5] also studied the behaviour of the Full Width at Half Maximum of the $ab$ peaks and found that it decreases with increasing field: at higher values of the field, indeed, the plateau extends up to 20°. Expecting the precision of the sample alignment to be better than 5°, one can conclude that, even with small errors on the angle, the critical current density will not be affected.

6.3.2 Changes in the current loop

The anisotropy of the critical current is not the only contribution that has to be taken into account: when the applied field is not perfectly perpendicular to the sample surface, the critical current loop stays no longer in the $ab$ plane. Instead of going into difficult calculations concerning the actual current’s path, I model this situation in the following way: let me split the applied field in its two components, one still perpendicular to the sample’s surface and one parallel to it. The weights of these components will obviously be $\cos \theta$ and $\sin \theta$, respectively, where $\theta$ is the mismatch angle, assumed
to be less than $5^\circ$. In this way, the sample will magnetize itself to exclude part of the applied field, in both directions: on the perpendicular direction (i.e. parallel to the $c$ axis) the induced moment will be the same as the one I would have with an applied field equal to $B \cos \theta$ in the $c$ direction, while in the parallel one I will have the moment produced in response to a parallel applied field equal to $B \sin \theta$, as shown in figure 6.8. The VSM is built to measure the vertical component of the moment, which is to say, with reference to figure 6.8, $\vec{\mu}$. The $y$ component of the moment is well known from the critical state model for a rectangular sample with perpendicular field, and the total measured moment will therefore be:

$$|\vec{\mu}| = \frac{|\vec{\mu}_y|}{\cos \theta}$$

(6.6)

This relation will hold also for the respective widths of the magnetization loops, that on the other hand are related to the critical current density, yielding:

$$\Delta M^{\text{meas}}(B \cos \theta) = \frac{1}{\cos \theta} \Delta M^{\text{cr}}(B \cos \theta) =$$

$$= \frac{1}{\cos \theta} J_c(B \cos \theta) \left[ w \left( 1 - \frac{w}{3l} \right) \right]$$

(6.7)

Let me define $(1/2)[w(1-(w/3l))] = G$ as a geometrical factor. All the differences between a perfectly matched experiment and one with deviations stay in the first two terms. As from the data gathered, which will be presented
in chapter 7, the \( J_c(B) \) behaviour followed well a Kim-like behaviour of the type (assuming \( B > 0 \)):

\[
J_c(B) = \frac{k}{B_0 + B}
\]  

(6.8)

I can write:

\[
\Delta M^{\text{meas}}(B \cos \theta) = \frac{1}{\cos \theta} \frac{k}{B_0 + B \cos \theta} G
\]

(6.9)

I now make use of the series expansion of the cosine function, as the angle \( \theta \) is small (5° corresponds to approximately 0.0873 rad), stopping at the second order correction, and I obtain:

\[
\Delta M^{\text{meas}}(B \cos \theta) = \frac{1}{1 - \frac{\theta^2}{2}} \frac{k}{B_0 + B - B \frac{\theta^2}{2}} G =
\]

\[
= \frac{k}{B_0 + B - (B_0 + 2B) \frac{\theta^2}{2} + B \frac{\theta^4}{4}} G
\]

(6.10)

Neglecting the term in \( \theta^4 \) and re-arranging the equations one obtains:

\[
\Delta M^{\text{meas}}(B \cos \theta) = \Delta M(B)_{\theta=0} \left[ 1 + \frac{B_0 + 2B}{B_0 + B} \frac{\theta^2}{2} \right]
\]

(6.11)

All corrections due to an angle mismatch lie in the second term. The term in \( \theta^2 \) is of the order of \( 10^{-3} \) and therefore small, it is necessary to study the term including \( B \) and \( B_0 \). Defining \( \beta = B/B_0 \) as a reduced field, the function

\[
f(\beta) = \frac{1 + 2\beta}{1 + \beta}
\]

(6.12)

has the following properties:

\[
f(0) = 1
\]

\[
\lim_{\beta \to \infty} f(\beta) = 2
\]

\[
f'(\beta) = \frac{1}{1 + \beta^2} > 0
\]

For any applied field (this result is general not only for positive but also for negative applied fields, as the Kim relation in equation 6.8 features the absolute value of \( B \) when taking into account negative values) the function remains between 1 and 2 and never diverges. This means that the term following 1 in equation 6.11 is small and one can write:

\[
\Delta M^{\text{meas}}(B \cos \theta) = \Delta M(B)_{\theta=0} \left[ 1 + \frac{B_0 + 2B}{B_0 + B} \frac{\theta^2}{2} \right]
\]

(6.13)

As I have shown before, the correction is of the order of \( 10^{-3} \) and therefore negligible, allowing me to conclude that the effect of a small angular mismatch in a VSM measurement does not drastically affect the final results.
6.4 References

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Chapter 7

Results and discussion

As illustrated in chapter 6, two different kinds of measurements have been performed: I will now summarize their features in order to have a clear starting point to understand the forthcoming paragraphs.

The first type of measurement is a transport one: these measurements can be performed either at 77 K and self-field or at 1.9 K and 4.2 K in applied perpendicular fields (\(B||c\)) up to 13 T. For the liquid nitrogen tests, the sample preparation is quite easy and fast, and the sample dimensions are around 14 cm, while for the in-field and low temperature measurements one needs a long and delicate soldering procedure and a length of the sample of around 30 cm. Besides, a current flowing in a conductor creates a magnetic field around the conductor, which is denoted as the self-field of the sample, which needs to be taken into account in these measurements. To finish with, the critical current is defined with a selected criterion among the ones introduced in chapter 2; I used the electric field criterion of 1 \(\mu V/cm\).

The second type of measurement is a magnetization one: for this measurement, small samples are needed, around 4 \(\times\) 6 mm\(^2\), and a wide range of temperatures, from 1.9 K to 77 K, and fields, from 0 to 10.5 T, is available. Also in this case the self-field has to be taken into account, in terms of the demagnetizing field of the sample under consideration, but taking into account the results presented in chapter 5 we can say that for thin film geometries this effect can be neglected.

7.1 Sample details

I performed transport and magnetization measurements for 4 different YBCO tapes with similar structure, but different microscopic pinning properties. They will be simply named sample 1, 2, 3 and 4. Sample 1 is a standard commercial YBCO tape provided by SuperPower inc. with no artificial pinning and a structure as the one shown in chapter 3. The other 3 samples are research tapes from different companies. Basic information concerning
the nominal values given by the producers can be found in table 7.1

<table>
<thead>
<tr>
<th>Sample</th>
<th>Width [mm]</th>
<th>YBCO thickness [µm]</th>
<th>$I_c(77K, sf)$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>~110</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>129</td>
</tr>
<tr>
<td>3</td>
<td>5.04</td>
<td>2.4</td>
<td>332</td>
</tr>
<tr>
<td>4</td>
<td>4.2</td>
<td>2</td>
<td>54</td>
</tr>
</tbody>
</table>

Table 7.1: Nominal relevant data for the measured samples; sf means self-field.

7.2 Optical imaging

In order to have a precise set of data concerning the dimensions of the samples we are investigating, I made images of a transversal section of the 4 tapes with the aid of an optical microscope. This is also important because some of the tapes are electroplated with copper all over their external surface, so the actual superconductor’s width is smaller than the total tape width. The samples were embedded in an epoxy resin and polished before the imaging. The results are shown in figures 7.1 - 7.4, and the measured values for the widths are summarized in table 7.2. As can be seen, for samples 1, 2 and 4 the actual width, which has been given by the one of the substrate as the YBCO layer is not directly visible optically, is slightly smaller than the total tape width, due to the copper on the edges. This is not the case for sample 3. One can also see from the pictures some delamination of the copper layer, which is due to mechanical stress during polishing and sample cutting.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Total width [mm]</th>
<th>YBCO width [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.90343</td>
<td>3.84231</td>
</tr>
<tr>
<td>2</td>
<td>4.03946</td>
<td>3.98820</td>
</tr>
<tr>
<td>3</td>
<td>4.96011</td>
<td>4.96011</td>
</tr>
<tr>
<td>4</td>
<td>4.24276</td>
<td>4.01777</td>
</tr>
</tbody>
</table>

Table 7.2: Measured total and YBCO widths for the 4 samples
Figure 7.1: *Optical image of the transverse cross-section of sample 1.*

Figure 7.2: *Optical image of the transverse cross-section of sample 2.*
Figure 7.3: Optical image of the transverse cross-section of sample 3.

Figure 7.4: Optical image of the transverse cross-section of sample 4.
7.3 Transport Measurements

7.3.1 Self-field effect estimation

Before presenting the results based on transport measurements, I would like to first discuss the impact of the self-field. I performed both analytical calculations, based on the formulas given by A. Brojeny et al. [2], and finite element calculations using the commercial software Comsol Multiphysics®, considering for simplicity a uniform current distribution in a 4 mm wide and 1 µm thick strip: this assumption actually overestimates the field produced by the flowing current, but since we just need to understand whether the self-field has a relevant effect on the total field distribution, rather than calculating precisely its magnitude, the approximation I used is legit. The value for the current to be injected in the superconductor has been chosen among the transport data I gathered as the highest value of current at the lowest field which could be reached due to the power supply limit, to evaluate the maximum self-field correction which one could have had. According to these criteria, I used the value obtained for sample 1 at an external applied field of 2 T, 763 A. As it can be seen from figure 7.5, the results with the two methods are very similar and give a field circulating around the tape with maximum magnitude along the c direction equal to \(\sim 20\) mT. As one superimposes an external field of 2 T along c, it is easy to see (figure 7.6) that the self-field contribution becomes negligible in both calculations. As this contribution is the maximum one could have had, I can conclude that the critical current values obtained during my in-field measurements do not need to be corrected to take into account self-field effects.

For the sake of completeness, I also calculated the self-field one gets for values of the injected current equal to the critical one at 77 K for all the samples, getting a value for the total magnitude of the field at the centre of the tape (i.e. mostly along ab) of \(\sim 10\) mT for a current of 54 A, as the one carried by sample 4 and \(\sim 200\) mT for a current of 320 A, like sample 3.
7.3.2 Critical currents at 77 K, self-field

As a first check, I measured the current performance of the 4 samples at 77 K and self-field with the straight sample holder described in the previous chapter. The measurements were performed with the current flowing in both directions, in order to remove the thermal voltages. The I-E curves obtained are shown in figure 7.7, and the results found for the critical currents and n-values for all the samples are summarized in table 7.3. As one can see, there is a big difference of performance among the 4 tapes, especially concerning sample 3 and 4, which have the best and worst performances, respectively.

As described in chapter 6, in order to test the samples in Liquid Helium, the sample needs to be soldered on the U-shaped holder. After every soldering, we tested the samples on the U-shaped support in Liquid Nitrogen as well, to see if there was any degradation in the performances of the tapes. We found out that basically the values remain the same as the one measured on the straight sample holder, within experimental error, meaning that the soldering procedure did not spoil the tapes.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Ic [A]</th>
<th>n-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>129</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>138</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>329</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 7.3: Measured performances for the samples at 77 K, self-field.
7.3.3 Critical currents in Liquid Helium, with applied magnetic field

We performed the soldering described in the previous chapter for all the tapes, and we measured their transport characteristics at Liquid Helium temperatures in a perpendicular (i.e. parallel to c) applied magnetic field up to 13 T. In figure 7.8 are shown as an example the I-E curves obtained from the data acquisition system for sample 1. We performed critical current transport measurements for the 4 tapes at 4.2 K and at 1.9 K just for the first 3 samples, obtaining the results shown in figure 7.9 and 7.10. Moreover, as a comparison, in figure 7.11 it can be seen that for sample 3 the improvement in critical current going from 4.2 K to 1.9 K is not so relevant (an average ratio of around 1.1). This feature is present for every sample investigated, and can be explained by the fact that YBCO is a high-temperature superconductor: these two temperatures, indeed, are both very far away from the critical one (∼ 90 K for YBCO) and so the scaling due to temperature does not change too much the superconducting properties between 4.2 K and 1.9 K. This result is in agreement with the one obtained by V. Lombardo et al. [1].

As shown in the previous chapters, the n-value gives a clue about the pinning potential, and then it could be a very important feature for understanding pinning. The calculation of the n-value is done via fitting a part of the I-V curve, therefore the n-value is sensitive to the experimental noise and the number of data points in the transition region. Due to this reason, the uncertainty the the experimentally derived n-values is rather high (∼ 3 ± 4), depending on the conditions of measurements and the sample. This does
Figure 7.8: I-E curves for 4 sample 1, at 4.2 K and external perpendicular fields up to 11 T. The straight black line corresponds to the electric field threshold of 1 μV/cm.

Figure 7.9: $I_c$-$B$ curves for the 4 samples at 4.2 K in perpendicular field.
Figure 7.10: $I_c$-$B$ curves for the 3 of the 4 samples at 1.9 K in perpendicular field.

Figure 7.11: $I_c$-$B$ curves for sample 3 at 1.9 K and 4.2 K
not allow to draw any particular conclusions from n-value data analysis.

7.3.4 Transport data analysis

The results obtained at 4.2 K and 1.9 K can be well fitted with either a Kim model or a power dependence of the kind:

\[ I_c(B) = k \frac{B}{B_0 + |B|} \quad \text{(Kim)} \tag{7.1} \]

\[ I_c(B) = I_c(0)B^{-\mu} \quad \text{(Power decay)} \tag{7.2} \]

Among them, only sample 3 features slight deviations from the Kim dependence, especially at low fields. Moreover, the Kim fit is slightly better at 4.2 K than at 1.9 K. Concerning the power decay behaviour, the fit is very good for every sample (also because the amount of points is not so big). Personally I find more physical the Kim behaviour, as it does not feature a divergence at \( B = 0 \) and the result \( J_c \sim B^{-1} \) for high fields has been derived also theoretically by A. V. Pan et al. in their Low Angle Boundaries pinning model [17], [18], [11], (see also appendix A) but the power law fit is more straightforward if one wants to see how strongly the critical current decreases as the field increases. In figure 7.12 are shown the experimental points at 4.2 K and the Kim fits for all the samples, while in table 7.4 and 7.5 the fitting parameters, respectively for the Kim and the power law fits, at 4.2 K are reported.

Usually, to achieve information concerning the pinning mechanisms, one makes use of the scaling law proposed by A. M. Campbell and J. E. Evetts [3] for the pinning force, namely:

\[ F_p = AH_{c2}^m(T) \left( \frac{B}{B_{irr}} \right)^\gamma \left( 1 - \frac{B}{B_{irr}} \right)^\delta \]

Where the exponents and the irreversibility field are fitting parameters. This function describes a bell-shaped curve, and the exponents can give much information about the pinning landscape. Unfortunately, in my case it is impossible to use this kind of study, as in YBCO and all high-temperature superconductors the irreversibility field is very high (see also [19]) and all my experimental points lie only on the ascending part of the curve without reaching any maximum, as can be seen in figure 7.13, making the fitting very difficult and not precise.

Therefore, the transport measurements I performed can give interesting results regarding the performances of the tapes, but not much can be said about the underlying pinning mechanisms. This is mainly due to two reasons: the first has been just discussed and concerns the impossibility of using the scaling law, the second one is that in my experimental setup for in-field transport measurements, only 2 temperatures are available. This precludes
Figure 7.12: $I_c$-$B$ curves for the 4 samples (points) and their fitting curves according to the Kim dependence (lines).

Table 7.4: Fitting parameters for the samples, Kim dependence.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$k$ [AT]</th>
<th>$B_0$ [T]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2455.48</td>
<td>1.274</td>
</tr>
<tr>
<td>2</td>
<td>3145.98</td>
<td>1.677</td>
</tr>
<tr>
<td>3</td>
<td>5508.97</td>
<td>3.532</td>
</tr>
<tr>
<td>4</td>
<td>7532.96</td>
<td>3.348</td>
</tr>
</tbody>
</table>

a very wide and interesting study which could be done for understanding pinning, that is the study of the temperature dependence of the critical current. Fortunately, magnetization measurements allow us to measure the magnetic moment (and indirectly the critical current density, making use of the models introduced in chapter 5) in a wide range of temperatures and fields, as it will be discussed in the next section.

7.4 Magnetization measurements

These measurements have been carried out with the VSM described in chapter 6. First, a pre cycle from $-3$ T to $+3$ T was performed at every temperature, to ensure that the sample was fully penetrated in the following measurements; then the applied field was cycled in this manner: $0$ T $\rightarrow -3$
Table 7.5: Fitting parameters for the samples, power law dependence.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$J_c(0)$ [A]</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1284.3</td>
<td>0.748</td>
</tr>
<tr>
<td>2</td>
<td>1511.3</td>
<td>0.736</td>
</tr>
<tr>
<td>3</td>
<td>1813.1</td>
<td>0.645</td>
</tr>
<tr>
<td>4</td>
<td>2972.9</td>
<td>0.724</td>
</tr>
</tbody>
</table>

Figure 7.13: Pinning force plot for the samples under study.
Table 7.6: Dimensions of the rectangular samples used for the VSM measurements.

<table>
<thead>
<tr>
<th>Sample</th>
<th>w [mm]</th>
<th>l [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.96</td>
<td>5.81</td>
</tr>
<tr>
<td>2</td>
<td>4.08</td>
<td>5.76</td>
</tr>
<tr>
<td>3</td>
<td>5.03</td>
<td>6.18</td>
</tr>
<tr>
<td>4</td>
<td>4.4</td>
<td>5.4</td>
</tr>
</tbody>
</table>

T → +10.5 T → 0 T. The selected temperatures were: 1.9 K, 4.2 K, 7.5 K, 10 K, 15 K, 20 K, 25 K, 30 K, 40 K, 50 K, 60 K, 70 K, 77 K. The three possible magnetic field ramp rates (magnetic field change per unit time) were 100 mT/min, 200 mT/min and 500 mT/min. For each temperature the 500 mT/min ramp rate has been used, adding also the 100 mT/min and 200 mT/min ramp rates to the measurement at 4.2 K and 1.9 K. The field was applied perpendicular to the samples’ surfaces, the dimensions of the samples are summarized in table 7.6. The vibrational frequency was 20.4 Hz.

The hysteresis loops measured are presented in figure 7.14 for a ramp rate of 500 mT/min for sample 2. The critical current density values were calculated using the extended critical state model formula described in chapter 5 [4], [5]:

\[ J_{c}(B, T) = \frac{2\Delta M(B, T)}{w(1 - \frac{w}{l})} \]  

Where \( \Delta M \) is the local width of the magnetization loop at a selected value of applied magnetic field, \( w \) and \( l \) are the transverse dimensions of the sample and \( w < l \). For the widths, the values achieved from the optical imaging have been used, as the signal to be taken into account is just the one coming from the superconductor, except for sample 4, where we used the nominal value given by the producer, 0.36 cm.

7.4.1 \( I_{c}(B) \) dependencies

Sample 1

Also for the magnetization data, in sample 1 it is possible to fit the results with either a Kim or a power decay fit, at low temperature. The fitting parameters slightly differ from the ones found for the transport because of the wider range of points present in the magnetization measurements (no lower limit due to the power supply) and a lower value for the critical current inferred with this technique, which will be deeply discussed in the forthcoming paragraphs. The Kim fit works quite well at 1.9 K, 4.2 K and 7.5 K, while for higher temperatures it is almost perfect. The model
starts to fail above 40 K, where the decay (not considering the 0 T and 0.5 T points) becomes more or less exponential. This gives us a clue about different pinning mechanisms which activate at different temperatures, even though fairly qualitatively. In table 7.7 are summarized the fitting parameters found for the Kim dependencies at the temperatures at which the fit works for sample 1.

### Sample 2

The results for sample 2 are qualitatively very similar to the ones found for sample 1: the Kim fit works almost perfectly between 10 K and 40 K, quite well at low temperatures and poorly above 40 K. In table 7.8 are summarized the fitting parameters for the sample under consideration. The
Table 7.8: Kim fit parameters for sample 2 at different temperatures.

<table>
<thead>
<tr>
<th>T [K]</th>
<th>Jc(0) [A]</th>
<th>µ</th>
</tr>
</thead>
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</table>

Table 7.9: Power law fit parameters for sample 3 at different temperatures.

same qualitative considerations can be done for this sample as well, but nothing more can be said from this study.

Sample 3

For what concerns sample 3, the Kim fit does not work very well. As it can be seen from figures 7.15 and 7.16, Where $1/J_c$ is plotted against $B$ for 4.2 K and 25 K, there is a conspicuous deviation at low fields from the Kim behaviour (i.e. the straight lines drawn in figures 7.15 and 7.16), this feature is present up to 40 K, above that temperature the dependence becomes more or less exponential. A power decay fit, instead, works fine up to 40 K, the fitting parameters are shown in table 7.9 for completeness.
Figure 7.15: $1/I_c$ plotted against $B$ for a temperature of 4.26 K. The black line is a guide to the eye in order to show the deviation from the Kim dependence at low fields.

Figure 7.16: $1/I_c$ plotted against $B$ for a temperature of 25 K. The black line is a guide to the eye in order to show the deviation from the Kim dependence at low fields.
Sample 4

In sample 4, instead, the Kim fit works very well for every temperature up to 30 K. As discussed previously and summarized in appendix A, the $I_c \sim B^{-1}$ dependence is related to correlated pinning by low angle grain boundaries, which gives a reason to think about a very well engineered pinning structure in this sample, which would explain the very high in-field critical current values for this sample. As before, no more precise conclusions can be drawn, though. The fitting parameters are summarized in table 7.10.

7.4.2 $I_c(T)$ dependencies

It is very interesting to study the $I_c(T)$ behaviour, as there are different theoretical predictions for them based on different pinning mechanisms. If one considers a weak pinning, which is characterized by single vortex pinning with elastic distortion of the flux lines and a small disorder parameter and can be related to pinning caused by point-like defects, the theoretical dependence has been derived by G. Blatter et al. [6] and follows the equation:

$$I_c^{\text{weak}}(T) = I_c^{\text{weak}}(0)e^{-\frac{T}{T_0}}$$  \hspace{1cm} (7.4)

The parameters $I_c^{\text{weak}}$ and $T_0$ indicate the weak pinning critical current at $T = 0$ and the weak pinning characteristic temperature, respectively. On the other hand, for a strong pinning given by correlated defects such as columnar pins and grain boundaries, which can pin more fluxons in the same defect and act on a relevant part of the thickness of the superconductor, the dependence has been derived by D. R. Nelson and V. M. Vinokour [7] and is namely:

$$I_c^{\text{strong}}(T) = I_c^{\text{strong}}(0)e^{-3\left(\frac{T}{T^*}\right)^2}$$  \hspace{1cm} (7.5)

A more detailed outline of these theoretical results is given in Appendix B. Clearly, it is very unlikely that in one sample there will be only one

<table>
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<th>$k$ [AT]</th>
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</table>

Table 7.10: Kim fit parameters for sample 4 at different temperatures.
type of these two pinning behaviours, as oxygen vacancies (acting as single vortex pinning sites) and grain boundaries are intrinsically present in every “macroscopic” sample which could be prepared. It is then legit to suppose that these two behaviours coexist, which has also been experimentally confirmed by Plain et al. [8] who showed that in melt textured YBCO films these two behaviours are both present. With the assumption that the currents actually sum together with some weights, $A$ and $B$, one gets:

$$I_c(T) = A\tilde{I}_c^{weak}(0)e^{-\frac{T}{T_0}} + B\tilde{I}_c^{strong}(0)e^{-3\left(\frac{T}{T^*}\right)^2} \quad (7.6)$$

The weights of the two currents can be inglobed in the pre-factors $\tilde{I}_c^{weak}(0)$ and $\tilde{I}_c^{strong}$, as they are both free parameters, giving:

$$I_c(T) = I_c^{weak}(0)e^{-\frac{T}{T_0}} + I_c^{strong}(0)e^{-3\left(\frac{T}{T^*}\right)^2} \quad (7.7)$$

The experimentally derived magnetization data have been with this theoretical expression.

**Sample 1**

None of the behaviours related to equation 7.4 and 7.5 could fit the data, and the complete equation 7.7 has been used, with $I_c^{weak}(0)$, $I_c^{strong}(0)$, $T_0$ and $T^*$ as free parameters. In the forthcoming discussion, the ratio of the 2 pre-factors, namely $I_c^{strong}(0)/I_c^{weak}(0)$, will be taken into account: this generalizes my conclusions as it eliminates the contribution deriving from the disagreement of these measurements with the transport data, which I anticipated before. The fit has been performed for all the experimentally determined $I_c(T)$ points at the applied magnetic fields from 1 T to 10 T. In figure 7.17 is possible to see the experimental points for all the fields and the fitted curves for some selected fields. I avoided to put all the fitting curves for clarity, but the fit works well at every applied field. The fit works very well, especially at high field, giving as a result the parameters summarized in table 7.11.

Moreover, it was possible to estimate the irreversibility field of the samples at 77 K, which lied between 5 T and 7 T. This is in agreement with the result found by J. Hanisch et al. [19] concerning the temperature dependence of the irreversibility field of YBCO. Due to the reaching of the irreversibility field, for high fields (i.e. above 7 T) there was one missing point for the fit.

As it can be seen from table 7.11, the values for $T_0$ and $T^*$ are in the range:

$$T_0 = 7 \div 21 \text{ K}$$
$$T^* = 66 \div 85 \text{ K}$$
This result is in agreement with the one of Ö. Polat et al. [9]. Their sample was a SuperPower tape similar to mine, but with Y-Gd additions which created (Y-Gd)$_2$O$_3$ precipitates acting as weak pinning centres. In the paper the ranges found for $T_0$ and $T^*$ were:

$$T_0 = 9 \div 13 \text{ K}$$

$$T^* = 78 \div 81 \text{ K}$$

In my sample there are no artificial pinning sites, while in theirs there are the nanoparticles - this can explain the slight difference found in the parameters.

**Sample 2**

As for sample 1, the single type of pinning functions could not fit the data on the whole range of temperatures, so I used the complete formula also in this case. The fit works well at every external field also for this sample, as it can be seen in figure 7.18, and the parameters are presented in table 7.12. Comparing the parameters with the previous sample, it can be seen that there is some difference in all of them, therefore the type of pinning is different between sample 1 and 2. A detailed discussion will be given in section 7.5.

**Sample 3**

Also in this sample I used the complete formula for the fitting. It works well for this tape, too, as it can be seen from figure 7.19, giving the parameters
Table 7.11: Fitting parameters for the $I_c - T$ dependence for sample 1.

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Figure 7.18: $I_c$ vs. $T$ plot for sample 2. The full lines represent the fitting curves for $B=1$ T, $B=3$ T, $B=4$ T and $B=10$ T.
Table 7.12: Fitting parameters for the $I_c - T$ dependence for sample 2.

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Table 7.13: Fitting parameters for the $I_c - T$ dependence for sample 3.

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reported in table 7.13. Already from the fitting curves one can see some sort of difference between this sample and sample 2 and some similarity with sample 1.

Sample 4

As for the other tapes, also for the last one the complete equation 7.7 has been used and works well, as shown in figure 7.20. In this sample, the point at 77 K is not present above 5 T, explaining the behaviour of the fitting parameters around 5 T. The parameters are summarized in table 7.14 and show some difference with respect to the previous samples. This will be discussed in detail in the forthcoming section.
Figure 7.19: $I_c$ vs. $T$ plot for sample 3. The full lines represent the fitting curves for $B=1$ T, $B=3$ T, $B=5$ T and $B=10$ T.

Figure 7.20: $I_c$ vs. $T$ plot for sample 4. The full lines represent the fitting curves for $B=1$ T, $B=3$ T, $B=5$ T and $B=10$ T.
Table 7.14: Fitting parameters for the $I_c - T$ dependence for sample 4.

### 7.5 Pinning properties discussion

Now that all the data have been presented, many points need to be discussed. First of all, it is easy to see that the study of the $I_c - T$ dependence allows us to gather much more information than any other kind of study, as $I_c - B$ dependence, pinning force or n-value. It is therefore the preferred way to study pinning in my experimental work. In order to follow the forthcoming discussion, one should refer to figure 7.21 and 7.22, where the fitting parameters $T_0$, $T^*$ and Strong/Weak are plotted for each sample against the applied field, $B$.

The first point I want to discuss about is the difference between sample 1 and 2. As it concerns sample 1 we know there are no artificial pinning sites present, so the flux pinning is mainly achieved by grain boundaries and point defects such as oxygen vacancies. As it can be seen from figure 7.21 for the Strong/Weak parameter, the weak pinning (point defects) has a considerable weight at low fields, while for high fields the strong pinning (grain boundaries) prevails. For what concerns sample 2, instead, one can see that the Strong/Weak weight always stays below 1, indicating a prevalence of the weak pinning. This can also be seen at the higher value of $T_0$, which gives a clue about the weak pinning potential, for sample 2 with respect to sample 1 at every field. The change of trend at 7 T is due to the already discussed fact that above that field the 77 K experimental point was missing from the fit. Since sample 2 showed a slightly better $I_c$ performance with respect to sample 1, it can be concluded that in this sample there have been added most probably some nanoparticles, such as, for example (Y-Gd)$_2$O$_3$ as in the sample used by Polat [9], which act as uncorrelated pinning centres enhancing the in-field properties of the sample and giving more weight to the weak pinning component of the critical current.
Figure 7.21: Dependence of the fitting parameters $T_0$ and $T^*$ on the magnetic field $B$ for samples 1 and 2.

Figure 7.22: Dependence of the fitting parameters $T_0$ and $T^*$ on the magnetic field $B$ for samples 3 and 4.
Concerning sample 3, one can see a trend for the parameters $T_0$ and $T^*$ which is very similar to sample 1: this indicates that the pinning properties of the two samples are similar. Looking at the values of the critical current at 77 K, self-field one notices that sample 3 clearly features a better transport performance, with a critical current higher by $\sim 200$ A than sample 1 (200% of improvement), but if one considers the low-temperature and in-field performances, this improvement abruptly drops, staying between 42% at high fields and 28% at low fields. This means that in this tape the excellent transport properties are very probably due to a very well controlled sample growth, layer uniformity and absence of defects rather than an artificial pinning structure. The pinning is achieved as well by natural defects, as oxygen vacancies and grain boundaries, like in sample 1, but in this case the mean size of the grain boundaries is bigger (i.e. better layer uniformity), so the strong pinning starts to act at higher fields with respect to sample 1, as it can be seen from the Strong/Weak parameter behaviour in figure 7.22. In the fashion of the pinning model presented in appendix A, this feature can be easily explained: in order to accommodate themselves in a defect, the flux lines of the lattice will have to strain: a vortex will be pinned if the energy gained by pinning is higher (in absolute value) than the strain energy necessary to accommodate the flux line in the pinning site. If the mean size of the grains is big, more strain energy will be necessary to accommodate a flux line, as the elastic energy can always be written as:

$$E_{\text{elastic}} = \frac{1}{2}k(\Delta x)^2$$

and $\Delta x$ is proportional to the grain size, in first approximation. Now, also the field dependence of the lattice has to be considered: as shown in chapter 1, the flux line lattice spacing has been proven theoretically to follow the equation:

$$d_{\text{latt}} = \sqrt{\frac{2\Phi_0}{\sqrt{3}B}}$$

where $\Phi_0$ is the flux quantum and $B$ is the magnetic flux density. As the field increases, the number of flux lines per unit area will increase (their lattice parameter decreases) and the lattice will become more and more dense. This will eventually energetically favour the pinning, as the flux lines will be statistically closer to the defect where they will accommodate into. This explains the different increase of the Strong/Weak parameter for sample 3 with respect to sample 1: in sample 1 there are more grain boundaries which means that their mean dimensions will be smaller, and it will be easier to accommodate the flux lines therein, thus the strong pinning will be effective at lower fields. For sample 3, instead, a higher field is necessary to make effective the grain boundaries pinning because of their bigger size, which also explains the better self-field current transport.
This idea is also supported by the $I_c - B$ behaviour found in the previous section. The main result of the pinning model proposed by Pan et al. [21], [22] and summarized in appendix A, which involves pinning caused by low angle grain boundaries, is that the $I_c(B)$ dependence at “high” fields follows a simple $I_c \sim B^{-1}$ law. The meaning of “high” for the field needs to be clarified, though: studies on the $I_c(B)$ functions derived in the model actually show that the crossover point between the $B^{-1}$ dependence and the complete one is given by the mean size of the grain boundaries; in figure 7.23 a normalized value of $1/I_c$ is plotted against $B$, using the complete formula given by the LAB pinning model, for different mean grain sizes, using all the other parameters found by the authors in [21]. It can be seen how, increasing the grain size, the linear dependence shifts to high fields and slowly disappears, and this feature has been seen to be present also for sample 3 (see figures 7.15 and 7.16).

Sample 4 is very interesting. Its performances at 77 K and self-field are quite poor compared to the other ones, while at low temperature and in-field it is the best tape. This indicates a very well engineered artificial pinning structure. Also, from figure 7.22 it can be seen that the field dependence of the fitting parameters for this sample is very different from the previous ones. The first feature I would like to point out is the extremely high Strong/Weak ratio, for low fields, which goes up to $\sim 2.5$, then decreases slightly below 1 at high fields. This is the first case among the 4 samples where the trend of the parameter Strong/Weak is a decreasing one, which indicates a strong difference in the pinning landscape for this last sample. The characteristic temperatures are as well quite different from the ones found previously, and so is the trend for $T^*$. What could be suggested is a material with both kinds of pinning: the strong pinning sites are probably introduced as columnar defects and very effective at low fields (up to $\sim 3-4$ T), the weak pinning sites as point-like nano-inclusions which become more effective at high field, implying a quite low density of point-like defects for the same reasoning made for the previous sample.

What can also be seen and is very important to point out is that, as the field increases, the ratio between the strong and weak pinning weight factors tends to a more or less constant value close to 1. Again, this can be explained with the same reasoning done for sample 3: as the flux line lattice gets more dense, the strain energy required to accommodate a flux line in a defect decreases, and the two types of defects tend to behave the same. In other words, when the fluxon lattice is very dense, there is basically no distinction between accommodating a flux line in a point defect or in a correlated one, as the strain is low and the energy scales of the two potential wells become more similar.

Another interesting feature to be underlined is the poor performance for this sample at 77 K, self-field, just 54 A of critical current, compared with the excellent performances achieved at low temperature and in-field. This
means that the intrinsic properties of the material are poor due to the fact that there are many defects used for pinning. The pinning properties of this sample, though, are exceptional, as the defects which limit the current in self-field become effective pinning centres as soon as an external field is applied. To stress again this point, namely the counterplay between the intrinsic transport properties and the pinning ones, I plotted in figure 7.24 the critical currents of the 4 samples, normalized to their value at 77 K and self-field, at an applied field of 1 T. This normalized value will be named the lift factor of the samples, namely

\[
\text{Lift Factor} = \frac{I_c(B,T)}{I_c(sf,77K)}
\]

It can be seen how sample 4 takes the high-ground over all the others and, even more important, how sample 3, which has the best performances in self-field due to its high degree of crystallinity, is the one with the worst lift factor. This trend is present at every external applied field.

On the other hand, if one normalizes the critical currents with respect to the critical current at 1.9 K, namely:

\[
\text{Normalized } I_c = \frac{I_c(B,T)}{I_c(B,1.9K)}
\]

the intrinsic pinning properties can be discussed, as the material-dependent transport features are cancelled out with this normalization and another
Figure 7.24: Normalized $I_c$ values with respect to $I_c(77K, sf)$ vs. $T$ dependencies, at an external field of 1 T.
Figure 7.25: Normalized $I_c$ values with respect to $I_c(1.9K, B)$ vs. $T$ dependencies, at an external field of 1 T.

An interesting characteristic can be found. At low fields, the 4 curves have slight different behaviours especially for what concerns sample 3 and 4 (figure 7.25). As the field is increased the 4 curves tend to collapse on the same trend (figure 7.26), suggesting an equilibration of the pinning properties (strong and weak pinning centres gain more or less the same weight) as the fluxon lattice becomes more and more dense, as already discussed previously referring, though, to other parameters.

One more comment can be made concerning the pronounced difference between sample 3 and 4. As the first one has the best performances in self field and at 77 K and the second one has its best at low temperature and in field, one wonders whether there is a crossover point at which one tape leaves the high-ground to the other. I therefore compared the performances of the two tapes at different temperatures and fields. The application of Bean’s formula on the magnetization loops is not legit when the external field is less than the full penetration field (in the order of hundreds of mT for these kind of samples), but just for comparison I calculated the self-field critical currents for samples 3 and 4 from their magnetization loops at various temperatures. The absolute value may not be correct, but as the geometry is more or less the same and the full penetration field will not change too much from one sample to another, some conclusions can be
Figure 7.26: Normalized $I_c$ values with respect to $I_c(1.9K, B)$ vs. $T$ dependencies, at an external field of 7 T.
anyway drawn. The interesting result is that, as it can be seen in figure 7.27, zero applied field, the critical current of sample 3 (rescaled to match the geometry of sample 4) is higher than the one sample 4 can carry. As soon as an external field is applied, though, sample 4 takes the high-ground up to a certain temperature at which the curves intersect. This crossover temperature increases as the field increases, underlining again how sample 4 is designed for in-field applications and all its performances are based on well engineered flux pinning, while sample 3 for self-field ones with no artificial pinning. An example is shown in figure 7.28 for an applied external magnetic field of 2 T.

As explained before, the $I_c(B)$ dependencies can be roughly divided into 3 regions for (more or less) all the samples: a region where there is a small low field deviation from the Kim behaviour, at temperatures lower than 10 K, a region where the Kim relationship works well, up to $\sim 30 \div 40$ K, and the last high-temperature region where something similar to an exponential decrease of $I_c$ can be found. If one compares these crossover temperatures with the characteristic ones found in the $I_c(T)$ analysis, namely $T_0$ and $T^*$, one finds a good agreement of the first crossover point with $T_0$, but a second crossover point is quite higher than $T^*$. With a closer look at equation 7.7, though, one can see that for the strong pinning function, there is a pre-factor of 3 in the exponent. If one incorporates this factor in the parameter $T^*$, as shown in equation 7.10, the parameter is scaled of a factor of $\sqrt{3}$:

$$I_c(T) = I_c^{weak}(0)e^{-\frac{T}{T_0}} + I_c^{strong}(0)e^{-\left(\frac{T}{(T^*/\sqrt{3})}\right)^2} \quad (7.10)$$
Figure 7.28: $I_c$ vs. $T$ plot for sample 3 and 4 in an external field of 2 T

The rescaled parameter $\tilde{T}^* = T^*/\sqrt{3}$ lie between 30 K and 50 K for all the samples, yielding a good agreement with the second crossover point also for this parameter. This brings even more correspondence between the $I_c(B)$ and $I_c(T)$ studies.

### 7.6 Correlation between transport and magnetization measurements

This paragraph will restrict the study just to the data at 1.9 K and 4.2 K, since these are the only temperatures for which both kinds of data are available. When one compares the results obtained with the two methods, the first thing that comes out is that the values obtained for the magnetization measurement are in general lower than the ones for the transport one, as it can be seen in figure 7.29. This difference stays between $\sim 15\% \div 30\%$.

Besides, the critical current values inferred from the magnetization loops are slightly lower for lower ramp rates. The first thing one could think about is a correction due to a different electric field. Indeed, the transport critical current is the current that develops inside that superconductor an electric field equal to the chosen critical one, in my case $1 \mu V/cm$. During the magnetization measurement, the current circulating in the sample and creating the magnetic moment is different from the transport one if the electric field inside the superconductor is different from the one chosen for the transport measurements. So the first thing to do is an evaluation of
the electric field in the sample during a magnetization measurement. From Lenz’s induction law one gets:

\[ E = s_p \frac{dB}{dt} \] (7.11)

Where \( s_p \) is the effective size of the sample, namely the ratio between its area and its perimeter. With this formula, one finds the values for the electric field in each sample as summarized in table 7.15. These electric fields are much lower than the critical field used during the transport measurements. I tried to scale down the values of the transport current using the calculated \( n \)-values and the power law \( E \sim \left( \frac{I}{I_c} \right)^n \), evaluating the \( I_c \) for a critical field of 0.1 \( \mu \)V/cm, which is close to the one corresponding to the 500 mT/min ramp rate. Unfortunately, the results were not satisfying both for the rescaled values and for the measured ones: the values are obviously closer than before, but as the \( n \)-values of the transition are very high (between 45 and 55) this correction is very small and cannot account for the quite large difference experimentally found. In some of their papers, A. V. Pan et al. [11], [23], were fitting their results of a VSM measurement with their theoretical Low Angle Boundaries pinning model, which takes into account also the electric field inside the superconducting volume. They found that with the electric field calculated according to the relationship I used they could
<table>
<thead>
<tr>
<th>Sample</th>
<th>E - 100 mT/min</th>
<th>E - 200 mT/min</th>
<th>E - 500 mT/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0193 µV/cm</td>
<td>0.0385 µV/cm</td>
<td>0.0963 µV/cm</td>
</tr>
<tr>
<td>2</td>
<td>0.0196 µV/cm</td>
<td>0.0392 µV/cm</td>
<td>0.0981 µV/cm</td>
</tr>
<tr>
<td>3</td>
<td>0.0229 µV/cm</td>
<td>0.0459 µV/cm</td>
<td>0.1146 µV/cm</td>
</tr>
<tr>
<td>4</td>
<td>0.018 µV/cm</td>
<td>0.036 µV/cm</td>
<td>0.09 µV/cm</td>
</tr>
</tbody>
</table>

Table 7.15: *Induced electric field in each sample for every ramp rate.*

not fit the experimental data. They claimed that the vibrational frequency of the VSM is an important factor as well, as different vibrating frequencies bring to a different reorganization of the fluxon structure across the sample. This is basically an experimental artifact, if the sample vibrated completely parallel to the applied field direction there would not be any change. Clearly, the sample could also move along the transverse direction and this would eventually change the field distribution. Up to now there is no more specific explanation for this effect. Changing the vibrational frequency from 2 Hz to 60 Hz, they found that the values for the critical electric field spanned between $0.1 \mu V/cm$ and $10^{-7} \mu V/cm$.

What has been done is to find the critical field which would lead the transport curves to collapse on the magnetization ones, scaling them with the power law and considering the same n-value for the original and scaled relationship. The fitted electric fields stay in the range proposed by Pan for the first 2 samples, which is consistent, as the vibrational frequency in my VSM is 20.4 Hz, an intermediate value of the two extreme ones used by the authors, but for sample 3 and 4 this fitted field can reach values of $10^{-9} \mu V/cm$, due to the high n-value and mismatch of the two curves. It can be said that for these samples, there could be a sum of corrections to be made, as the self-field corrections could be more relevant due to the high value of current they can carry. A self-field correction would anyway shift the curves on the $B$ axis, but the difference along that axis increases with the field: since the critical current decreases with the field, the self-field effect should decrease as well with the field, so it can be said that there could be a contribution due to the self-field, but it is not for sure the only one. The hypothesis of Pan et al. about the vibrational frequency contribution needs to be tested, performing magnetic measurements at different frequencies. As one tends to zero frequency, the reorganization of the flux lines will be less predominant and one would eventually reach the values obtained with the transport measurements. So the point would be to lower the VSM frequency, but this, as shown in chapter 6, would reduce the Signal-to-Noise Ratio, thus making the measurement more difficult. Fortunately, as already said, all the previous analysis made on the $I_c(T)$ dependencies from the magnetization measurements just involves the ratio of the two different currents due to
the different types of pinning, and so every quantitative error on the critical current does not affect the conclusions I drew.

Another source of mismatch between these two types of measurements could be an uncertainty of the width of the superconducting layer (which comes into play linearly in the critical current): in figure 7.30 it is possible to see the comparison between transport and magnetization presented before (see figure 7.29), adding error bars to the magnetization measurements assuming an error of ±10% on the width of the superconducting layer. As it can be noticed, this effect alone would not lead to a complete agreement of the results.

7.7 Further comments

In this section I will point out some other interesting features noticed during data analysis, which will be investigated with future measurements. One thing that I found out during the analysis of the magnetization loops is that these cycles are not fully symmetric with respect to the x axis. This feature can be clearly seen from the hysteresis loop of sample 4 (the one in which this effect was present the most) shown in figure 7.31, measured at 77 K, as it was the temperature at which the samples featured the smallest magnetic moment (i.e. smallest critical current due to the high temperature) and thus also small asymmetries can be seen well.
This contribution is most likely due to the substrate magnetic moment: all the substrates are made of Hastelloy C-276 or similar materials. The magnetic properties of Hastelloy have been investigated by J. Lu et al. [10], and they found out that this material behaves as a paramagnet for all temperatures above 3 K, while it has a spin glass behaviour below 3 K. The asymmetry in the loops can be related to the paramagnetic signal of the substrate, but the spin glass properties, which in theory should be present at 1.9 K, are difficult to infer from the data, as at this temperature the superconducting signal is very high. Magnetic measurements on the substrates will be a very interesting study to be performed, also to see if they have any other contributions which could alter the VSM measurements. The presence of the paramagnetic substrate should not change substantially the field distribution in the volume of the superconductor when the field is applied perpendicular to the sample, while its influence will be more pronounced when the field is applied parallel to the tape. A scheme is given in figure 7.32 in order to better understand this reasoning.

Another interesting study which should be performed is a Scanning Electron Microscope imaging of the cross section of the samples in order to find out the exact thickness of the superconducting layer and in case achieve information about the microstructure of the superconducting layer and the possible inclusions and defects which act as pinning sites. The exact thickness value will not anyway make any change concerning the difference found between transport and magnetization data, as the thickness cancels out in the equations when calculating the critical current from the Bean’s model critical current density. Sample preparation is a very delicate issue in this
regard, due to the layered structure of the tapes, as shown in chapter 3. The best technique would be a polishing obtained via Focused Ion Beam, which is unfortunately not available at the CERN laboratories.

7.8 References


Figure 7.32: Reference scheme to better understand the reasoning made on the paramagnetic substrate effect in the two different field configurations.


Chapter 8

Conclusions

In this work, a wide study of the pinning properties of YBCO tapes is presented. It has been carried out via two types of measurements: transport I-V characteristics at liquid helium and liquid nitrogen temperatures and magnetization measurements in a VSM. The magnetization measurements turned out to be the ones giving the most information about the pinning landscape in the samples. This is basically due to the possibility of performing magnetization measurements in a wide range of temperatures, while for the transport ones only 1.9 K and 4.2 K are available. Studies of the temperature dependence of the critical current allowed me to draw a picture of the underlying pinning mechanisms, according to existing theoretical models which provide predictions on the $I_c(T)$ dependence that have been used to fit the data. The fitting equation was:

$$I_c(T) = I_{c, weak}^{(0)} e^{-\frac{T}{T_0}} + I_{c, strong}^{(0)} e^{-3\left(\frac{T}{T^*}\right)^2}$$ (8.1)

Although the field dependences of the critical current could not give quantitative information concerning the pinning, they are anyway the main tool for evaluating and comparing the performances of the tapes.

Nevertheless, a difference between the critical current evaluations found via transport measurements and the magnetization ones has been found, which still needs to be explained and will be part of the future work. The main reason for this disagreement is, in my personal opinion, a geometrical correction taking into account demagnetization effects and the effective area of the superconductor, which could be smaller due to damaging during sample preparation.

The results based on fitting the theoretical models to the measured data are, however, independent from this mismatch, due to a suitable choice of the parameters that eliminates all the contributions arising from the magnitude of the critical current deduced by the magnetization loops via the Bean’s model. The results found therefore do not lack generality.
One commercial and three research tapes provided by different producers have been analyzed using the experimental techniques summarized previously. The first sample was a SuperPower standard tape with no artificial pinning structures, which gave a reference for the other samples. Studies of the field dependence of the fitting parameters for the $I_c(T)$ showed characteristic weak and strong pinning temperatures in the range of, respectively:

$$T_0 = 7 \div 21 \text{ K}$$

$$T^* = 66 \div 85 \text{ K}$$

with a decreasing trend as the field increases. Sample 2 showed a higher $T_0$ and a higher Strong/Weak ratio, which, together with the better current carrying performance with respect to sample 1, suggests an engineered weak pinning structure, via nanoparticle or point defect addition.

On the other hand, the trends obtained for sample 3, together with studies on the $I_c(B)$ characteristic and existing models, suggest a structure without artificial pinning sites for that tape. The difference in the Strong/Weak parameter with respect to sample 1, though, indicates a better layer uniformity, thus explaining also the excellent current carrying capability at 77 K and self-field for sample 3.

The dependence of the fitting parameters on the applied field for sample 4 is completely different from the previous ones and its performances in an applied field and at low temperatures are by far the best among all the samples. This leads to the conclusion of a very well engineered pinning structure, most probably containing both columnar defects acting as strong pinning sites and point-like ones acting as weak pinning sites.

Moreover, the fitting parameters are in a quantitative agreement with the $I_c(B)$ trend changes with temperature, giving more credit to the results obtained.

Another interesting feature which has been pointed out is the equilibration of the pinning mechanisms at high applied fields. The Strong/Weak parameter, indeed, always tends to a value close to 1 as the field is increased, indicating that the two possible types of pinning sites acquire more or less the same energy scale. This has been explained considering the high fluxon density for high fields, at which the elastic displacement on the two different types of defects becomes energetically more or less the same. This characteristic has also been shown via a study of the $I_c(T)$ renormalized with respect to the $I_c$ at 1.9 K. In this way, the differences in the intrinsic transport properties of different tapes are compensated and only the differences due to pinning features remain visible. It has been shown that at high fields the curves for all the samples tend to collapse to a single curve, which indicates that in this field region the effective pinning landscape is very similar for all the samples.
The comparison between sample 3 and 4 has been a source of many interesting discussions. While at 77 K, self-field sample 3 has a critical current approximately 6 times higher than sample 4, when the field is applied and the temperature is lowered the sample 4 takes the high-ground over sample 3 for a certain range of temperature. There is a crossover temperature at which the $I_c(T)$ curves for the 2 samples intersect, and this temperature increases as the field increases. The high degree of crystallinity inferred from the self-field critical currents and pinning properties of sample 3 does not provide many pinning centres, but allows a good current transport in self-field conditions. On the other hand, for sample 4 a well engineered defect structure provides effective pinning centres, although depressing the intrinsic transport properties. This idea is also supported by the zero-field critical currents inferred from the magnetization loops, which are always higher for sample 3. Bean’s model is not valid for zero applied field, but the corrections should come into play with the same weight in both samples, so general qualitative conclusions can be drawn anyway. These features point out the counterplay between the intrinsic transport properties of the samples, governed by the layer uniformity and crystallinity, and the pinning ones, governed by the defect structure which is effective for pinning.

An interesting test which will be performed in the future will be a measurement concerning the performances of the tapes for an externally applied field along $ab$ (i.e. parallel field). In this configuration the Lorentz force acting on the fluxons is zero and the situation is close to the self-field conditions. Moreover, for this geometry the CuO$_2$ planes provide natural correlated pinning sites for both tapes, so one would expect a very different critical current behaviour due to the different level of crystallinity of the YBCO layer in the two tapes. Usually the currents for $B$ applied along $ab$ are much ($\sim 4$ times) higher than the ones for $B$ applied along $c$, and so the power supply limit for transport measurements will be an issue. Also the magnetization measurements will be difficult to perform, due to the paramagnetic contribution of the substrate and the high requirements on the precise alignment between the sample’s surface and the applied field. In this fashion it will become very important to find a good fitting function to extrapolate the critical current values at low fields for the transport measurements.
Appendix A

The low angle boundaries pinning model

I will introduce here a theoretical model developed by V. Pan et al. [1] concerning pinning caused by low angles grain boundaries in YBCO thin films. This is probably the most suitable for the case in which artificial pinning is not present and the pinning centres are mainly due to edge dislocations, that seem to have a radius similar to the coherence length in the $ab$ plane.

A.1 Small field applied along $c$

First, a thin film geometry, where the demagnetization factor is close to unity, is considered. In this approximation, $B \simeq H$ (I will use here cgs units to stick to the paper’s notation) and the mean density of vortices can be expressed as:

$$n_v = \frac{H}{\Phi_0}$$  \hspace{1cm} (A.1)

Where $\Phi_0 = \frac{hc}{2e}$ is the flux quantum, with $h$ the Planck’s constant and $e$ the electron charge. For low fields, one can neglect the interaction between vortices, and think about a single vortex pinning picture. Theoretical results from Blatter et al. [2] can give expressions for the maximum pinning force for low temperatures, at which the edge dislocations’ radius is similar to the coherence length in the $ab$ plane. In formulae:

Characteristic vortex energy \hspace{1cm} $\epsilon_0 = \left( \frac{\Phi_0}{4\pi\lambda} \right)^2$  \hspace{1cm} (A.2)

Reduced temperature \hspace{1cm} $\tau = 1 - \frac{T}{T_c}$  \hspace{1cm} (A.3)
Temperature dependent coherence length \[ \xi_{ab}(T) = \frac{\xi_{ab}(0)}{\sqrt{T}} \] (A.4)

Maximum pinning force \[ f_{\text{pin}}^{\text{max}} = \frac{\epsilon_0}{\xi_{ab}} \] (A.5)

For higher temperatures, where the defect radius is smaller than \( \xi_{ab} \), the energy of a vortex uniformly displaced from the centre of the dislocation core by a distance \( u \) can be expressed as:

\[ \epsilon_{(\text{pin})}(u) \simeq -\frac{\epsilon_0}{2} \frac{r_0^2}{u^2 + 2\xi^2} \] (A.6)

Taking the first derivative with respect to \( u \), the pinning force can be found:

\[ f_{\text{pin}}(u) = \epsilon_0 \left( \frac{u r_0^2}{u^2 + 2\xi^2} \right)^2 \] (A.7)

Calculating the derivative with respect to \( u \) of the pinning force and putting it equal to zero one can find the value of \( u \) at which the pinning force density reaches its maximum, with the result:

\[ \bar{u} = \sqrt{\frac{2}{3}} \frac{\epsilon_0}{r_0^2} \xi \] (A.8)

\[ f_{\text{pin}}^{\text{max}} = \frac{9}{32} \sqrt{\frac{2}{3}} \epsilon_0 \frac{r_0^2}{\xi^3} \] (A.9)

### A.2 Higher field

The distance between vortices can be found in the Abrikosov theory [3] to be \( a = \sqrt{\frac{\Phi_0}{\pi}} \). When the field increases, the distance decreases and the interaction between the vortices must be considered. If there were not pinning centres, the fluxons would tend to form the well-known triangular lattice, but as far as the pinning regions have been introduced, the picture will change. Let us consider a mean density of pinning sites \( n_d \), much higher than the density of vortices \( n_v \) introduced before. This assumption is valid up to a field of 2 T, provided that \( n_d \) is in the order of magnitude of \( 10^{11} \) cm\(^{-2} \), so that it gives a mean distance between defects around \( 10^{-6} \) cm. However, if the edge dislocations are assumed to be located mainly within low angle boundaries, the average distance between them is much shorter than the one calculated above, and the condition \( n_d > n_v \) is satisfied up to 20 T. To find the relation linking the critical current with the applied field, one should perform a calculation on the energy gained by pinning. The fluxons will gain the maximum energy by pinning if the edge dislocation is located at a node of the triangular fluxon lattice, if not in order to successfully pin the vortex, the gain in energy due to pinning should be greater than the
energy due to the elastic deformation of the lattice. The pinning energy relation written previously is good on a length scale around $\xi$. On a larger scale, i.e. $\lambda$, the interaction between fluxons and pinning centres can be approximated as follows: $\epsilon_{\text{pin}}(u) \simeq \epsilon_{\text{pin}}(0)\delta(u)$. This means that if the vortex is pinned, it will gain an energy $\epsilon_{\text{pin}}(0)$, while it will not gain anything if it is unpinned. The general picture is the following: a density of vortices will be pinned by the lattice, $n_p$, while the ones which are too far away from the defect’s position so that the strain energy is too big will remain in their position and unpinned. The ratio between the pinned vortices and the total number of vortices is named accommodation function, and is the core of the theory. When a macroscopic current passes through the superconductor, the distorted lattice will rearrange due to the Lorentz force. The main assumption of the theory is that the accommodation function averaged over a macroscopic area is invariant with or without the presence of currents. When a current is applied, the free fluxons will remain in their position, due to the electromagnetic interactions that build the lattice, while the pinned ones will remain in the pinning sites if the total Lorentz force will not exceed the maximum pinning force.

\[ F_{\text{Lorentz}} = n_v f_{\text{Lorentz}} = n_v \frac{\Phi_0}{c} J \]  
\hspace{1cm} (A.10)

\[ F_{\text{pin}} = n_p(H,T) f_{\text{pin}} \]  
\hspace{1cm} (A.11)

\[ F_{\text{pin}}^{\text{max}} = F_{\text{Lorentz}} \]  
\hspace{1cm} (A.12)

\[ n_p(H,T) f_{\text{pin}}^{\text{max}} = n_v \frac{\Phi_0}{c} J_c \]  
\hspace{1cm} (A.13)

Which becomes:

\[ \frac{n_p(H,T)}{n_v(H,T)} f_{\text{pin}}^{\text{max}} = n_v \frac{\Phi_0}{c} J_c \]  
\hspace{1cm} (A.14)

In the left member of the last equation one can find the accommodation function defined above, the maximum pinning force is defined by the previous relation, and all the others variables are known. Therefore, by knowing the functional form of the accommodation function one can find the critical current behaviour with the applied field.

### A.3 The accommodation function

From reference [2], the energy due to a vortex line displaced from its equilibrium position of a quantity $\delta$ is:

\[ \epsilon_d(\delta) = c_{66}(H)\delta^2 \]  
\hspace{1cm} (A.15)
Where $c_{66}$ is the elastic shear stress modulus, that in the fluxon case takes the form:

$$c_{66} = \left( \frac{\Phi_0 H}{8\pi \lambda} \right)^2 = \frac{\epsilon_0}{4a^2} \quad (A.16)$$

While the maximum absolute value of the negative potential energy for pinning of normal edge dislocation cores for $u = 0$ is:

$$\epsilon_{pin} = -\frac{\epsilon_0}{2} \ln \left( 1 + \frac{r_0^2}{2\xi^2} \right) \quad (A.17)$$

The pinning will be successfully achieved when the sum of these two energies will be lower than 0. Then, equalising the two equations above one can find the critical displacement value above which the vortex is no longer pinned, obtaining:

$$\delta_c = \begin{cases} \frac{1}{2}a = \frac{1}{2} \sqrt{\frac{\Phi_0}{\pi H}} & \text{if } r_0 \simeq 2\xi \\ \frac{r_0}{\xi} \sqrt{\frac{\Phi_0}{\pi H}} & \text{if } r_0 < \xi \end{cases} \quad (A.18)$$

The probability of vortex capture will be the product of the probability of finding a vortex line lattice node inside the considered domain and the probability of finding it at a distance smaller than $\delta_c$. To begin with, consider that the domains are squares of random size $L$; the normalized $\Gamma$ distribution can be used to describe the statistical distribution of the domain length:

$$P(L) = \frac{\mu^\nu}{\Gamma(\nu)} L^{\nu-1} e^{-\mu L} \quad (A.19)$$

Where:

$$\nu = \left( \frac{\langle L \rangle}{\sigma} \right)^2 \equiv k^2 \quad (A.20)$$

$$\mu = \frac{\langle L \rangle}{\sigma^2} = \frac{k^2}{\langle L \rangle} = \frac{k}{\sigma} \quad (A.21)$$

And $\sigma$ is the dispersion of the size $L$ around the mean value $\langle L \rangle$. The probability density for a node to fall inside a domain of area $L^2$ is equal to the product of the area of the domain with the distribution function of the domains, then:

$$W(L) = L^2 P(L) \quad (A.22)$$

The probability of finding a flux line within a distance $\delta_c$ from the pinning centre is given by the ratio of the area of a $\delta_c$-wide stripe and the area of the domain, giving:

$$\tilde{P}(L, \delta_c) = \begin{cases} 1 - \frac{(L-2\delta_c)^2}{L^2} & \text{if } L \leq 2\delta_c \\ 1 - \frac{(L-2\delta_c)^2}{L^2} & \text{if } L > 2\delta_c \end{cases} \quad (A.23)$$
The first term comes out when the stripe occupies the whole domain area, the second one is the ratio of the areas, the nominator calculated as the total domain area minus the area not occupied by the strip. It can be seen that the accommodation function is a direct description of the critical current density behaviour, normalized to the zero field depinning current density, $J_{dep}^c(0, \tau) = c/\Phi_0 f_{pin}^{max}$. Putting all together the expression for the accommodation function becomes:

$$\frac{J_c(H, \tau)}{J_{dep}^c(0, \tau)} = \frac{n_p(H, T)}{n_v(H, T)} = \int_0^{2\delta_c} L^2 P(L) dL + \int_{2\delta_c}^{+\infty} L^2 P(L) \left[ 1 - \frac{(L - 2\delta_c)^2}{L^2} \right] dL \quad (A.24)$$

It can be seen that the dependence on the magnetic field is present only inside $\delta_c$, that goes like $\sim 1/\sqrt{H}$. The integrals can be expressed with the aid of Euler’s complete and incomplete $\Gamma$ functions, respectively defined as:

$$\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t} dt \quad (A.25)$$

$$\Gamma(x, \alpha) = \int_{\alpha}^{+\infty} t^{x-1} e^{-t} dt \quad (A.26)$$

And the accommodation function takes the form:

$$\frac{n_p(H, T)}{n_v(H, T)} = 1 - \frac{1}{\Gamma(2 + \nu)} \left[ 4\mu^2 \delta^2 \frac{1}{\mu \delta_c} - 4\mu \delta_c \Gamma(1 + \nu, 2\mu \delta_c) + \Gamma(2 + \nu, 2\mu \delta_c) \right] \quad (A.27)$$

Extending the model to domains with a rectangular shape, the $\tilde{P}$ distribution changes as follows (with $L_{min}$ the minimum value of the length of the domains along $x$ and $y$):

$$\tilde{P}(L, \delta_c) = \begin{cases} 
1 & \text{if } L_{\min} \leq 2\delta_c \\
1 - \frac{(L_x - 2\delta_c)(L_y - 2\delta_c)}{L_x L_y} & \text{if } L_{\min} > 2\delta_c 
\end{cases} \quad (A.28)$$

And the accommodation function becomes:

$$\frac{n_p(H, T)}{n_v(H, T)} = 1 - \frac{1}{\Gamma(2 + \nu)} \left[ \Gamma(\nu, 2\mu \delta_c) - 2\mu \delta_c \Gamma(\nu - 1, 2\mu \delta_c) \right]^2 \quad (A.29)$$

Which turns out not to be so different from the one obtained with squared domains, indicating a weak effect of domain shape on the $J_c(B)$ behaviour. Another aspect to be taken into account is the granularity of low angle boundaries, separated by a distance $d$, which can be treated with the aid of a geometrical function defined as:

$$f(x) = \begin{cases} 
\frac{1}{2} \left[ \sqrt{1 - x^2} + \frac{\arcsin x}{x} \right] & \text{if } x < 1 \\
\frac{\pi}{4x} & \text{if } L_{\min} > 2\delta_c 
\end{cases} \quad (A.30)$$
With $x = d/(2\delta_c)$. Putting this function inside the $\tilde{P}$ distribution, a final result for the accommodation function is found:

$$
\frac{n_p(H,T)}{n_v(H,T)} = 1 - \left[ 1 - f\left(\frac{d}{2\delta_c}\right) \left[ \frac{\Gamma(\nu, 2\mu\delta_c)}{\Gamma(\nu)} \right]^2 \right] \left[ 1 - f\left(\frac{d}{2\delta_c}\right) \left[ \frac{\Gamma(\nu, 2\mu\delta_c) - 2\mu\delta_c\Gamma(\nu - 1, 2\mu\delta_c)}{\Gamma(\nu)} \right]^2 \right] \tag{A.31}
$$

This function was found to be consistent with the experimental data. Moreover, it is to be noted that at high applied fields has an asymptotic $H^{-1}$ behaviour, similar to the dependence proposed by Kim.

### A.4 The flux creep contribution

The model presented previously does not take into account the effect of temperature. In a recent work [4] the theory has been extended including the flux creep contribution derived by P. W. Anderson and Y. B. Kim [5]. In this extension, a slightly different geometrical function is introduced, defined as:

$$
K_{sh} = \begin{cases} 
\frac{\pi \delta}{d} d^{d/2} & \text{if } \frac{2\delta}{d} < 1 \\
\frac{1}{2} \int_0^{\frac{2\delta}{d}} \left(1 - \left(\frac{x}{\delta}\right)^m\right)^{1/n} dx & \text{if } \frac{2\delta}{d} > 1
\end{cases} \tag{A.32}
$$

Moreover, the temperature dependence of the coherence length was taken to be:

$$
\xi_{ab}(T) = \frac{\xi_{ab}(0)}{\sqrt{1 - \left(\frac{T}{T_c}\right)^4}} \tag{A.33}
$$

The normalized critical current is equal to the accommodation function times the geometrical factor. To take into account the flux creep contribution, we can start from the expression of the vortex creep speed by Anderson and Kim [5], [4] and references therein, which is:

$$
v = a\nu_0 e^{-\frac{\nu}{KT}} \sinh\left(\frac{UL}{KT}\right) \tag{A.34}
$$

Here $a$ is the usual vortex lattice parameter, $U$ is the pinning potential, $UL$ is the Lorentz force potential, $\nu_0$ is the vibration frequency of the lattice. The Lorentz force potential was taken equal, with $\alpha$ a free parameter, to:

$$
U_L = \frac{1}{\alpha} J \tag{A.35}
$$

The accommodation function times the geometrical factor can be seen as the probability for a fluxon to be pinned, so one can assume the form of the pinning potential energy to be like:

$$
U = U_0 \frac{n_p}{n_v} K_{sh} \tag{A.36}
$$
The parameter $U_0$ indicates the probability for a single vortex to be pinned, that is taken to be $U_0 = \beta KT$ with $\beta$ another free parameter. Putting all this in the flux line velocity formula, using the fact that the electric field developed inside the material is $E = \mathbf{v} \times \mathbf{H}$ and considering the magnetic field applied perpendicular to the current flow direction, one easily obtains:

$$J = \alpha KT \text{arcsinh} \left[ \frac{E}{a(H) v_0(T)} e^{\beta \frac{\mu_0 H}{\pi v_0(H,T)}} K_{sh} \right]$$  \hspace{1cm} (A.37)

Although $\alpha$ and $\beta$ have been introduced as free parameters, with some physical reasoning one can have an idea about at least the order of magnitude of their values. The Lorentz force that affects one vortex is $F = \Phi_0 J_c d_p$, with $d_p$ the film thickness; assuming that the distance for a single vortex jump is in the order of $d$, the Lorentz force potential can be taken to be $U_L = \Phi_0 J_c d_p d$, giving $\alpha = \frac{1}{\Phi_0 d_p d}$. This parameter can be seen as the depinning current density giving a unit of potential pinning energy. To estimate $U_0$, and then $\beta$, one can use the expression for columnar defects [6]:

$$U_0(T) \sim \frac{U_v}{2} \ln \left( 1 + \frac{r_0^2}{2\xi^2} \right)$$  \hspace{1cm} (A.38)

With:

$$U_v = \frac{\Phi_0^2 d_p}{4\pi\mu_0\lambda^2}$$  \hspace{1cm} (A.39)

With these values one can easily calculate $\beta$. This model provides an analytical expression for the $J_c(B,T)$ behaviour for YBCO thin films with the field applied along the $c$ direction for a wide range of magnetic fields. Experimental results show slightly deviations, mainly due to other pinning mechanisms that can take place at certain conditions, and to the fact that at low fields single vortex pinning takes place, meaning that there is no longer a flux lattice and the elastic theory cannot be applied. Besides, the theory also takes into account the electric field inside the superconductor, a very important feature that needs to be taken into account as it experimentally defines the critical current.

A.5 References

Appendix B

Predictions of pinning theories for the $J_c(T)$ dependencies

In this section, an overview of some of the pinning theories is given, in order to better understand the origin of the fitting formulas used for the $I_c(T)$ dependencies discussed in chapter 7.

B.1 Weak pinning

Following the work by Blatter et al. [1], let me first consider a single flux line directed along $z$, in the presence of a current flowing in the $y$ direction and a weak pinning potential $\epsilon_{\text{pin}}$. The free energy can be written as (cgs units will be used to stick to the original paper’s notation):

$$F[u] = \int dz \left[ \frac{\ell}{2} (\partial_z u)^2 + \epsilon_{\text{pin}}(z, u) - f_L \cdot u \right]$$  \hspace{1cm} (B.1)

where the Lorentz force is written as

$$F_L = \frac{\Phi_0}{c} j \times e_z$$  \hspace{1cm} (B.2)

and $u(z)$ is the displacement vector of the flux line. One characterizes the random pinning potential by means of a correlation function, namely:

$$K(z - z', u - u') = \langle \epsilon_{\text{pin}}(z, u) \epsilon_{\text{pin}}(z', u') \rangle$$  \hspace{1cm} (B.3)

This correlation function depends on the nature of the disorder: if one assumes that pinning is due to point defects, which perturb the superconductor on a scale smaller than the coherence length $\xi$, the disorder correlation length is given by the size of the defect, which can be assumed to be zero.
But the smallest distance that can be resolved by the vortex core is the coherence length $\xi$, so $r_p \simeq \xi$ defines the disorder correlation length in the transverse direction.

The pinning energy can be expressed in terms of a pinning potential $U_{\text{pin}}$ and a shape factor $p$, depending on the type of disorder, namely:

$$\epsilon_{\text{pin}}(z, u) = \int d^2 R \ U_{\text{pin}}(r) \ p(R - u) \quad (B.4)$$

With this starting point, some calculations and early theoretical results and assumptions, one can finally get an expression for the correlator:

$$\mathcal{K}(z, u) = \gamma \xi^4 \delta(z) k(u) \quad (B.5)$$

where $\gamma$ is a disorder parameter and $k$ a geometrical function, given by:

$$\gamma = \begin{cases} 
2\pi \frac{\gamma_m}{\alpha} \left[ \frac{H^2}{4\pi} \right]^2 & \text{for disorder due to } \delta T_c \\
\frac{14\pi}{15} \frac{\gamma_m}{m^2} \left[ \frac{H^2}{4\pi} \right]^2 & \text{for disorder due to } \delta m 
\end{cases} \quad (B.6)$$

$$k(u) = \begin{cases} 
\frac{1}{\xi^2} & \text{for } u = 0 \\
\sim \left[ \frac{1}{u} \right]^2 \ln \frac{u}{\xi} & \text{for } u \to \infty 
\end{cases} \quad (B.7)$$

Now, if one considers a stiff vortex, the average pinning energy $\langle E_{\text{pin}}(L) \rangle$ is zero, but the fluctuations will be:

$$\langle E_{\text{pin}}^2(L) \rangle = \int dz \ dz' \langle \epsilon_{\text{pin}}(z, 0) \epsilon_{\text{pin}}(z', 0) \rangle = \gamma \xi^2 L \quad (B.8)$$

Now, considering the individual pinning force $f_{\text{pin}}$, the density of pins $n_i$ and the fact that the pinning is effective only for defects lying at a maximum distance $\xi$ from the flux line one can write:

$$\sqrt{\langle E_{\text{pin}}^2(L) \rangle} \simeq \sqrt{f_{\text{pin}}^2 n_i \xi^2 L} \xi \quad (B.9)$$

This allows relating the disorder parameter $\gamma$ to more familiar quantities, as the pinning force and the pin density, yielding:

$$\gamma \simeq f_{\text{pin}}^2 n_i \xi^2 \quad (B.10)$$

In this limit, one can see that the pinning force increases sublinearly with respect to $L$. Since the Lorentz force increases linearly, a stiff vortex is never pinned and the critical current density is zero. Taking into account vortices with finite elasticity, one has to cut-off the sublinear growth of $\sqrt{\langle E_{\text{pin}}^2(L) \rangle}$ at the collective pinning length $L_c$, above which the displacement $u$ increases beyond the characteristic length of fluctuations in the random potential. Each flux line is pinned independently. In order to find out this collective
pinning length, one has to write down the energy functional of a flux line, taking into account the elastic, pinning and Lorentz energies:

$$ \mathcal{F}(u, L) = \epsilon_0 u^2 - \sqrt{\gamma \xi^2 L} - j \frac{\Phi_0}{c} Lu \quad (B.11) $$

Minimizing this energy functional per unit length at $j = 0$ with respect to $L$ allows finding the collective pinning length, namely:

$$ L_c = \sqrt[3]{\frac{\epsilon_0^2 \xi^2}{\gamma}} \quad (B.12) $$

With this expression, one can find a relationship for the critical current density equaling the pinning force with the Lorentz one at this cutoff length:

$$ f_p = \sqrt{\gamma L_c} \quad (B.13) $$

$$ f_L = \frac{\Phi_0 L_c}{c} j_c \quad (B.14) $$

$$ j_c \approx \frac{c}{\Phi_0} \sqrt{\gamma \frac{\xi}{L_c}} = j_0 \left[ \frac{\xi}{L_c} \right]^2 \quad (B.15) $$

With $j_0$ the depairing current density:

$$ j_0 = \frac{4}{3\sqrt{3}} \frac{c \epsilon_0}{\xi \Phi_0} \quad (B.16) $$

In order to find the critical current density vs. temperature behaviour, I apply the model of classical flux creep for $B$ applied along $z$ and a current flowing along $y$. The electric field generated by vortex motion is:

$$ E = \frac{1}{c} B \times v \quad (B.17) $$

The quantities $B$ and $v$ are perpendicular to one another, and I will therefore use their moduli from now on. Making use of the third and fourth Maxwell equation,

$$ \frac{\partial B}{\partial t} = -c \frac{\partial E}{\partial x} \quad (B.18) $$

$$ \nabla \times B = \frac{4\pi}{c} j \quad (B.19) $$

I get two non-linear diffusion equations for the current and the field:

$$ \frac{\partial B}{\partial t} = -c \frac{\partial (vB)}{\partial x} \quad (B.20) $$

$$ \frac{\partial j}{\partial t} = \frac{c}{4\pi} \frac{\partial^2 (vB)}{\partial x^2} \quad (B.21) $$
The factor to be discussed now is the velocity of the vortices: one can model its behaviour considering the fact that the motion of the flux lines is due to their thermal activation, which unpins them from their pinning potential well. Thus, one can write for the velocity a simple Boltzmann-like relation (the energy is measured in Kelvin):

\[ v = v_0 e^{-U(j)/T} \]  

(B.22)

Putting this in equation B.21 one gets:

\[ \frac{\partial j}{\partial t} = -\frac{j_c}{\tau_0} e^{-U(j)/T} \]  

(B.23)

with \( \tau_0 = (4\pi \sqrt{\gamma/L_C})/(v_0 c \partial^2 B) \). This equation can be solved with logarithmic accuracy, yielding:

\[ U(j) = T \ln \left[ 1 + \frac{t}{t_0} \right] \]  

(B.24)

where \( t_0 = (\tau_0 T)/(j_c |\partial_j U|) \). If now one wants to find the time (but also temperature) dependence of the critical current density, an assumption on the functional form of \( U(j) \) needs to be done in order to invert equation B.24. Assuming a simple Anderson model [2]

\[ U(j \rightarrow j_c) \approx U_c \left[ 1 - \frac{j}{j_c} \right] \]  

(B.25)

one gets, for the current density:

\[ j(t) = \left[ 1 - \frac{T}{U_c} \ln \left[ 1 + \frac{t}{t_0} \right] \right] \]  

(B.26)

In a relaxation experiment the time \( t \) is a variable, but for other kinds of experiments it can be considered as a parameter, and for small enough temperatures with respect to the pinning potential one can then write:

\[ j(t) \approx j_c e^{-\frac{T}{T_0} \ln \left[ 1 + \frac{t}{t_0} \right]} \]  

(B.27)

If one regards the time as a parameter, as it can be done for the experiments I have been performing during this work, this equation also gives the temperature dependence of the current density, with \( T_0 = U_c/(1 + \ln(t/t_0)) \):

\[ j(T) \approx j_c e^{-\frac{T}{T_0}} \]  

(B.28)

This relation is valid for any current density in the superconductor: if one considers the critical current density, equation B.28 can be rewritten in the form:

\[ j_c(T) \approx j_{c,weak}(0) e^{-\frac{T}{T_0}} \]  

(B.29)

which is the same form of the one used to fit the data (namely, equation 7.4).
### B.2 Strong pinning

This section follows the work by D. R. Nelson and V. M. Vinokour [3], providing some theoretical basis in order to model pinning achieved by correlated disorder, such as twin or grain boundaries and columnar pins. The main task is to map the physics of flux lines onto the problem of a 2D boson localization. This implies the comprehension of the coefficients of the stiffness tensor for an elastic medium, \( C_{ijkl} \), which can be reduced to the stiffness matrix elements \( C(J) \) via the Voigt notation summarized in table B.1 [4].

The elements one should focus on are the \( c_{11} \) and \( c_{44} \) which respectively indicate the compressional and tilt moduli, a schematic picture is given in figure B.1.

At low temperatures, a Bose glass is present, where the flux lines are localized on the columnar pins, and the tilt modulus (i.e. the \( c_{44} \) coefficient) tends to infinity. Moreover, a Mott-insulator phase is also predicted, at low temperatures, when the fluxon density matches the columnar pin density. In this phase both \( c_{11} \) and \( c_{44} \) tend to infinity, and the magnetic field is locked at the matching field, namely:

\[
B = B_\Phi = n_{\text{pin}}\Phi_0
\]  

(B.30)

Besides, when the flux lines are free to move, which means that the irreversibility line has been overcome, the fluxon structure can be seen as an entangled-flux-liquid or a superfluid. The transition temperature from the Bose-glass phase to this superfluid one will be indicated as \( T_{BG} \), as it will be useful in the forthcoming paragraphs, even if there is not any exact theory for this transition. The fluxons can be then treated as 2D quasi particles, and the 2D quantum mechanics can be used: a columnar pin is modeled as a cylindrical well of radius \( b_0 \), as shown in figure B.2; the binding energy calculated using Ginzburg-Landau is:

\[
U_0 \simeq \begin{cases} 
\epsilon_0 \ln \left[ \frac{b_0}{\sqrt{2}\xi_{ab}} \right]^2 & \text{if } b_0 \gg \sqrt{2}\xi_{ab} \\
\epsilon_0 \left[ \frac{b_0}{2\xi_{ab}} \right]^2 & \text{if } b_0 \ll \sqrt{2}\xi_{ab}
\end{cases}
\]  

(B.31)
Figure B.1: Schematic picture for the $c_{11}$ and $c_{44}$ coefficients; source [5].
Figure B.2: a: parameters describing a flux line in a well. b: parameters describing the potential well modeling the pin; source [3].

In which $\epsilon_0 = (\Phi_0 / 4\pi \lambda_{ab})^2$ and $\xi_{ab}$ is the coherence length. An interpolating formula is given by:

$$U_0 \simeq \frac{1}{2} \epsilon_0 \ln \left[ 1 + \left( \frac{b_0}{\sqrt{2} \xi_{ab}} \right)^2 \right]$$  \hspace{1cm} \text{(B.32)}

The critical current can be calculated as always equaling the maximum pinning force with the Lorentz one, finding, at zero temperature:

$$j_c \simeq \begin{cases} \frac{3\sqrt{3}}{4\sqrt{2}} j_{pb} & \text{if } b_0 > \sqrt{2} \xi_{ab} \\ \frac{27\sqrt{3}}{64} \left( b_0 \sqrt{2} \xi_{ab} \right)^2 j_{pb} & \text{if } b_0 < \sqrt{2} \xi_{ab} \end{cases}$$  \hspace{1cm} \text{(B.33)}$$

Where $j_{pb}$ is the pair-breaking current density, given by:

$$j_{pb} = \frac{c\Phi_0}{12\sqrt{3}\pi^2 \lambda_{ab}^2 \xi_{ab}}$$  \hspace{1cm} \text{(B.34)}

The critical current renormalization is the given by considering a reduction of the effective pinning barrier due to temperature $U_0 \rightarrow U(T)$, as shown in figure B.2b, and also the coherence length $\xi_{ab}$ should be replaced by the temperature-dependent spatial extent of the ground-state wavefunction of a particle confined in a cylindrical well (namely $l_\perp$) as shown in figure B.2a. This temperature renormalization yields the following relationship for the
critical current vs. temperature dependence:

\[ j_c \simeq j_c(0) e^{-3(T/T^*)^2} \]  \hspace{1cm} (B.35)

In this equation, the energy scale of the pin is given by:

\[ T^* = \sqrt{\tilde{\epsilon}_1 U_0} b_0 \]  \hspace{1cm} (B.36)

where \( \tilde{\epsilon}_1 \) is the tilt modulus of the flux line lattice, namely:

\[ \tilde{\epsilon}_1 = \frac{M_\perp}{M_z} \epsilon_0 \ln \left( \frac{\lambda_{ab}}{\xi_{ab}} \right) \]  \hspace{1cm} (B.37)

It can be seen how equation B.35 is analogous to the relationship used in chapter 7.

**B.3 References**


Ringraziamenti

È arrivato il momento della parte più difficile di tutta la tesi, i ringraziamenti. All’inizio pensavo sarebbero stati una passeggiata, finiti in una mezz’oretta, invece questa è la terza o quarta versione che provo a scrivere. C’è anche da dire che non sono molto avvezzo a farli, mi sono sempre risparmiato quest’incombenza in tutti i lavori precedenti con la scusa “tanto li faccio per la tesi magistrale fatti bene”. Quindi ora ho un sacco di pressione addosso, capitemi se alla fine usciranno in maniera un po’ ignobile.

Ah, ho scelto di farli in italiano, chiedo scusa a tutti quelli che non lo parlano, prendetelo come un ottimo motivo per cominciare a studiarlo.

In primis vorrei ringraziare i miei supervisori del CERN, Amalia, Eugen e Lucio, per tutto l’aiuto e sostegno che mi hanno dato durante il mio lavoro, e senza i quali nulla di tutto ciò sarebbe stato possibile. Un sentito ringraziamento anche al mio relatore, Ezio, per la grande occasione che mi ha dato.

Qui a Ginevra ho avuto la possibilità di conoscere tantissime persone che (nel bene o nel male) hanno reso la mia esperienza indimenticabile. Potrei fare un bell’elenco di nomi, ma dubito sull’utilità di tale soluzione e, soprattutto, finirei col dimenticarmi di qualcuno offendendolo profondamente. Tra l’altro, non ho bisogno di segnarmi i nomi per ricordarmi di tutti voi, sarete sempre presenti nel mio cuore e nei miei ricordi. D’altro canto, per mediare tra queste due estreme soluzioni e rendere questi ringraziamenti un po’ meno sterili, nominerò in ordine casuale i vari gruppi di persone, cosicché ognuno possa ritrovarvisi. Ringrazio quindi gli amici incontrati il primo giorno di lavoro, impauriti ed eccitati come me, il gruppo gigante di italiani al CERN, le ragazze delle Nazioni Unite, i russi del CERN, gli amici della pallavolo, la squadra di calcetto, i coinvilupi, i musicisti, le ragazze di traduzione, gli amici erasmus e “quelli del pranzo a R1”. Spero di rivedervi tutti un giorno, in un modo o nell’altro.

Da questo paragrafo in poi le cose si fanno un po’ più complicate. Come ho detto, ho sempre posticipato la scrittura dei ringraziamenti, quindi ora devo tentare di scrivere qualcosa che compensi tutto ciò. D’ora in poi quello che scriverò non riguarderà semplicemente la stesura di questa tesi, ma spazierà su lasso di tempo parecchio più ampio, diciamo tutto il periodo universitario.

Un grazie ad Enrica, che anche se si arrabbia come una belva quando le dico di non studiare e uscire a fare festa, alla fine nei momenti importanti
c’è sempre.

Un grazie a Oleg, per tutti i pomeriggi passati a studiare e resi meno pesanti grazie alla tua presenza, e anche per due o tre seratone per le quali, forse, l’aggettivo indimenticabili non è troppo adatto. (Indizio: salti dalla pedana del fioraio).

Un grazie a Simone, per tutte le risate che ci siamo fatti in questi anni di Università, per avermi fatto entrare di straforo nella tua marching band facendo finta di suonare il sax, per i gelati invernali all’Etna e per la tua amicizia.

Grazie a David, per tutte le discussioni e il supporto reciproco che ci siamo dati in questi anni. E anche per l’enorme fonte di prese in giro che ci fornisce, Simone dovrebbe ancora avere il tuo poster appeso a casa.

Un grazie anche a Livio, con il quale abbiamo resistito per anni contro i coinquilini più disparati e stravaganti.

E grazie anche a tutti gli altri compagni di università, che hanno condiviso con me le gioie e i dolori di questi lunghi anni.

Il mio percorso universitario e di vita non sarebbe mai stato lo stesso se non fosse stato perennemente accompagnato dalla musica. Nel corso di questi anni ho avuto modo di suonare con moltissime persone, tra le tante vorrei ringraziare soprattutto Teo, che oltre ad essere un fior fior di batterista, è anche un grande amico.

Il mio cammino sta ora continuando con gli Alchemy, il progetto musicale che mi sta dando il maggior numero di soddisfazioni, sia perché i componenti sono veramente dei bravi musicisti, sia perché sono anche dei grandi amici. Perciò ci tengo a ringraziarli uno per uno, in ordine casuale, come sempre.

Un ringraziamento particolare a Cristiano, che mi ha fatto tornare la voglia di scrivere pezzi inediti, e ci regala ogni giorno perle di saggezza non indifferenti.

Un grazie a Luca, che ha creduto in me dal primo giorno in cui ci siamo incontrati, e mi ha dato l’ispirazione per andare avanti a comporre.

Un grazie ad Andrew, che quel giorno in stazione si è trasformato in tastierista e ha dato una svolta decisiva al nostro progetto.

Un grazie a Matteo, che ogni volta fa da mediatore e da organizzatore tra i tre sovracitati, senza di te ci saremmo già presi a pugni.

Nella vita si conoscono tante persone, si stringono tante amicizie, ma
nessuna di quelle è come quella che si ha con gli amici “storici”, quelli che, in un modo o nell’altro e anche se stanno dall’altra parte del mondo, sai che ci saranno sempre. Grazie Bremba, Fabio, Paolo e Simone.

Grazie Maestro Marco, per tutte le cose che mi hai insegnato in palestra e nella vita, e grazie a tutti i compagni di squadra del Centro Studi Wushu Brescia, in particolare Daniele, Emma, Elisabetta e Vittorio.

Ed infine, un ringraziamento va alla mia famiglia, che mi ha aiutato e supportato per tutta la mia vita.

Grazie Papà, per tutte le volte che mi hai aiutato in analisi, per le suonate serali in acustica, per quella settimana passata a Ginevra in cerca di una casa e per tutto quello che mi hai insegnato.

Grazie Mamma, per tutte le volte che mi hai aiutato in algebra, per le foto che mi fai ai concerti, per i pranzi infiniti che mi prepari quando torno e per esserti sempre quando ho bisogno di qualunque cosa.

Un grazie a tutti e due per la prontezza con la quale sareste venuti a Ginevra in piena notte quando mi hanno rubato le chiavi della macchina.

Grazie a Nonna Piera, che pensa già che sia un professore, e a Zia Maria, a cui ho rubato gli occhiali da sole prima di partire. Mi stanno benissimo.

Grazie anche a Zia Miriam, Zia Cecilia, Zio Renato e al cuginetto Giona: siamo sempre più distanti ma ciò non vuol dire che non pensi a voi.

Il ringraziamento finale va a Nonna Elena: anche se sei lontana, so che sei sempre stata con me.