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Chiral Lagrangians as a tool to probe the Symmetry Breaking Sector of the SM at LEP

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ABSTRACT

An effective $SU(2)_L \times U(1)_Y$ invariant chiral lagrangian can accommodate *any* symmetry breaking scheme in the Standard Model. It is shown that all process-independent quantities relevant to LEP precision measurements ($\Delta\rho$, $\Delta\kappa$ and Δr) depend only on one coefficient of the weak chiral lagrangian, taking different values depending on the precise symmetry breaking scheme. We remove any ambiguity due to the use of an effective theory by a careful choice of observables and make definite predictions that can be tested in the foreseeable future. Our analysis complements nicely that of longitudinal W and Z scattering.

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1 Introduction

Can one tell the standard Higgs picture from alternative symmetry breaking schemes in the electroweak theory? This is probably the most exciting question yet to be answered in the Standard Model (SM), and this is the issue that we want to address in the present work.

We can envisage the following three “minimal” possibilities for the symmetry breaking sector of the SM : a) A light Higgs particle. If its mass is less than about 80 GeV, it will be found at LEP. Otherwise, we will have to wait until the new generation of pp colliders is commissioned. b) A strongly interacting Higgs, with a mass in the TeV scale. This is beyond the reach of present e^+e^- colliders and, depending on the actual value of its mass, it may be quite hard to detect in future pp machines. However, the mass of an elementary Higgs is not likely to be too large. There are theoretical arguments, based on the triviality of $\lambda\Phi^4$, suggesting that a multi-TeV elementary Higgs might be inconsistent. Nevertheless, none of these arguments is absolutely compelling. c) No elementary Higgs exists. If this is the alternative chosen by Nature, the intermediate vector bosons must get their mass through dynamical symmetry breaking. Composite models or some variant of technicolor theories would be required to provide the dynamics leading to the symmetry breaking. The typical scale involved must again lie somewhere in the TeV region.

Although the last two possibilities we have just discussed are physically very different, there exists a systematic way in which we can pose and answer the question with complete generality, namely with the help of an effective lagrangian [1] involving only the physical degrees of freedom. As it is well known these can be incorporated into a weak chiral lagrangian. Different possibilities, such as a strongly interacting Higgs [2] or a technicolor model would simply lead to different values for the parameters appearing in the chiral lagrangian [3].

With the increasing level of accuracy that is being attained in e^+e^- colliders, singularly at LEP, it seems appropriate to investigate at what level the differences between possibilities a), b) and c) would set in and whether they are within experimental reach in the near future. For instance, regarding the parameter Δr , the present accuracy is set to the level of 1.3×10^{-2} and is essentially due to the uncertainty in the measurement of M_W , which in turn implies $\delta \sin^2 \theta_W = 4.5 \times 10^{-3}$ [4]. Hopefully, the future measurement of M_W at LEP II and CDF will reduce considerably the uncertainty in $\sin^2 \theta_W$. This would lead, in the most favourable case, to an accuracy of $\delta(\Delta r) = 3 \times 10^{-3}$. As for the effective Weinberg angle [5] ($\overline{\sin^2 \theta_W}$) the expected errors range from $\delta \sin^2 \theta_W = 2 \times 10^{-3}$ (from the various asymmetries) to $\delta \sin^2 \theta_W = 4 \times 10^{-4}$ (the last being possible only if the polarization program is finally achieved).

This fantastic accuracy can be used to constrain the parameters of the effective chiral lagrangian and discriminate between different symmetry breaking alternatives. In fact, as we will see, the information that can be obtained from LEP data complements nicely what can be learned from the scattering of longitudinal W and Z bosons. The latter is the most efficient way of probing a strongly interacting symmetry breaking sector in pp machines [6]. Given sufficient experimental precision, we can constrain from future pp supercolliders

such as LHC and SSC and from LEP and SLC all but one of the physically relevant $O(p^4)$ coefficients in the weak chiral lagrangian. The remaining one could be constrained by a future measurement of the ZWW vertex at LEP II.

2 Weak Chiral Lagrangian

Either because one has integrated out the Higgs field or because this does not exist or it is not elementary, one is led to consider the most general effective lagrangian compatible with the $SU(2)_L \times U(1)_Y$ local symmetry of the SM and the breaking pattern $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$. As it is well known, one can describe the would be longitudinal components of the intermediate vector bosons and their interactions by means of an effective chiral lagrangian of the form $\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \dots$. The first term of this infinite expansion is given by

$$\mathcal{L}^{(2)} = \frac{v^2}{4} \text{tr} D_\mu U D^\mu U^\dagger \quad (1)$$

where $v = 246 \text{ GeV}$ and the U field is an unitary matrix that can be parametrized as $U = \exp(i\sigma^a w^a/v)$ where the three w^a fields are the would be longitudinal components of the intermediate bosons. The covariant derivative $D_\mu U$ is defined as

$$D_\mu U = \partial_\mu U + L_\mu U - U R_\mu \quad (2)$$

Here, L_μ and R_μ are the left and right-handed gauge fields $L_\mu = ig\sigma^a W_\mu^a/2$ and $R_\mu = ig'\sigma^3 B_\mu/2$.

Expanding U in eq.1 we recognize some of the interaction vertices of the SM. Of course, given the non-linear nature of $\mathcal{L}^{(2)}$, at some point modified vertices appear. Choosing the unitary gauge, ($U = 1$), $\mathcal{L}^{(2)}$ transforms into

$$\mathcal{L}^{(2)} = \frac{v^2}{4} g^2 W_\mu^+ W_\mu^- + \frac{v^2}{8} g^2 W_\mu^3 W_\mu^3 + \frac{v^2}{8} g'^2 B_\mu B_\mu - \frac{v^2}{4} g g' W_\mu^3 B_\mu \quad (3)$$

or equivalently

$$\mathcal{L}^{(2)} = \frac{v^2}{4} g^2 W_\mu^+ W_\mu^- + \frac{v^2 g^2}{8 \cos^2 \theta_W} Z_\mu Z_\mu \quad (4)$$

This makes apparent that eq.1 indeed corresponds to the mass term in the SM. At this order in the chiral expansion, and at tree level, there is no difference in the effective lagrangian description of an elementary heavy Higgs or a “minimal” composite model. In either case, the w^a fields can be interpreted as the Goldstone bosons corresponding to the global symmetry breaking pattern $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$. The residual symmetry $SU(2)_{L+R}$, the so-called custodial symmetry [7], keeps the ρ parameter equal to one. Deviations from $\rho = 1$ are due to the explicit breaking of the custodial symmetry once the $U(1)_Y$ group is gauged. However, when one-loop corrections are included, differences between the several symmetry breaking possibilities do appear. At order $O(p^4)$ in the chiral expansion we have to include two type of contributions. Firstly, one-loop diagrams coming from $\mathcal{L}^{(2)}$ and, secondly, the terms of $\mathcal{L}^{(4)}$ at tree level. These terms are the ones

that contain all the information about the nature of the symmetry breaking sector of the SM. A complete basis for $\mathcal{L}^{(4)}$ is given in [1] (second ref.).

$$\begin{aligned} \mathcal{L}^{(4)} = & L_1 \text{tr} D_\mu U D^\mu U^\dagger \text{tr} D_\nu U D^\nu U^\dagger + L_2 \text{tr} D_\mu U D_\nu U^\dagger \text{tr} D^\mu U D^\nu U^\dagger \\ & - i L_3 \text{tr} (F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U D_\nu U^\dagger) \\ & + L_{10} \text{tr} (F_R^{\mu\nu} U F_{L\mu\nu} + H_1 \text{tr} (F_R^{\mu\nu} F_{R\mu\nu} + F_L^{\mu\nu} F_{L\mu\nu})) \end{aligned} \quad (5)$$

with $F_{L,R}^{\mu\nu}$ being the field strengths associated to the left and right gauge fields. Terms which vanish by the equations of motion can be neglected at this order. Notice that the gauge fields always appear through covariant derivatives or field strengths. This reflects the requirement that the unknown symmetry breaking sector, whose dynamics determines the $O(p^4)$ coefficients, is $SU(2)_L \times U(1)_Y$ gauge invariant. Of course, there are also other $SU(2)_L \times U(1)_Y$ gauge invariant terms, which may be used as counterterms in the chiral lagrangian when $SU(2)_L \times U(1)_Y$ gauge loops are taken into account [8]. They do not have to be included in our approach. The reason is that we are interested in an effective theory description only for the symmetry breaking sector, and we will treat gauge boson loops explicitly (including, of course, finite parts). For instance, terms like $(\text{tr} U \sigma^3 U^\dagger D_\mu U U^\dagger)^2$ will never appear because they are generated only as a gauge boson loop counter-term and they contain no new information about the Higgs from the theory or because we are providing an effective description of a more complex dynamics, the theory ceases to be renormalizable in the usual sense. It might seem then that we lose any predictive power but this is not the case. We will find combinations of observables that are completely free of one-loop divergences, i.e. unambiguous and predictable.

Either because we have removed the Higgs from the theory or because we are providing an effective description of a more complex dynamics, the theory ceases to be renormalizable in the usual sense. It might seem then that we lose any predictive power but this is not the case. We will find combinations of observables that are completely free of one-loop divergences, i.e. unambiguous and predictable.

3 One-loop radiative corrections

When computing radiative corrections in the SM one must be careful with the choice of parameters to define the model. Here we will use the on-shell renormalization scheme, first proposed by Marciano and Sirlin [9]. In this scheme the set of input parameters are the muon decay constant G_μ , the fine structure constant α and M_Z . The clearest way to proceed is to express all the observables one is interested in in terms of unrenormalized quantities, according to the on-shell prescriptions. The unrenormalized Green functions will then have contributions from the SM Feynman diagrams not involving the symmetry breaking sector and contributions from diagrams involving this sector. We will be concerned with these last ones and we will assume that they are described, in the philosophy of chiral perturbation theory, by the effective lagrangian given in eq. 1 (tree level plus one-loop corrections) and eq. 5 (tree level only).

All the effects in e^+e^- colliders that are sensitive to the symmetry breaking sector can be incorporated into three basic process-independent quantities:

- 1) The relation between G_μ (obtained from muon decay) and the W mass

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2 \theta_W} \frac{1}{1 - \Delta r} \quad (6)$$

Here $\sin^2 \theta_W$ is defined as

$$\sin^2 \theta_W \equiv 1 - \frac{M_W^2}{M_Z^2} \quad (7)$$

and g and g' used in eqs. 3 and 4 are defined at any order as $g = e/\sin \theta_W$ and $g' = e/\cos \theta_W$. In terms of Green functions [9,10]

$$\begin{aligned} \Delta r = & \frac{\Sigma_{WW}(0) - \Sigma_{WW}(M_W^2)}{M_W^2} + \Sigma'_{\gamma\gamma}(0) \\ & + \frac{c^2}{s^2} \left(2 \frac{s}{c} \frac{\Sigma_{\gamma Z}(0)}{M_Z^2} - \frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Sigma_{WW}(M_W^2)}{M_W^2} \right) \\ & + \text{vertex corrections} + \text{box corrections} \end{aligned} \quad (8)$$

where from now on we use the notation $s = \sin \theta_W$ and $c = \cos \theta_W$.

- 2) The ratio between neutral and charged currents at $q^2 = 0$

$$\rho = 1 + \Delta\rho = \frac{G_{NC}(0)}{G_{CC}(0)} \quad (9)$$

where

$$\Delta\rho = \frac{\Sigma_{ZZ}(0)}{M_Z^2} - \frac{\Sigma_{WW}(0)}{M_W^2} - 2 \frac{s}{c} \frac{\Sigma_{\gamma Z}(0)}{M_Z^2} \quad (10)$$

- 3) Finally, the ratio between J_{em} and J^3 in the one-loop corrected Born approximation. This is the so called effective Weinberg angle

$$\tilde{s}^2(q^2) = \kappa s^2 = s^2(1 + \Delta\kappa(q^2)) \quad (11)$$

with

$$\begin{aligned} \Delta\kappa = & -\frac{c}{s q^2} (\Sigma_{\gamma Z}(q^2) - \Sigma_{\gamma Z}(0)) \\ & - \frac{c^2}{s^2} \left(2 \frac{s}{c} \frac{\Sigma_{\gamma Z}(0)}{M_Z^2} - \frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Sigma_{WW}(M_W^2)}{M_W^2} \right) \end{aligned} \quad (12)$$

This equation contains only the self-energy ("oblique") corrections. In addition, $\Delta\kappa$ contains vertex, box, and external leg corrections which are, of course, process dependent, and have to be subtracted when comparing with the experimental results. This poses no problem because the contribution from the symmetry breaking sector to these vertex corrections is negligible.

The radiative corrections to all these quantities (excluding QED corrections) are dominated by the top quark contributions. This is, therefore, the main uncertainty within the SM. A combined analysis of the determinations of s^2 coming from different experiments (neutrino deep inelastic scattering, measuring the neutral to charged current ratio; UA2 and CDF collider experiments, that measure M_Z and M_W ; and the more accurate measurement of M_Z at LEP 1) leads to an upper and lower bound for the top quark mass. The estimated upper bound is now in the region of 200 GeV and the preferred central value is around 130 GeV (see for instance first reference in [4]). Besides, the CDF experiment sets

a lower limit for the top quark mass of 89GeV [11]. It seems likely that in next years of run CDF will be able to determine its mass (the discovery limit is around 240GeV) [12]. This will allow to set more stringent bounds on the symmetry breaking contribution to $\Delta\rho$, $\Delta\kappa$ and Δr . As working hypothesis we shall assume that the top mass is known with infinite accuracy and later on we will comment on the experimental resolution one needs to really test the symmetry breaking sector. From now on, we restrict ourselves to the bosonic sector and when we write $\Delta\rho^{(b)}$, $\Delta\kappa^{(b)}$ or $\Delta r^{(b)}$ we understand that only this contribution is taken into account.

4 Computations and Results

In order to carry out the computation of the bosonic contribution to the unrenormalized self-energies $\Sigma_{\tau\tau}(q^2)$, $\Sigma_{WW}(q^2)$, $\Sigma_{ZZ}(q^2)$ and $\Sigma_{\tau Z}(q^2)$ in the effective lagrangian approach we have used the 't Hooft-Feynman gauge. This is the customary choice, allowing us to make contact with the existing literature. In particular, with the original computation of Marciano and Sirlin [9], made in the framework of the Minimal SM with a single Higgs doublet (MSM), from whose work we borrow some notation. The integrals appearing in the loop computations have been regularized by dimensional regularization.

For a given unrenormalized self-energy Σ let us distinguish between the contributions coming from bosonic loops (Σ_{loop}) that involve gauge bosons, ghosts and Goldstone bosons, and the contributions coming from the $O(p^4)$ terms in eq.5 ($\Sigma^{(4)}$),

$$\Sigma = \Sigma_{loop} + \Sigma^{(4)} \quad (13)$$

In order to compute Σ_{loop} we use the lagrangian $\mathcal{L}^{(2)}$ describing the interactions among the Goldstone and gauge bosons. We have to include the pure Yang-Mills, gauge-fixing and Fadeev-Popov terms as well.

$$\mathcal{L}_G = \mathcal{L}_{YM} + \mathcal{L}_{FP} + \mathcal{L}_{GF} \quad (14)$$

The set of Feynman rules used in the one loop calculation of $\Sigma_{\tau\tau}^{loop}(q^2)$, $\Sigma_{WW}^{loop}(q^2)$, $\Sigma_{ZZ}^{loop}(q^2)$ and $\Sigma_{\tau Z}^{loop}(q^2)$ are readily obtained. They are *not* the ones of the MSM. Apart from the fact that the diagrams involving the physical Higgs do not appear, several other vertices are different. This can be traced back to the non-linear realization of the gauge symmetry in the chiral lagrangian. By explicit calculation in the 't Hooft-Feynman gauge we find for $\Delta\rho_{loop}^{(b)}$, $\Delta\kappa_{loop}^{(b)}$ and $\Delta r_{loop}^{(b)}$ defined as

$$\begin{aligned} \Delta\rho^{(b)} &= \Delta\rho_{loop}^{(b)} + \Delta\rho^{(4)} \\ \Delta\kappa^{(b)} &= \Delta\kappa_{loop}^{(b)} + \Delta\kappa^{(4)} \\ \Delta r^{(b)} &= \Delta r_{loop}^{(b)} + \Delta r^{(4)} \end{aligned} \quad (15)$$

the following results

$$\begin{aligned} \Delta\rho_{loop}^{(b)} &= \left(\frac{g^2}{16\pi^2}\right)\left(-\frac{3s^2}{4c^2}N_c - \frac{5s^2}{8c^2}\right. \\ &\quad \left. + \frac{1}{c^2}\log c^2\left(\frac{17}{4s^2} - 7 + 2s^2\right) + \frac{17}{4} + \frac{3s^2}{4c^2}\log\left(\frac{M_W^2}{\mu^2}\right)\right) \end{aligned} \quad (16)$$

$$\begin{aligned} \Delta\kappa_{loop}^{(b)}(q^2) &= \left(\frac{g^2}{16\pi^2}\right)\left(-\frac{5}{6}N_c + \frac{1}{18} + \log\left(\frac{M_W^2}{\mu^2}\right)\left(7c^2 + \frac{1}{6} - \frac{5c^2}{6s^2}\right)\right. \\ &\quad \left. + \log\left(\frac{M_Z^2}{\mu^2}\right)\left(-7c^2 - \frac{1}{6} + \frac{5}{6s^2}\right) + \frac{1}{18}(157 - 154s^2)\right. \\ &\quad \left. - \frac{c^2}{s^2}\log c^2\left(\frac{59}{6} - 7c^2 - \frac{2}{3c^2}\right) - \frac{c^2}{s^2}(I_1(c^2) - I_2(c^2))\right. \\ &\quad \left. - (3c^2 + \frac{1}{6} + (4c^2 + \frac{4}{3})\frac{M_W^2}{q^2})F(q^2, M_W, M_W)\right) \end{aligned} \quad (17)$$

$$\begin{aligned} \Delta r_{loop}^{(b)} &= \Delta\alpha + vertex + box \\ &\quad + \left(\frac{g^2}{16\pi^2}\right)\left(\frac{11}{12}N_c - \frac{11}{12}\log\frac{M_W^2}{\mu^2} + \log c^2\left(\frac{5}{c^2} - 1 + 3\frac{c^2}{s^2} - \frac{17}{4s^2c^2}\right)\right. \\ &\quad \left. - s^2(3 + 4c^2)F(M_Z^2, M_W, M_W) + I_2(c^2)\left(1 - \frac{c^2}{s^2}\right) + \frac{c^2}{s^2}I_1(c^2)\right. \\ &\quad \left. + \frac{1}{8c^2}(43c^2 - 38) + \frac{1}{18}(154s^2 - 166c^2) + \frac{1}{4c^2} + \frac{1}{6}\right) \end{aligned} \quad (18)$$

where

$$\begin{aligned} N_c &= \frac{2}{\epsilon} - 7_E + \log 4\pi \\ \Delta\alpha &= 0.0602 + \frac{40}{9}\frac{\alpha}{\pi}\log\frac{M_Z}{92(\text{GeV})} \\ vertex + box &= \left(\frac{g^2}{16\pi^2}\right)\left(6 + \frac{7 - 4s^2}{2s^2}\log c^2\right) \end{aligned} \quad (19)$$

and explicit expressions for the I_1, I_2 and F functions can be found in eqs.(A7) and (A8) of ref.[9] and in [10] (Burgers and Hollik, page 139) respectively. On the other hand, by using $\mathcal{L}^{(4)}$ we find the following results:

$$\begin{aligned} \Delta\rho_{(4)}^{(b)} &= 0 \\ \Delta\kappa_{(4)}^{(b)} &= g^2 L_{10} \\ \Delta r_{(4)}^{(b)} &= -2g^2 L_{10} \end{aligned} \quad (20)$$

Notice that out of the five parameters appearing in $\mathcal{L}^{(4)}$ there is just one, L_{10} , contributing to the self-energies and therefore the measurement of $\Delta\rho^{(b)}$, $\Delta\kappa^{(b)}$ and $\Delta r^{(b)}$ will constrain only this parameter. Besides, the parameters L_1 and L_2 can, in principle, be determined from the scattering of the longitudinal gauge bosons, whereas the parameter L_9 is expected to contribute to the triple gauge boson vertex [13].

As expected, going to the non-linear theory introduces ultraviolet divergences in $\Delta\rho^{(b)}$, $\Delta\kappa^{(b)}$ and $\Delta r^{(b)}$. In fact, we recover the coefficients of the $\log M_H$ terms in the MSM by just reading the factors in front of the $(1/\epsilon)$ terms of our formulae (16), (17) and (18). As we have already announced, one may build quantities that are free of one-loop divergences out of $\Delta\rho^{(b)}$, $\Delta\kappa^{(b)}$ and $\Delta r^{(b)}$ as given in the above equations. It is a simple exercise to form combinations that cancel the N_c -terms. For instance

$$\begin{aligned} O_1 &\equiv \Delta\rho - \frac{9s^2}{10c^2}\Delta\kappa \\ O_2 &\equiv \Delta r + \frac{11}{10}\Delta\kappa \\ O_3 &\equiv \Delta\rho + \frac{9s^2}{11c^2}\Delta r \end{aligned} \quad (21)$$

Notice that only two of these observables are independent, being related by

$$O_2 = \frac{11c^2}{9s^2}(O_3 - O_1) \quad (22)$$

By using the expressions for $\Delta\rho^{(b)}$, $\Delta\kappa^{(b)}$ and $\Delta r^{(b)}$ given in eqs.15 through 20 one gets the bosonic contribution to these observables, $O_1^{(b)}$, $O_2^{(b)}$ and $O_3^{(b)}$, in terms of just one unknown quantity i.e. the parameter L_{10} (we have checked explicitly the cancellation of the μ -scale dependence).

One of the most interesting aspects of these chiral lagrangian predictions relies in that they are completely general and, therefore, for specific choice of the parameters, they must reproduce the results of different underlying theories. In particular, in this work we are concerned with two of them; the MSM with a very heavy Higgs boson and a $SU(N_{TC})$ gauge theory within the spirit of minimal technicolor theories, N_{TC} being the number of technicolors. (Here by minimal we understand a theory with the global symmetry breaking pattern $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$ triggered by a QCD-like chiral transition. Therefore, we do not include additional Goldstone bosons. An extension to theories with an enlarged matter content is of course possible.) The parameter to be adjusted in the case we are dealing with is obviously L_{10} .

Let us start by comparing the MSM predictions for $O_1^{(b)}$, $O_2^{(b)}$ and $O_3^{(b)}$ with the corresponding ones from the effective lagrangian description. In the MSM $\Delta\rho^{(b)}$, $\Delta\kappa^{(b)}$ and $\Delta r^{(b)}$ are well known functions of the Higgs mass M_H and so are the three observables above. An explicit computation of $O_1^{(b)}$, $O_2^{(b)}$ and $O_3^{(b)}$ in the MSM shows, however, that in the limit of large M_H , they tend to a constant asymptotic value. This is expected since, for large values of M_H , all the dependence in M_H is logarithmic and the particular combinations entering in $O_1^{(b)}$, $O_2^{(b)}$ and $O_3^{(b)}$ cancel out such logs. (Recall that the coefficients of the logs match those of the divergent parts).

The evaluation of these observables in the MSM has been done by using the analytic formulae of the non-renormalized bosonic-self-energies presented in [9]. We will omit the final expressions here for the sake of brevity. The numerical values we have found are collected in the first column of table 1.

In figs. 1,2 and 3 we have plotted $\Delta\kappa^{(b)}$ versus $\Delta\rho^{(b)}$, $\Delta\kappa^{(b)}$ versus $\Delta r^{(b)}$ and $\Delta r^{(b)}$ versus $\Delta\rho^{(b)}$ respectively. Each point on the solid line of these plots corresponds to a particular choice for M_H . In particular, the marked crosses on the solid line correspond to taking $M_H = 50, 100, \dots, 1500 \text{ GeV}$. The specific values of $O_{1MSM}^{(b)}$, $O_{2MSM}^{(b)}$ and $O_{3MSM}^{(b)}$ are trivially related to the intercept of the straight solid lines in figs. 1,2 and 3 respectively, whereas the slopes are fixed to the values $10c^2/9s^2$, $-10/11$ and $-11c^2/9s^2$ respectively.

We next compare these solid lines with the outcome for $O_1^{(b)}$, $O_2^{(b)}$ and $O_3^{(b)}$ as evaluated from eqs. 15 through 20 in the effective lagrangian description. The dotted lines in figs.1, 2 and 3 correspond to their values for the choice $L_{10} = 0$. Since the solid and dotted lines overlap for large M_H then we conclude that the appropriate choice for L_{10} as for the effective lagrangian description to correctly mimic the MSM with a heavy Higgs (say $M_H > 300 \text{ GeV}$) is given by

$$L_{10}^{MSM} \simeq 0 \quad (23)$$

In contrast, the values for L_1 and L_2 , relevant in longitudinal W scattering [3], that reproduce the MSM are non zero.

On the other hand, and in order to mimic the results for $O_1^{(b)}$, $O_2^{(b)}$ and $O_3^{(b)}$ in a $SU(N_{TC})$ -technicolor theory we choose the value of L_{10} as given by its corresponding value in QCD but conveniently rescaled by the number of technicolors. The leading contribution to L_{10} is proportional to N_{TC} [1,14] and is scale independent [14]

$$L_{10}^{TC} = L_{10}^{QCD} \frac{N_{TC}}{3} \quad (24)$$

For L_{10}^{QCD} we will take here its experimental value. This gives a sufficiently good estimate for our purposes. (See the second reference in [1] and the discussion in [14] for the relation between the bare chiral coefficients, which are the ones relevant in the present analysis where we treat chiral and gauge loops explicitly, and the renormalized ones.)

$$L_{10}^{QCD} \simeq -5.2 \times 10^{-3} \quad (25)$$

The resulting values of $O_{1TC}^{(b)}$, $O_{2TC}^{(b)}$ and $O_{3TC}^{(b)}$ for different values of N_{TC} are collected in table 1. The dashed lines in figs.1, 2 and 3 show the outcome in technicolor theories for $N_{TC} = 3$, $N_{TC} = 5$ and $N_{TC} = 10$.

The important thing to have in mind when looking at figs.1, 2 and 3 is the fact that an experimental measurement of $\Delta\rho$, $\Delta\kappa$ and Δr gives just one point in each plot so that, in principle, by using our results, one should be able to favour or disfavour the various possibilities for the symmetry breaking sector. Unfortunately this is only true for ideal experimental data with no error bars in the measurement of $\Delta\rho$, $\Delta\kappa$ and Δr and also in the ideal situation where one can extract their bosonic contributions without introducing additional uncertainties. In other words, we are assuming that the fermionic contribution, in particular, the one from the top quark is known with infinite precision. Obviously, this is not the case at present since the top quark has not been even discovered. However, the situation for the near future can change drastically if, as expected, the top quark is found at CDF and its mass can be measured (either by direct measurement or by combined analysis of the current experiments) with an uncertainty of, say, $\delta m_t = 10 \text{ GeV}$. Remarkably, in the

observables O_1 , O_2 and O_3 (constructed to eliminate the ultraviolet divergences due to the non-linearity of the theory) there is an almost exact cancellation of the leading quadratic dependence on m_t . A simple exercise shows that an experimental uncertainty of 10GeV in the top mass propagates, for instance in $O_3^{(6)}$, to an uncertainty around $\delta O_3^{(6)} \simeq 1 \times 10^{-4}$, which is below the foreseen experimental errors in direct measurements of $O_3^{(6)}$.

Corrections to our results have different origins, but all of them are expected to be small. First, of course, we have two loop corrections (Both from the pure gauge + ghost part and from $\mathcal{L}^{(2)}$). They are known to be completely negligible. Second, we have one loop corrections from $\mathcal{L}^{(4)}$. By naive counting these are down by a factor $q^2/16\pi^2 v^2$ where q^2 is M_Z^2 for our observables. Finally, we would have to consider $\mathcal{L}^{(6)}$ at tree level. These terms have been discussed with some detail in [14] and their contribution is also very small. The outcome is that the parameters L_1 , L_2 , L_9 and L_{10} give a very good description of the symmetry breaking sector up to energies in the TeV domain.

In conclusion, by the use of a weak chiral lagrangian, we have been able to parametrize the contribution from *any* symmetry breaking sector of the SM to the relevant process-independent quantities measured at LEP in terms of just one quantity, L_{10} . Different symmetry breaking dynamics lead to very different values for L_{10} . For instance, the heavy Higgs case corresponds to $L_{10} = 0$, while the minimal technicolor scenario leads to values for L_{10} in the range described in equation 24. As is apparent from the uncertainty bars in fig.2, moving from the present accuracy in $\Delta\kappa$, Δr (large bars) to the expected accuracy of, say, 3×10^{-3} (small bars) one will be able to discern between the different ways of breaking $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$. The uncertainty bars given in the figures are only indicative. A proper error analysis would require to take into account that the uncertainties in Δr and $\Delta\kappa$ are correlated as they come mainly from the uncertainty in the measurement of M_W . One could, for instance, falsify the MSM, set a bound on the number of technicolors, give some estimation for the Higgs mass or even find some new unexpected result!

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When this paper was being written up we received, almost at the same time, the works quoted in [15], where contributions to some of the parameters relevant to precision tests of the SM are considered in the context of technicolor theories.

References

- [1] S. Weinberg, *Physica* **96 A**(1979)327.
J. Gasser and H. Leutwyler, *Ann. of Phys.* **158**(1984)142.
- [2] T. Appelquist and C. Bernard, *Phys. Rev. D* **22** (1980) 200.
A.C. Longhitano, *Phys. Rev. D* **22** (1980) 1166.
A.C. Longhitano, *Nucl. Phys. B* **188** (1981)118.
O. Cheyette, *Nucl. Phys. B* **297** (1988)183.
- [3] A.Dobado and M.J. Herrero, *Phys.Lett.* **B228**(1989)495; **B233**(1989)505.
J.F. Donoghue and C. Ramirez, *Phys.Lett.* **B234**(1990)361.
- [4] E. Fernandez, Talk given at Neutrino 90 Conference. (1990)
The Aleph Collaboration, D. Decamp et al., preprint CERN-PPE/90-104
- [5] See for instance, G. Altarelli in *Polarization at LEP*, CERN Yellow Book (1988), Ed. by G. Alexander et al.
- [6] O. Cheyette and M. K. Gaillard, *Phys. Lett. B* **197**(1987)205.
A. Dobado, M.J. Herrero and J. Terrón, CERN preprints TH-5670/90 (1990) and TH-5813/90 (1990).
- [7] P. Sikivie, L. Susskind, M. Voloshin and V. Zakharov, *Nucl. Phys.* **173** (1980) 189.
- [8] See third reference in [2].
- [9] W.J. Marciano and A. Sirlin, *Phys. Rev. D* **22** (1980)2697.
- [10] G. Burgers and W. Hollik in *Polarization at LEP*, CERN Yellow Book (1988), Ed. by G. Alexander et al.
M. Consoli and W. Hollik in *Z Physics at LEP 1*, CERN Yellow Book (1989), Ed. by G. Altarelli.
G. Burgers and F. Jegerlehner, *ibid.*
- [11] The CDF Collaboration, F. Abe et al.; *Phys. Rev. Lett.* **64**, (1990) 142.
- [12] For a recent review see for instance, E.J.Eichten, FERMILAB-CONF-90/29-T.
- [13] A. Dobado, D. Espriu and M. J. Herrero, *Work in progress.*
- [14] D. Espriu, E. de Rafael and J. Taron, CERN preprint TH 5599/89, to appear in *Nucl. Phys. B*
- [15] M. Golden and L. Randall, *Fermilab-Pub-90/83-T*, LBL-29050, NSF-ITP-90-82 (1990).
M. Peskin and T. Takeuchi, *SILAC-PUB-5272*(1990).
B. Holdom and J. Terning, *NSF-ITP-90-108*(1990).

Figure Captions

Fig. 1: Relation between $\Delta\kappa^{(b)}$ and $\Delta\rho^{(b)}$ as predicted in different underlying theories. The solid line is the prediction in the MSM. The marked crosses on the solid line correspond to taking $M_H = 50, 100, \dots, 1500 \text{ GeV}$, with the larger values of M_H lying on the straight part of this solid curve. The dotted line is the prediction in the effective chiral Lagrangian description for $L_{10} = 0$. The dashed lines are the predictions in technicolour theories for $N_{TC} = 3$ (the closest line to the solid curve), $N_{TC} = 5$ (the dashed line in between) and $N_{TC} = 10$ (the farthest line to the solid curve) respectively.

Fig. 2: Relation between $\Delta\kappa^{(b)}$ and $\Delta r^{(b)}$ as predicted in different underlying theories. The detailed description is as in fig.1. The large error bars refer to the present uncertainty in the measurement of $\Delta\kappa$ and Δr . The small error bars represent the size of the expected uncertainties for these two parameters in future precision measurements. The position of all these error bars are chosen arbitrarily in this plot.

Fig. 3: Relation between $\Delta r^{(b)}$ and $\Delta\rho^{(b)}$ as predicted in different underlying theories. The detailed description is as in fig.1.

Table Captions

Table 1: Predicted values for the bosonic contributions to the observables O_1 , O_2 and O_3 as defined in the text in different underlying theories.

	MSM	$N_{TC} = 3$	$N_{TC} = 5$	$N_{TC} = 10$
$O_1^{(b)}$	-1.69×10^{-3}	-1.13×10^{-3}	-7.62×10^{-4}	1.67×10^{-4}
$O_2^{(b)}$	6.91×10^{-2}	7.10×10^{-2}	7.22×10^{-2}	7.53×10^{-2}
$O_3^{(b)}$	1.52×10^{-2}	1.62×10^{-2}	1.69×10^{-2}	1.86×10^{-2}

Table 1:

Figure 3

Figure 2