CERN-TH-5829/90
UMN-TH-903/90
LAPP-TH-298/90

Errata for: Axion Hair For Kerr Black Holes.

Bruce A. Campbell
CERN, Theory Division, CH-1211 Geneva 23, SWITZERLAND
and
Department of Physics, University of Alberta,
Edmonton, Alberta, CANADA T6G 2J1

and

M.J. Duncan
and
Nemanja Kaloper
School of Physics and Astronomy, University of Minnesota
Minneapolis MN 55455, USA

and

Keith A. Olive
Laboratoire d'Annecy-le-Vieux de Physique des Particules,
IN2P3-CNRS, BP 110,
F-74941 Annecy-le-Vieux Cedex
FRANCE
and
School of Physics and Astronomy, University of Minnesota
Minneapolis MN 55455, USA

CERN-TH-5829/90
UMN-TH-903/90
LAPP-TH-298/90

1) Eq. (9) should be:

$$\begin{aligned}
 ds^2 = & \left(\frac{r^2 + A^2 \cos^2 \theta - 2Mr}{r^2 + A^2 \cos^2 \theta} \right) dt^2 - \left(\frac{r^2 + A^2 \cos^2 \theta}{r^2 + A^2 - 2Mr} \right) dr^2 \\
 & - (r^2 + A^2 \cos^2 \theta) d\theta^2 - \frac{[(r^2 + A^2)^2 - A^2 \sin^2 \theta (r^2 + A^2 - 2Mr)] \sin^2 \theta}{r^2 + A^2 \cos^2 \theta} d\phi^2 \\
 & - \frac{4 MA \sin^2 \theta r}{r^2 + A^2 \cos^2 \theta} dt d\phi
 \end{aligned} \quad (9)$$

2) Eq. (15) should be:

$$\begin{aligned}
 & \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \left(1 - \frac{2M}{r} \right) \frac{\partial G}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial G}{\partial \theta} \\
 & = \frac{\delta(r_0 - b) \delta(\cos \theta_0 - 1) \delta(\phi_0)}{r_0^2}
 \end{aligned} \quad (15)$$

3) The text following Eq. (15) up to Eq. (27) should be replaced by:

which has as a solution for the axion field strength around a slowly rotating Kerr black hole ($r > 2M$):

$$a(r, \theta) = \frac{5}{48} M^{-3} A \cos \theta \left[\frac{4M^2}{r^2} + \frac{8M^3}{r^3} + \frac{72M^4}{5r^4} \right] \quad (16)$$

Note that Eq. (27) has been replaced by Eq.(16).

4) Eq. (28) should be replaced by Eq(17):

$$a(r, \theta) \sim \frac{5A}{12M} \cos \theta r^{-2} \quad (17)$$

5) We add the following acknowledgement:

We wish to thank W. Israel and E. Poisson for helpful comments on the manuscript. We wish to express special thanks to D. Page for a careful reading of the manuscript and for pointing out an error in our Laplacian which led to the corrections in Eqs. (16) and (17). The work of BAC was supported in part by the Natural Sciences and Engineering Research Council of Canada. The work of MJD, NK, and KAO was supported in part by DOE grant DE-AC02-83ER-40105. In addition the work of KAO was also supported in part by a Presidential Young Investigator Award.

6) References 1,3,7, and 10 have changed to the following:

1. see e.g.: R. Ruffini and J.A. Wheeler, *Physics Today* **24** (1971) 30.
3. B. Carter, *Phys. Rev. Lett.* **26** (1971) 331;
R. Price, *Phys. Rev. D* **5** (1972) 2419 and 2439;
D.C. Robinson, *Phys. Rev. Lett.* **34** (1975) 905;
for a review, see: B. Carter in General Relativity, an Einstein Centenary Survey,
ed. S. W. Hawking and W. Israel (Cambridge Univ. Press, Cambridge 1979).
7. J.D. Bekenstein, *Phys. Rev. Lett.* **28** (1972) 452; *Phys. rev. D* **5** (1972) 1239;
Phys. Rev. **5** (1972) 2403;
C. Teitelboim, *Lett. al. Nuovo Cimento* **3** (1972) 326; *Phys. Rev. D* **5** (1972) 2941.
10. R. Bartnik and J. McKinnon, *Phys. Rev. Lett.* **61** (1988) 141;
P. Bizon, *Phys. Rev. Lett.* **64** (1990) 2844;
H. P. Kunzel and A. K. M. Masood-ul-Alam, *J. Math. Phys.* **31** (1990) 928.

These changes do not affect any of the qualitative results of this paper.

CERN-TH-5829/90
UMN-TH-903/90
LAPP-TH-298/90

Axion Hair For Kerr Black Holes.

Bruce A. Campbell
CERN, Theory Division, CH-1211 Geneva 23, SWITZERLAND
and
Department of Physics, University of Alberta,
Edmonton, Alberta, CANADA T6G 2J1

and

M.J. Duncan
and
Nemanja Kaloper
School of Physics and Astronomy, University of Minnesota
Minneapolis MN 55455, USA

and

Keith A. Olive
Laboratoire d'Annecy-le-Vieux de Physique des Particules,
IN2P3-CNRS, BP 110,
F-74941 Annecy-le-Vieux Cedex
FRANCE
and
School of Physics and Astronomy, University of Minnesota
Minneapolis MN 55455, USA

ABSTRACT

We consider axion fields with Lorentz Chern-Simons coupling to gravity. In Kerr black hole background metrics this coupling gives a non-vanishing source term for the axion field. For the case of a slowly rotating black hole, we explicitly calculate the long range axion field strength "hair" produced. We believe that this represents the first known example of stable, dynamical, non gauge "hair" exhibited by a black hole.

CERN-TH-5829/90
UMN-TH-903/90
LAPP-TH-298/90
July 1990

The "no hair" conjecture¹⁾ has been a useful paradigm in understanding black hole physics. It maintains that the only dynamical field strengths present in exterior black hole solutions are those required by a local gauge invariance. In particular, general coordinate invariance implies that the non-trivial exterior metric field will depend on the mass and angular momentum of the black hole as measured at spatial infinity. If one includes Abelian, or non-Abelian, gauge fields in the theory, then their local gauge invariance implies that one should expect external gauge fields around the black hole, carrying the gauge flux that can be measured on a Gaussian surface at spatial infinity.

In the last twenty-five years, proofs of the "no-hair" conjecture have been adduced in a variety of physical circumstances. In the case of pure gravity, without any other fields, Israel²⁾ showed for the non-rotating black holes the Schwarzschild solution was the unique static exterior metric solution. Subsequent work³⁾ has extended his result to the demonstration that for rotating black holes, the unique stationary exterior metric solutions are those of Kerr, parametrized by their angular momentum and mass.

For the case where gravity is coupled to an Abelian U(1) gauge theory (electromagnetism), Israel further proved⁴⁾ that the exterior solution for a non-rotating black hole must be that of Reissner-Nordstrom. Again, this result has been extended⁵⁾ to the case of rotating black holes, with the demonstration that the solution must be in the Kerr-Newman family, parametrized by the mass, angular momentum, and charge.

For the case of bosonic fields with minimal coupling to gravitation, it can be shown^{6,7)} that a black hole cannot support exterior fields of massless scalars, or of massive bosons of spin zero, one, or two. Nor can⁸⁾ spin one-half fermionic fields, with minimal gravitational coupling, give non-vanishing fermionic hair.

As well as direct proof in these circumstances, evidence for the "no hair" conjecture has come from the failure of efforts to produce counterexamples. In the case of non-Abelian gauge fields coupled to gravity, there have been attempts to find alternatives to the generalized Reissner-Nordstrom solution⁹⁾. While a discrete family of solutions with vanishing Yang-Mills charges, discrete values of the ADM mass, and non-trivial SU(2) Yang-Mills field strength, has been found numerically¹⁰⁾, these solutions are unstable¹¹⁾ against a growing radial perturbation, and are expected to decay to the Schwarzschild solution.

Non-minimal (conformal) coupling of a scalar field to gravity has been considered, in an attempt to use the exterior gravitational field to support scalar hair. While a non-trivial solution

of this form has been found^{12,13)}, it has been shown¹⁴⁾ to be unstable against radial perturbations, and is expected to decay to the Schwarzschild or Reissner-Nordstrom solution.

Recently there has been much interest in black hole solutions with a Kalb-Ramond axion field. The case of non-rotating black holes, with minimally coupled axions, was considered in references [15-17]. As we discuss below, the Kalb-Ramond axion is the pseudoscalar which represents the dynamical degrees of freedom in a three-form field strength H, derived from a two-form potential B, $H = dB$. As such the theory has a local gauge invariance under $B \rightarrow B + d\Lambda$, and we may then expect that black holes will carry a gauge charge defined by a surface integral at spatial infinity. For the solution obtained this is indeed the case, but the effect of the axion charge is purely topological¹⁵⁾, and it does not result in any external field "hair". Since the dynamical degree of freedom is a minimally coupled spinless field, this is exactly what we would expect in view of previous results⁶⁾.

In general, however, we do not expect two-form Kalb-Ramond axion fields to be minimally coupled. Axions normally have axial fermionic couplings and induced anomalous couplings to gauge bosons which are of the Chern-Simons form. Certainly this is the case for the two-form axion in the graviton multiplet in string theory, whose low energy effective action includes Lorentz and Yang-Mills, Chern-Simons contributions to the form field strength.

Let us begin our treatment, then, with a discussion of the dynamics of form fields. We begin with a two-form potential B, and define a three-form field strength :

$$H = dB + \omega_L - \omega_Y \quad (1)$$

where ω_L , ω_Y are the Chern-Simons forms for the Lorentz spin connection and Yang-Mills gauge connection respectively

$$\omega_L = \text{Tr}(\omega \wedge d\omega + \frac{2}{3} \omega \wedge \omega \wedge \omega) \quad (2a)$$

$$\omega_Y = \text{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A) \quad (2b)$$

with $\omega_{ab\mu} = -e_b^\alpha e_{a\alpha\mu}$ the spin connection and e^a_μ the vierbein. Thus H satisfies the Bianchi identity

$$dH = \text{Tr}(R \wedge R) - \text{Tr}(F \wedge F) \quad (3)$$

Since we are interested in gravitational backgrounds only, hereafter we set the Yang-Mills field strength to zero, and drop it from our equations.

The action for our coupled gravity-axion system is (in components)

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right\} \quad (4)$$

which results in the equations of motion¹⁸⁾

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa^2 \left[6 H^{\mu\lambda\rho} H^{\nu\lambda\rho} - g^{\mu\nu} H^{\lambda\rho\sigma} H_{\lambda\rho\sigma} + 4 (H^{\lambda\alpha} \omega^{\nu\rho\sigma} R_{\alpha\lambda}{}^{\nu\sigma}) \right] \quad (5)$$

$$H^{\mu\nu\lambda}{}_{;\lambda} = 0 \quad (6)$$

(We have neglected here the contributions to the action and hence to the equations of motion due to the dilaton. For the complete set of equations of motion see ref.18.) In terms of the dual form $*H$ with components

$$(*H)_\mu = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} \quad (7)$$

the H equation of motion (6) gives $d(*H)=0$, which means that $*H = da$, at least locally. Then the Bianchi identity (6) : $*(dH) = *\text{Tr}(R \wedge R)$ may be written

$$\square a = \frac{1}{4!} \epsilon^{\mu\nu\rho\lambda} R_{\alpha\beta\mu\nu} R_{\rho\lambda}{}^{\beta\alpha} \quad (8)$$

Equations (5) and (8) represent the dynamical equations for the degrees of freedom in our problem.

We see that due to the Chern-Simons term in the form field strength, the dual of the Hürzbruch signature density $*\text{Tr}(R \wedge R)$ acts as a source for the axion field. We now consider whether this coupling could produce axionic black hole hair. For the case of nonrotating black holes, the answer is negative. In our previous paper¹⁸⁾ we have proven generally that spacetimes conformal to a spacetime with a maximally symmetric two-dimensional subspace have exact Lorentz Chern-Simons forms, which only modify the axion potential $B_{\mu\nu}$ and do not contribute to either equations of motion or Bianchi identities. Thus spherically symmetric black hole solutions do not receive any corrections from the Lorentz Chern-Simons terms and reduce to the minimal coupling case. Thus we may take over all the results of references [15-17] and extend them to the case of axions with Chern-Simons couplings, as in string theory. In particular, it remains the case that nonrotating black holes do not have axion field strength "hair".

For rotating black holes, on the other hand, the Lorentz Chern-Simons induced couplings do indeed produce axion field strength "hair". We will demonstrate this explicitly for the case of slowly rotating black holes, and compute the asymptotic long range form of the axion field produced. First note that as the angular momentum of the black hole goes to zero we return to the spherically symmetric case, and all Lorentz Chern-Simons coupling effects must vanish. For sufficiently small rotation then we will produce a weak source for the axion field, in the axion field equation, which in turn will only slightly perturb the background metric due to its stress energy. So to first order in the angular momentum of the black hole we may consider the unperturbed black hole metric as our metric background. This means we will take as our metric background the Kerr metric^{19,20)}:

$$ds^2 = \left(\frac{r^2 + A^2 \cos^2 \theta - 2Mr}{r^2 + A^2 \cos^2 \theta} \right) dt^2 - \left(\frac{r^2 + A^2 \cos^2 \theta}{r^2 + A^2 - 2Mr} \right) dr^2 - (r^2 + A^2 \cos^2 \theta) d\theta^2 - \frac{[(r^2 + A^2)^2 - A^2(r^2 + A^2 - 2Mr)] \sin^2 \theta}{r^2 + A^2 \cos^2 \theta} d\phi^2 \quad (9)$$

$$- \frac{4MA \sin^2 \theta r}{r^2 + A^2 \cos^2 \theta} dt d\phi$$

which reduces to the simpler form :

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) - \frac{4MA \sin^2 \theta}{r} dt d\phi \quad (10)$$

in the limit of small rotation, $A \ll M$ (we use $G=1$ units).

From equation (10) one can calculate directly the dual of the Hirzebruch density. To first order in expansion in A one gets :

$$\frac{1}{4!} \epsilon^{\mu\nu\lambda\sigma} R_{\alpha\beta\mu\nu} R_{\lambda\sigma\beta\alpha} = \frac{-48 M^2 A \cos\theta \sin\theta}{r^5} + \dots \quad (11)$$

We now wish to solve the axion equation of motion (8) in this background to order $O(A)$. We do this by constructing a Green's function (for a timelike line source) satisfying

$$\square_x G(x,y) = \frac{\delta^3(x-y)}{\sqrt{-g(y)}} \quad (12)$$

and then writing

$$a(x) = \int d^3y \sqrt{-g(y)} G(x,y) J(y) \quad (13)$$

where $J(y)$ is the inhomogeneous source term appearing on the right hand side of equation (8). To solve equation (13) to order $O(A)$ we only need to keep terms to linear order in A on the right hand side. Since the expansion in A of $J(y)$ starts at $O(A)$, to find a to linear order in A we only need to keep terms of order $O(A^0)$ in the Green's function; i.e. to obtain the axion field strength to linear order in A we may use the Green's function and square root of metric determinant, of the nonrotating (Schwarzschild) metric! Indeed the metric factors $\sqrt{-g}$ on the right hand sides of equations (8) and (13) cancel from the final answer and may be consistently assumed to be Schwarzschild, so (8) can be written

$$\square a = \frac{-48 M^2 A \cos\theta}{r^7} \quad (14)$$

to the order at which we work, and will be solved as just described.

The problem of finding the Green function for the scalar in the Schwarzschild background is very similar to that of the Green function for the electrostatic potential of a point charge in this background, and may be solved by similar methods^{21,22}. Because our solution is derived by linear response of an elliptic operator to an external field independent source, as in the electrostatic case^{21,22}, the resulting solution will be stable.

Evaluating equation (12) to $O(A^0)$ (i.e., in a Schwarzschild background for a point source at $(r_0 = b, \theta_0 = 0 = \phi_0)$ we get

$$\begin{aligned} \left(1 - \frac{2M}{r}\right) \left[\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial G}{\partial r} + \frac{1}{\left(1 - \frac{2M}{r}\right)} \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \sin\theta \frac{\partial G}{\partial \theta} \right] \\ = \frac{\delta(r_0 - b) \delta(\cos\theta_0 - 1) \delta(\phi_0)}{r_0^2} \end{aligned} \quad (15)$$

In order to construct solutions of this equation, we consider solutions of the homogeneous equation $\square g=0$, which are of the form (when expanded in spherical harmonics)

$$g(r, \theta) = \sum_{\ell=0}^{\infty} R_{\ell}(r) P_{\ell}(\cos\theta) \quad (16)$$

where

$$\left(1 - \frac{2M}{r}\right) \frac{d}{dr} \left(r^2 \frac{dR_{\ell}}{dr}\right) - \ell(\ell+1) R_{\ell}(r) = 0 \quad (17)$$

A basis for the space of linearly independent solutions of (17) is given by²¹

$$gg(r) = \begin{cases} 1, (\ell=0) \\ \frac{2\ell! (\ell-1)! M^{\ell}}{(2\ell)!} (r-2M)^{\ell} \frac{dP_{\ell}}{dr} \left(\frac{r}{M} - 1\right), (\ell \neq 0) \end{cases} \quad (18a)$$

and

$$f_\ell(r) = \frac{-(2\ell+1)!}{2^\ell (\ell+1)! \ell! M^{\ell+1}} (r-2M) \frac{d}{dr} Q_\ell \left(\frac{r}{M} - 1 \right) \quad (18b)$$

where P_ℓ and Q_ℓ are the two types of associated Legendre functions. So for our Green's function $G(r, \theta)$ we write

$$G(r, \theta) = \begin{cases} \sum_{\ell=0}^{\infty} [A_\ell f_\ell(r) + A'_\ell g_\ell(r)] P_\ell(\cos\theta) & (r > b) \\ \sum_{\ell=0}^{\infty} [B_\ell g_\ell(r) + B'_\ell f_\ell(r)] P_\ell(\cos\theta) & (r < b) \end{cases} \quad (19)$$

Then finiteness of the axion field at $r \rightarrow \infty$ gives $A'_\ell = 0$, and finiteness of the axion action at $r \rightarrow 2M$ gives $B'_\ell = 0$. Combined with the requirement of continuity of the axion field at $r=b$ this implies :

$$G(r, \theta) = \begin{cases} \sum_{\ell=0}^{\infty} C_\ell g_\ell(b) f_\ell(r) P_\ell(\cos\theta) & (r > b) \\ \sum_{\ell=0}^{\infty} C_\ell f_\ell(b) g_\ell(r) P_\ell(\cos\theta) & (r < b) \end{cases} \quad (20)$$

To fix the magnitude of the constants C_ℓ , we insert the expansion (20) in equation (15), multiply the resulting equation by $P_n(\cos\theta)$, and integrate the resulting equation over the source region.

After standard manipulations²¹⁾ this gives :

$$C_\ell = \left[4\pi \left(1 - \frac{2M}{b} \right) \right]^{-1} \quad (21)$$

So that our Green's function becomes

$$G(r, \theta) = \frac{1}{4\pi \left(1 - \frac{2M}{b} \right)} \begin{cases} \sum_{\ell=0}^{\infty} g_\ell(b) f_\ell(r) P_\ell(\cos\theta) & (r > b) \\ \sum_{\ell=0}^{\infty} f_\ell(b) g_\ell(r) P_\ell(\cos\theta) & (r < b) \end{cases} \quad (22)$$

Now we note that we may immediately generalize (22) to give the Green's function for an axion point source at (b, θ_0, ϕ) as :

$$G(r, \theta, \phi; b, \theta_0, \phi_0) = \frac{1}{4\pi \left(1 - \frac{2M}{b} \right)} \begin{cases} \sum_{\ell=0}^{\infty} g_\ell(b) f_\ell(r) P_\ell(\lambda) & (r > b) \\ \sum_{\ell=0}^{\infty} f_\ell(b) g_\ell(r) P_\ell(\lambda) & (r < b) \end{cases} \quad (23)$$

where : $\lambda(\theta, \phi) \equiv \cos\theta \cos\theta_0 + \sin\theta \sin\theta_0 \cos(\phi - \phi_0)$ is the angle between source and detector.

In terms of this Green's function we may now write the external axion field solution for (14) as :

$$a(r, \theta, \phi) = \int_{2m}^{\infty} db \int_0^\pi d\theta_0 \int_0^{2\pi} d\phi_0 b^2 \sin\theta_0 G(r, \theta, \phi; b, \theta_0, \phi_0) J(b, \theta_0, \phi_0) \quad (24)$$

where the source term

$$J(b, \theta_0, \phi_0) = \frac{-48 M^2 A}{b^7} \cos\theta_0 \quad (25)$$

The first thing to note is that the source term is a pure dipole ($\sim P_1(\cos\theta_0)$); with a spherically symmetric Green's function this implies that the axion field set up will be of dipole form. Expanding the terms $P_\ell(\lambda)$ in the Green's function by the spherical harmonics addition

theorem, and doing the angular integration using the orthogonality property of the P_ℓ we will only get a contribution from the $\ell=1$ spherical harmonic. So

$$a(r, \theta, \phi) = \frac{-48M^2A}{4\pi} \times \left(\int_{2M}^r \int_0^\pi \int_0^{2\pi} db \, d\theta_0 \, d\phi_0 \, b^2 \sin\theta_0 \, g_1(b) \, f_1(r) \, P_1(\lambda) \cos\theta_0 \, b^{-6} (b-2M)^{-1} \right. \\ \left. + \int_r^\infty \int_0^\pi \int_0^{2\pi} db \, d\theta_0 \, d\phi_0 \, b^2 \sin\theta_0 \, g_1(r) \, f_1(b) \, P_1(\lambda) \cos\theta_0 \, b^{-6} (b-2M)^{-1} \right) \quad (26)$$

Inserting the explicit forms for g_1 , f_1 and P_1 , and doing the ϕ_0 , θ_0 , and r_0 integrals, finally we get for the axion field strength around a slowly rotating Kerr black hole ($r > 2M$):

$$a(r, \theta) = \frac{3}{2} M^4 A \cos\theta (r-2M) \left\{ \frac{1}{2} \ln \left(\frac{r}{r-2M} \right) - \frac{1}{9r^4} [6M^4 + 8M^3r + 9M^2r^2 + 9Mr^3] \right\} \quad (27)$$

Let us now turn to the physical features of our result. First we note that the result is ϕ independent and only involves $\cos(\theta)$, as the field form is dipole, aligned with the black hole rotation axis. It is regular and finite for all $r \geq 2M$. If we extract the leading r behaviour of our answer (27) we find

$$a(r, \theta) \sim \frac{-2A}{3} \cos\theta \, r^{-2} \quad (28)$$

which exhibits the r^{-2} long range fall off one would expect of a dipole (recall that our scalar field is mathematically similar to an electrostatic potential). Also the result is linearly proportional to the angular momentum parameter A as we expect for slow rotation. In short, our solution has all the qualitative features one would expect for a massless field produced by a short range dipole source.

The dipole form of the source term arising from the Hirzebruch signature density means that there is no net total axion charge (monopole moment) carried by the black hole. Indeed the integral of the Hirzebruch signature density in Kerr spacetime is a topological invariant and is zero. It must be zero since the Kerr family of solutions is connected to the Schwarzschild solution by continuous variation of the angular momentum, and we know for Schwarzschild the Hirzebruch density vanishes locally¹⁸. So we have dipole axion "hair", without a net axion source or net axion charge. This "hair" without axion charge is complementary to the effect of charge without "hair" seen previously^{16,17}. But there is no reason that axion charge could not be added to the solution we demonstrate. Then we would have dipole axion "hair" determined by the mass and angular momentum with no net total charge, plus axion charge with only topological effects.

As for the physical effects produced by this axionic "hair" we must recall that the physical couplings of an axion should not be altered by a constant field shift, as it is a Goldstone boson. This implies a derivative coupling of the axion to the (axial) current associated with the global symmetry of which it is the Goldstone mode: $(L_{int} \sim \partial^\mu a J_\mu)$. For our time independent solution, then, we would have coupling of the spatial components of the symmetry current to the spatial gradient of our field. This would give, for example spin dependent potentials for fermions contributing to the axial current, with an asymptotic r^{-3} long range behaviour around the black hole. The detailed form of the physical effects thus produced, would depend on the contributions of the various fields in the theory to the axial current which couples to the axion. Finally, we should recall that all our considerations are classical. When we include quantum effects, our axion may get a mass from its Yang-Mills Chern-Simons coupling, if there is a vev for the $F \wedge F$ combination to which it couples. In this case our treatment would only be a good approximation at a radial distances from the black hole that were smaller than the Compton wavelength of our now massive axion. At distances large compared with the Compton wavelength of the axion, axion propagation is effectively short range and the axion field will die off at the same rate as the source (cf. eq. 14).

In conclusion, we have shown that rotating black holes act as sources for axions which have a Lorenz Chern-Simons induced coupling to gravity. For the case of slowly rotating black holes, we have explicitly evaluated the axion field strength "hair" that this coupling produces. We believe that our result is the first known example of stable, non-gauge field strength hair around a black hole. Our result suggests that non-minimal gravitational couplings may produce interesting new effects in black hole backgrounds, and merit further exploration.

REFERENCES

1. J.A. Wheeler, in Relativity, Groups and Topology, Gordon and Breach (1964).
2. W. Israel, *Phys. Rev.* **164** (1967) 1776.
3. B. Carter, *Phys. Rev. Lett.* **26** (1971) 331;
R. Price, *Phys. Rev. D* **5** (1972) 2419 and 2439;
D.C. Robinson, *Phys. Rev. Lett.* **34** (1975) 905.
4. W. Israel, *Comm. Math. Phys.* **8** (1968) 245.
5. R.M. Wald, *Phys. Rev. Lett.* **26** (1971) 1653;
E.D. Fackerell and J.R. Ipser, *Phys. Rev. D* **5** (1972) 2455.
6. J. Chase, *Comm. Math. Phys.* **19** (1970) 276;
T. Koikawa and M. Yoshimura, *Phys. Lett.* **B189** (1987) 29.
7. J.D. Bekenstein, *Phys. Rev. Lett.* **28** (1972) 452; *Phys. rev. D* **5** (1972) 1239;
Phys. Rev. **5** (1972) 2403;
C. Teitelboim, *Lett. al. Nuovo Cimento* **3** (1972) 326
8. J.B. Hartle, *Phys. Rev. D* **3** (1971) 2938.
9. P.B. Yasskin, *Phys. Rev. D* **12** (1975) 2212.
10. R. Bartnik and J. Minkinson, *Phys. Rev. Lett.* **61** (1988) 141;
P. Bizon, *Phys. Rev. Lett.* **64** (1990) 2844.
11. N. Straumann and Z. Zhou, *Phys. Lett.* **B243** (1990) 33.
12. N.M. Bocharova, K.A. Bronnikov and V.N. Melnikov, *Vestn. Mosk. Univ. Fiz. Astron.* (1970) n°6, 706.
13. J.D. Bekenstein, *Ann. Phys. (N.Y.)* **91** (1975) 75.
14. K.A. Bronnikov and Yu. N. Kiraev, *Phys. Lett.* **A67** (1978) 95.
15. M.J. Bowick, S.B. Giddings, J.A. Harvey, G.T. Horowitz and A. Strominger, *Phys. Rev. Lett.* **61** (1988) 2823.
16. T.J. Allen, M.J. Bowick and A. Lahiri, Syracuse preprint SU-HEP-4238-405 (1989).
17. B.C. Xanthopoulos and T. Zannias, Queen's University preprints (1989).
18. B.A. Campbell, M.J. Duncan, N. Kaloper and K.A. Olive, preprint :
CERN-TH-5801/90, LAPP-TH-293/90, UMN-TH-901/90, (1990).
19. R. Adler, M.J. Bazin and M. Schiffer, Introduction to General Relativity
(Mc Graw-Hill, New York) 1975.
20. C.W. Misner, K.S. Thorne and J.A. Wheeler, Gravitation (Freeman, San Francisco 1973).
21. J.M. Cohen and R.M. Wald, *J. Math. Phys.* **12** (1971) 1845.
22. R.S. Hanni and R. Ruffini, *Phys. Rev. D* **8** (1973) 3259;
B. Linet, *J. Phys. A* **9** (1976) 1081.