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PRECISION LEP DATA, SUPERSYMMETRIC GUTs AND STRING UNIFICATION

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ABSTRACT

The precision value of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ extracted from LEP data (0.233 ± 0.001) confirms the prediction of minimal supersymmetric GUTs (0.235 ± 0.004) within the errors of about 2%. Moreover, supersymmetric GUTs with three generations and a heavy top quark also predict $m_b = 5.2 \pm 0.3$ GeV in perfect agreement with potential model estimates (5.0 ± 0.2 GeV). String unification would require that the effective grand unification scale m_{GUT} be no larger than the effective string unification scale m_{SU} , which is indeed consistent with the LEP data, which indicate $m_{GUT} \sim 2 \times 10^{16}$ GeV in a minimal supersymmetric GUT, compared with the theoretical estimate $m_{SU} \sim 10^{17}$ GeV. Specific choices of the string model moduli could enforce $m_{GUT} = m_{SU}$ even in minimal supersymmetric GUTs, whilst non-minimal supersymmetric GUTs can reconcile the successful predictions of $\sin^2 \theta_W$ with $m_{GUT} = m_{SU}$ for generic values of the moduli, but tend to have m_b too large.

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Physicists agree that the Standard Model is unsatisfactory and incomplete, and is presumably embedded in a larger and more unified framework. The three independent $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge group factors of the Standard Model have three independent couplings. The strong coupling $\alpha_3(Q^2)$ is often parametrized in terms of a scale parameter Λ_{QCD} , whilst the electroweak couplings $\alpha_2(Q^2)$ and $\alpha_1(Q^2) = \frac{5}{3}\alpha'(Q^2)$ are often re-expressed in terms of the fine structure constant α and $\sin^2\theta_W = g'^2/g^2 + g^2$. Any framework for unification should reduce the number of these coupling parameters. A conventional Grand Unified Theory (GUT) [1] has just two relevant free parameters, which can be taken as a grand unification scale m_{GUT} at which the three Standard Model couplings become equal, and their common value $\alpha_{GUT}(m_{GUT}^2)$ at this scale. Thus a typical GUT makes one prediction, usually expressed as the value of $\sin^2\theta_W$ for given input values of α_m and α_3 , with the grand unification scale being roughly proportional to Λ_{QCD} , other things being equal. Already in the earliest days of GUTs the predictions [2] for $\sin^2\theta_W$ were known to be below the symmetry value of $\frac{3}{8}$ when $\alpha_1 = \alpha_2$, and typically in the range 0.20 to 0.25. Conventional GUTs gave $\sin^2\theta_W \sim 0.215$ to 0.220 and $m_{GUT} \sim 10^{14}$ to 10^{15} GeV, whereas minimal supersymmetric GUTs give $\sin^2\theta_W \sim 0.230$ to 0.235 and $m_{GUT} \gtrsim 10^{16}$ GeV [3].

It has been known for some time that the low-energy electroweak data [4,5] favoured a value of $\sin^2\theta_W$ larger than that predicted by minimal non-supersymmetric GUTs, which were also apparently ruled out by the experimental lower limits on the proton lifetime. On the other hand, supersymmetric GUTs were impressively close [4,5] to the pre-LEP range of $\sin^2\theta_W$.

Some time after GUTs were first proposed, it was realized [6] that minimal models also made predictions for the bottom quark mass m_b that agreed qualitatively with experiment. Subsequently, it was pointed out [7] that the success of this prediction depended on the number of generations N_G , and $N_G = 3$ was predicted on the basis of a quantitative analysis of m_b [8]. The prediction $N_G = 3$ held also in supersymmetric GUTs [9], and was of course confirmed by LEP [10]. The precision LEP data now enable the value of m_b extracted from potential models [11] to be confronted more accurately with the supersymmetric GUT prediction.

Supersymmetric GUTs have a grand unification scale m_{GUT} quite close to the Planck scale, and is natural to ask if these should be identified. The only plausible candidate for a Theory of

Everything (TOE) encompassing gravity and other Planck scale physics is the superstring [12]. In this framework, one would expect a unification scale m_{SU} close to the Planck mass with $m_{GUT} = m_{SU}$ if there is no intermediate scale of field-theoretic symmetry breaking. In this case, all the three Standard Model gauge couplings would be given in terms of one parameter, and one could make two predictions, most naturally phrased as those for $\sin^2\theta_W$ and α_3 or Λ_{QCD} . On the other hand, there could be an intermediate scale in the TOE, in which case m_{GUT} could be less than m_{SU} and there is less predictivity.

In this paper we re-examine these supersymmetric GUT and string predictions for Standard Model parameters in the light of recent precision data from LEP on $\sin^2\theta_W$ [13] and α_3 [14]. We use the LEP value of α_3 to calculate $\sin^2\theta_W$ within minimal supersymmetric GUTs, finding agreement with the LEP value of $\sin^2\theta_W$ within 1% and a grand unification scale $m_{GUT} \sim 2 \times 10^{16}$ GeV. We then re-evaluate the three-generation supersymmetric GUT prediction for m_b , using LEP constraints on m_t , m_H and $\sin^2\theta_W$ [15], finding 5.2 ± 0.3 GeV to be compared with the estimate $m_b = 5.0 \pm 0.2$ GeV given by phenomenological analyses of bottomonium spectroscopy using potential models [11]. Supersymmetric GUTs are compatible with string unification only if $m_{GUT} \leq m_{SU}$, the scale at which string expects gauge couplings to be equal. String calculations indicate [16,17] that $m_{SU} = g \times \hat{M} \times 1.93 \times 10^{17}$ GeV, where g is the unified gauge coupling constant and $\hat{M}$ is a model-dependent constant that is expected to be 0(1). If $\hat{M} \sim 1/10g$, $m_{GUT} = m_{SU}$ and there is no need for new physics below the Planck scale. If $\hat{M} > 1/10g$, either there is some intermediate scale of gauge symmetry breaking as in flipped $SU(5)$ [18], or extra matter supermultiplets are needed to increase m_{GUT} . We investigate what combinations of supermultiplets preserve the successful LEP correlation between $\sin^2\theta_W$ and α_3 , and also see if they preserve the successful calculation of m_b . We find that extra (3,1) and (3,2) representations of $SU(3) \times SU(2)$ at some intermediate scale could easily enable $m_{GUT} = m_{SU}$ without losing the successful minimal supersymmetric GUT prediction for $\sin^2\theta_W$, while increasing the predicted range of m_b by about 0.5 GeV.

We start by discussing the low-energy values of $\sin^2\theta_W$ and α_3 . The former is often discussed [5,15] in terms of $\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2$, but for our unification studies it is more convenient to use the $\overline{MS}$ definitions of both quantities. The value of $\bar{s}^2 \equiv \sin^2\theta_W^{\overline{MS}}(m_Z)$ is

derivable from m_Z by:

$$\bar{s}^2 = \frac{1}{2} \left\{ 1 - \left[1 - \left(\frac{74.561 \text{ GeV}}{m_Z} \right)^2 \frac{1}{(1 - \Delta r)} \right]^{\frac{1}{2}} \right\} \quad (1)$$

where the mass-scale is determined from μ decay and Δr is a well-known [19] radiative correction for which the principal uncertainties in the Standard Model are m_H and m_t . Taking $m_H \simeq m_Z$ and $m_t = 127 \text{ GeV}$ (the central value [15] for this choice of m_H), the LEP value of $m_Z = 91.17 \pm 0.03 \text{ GeV}$ [13] corresponds to $\bar{s}^2 = 0.2328 \pm 0.0002$. The $1 - \sigma$ range of m_t increases (decreases) as the chosen value of m_H is increased, and the central value of $\bar{s}^2$ decreases when m_H decreases (m_t increases). Thus the minimum possible value of $\bar{s}^2$ is obtained by taking m_H small and m_t at the top end of its range for this value of m_H . For the minimum value $m_H \simeq 40 \text{ GeV}$ [20] and correspondingly $m_t(\text{max}) = 148 \text{ GeV}$ [15], we find $\bar{s}^2 = 0.2318 \pm 0.0002$. Analogously for $m_H = 1 \text{ TeV}$ and correspondingly $m_t(\text{min}) = 127 \text{ GeV}$ [15], we find the maximum values $\bar{s}^2 = 0.2340 \pm 0.0002$. We therefore take as our allowed range

$$\begin{aligned} \sin^2 \theta_W^{\overline{MS}}(m_z) &= 0.2328 & +0.0014 & (m_t = 127 \text{ GeV}, \quad m_H = 1 \text{ TeV}, \quad m_Z = 91.14 \text{ GeV}) \\ & & -0.0012 & (m_t = 148 \text{ GeV}, \quad m_H = 40 \text{ GeV}, \quad m_Z = 91.20 \text{ GeV}) \end{aligned} \quad (2)$$

This is the allowed range in the Standard Model, which is unaltered if the effective global supersymmetry breaking scale $\tilde{m}$ is large ($\gg 100 \text{ GeV}$), but could in principle be altered if $\tilde{m} \sim 100 \text{ GeV}$. In fact, it was found in Ref. [21] that the variation in the low-energy analysis of $\bar{s}^2$ due to varying $\tilde{m}$ is at most comparable that induced by varying m_H , so we retain (2) as our input range of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$.

There are in principle two methods of measuring α_3 at LEP, via the total cross-section and via hadronic final state event shapes [14]. The former method is as yet relatively inaccurate, whereas the latter method gives central values of $\alpha_3 \sim 0.11$ to 0.12 with statistical errors less than ± 0.01 . To these should be added systematic errors, which are smaller than they were in lower-energy experiments but still larger than the statistical errors. A recent review [14] gives

$$\alpha_3^{\overline{MS}}(m_Z) = 0.115 \pm 0.016 \quad (3)$$

which we take as our allowed range.

The two-loop renormalization group equations for the minimal supersymmetric extension of the Standard Model, both without and with the top quark Yukawa coupling, are well known [22]. In Fig. 1 we show the results of integrating them from $\tilde{m} = m_Z$ up to the neighbourhood of the unification scale using as a fixed input the value $\alpha_{em}(m_Z) = 1/127.9$ for the electromagnetic fine structure constant. We see that the low-energy data (2), (3) are compatible with a supersymmetric GUT unification scale $m_{GUT} \sim 2 \times 10^{16} \text{ GeV}$:

$$m_{GUT} \simeq 1.3 \times 10^{16} \text{ GeV} \exp[63(\alpha_3 - 0.115)] \quad (4a)$$

$$m_{GUT} \simeq 1.3 \times 10^{16} \text{ GeV} \times 2.7^{0 \pm 1} \quad (4b)$$

One way of phrasing this agreement is to use the value (3) of α_3 to predict $\sin^2 \theta_W$:

$$\simeq 0.235 + 0.267(0.115 - \alpha_3) \quad (5a)$$

$$\sin^2 \theta_W^{\overline{MS}}(m_Z)$$

$$\simeq 0.235 \pm 0.004 \quad (5b)$$

in perfect agreement with the low-energy data (2). These results are essentially unchanged whether one includes the top quark Yukawa coupling or not. Therefore, this analysis gives no new indication on m_t beyond that from the low-energy data.

We have also illustrated on Fig. 1 the effect of changing the supersymmetry threshold $\tilde{m}$, in the form of a “slop” in the $(\alpha_3, \sin^2 \theta_W^{\overline{MS}}(m_Z))$ plane of the $m_{GUT} = 2 \times 10^{16} \text{ GeV}$ point. Two supersymmetry-breaking parameters m_0 and $m_{\frac{1}{2}}$ suffice to characterize most sparticle masses [23]. We see that increasing $m_{\frac{1}{2}}$ from 25 GeV to 200 GeV has the effect of increasing α_3 by about 0.01 while reducing $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ by about 0.001. Increasing m_0 from 0 to 20 $m_{\frac{1}{2}} : m_{\frac{1}{2}} = 25 \text{ GeV}$ decreases α_3 by at most 0.002 and has no discernible effect on $\sin^2 \theta_W^{\overline{MS}}(m_Z)$. We will continue to work with $\tilde{m} = m_Z$ in what follows, but this “slop” should be borne in mind when interpreting our results.

We note in passing that the precision LEP data also leave open the possibility that the $SU(2)$ and $SU(3)$ gauge couplings of the Standard Model become equal ($g_3 = g_2$) at a scale m_3 somewhat below the one at which their common value equals that of the $U(1)$ gauge coupling

* as also seen in Fig. 1. This option is not allowed in minimal $SU(5)$, but could occur in models such as flipped $SU(5) \times U(1)$ [18]. However, in the rest of this paper we will assume that $g_3 = g_2 = g_1$ at the same scale m_{GUT} , and will give no further numerical analysis of more general models.

Emboldened by this success of minimal supersymmetric GUTs, we now turn to a careful evaluation of the GUT prediction [6-9] for m_b . Already before the b quark was discovered, it was pointed out [6] that in minimal $SU(5)$ and other GUTs the b quark and τ lepton Yukawa couplings were equal at the grand unification scale, and that the ratio of the physical masses could be computed via the renormalization group. The range $m_b/m_\tau = 2$ to 10 was predicted [6], and shortly afterwards the b quark was found [24] with $m_b \simeq 3m_\tau$. More refined calculations then gave $m_b \simeq 5$ GeV IF THERE WERE ONLY THREE GENERATIONS [7]. It was shown that larger numbers of generations gave unacceptably large values of m_b [8], and hence $N_G = 3$ was predicted long before cosmological nucleosynthesis reached the necessary precision [25] and LEP settled the issue [10]. The prediction for m_b was unchanged in supersymmetric GUTs [3,9], and reasons were found (e.g., non-renormalizable effective interactions or radiative corrections) why the analogous predictions for lighter quark/lepton mass ratios might fail. Phenomenological estimates of m_b using non-relativistic potential models for the bottomonium system now lie in the range [11]

$$m_b = 5.0 \pm 0.2 \text{ GeV} \quad (6)$$

and this is the small target with which we now compare the supersymmetric GUT prediction.

We use the full two-loop renormalization group equations for the b quark, t quark and τ lepton Yukawa couplings between m_{GUT} and m_t [26]. The latter enters because the predicted value of m_b depends on the t quark Yukawa coupling, which in turn depends on the physical value of m_t and on the assumed value of $\tan\beta \equiv v_2/v_1$, where $v_{1,2} \equiv \langle 0|H_{1,2}^0|0 \rangle$ are the v.e.v.'s of the Higgs giving masses to the (b, τ) and t respectively. Below m_t , the effects of the Yukawa couplings in the renormalization group equations are dropped, because the largest Yukawa coupling is that for the t quark, which does not contribute below m_t . However, the full

* Assuming the suitable Grand Unification normalization of the $U(1)$ hypercharge $Y_G : \text{Tr } Y_G^2 = \text{Tr } I_3^2$ for each generation.

two-loop renormalization group equations for the gauge couplings are used from m_t down to m_b , and one-loop corrections suffice for the running of α_1 and α_2 between m_b and m_τ . However, we do not tackle the one-loop GUT or string threshold corrections to the equality of b quark and τ lepton Yukawa couplings, since these are model-dependent.

Our results are shown in Fig. 2 in the (α_3, m_t) plane for different values of $\tan\beta$. (The corresponding values of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ are those given by the m_{GUT} line in Fig. 1.) Let us restrict our attention to the range of α_3 permitted by the LEP data, either directly via (3) or indirectly via the range (2) of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ and the GUT relation (5a). We further note that for any given value of α_3 there is a restricted range of m_t that is allowed by the LEP data, determined by the value of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ corresponding to α_3 via (5a), as also shown in Fig. 2. Within the allowed range of α_3 and m_t , and considering all values of $\tan\beta$ above the LEP lower limit 1.5, we find

$$m_b = 5.2 \pm 0.3 \text{ GeV} \quad (7)$$

whose central value almost coincides with that found (6) in potential models, and whose range is only slightly larger. It is clear from Fig. 2 that the agreement between the supersymmetric GUT calculation and experiment for m_b has been improved by adopting the high range of m_t values deduced [15] indirectly from LEP data, compared with lower m_t values. Is this just a second remarkable coincidence, or are LEP and m_b trying to tell us something?

Taking the second, optimistic, view, we now check for compatibility with string unification. It has been shown [16,17] that in a wide class of string models, including orbifolds and free fermionic constructions, the low-energy gauge couplings g_i appear to become equal at a scale [16]

$$m_{SU} \simeq \frac{1}{4\pi\sqrt{\alpha'}} \hat{M} \quad (8)$$

where the string tension $(\alpha')^{-1} = 8\pi G_N/g^2$ and $\hat{M}$ is a model-dependent number which can be expressed [16] in the form

$$\hat{M} = \prod_{\alpha=1}^3 [\sqrt{\text{Im } T_\alpha} |\eta(T_\alpha)|^2]^{\lambda_\alpha} : \sum_{\alpha=1}^3 \lambda_\alpha = -1 \quad (9)$$

where the λ_a are related to the β -functions of intermediate $N = 2$ supersymmetric models. The quantities T_a have the interpretations of moduli of compactification in the orbifold language, and are related to v.e.v.'s of bilinears of world-sheet fermions in the free fermionic constructions. It is generally thought that $\langle 0|T_a|0\rangle = 0(1)$, and the modulus $|\eta(q)|$ of the Dedekind η -function is also $0(1)$, so one could expect that $\hat{M} = 0(1)$ in a generic string model^{*}. Numerically, eq. (8) gives the estimate

$$m_{SU} = 1.93 \times 10^{17} \text{ GeV} \times g \times \hat{M} \quad (10)$$

which is startlingly close to the value (4) of m_{GUT} predicted on the basis of LEP data and minimal supersymmetric GUTs, if one takes $g \sim 0.7$ as indicated by these theories!

It is a non-trivial consistency check on the supersymmetric GUT philosophy that we found $m_{GUT} \leq m_{SU}$ in minimal supersymmetric GUTs. If we had found $m_{GUT} > m_{SU}$, string effects would have intervened before the field-theoretical gauge couplings came together, and the above estimates of $\sin^2 \theta_W^{MS}(m_Z)$ and m_b would have been invalidated. If $m_{GUT} = m_{SU}$, then there is no field-theoretical GUT, but there is grand unification at the string scale, and the above estimates of $\sin^2 \theta_W^{MS}(m_Z)$ and m_b may remain valid, modulo a more complete study of string threshold effects. If $m_{GUT} < m_{SU}$, then grand unification may occur in the traditional field-theoretical GUT framework, for example via flipped $SU(5) \times U(1)$ [18].

There are two ways to obtain $m_{GUT} = m_{SU}$ that we have considered. One is to retain the minimal GUT spectrum, but specify the moduli T_a such that $\hat{M} = 0(\frac{1}{10})$:

$$\prod_{a=1}^3 [\sqrt{\text{Im } T_a} |\eta(T_a)|^2]^{\lambda_a} = \left(\frac{19.3g}{2}\right) \times 2^{0 \pm 1} \simeq \frac{1}{3} \text{ to } \frac{1}{15} \quad (11)$$

According to this philosophy, LEP has initiated "Modulus Phenomenology". The other way to obtain $m_{GUT} = m_{SU}$ is to add some matter supermultiplets to the GUT spectrum at intermediate scales so as to increase m_{GUT} to equal m_{SU} for some generic value of the moduli T_a : $\hat{M} \sim 1$. We now explore this latter approach.

^{*} We neglect contributions to (9) that could come from fields appearing at the $N = 1$ level, since these are generally expected [18] to be smaller.

We introduce additional supermultiplets with masses m_I intermediate between m_Z and m_{GUT} in representations $R(i, j)$ of $SU(3) \times SU(2)$. We find that in the domain of interest their effects on $\log m_{GUT}$ and $\sin^2 \theta_W$ can be adequately expressed in terms of linear dependences on the quantities

$$L_{(i,j)} \equiv \sum_{R(i,j)} \log m_{GUT}/m_{R(i,j)} \quad (12)$$

In the case of m_{GUT} , we find that Eq. (4a) generalizes to

$$\begin{aligned} \log(m_G/1 \text{ GeV}) = & \log(1.3 \times 10^{16}) + 63(\alpha_3 - 0.115) \\ & + (-0.051)L_{(1,2)} + (0.052)L_{(3,1)} + (0.053)L_{(3,2)} \end{aligned} \quad (13)$$

when extra (1,2), (3,1) and (3,2) representations are added. It is clearly easy to choose additional representations that imply $m_{GUT} = m_{SU}$ (11). However, the extra representations in general also change the successful prediction (5) of $\sin^2 \theta_W^{MS}(m_Z)$:

$$\begin{aligned} \sin^2 \theta_W^{MS}(m_Z) \simeq & 0.235 + 0.267(0.115 - \alpha_3) + \\ & (3.8 \times 10^{-4})L_{(1,2)} - (3.8 \times 10^{-4})L_{(3,1)} + (9 \times 10^{-4})L_{(3,2)} \end{aligned} \quad (14)$$

Nevertheless, with a suitable choice of $L_{(1,2)}$, $L_{(3,1)}$ and $L_{(3,2)}$ the successful prediction (5) can be retained.

We have looked for specific extensions of the minimal supersymmetric GUT model in which $m_{SU} = 5.3g \times 10^{17} \text{ GeV} \simeq 4 \times 10^{14} \text{ GeV}$ (this being a previous estimate [17] of the string unification scale that is in the range [16] allowed by (8)), checking their predictions for α_3 and $\sin^2 \theta_W^{MS}(m_Z)$. Our results are shown in Fig. 3, where we see that α_3 in the range (2) and $\sin^2 \theta_W^{MS}(m_Z)$ in the range (1) are compatible with

$$L_{(3,1)} \simeq 36 \quad \text{and} \quad L_{(3,2)} \simeq 18 \quad (15)$$

This option could be realized in many ways, e.g., by having two vector-like pairs of (3,1) chiral supermultiplets with common mass $m_I \simeq 6 \times 10^{13} \text{ GeV}$, and two vector-like pairs of (3,2)

chiral supermultiplets with common mass $m_I \simeq 5 \times 10^{15}$ GeV. Similar effects can be obtained by adding to the minimal supersymmetric GUT spectrum complete GUT supermultiplets with intermediate masses, for example four vector-like 10 representations of $SU(5)$ with intermediate masses $m_I \simeq 2 \times 10^{15}$ GeV would give $\sin^2 \theta_W^{\overline{MS}}(m_Z) = 0.235$ and $\alpha_3(m_Z) = 0.105$.

The values of m_b for models with four (3,1) chiral supermultiplets and two (3,2) supermultiplets at a common mass m_I are shown in Fig. 3 as tick marks. We see that $m_b \simeq 5.6$ to 6.0 GeV when $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ and $\alpha_3(m_Z)$ are in the ranges (2,3) allowed by LEP. These values are about 0.5 GeV higher than those in the minimal GUT model, and indicate that the successful prediction of m_b in that model was a non-trivial success. It is clear that a generic extension of the minimal GUT spectrum does not reproduce the successful minimal supersymmetric GUT predictions (4),(5). This may be circumstantial evidence for the possibility that the superstring moduli are such that $\hat{M} \simeq \frac{1}{3}$ to $\frac{1}{15}$, as suggested above (11). Alternatively, one should re-examine GUTs such as flipped $SU(5)$ [18] in which $\alpha_2 = \alpha_3$ at some scale below the scale m_{SU} at which their common value equals α_1 .

The following are the major conclusions of our analysis.

- 1) It is remarkable that the LEP determination of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$, which has a precision of $\pm \frac{1}{2}\%$, agrees to within 1% with the prediction of the minimal supersymmetric GUTs, which has a precision of $\pm 2\%$. Moreover, the observed and calculated values of m_b also agree within the estimated errors of $\pm 5\%$. We strongly believe that these successes are not coincidences, and take them as strong circumstantial evidence for the twin ideas of supersymmetry and grand unification.
- 2) Contrary to general belief, string theory is not immune to experimental test. String unification requires that the low-energy gauge couplings become equal at a common energy scale $\sim 10^{17}$ GeV. This imposes constraints on the effective low-energy couplings which depend on the possible existence of intermediate-scale states, but are nevertheless quite restrictive. It is non-trivial that the precision LEP values of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ and $\alpha_3(m_Z)$ are consistent with these restrictions. It is even possible that there are no intermediate mass scales, and that the supersymmetric GUT scale can be identified with the string unification scale, thanks to a particular, but not implausible choice of string moduli. In this respect, LEP data give string

modellers a worthy target.

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FIGURE CAPTIONS

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- 1) Unification scales estimated by integrating the two-loop renormalization group equations from $\bar{m} = m_Z$ upwards. The dotted lines are those of m_{32} , where $\alpha_3 = \alpha_2$. The solid line is that of m_{GUT} , where $\alpha_3 = \alpha_2 = \alpha_1$. The LEP ranges of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ (2) and $\alpha_3(m_Z)$ (3) are shown on the vertical and horizontal axes respectively. The error bars emanating from the $m_{GUT} = 2 \times 10^{16}$ GeV point indicate the effects of varying the supersymmetry breaking parameters m_0 and $m_{\frac{1}{2}}$.
- 2) Values of m_b estimated by integrating the two-loop renormalization equations including Yukawa couplings, assuming equal couplings for the b quark and the τ lepton at m_{GUT} . The solid (dashed) lines are for $\tan\beta = 2(5)$. Also shown is the domain of $(\alpha_3(m_Z), m_t)$ allowed by the LEP data for $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ and m_t , and assuming the GUT relation (5) between $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ and $\alpha_3(m_Z)$.
- 3) Values of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ and $\alpha_3(m_Z)$ obtained assuming $m_{GUT} = 4 \times 10^{17}$ GeV and incorporating $[n, m]$ extra (3,1) and (3,2) representations of $SU(3) \times SU(2)$ at some intermediate scale m_I . The tickmarks on the [4,2] line indicate the corresponding values of m_b . The LEP ranges of $\sin^2 \theta_W^{\overline{MS}}(m_Z)$ and $\alpha_3(m_Z)$ are indicated on the axes.

- Figure 1 -

- Figure 2 -

- Figure 3 -