CERN-TH.5808/90
ITP-SB-90-54

Strong CP Violation and the Neutron Electric Dipole Moment Revisited

Sinya AOKI¹

Institute for Theoretical Physics,
State University of New York at Stony Brook,
Stony Brook, NY 11794, USA

and

Tetsuo HATSUDA²

CERN, CH-1211 Geneva 23, Switzerland.

Abstract

The relation between the CP violating operator $\theta F\tilde{F}$ and its effective version L_{CP}^{eff} is reconsidered on the basis of the anomalous Ward-Takahashi (WT) identity. The consistency of the previous phenomenological calculations of the neutron electric dipole moment (NEDM) with the WT identity is critically examined, and a new consistent evaluation of NEDM is given. We found NEDM at the tree level of the hadron fields, which is comparable in magnitude with the result of the one-loop calculation.

¹AOKI@SUNYSBNP.BITNET

²THATSUDA@CERNVM.BITNET. On leave of absence from Department of Physics, State University of New York at Stony Brook, Stony Brook, NY 11794, USA. Address after September 1, 1990; Institute for Nuclear Theory, University of Washington, Seattle 98185, USA.

1 Introduction

The absence of the $\theta F\tilde{F}$ term is one of the long-standing problems in QCD, which have stimulated the search for the rationale such as the Peccei-Quinn symmetry [1] and its variations (see the reviews, [2,3]). Experimentally, the most stringent bound on θ comes from the measurement of the neutron electric dipole moment (NEDM) [4] which should vanish if there is no CP violation. On the theoretical side, the precise calculation of NEDM is rather difficult since it requires the non-perturbative evaluation of the nucleon matrix element of $F\tilde{F}$. Instead of taking such a matrix element, it has been proposed to replace $F\tilde{F}$ by an effective CP violating operator L_{CP}^{eff} written only by the quark fields[5], which makes one possible to apply the current algebra techniques and the effective Lagrangians of QCD at low energies. So far, a number of papers has been published based on this idea. (See e.g. the reviews [2,3,6].) One of the problems of these approaches is that it is not clear whether the results and the approximations adopted are consistent with the anomalous Ward-Takahashi (WT) identity. Actually, some of the previous calculations explicitly break the constraint from the WT identity as we will show.

The purpose of this paper is to clarify the above issue in view of a general principle and also to give a new consistent evaluation of NEDM. First we carefully examine the relation of $F\tilde{F}$ and L_{CP}^{eff} on the basis of the anomalous WT identity for the axial current in section 2. The analysis tells us about the danger in the naive use of L_{CP}^{eff} , and clarifies the problematic points in the previous calculations of NEDM. Then, in section 3, we reanalyse NEDM based on an effective Lagrangian which is consistent with the WT identity and gives a large and non-vanishing contribution at the tree level of the hadron fields. Section 4 is devoted to the summary and concluding remarks. Throughout this paper, we limit ourselves to the discussion of the continuum QCD and its effective Lagrangians. As for the calculation of NEDM on the lattice[7], it has been discussed in the recent paper by Aoki, Gocksch, Manohar and Sharp [8].

2 Anomalous WT Identity

We first assume the following anomalous WT identity for QCD without a θ term:

$$((\partial^\mu J_\mu^5(x) - 2m\bar{\psi}\gamma_5\psi(x) - N_f \frac{g^2}{16\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a(x))\mathcal{O} + \delta_\varepsilon \mathcal{O}) = 0, \quad (2.1)$$

where J_μ^5 is the (flavor singlet) axial current, m is the quark mass¹, g is the QCD coupling constant, N_f is the number of flavors, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ with $\epsilon_{0123} = 1$, and δ_ε is the (infinitesimal) variation under the local chiral rotation at site x , $\mathcal{O}$ is an arbitrary operator and $\langle \cdot \rangle$ denotes the average over all the configuration of quarks and gluons.² If one recalls the derivation of the axial anomaly [9], it is easy to see that the above WT identity indeed implies the following two independent equations when $\mathcal{O}$ contains the quark operators:

$$((\partial^\mu J_\mu^5(x) - 2m\bar{\psi}\gamma_5\psi(x))\mathcal{O})_{dis} = N_f \frac{g^2}{16\pi^2} (F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a(x)\mathcal{O}), \quad (2.2)$$

$$((\partial^\mu J_\mu^5(x) - 2m\bar{\psi}\gamma_5\psi(x))\mathcal{O})_{con} = -(\delta_\varepsilon \mathcal{O}), \quad (2.3)$$

where $(\mathcal{O})_{dis}$ denotes the *disconnected* insertions of A (where $A = \partial^\mu J_\mu^5$ or $2m\bar{\psi}\gamma_5\psi$) into the operator $\mathcal{O}$ and $(\mathcal{O})_{con}$ denotes the *connected* insertions of A into the operator $\mathcal{O}$ [8]. The famous triangle diagram which produces the axial anomaly is a former example. In other words, Eq. (2.2) means that, if we connect the fermion and anti-fermion operators in A to create the fermion loop, the sum of $\partial^\mu J_\mu^5$ and $2m\bar{\psi}\gamma_5\psi$ produces the anomaly at the lowest order (g^2), and the radiative corrections to these diagrams are always cancelled between $\partial^\mu J_\mu^5$ and $2m\bar{\psi}\gamma_5\psi$ (Adler-Bardeen Theorem [10]). Eq. (2.3) says that the sum of the insertions of $\partial^\mu J_\mu^5$ and $2m\bar{\psi}\gamma_5\psi$ into the fermion line, which starts from or ends at the operator $\mathcal{O}$, is always equal to the local chiral variation of the operator $\mathcal{O}$.

Now we consider the space-time integration of the anomalous WT identity eqs. (2.2-3). Since there are no massless particles which couple to

¹For simplicity, all the current mass is assumed to be m . However, it is easy to generalize the argument to the asymmetric quark masses.

²We use Bjorken-Drell notation for metric and γ -matrices throughout this paper.

the axial current J_μ^5 as long as $m \neq 0$, we can safely use the integration by parts:

$$\int d^4x (\partial^\mu J_\mu^5(x) \mathcal{O})_{dis} = \int d^4x (\partial^\mu J_\mu^5(x) \mathcal{O})_{con} = 0. \quad (2.4)$$

Thus we obtain

$$\int d^4x 2m (\bar{\psi} i \gamma_5 \psi(x) \mathcal{O})_{dis} = -2N_f \langle Q \mathcal{O} \rangle, \quad (2.5)$$

$$\int d^4x 2m (\bar{\psi} i \gamma_5 \psi(x) \mathcal{O})_{con} = \int d^4x (\delta_z \mathcal{O}), \quad (2.6)$$

where the topological charge Q is defined as

$$Q = \frac{g^2}{32\pi^2} \int d^4x F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a(x) \equiv \int d^4x q(x). \quad (2.7)$$

Equation (2.5) implies that the *disconnected* insertion of $\int d^4x m \bar{\psi} i \gamma_5 \psi$ is identical to the insertion of $-N_f Q$.

One can use the above relations to evaluate the effect of small θ on the physical quantities:

$$\begin{aligned} \langle \mathcal{O} \rangle_\theta &\equiv \langle \mathcal{O} \exp(-i\theta Q) \rangle \\ &\simeq \langle \mathcal{O} \rangle - i\theta \langle Q \mathcal{O} \rangle \\ &= \langle \mathcal{O} \rangle + i\theta \frac{m}{N_f} \int d^4x (\bar{\psi} i \gamma_5 \psi(x) \mathcal{O})_{dis} \\ &\simeq \langle \mathcal{O} \exp[i\theta \frac{m}{N_f} \int d^4x (\bar{\psi} i \gamma_5 \psi)] \rangle_{dis} \\ &\simeq \langle \mathcal{O} \exp[im \int \tilde{\psi} (e^{i\theta \gamma_5 / N_f} - 1) \psi] \rangle_{dis}. \end{aligned} \quad (2.8)$$

Eq.(2.8) represents the precise meaning of the usual statement that the effect of $-i\theta Q$ can be replaced by the complex fermion mass term. (Note that one can prove this equivalence even if θ is not infinitesimal.) However one should remark that the *connected* insertion of $\int d^4x 2m \bar{\psi} i \gamma_5 \psi(x)$ has nothing to do with the θ term as eq.(2.6) tells us; this produces only the chiral variation of the external fermions in the operator $\mathcal{O}$. This chiral rotation cannot produce any physical effect as the equivalence theorem [11] tells us (we show it explicitly in the last part of this section), which means that *the*

connected insertions are essentially zero for the physical processes once the on-shell condition are imposed on the external lines. In some papers (e.g. [5,12]), the *connected* insertion of $\int d^4x 2m \bar{\psi} i \gamma_5 \psi(x)$ is considered and the finite results are obtained for NEDM. However, they cannot be correct since such a contribution has to be zero. One can imagine two possible origins of such a fake result³: (i) Breaking of the chiral invariance in the intermediate states and (ii) an inadequate use of the on-shell condition for the external lines. In the former, if the chirally non-invariant intermediate states were used to calculate NEDM, it would break the equality (2.6) and a non-zero result would be obtained. However, there always exist contributions from the other intermediate states which ensure the chiral invariance and tend to cancel the fake contribution. (Note that the identity operator $1 = \sum_n |n\rangle \langle n|$ is chiral invariant.) In the latter case, if the correct on-shell condition for the nucleon is not imposed (which often occurs when one uses the extended models of the nucleon which does not have an exact covariance), a fake contribution remains since r.h.s. of eq.(2.6) does not vanish.

Here let us briefly see what happens when one takes the chiral limit ($m \rightarrow 0$). One is not allowed to take the limit separately in eq.(2.5) and (2.6) since each matrix element can have a massless pole, while it does not appear in the sum.⁴

$$\int d^4x 2m \langle \bar{\psi} i \gamma_5 \psi(x) \mathcal{O} \rangle = -2N_f \langle Q \mathcal{O} \rangle + \int d^4x (\delta_z \mathcal{O}). \quad (2.9)$$

By taking the chiral limit in this formula, one gets

$$2N_f \langle Q \mathcal{O} \rangle = \int d^4x (\delta_z \mathcal{O}), \quad (2.10)$$

which is a well-known relation telling us that the effect of the θ term is equivalent to the chiral rotation with the angle $-\theta/2N_f$ in the chiral limit.[13] However, one should remember that it does not imply that there

³In the lattice calculation[7], the violation of the WT identity by the finite lattice spacing causes the fake result.[8]

⁴Remember the structure of the connected part is flavor independent in the chiral limit. Then a Nambu-Goldstone pole certainly appears in (2.6) which is cancelled by the same but opposite contribution arising from (2.5).

is no effect of θ in the matrix elements. In fact, off-shell matrix elements may be non-zero for the chiral non-invariant operators, e.g.,

$$\langle \bar{\psi} \gamma_5 \psi(y) \rangle_\theta \propto \int d^4x \langle \delta_x (\bar{\psi} \gamma_5 \psi(y)) \rangle = -2 \langle \bar{\psi} \psi(y) \rangle \neq 0. \quad (2.11)$$

Of course, as we mentioned and will show soon, the *on-shell* quantities for $\int d^4x \delta_x \mathcal{O}$ are always zero, which implies that one cannot observe the physical effect of θ in the chiral limit. This gives a constraint on the structures of the low energy effective theory of QCD; the terms induced by the $U_A(1)$ anomaly of QCD should satisfy e.g. (2.10) which is violated in some previous calculation of NEDM (e.g., ref.[14]). We will return to this point in section 3.

The above consideration has to be modified if there is no dynamical breaking of chiral symmetry. Since there are no Nambu-Goldstone modes in this phase, (2.5) and (2.6) are satisfied separately in the chiral limit;

$$\begin{aligned} \langle Q\mathcal{O} \rangle &= 0, \\ \int d^4x \langle \delta_x \mathcal{O} \rangle &= 0. \end{aligned} \quad (2.12)$$

This implies that the effect of the anomaly is completely suppressed in this phase irrespective of the on-shell or off-shell nature of the matrix element. It would be interesting to generalize this statement to finite temperature and/or density, which are relevant to the physics of the quark-gluon plasma.

Now we prove that the $\delta\mathcal{O}$ term cannot contribute to the on-shell quantities as we promised.⁵ (We have defined $\delta = \int d^4x \delta_x$ as the $U(1)$ global chiral rotation.) Let us consider the following general operator:

$$\mathcal{O} = \prod_{i=1}^l B_i \prod_{j=1}^m F_j \prod_{k=1}^n \bar{F}_k, \quad (2.13)$$

where B_i is a bosonic composite operator made of quarks, which produces a physical meson with mass M_i , and F_j ($\bar{F}_j$) is a fermionic composite operator made of quarks, which produces a physical fermion (anti-fermion) with

⁵This proof can be regarded as a special case of the invariance of the S-matrix under the change of variables.[11]

mass m_j .⁶ For example, $B_i = \bar{\psi} \gamma_5 \psi$ or $\bar{\psi} \psi$, $F_j = \bar{\psi} \psi \psi$ and $\bar{F}_j = \bar{\psi} \bar{\psi} \bar{\psi}$. The on-shell amplitude $S(\mathcal{O})$ is proportional to the pole residue of the vacuum expectation value of $\mathcal{O}$:

$$S(\mathcal{O}) = \frac{1}{(2\pi)^{3(l+m+n)/2}} \prod_{i,j,k} \sqrt{\frac{m_k}{2E_i}} \sqrt{\frac{m_k}{E_j}} \bar{u}(p_k) T(\mathcal{O}) u(p_i), \quad (2.14)$$

where the external momentum p_a satisfies the on-shell conditions, E_a is the energy, and $\bar{u}$ and u are the on-shell spinors which satisfy the Dirac equation. Since $\mathcal{O}$ has a pole where all external momenta satisfy their on-shell conditions, we have defined the residue $T(\mathcal{O})$ such that

$$\langle \mathcal{O} \rangle = \prod_{i,j,k} \frac{1}{p_i^2 - M_i^2} \frac{1}{\gamma \cdot p_k - m_k} T(\mathcal{O}) \frac{1}{\gamma \cdot p_j - m_j}, \quad (2.15)$$

for positive energy particles. Now we will prove

$$S(\delta\mathcal{O}) = 0, \quad (2.16)$$

provided that the external momenta satisfy the *same* on-shell conditions with those of $S(\mathcal{O})$. From eq. (2.12),

$$\delta\mathcal{O} = \sum_i \delta B_i \mathcal{O}_{B_i} + \sum_j \delta F_j \mathcal{O}_{F_j} + \sum_k \delta \bar{F}_k \mathcal{O}_{\bar{F}_k}, \quad (2.17)$$

where $\mathcal{O}_A$ is defined as $\mathcal{O} = A \mathcal{O}_A$ with $A = B_i, F_j$ or $\bar{F}_k$. The $U_A(1)$ chiral rotation among the fields B_i, F_j and $\bar{F}_k$ can be written using the c-number matrices C, D and D' ,

$$\begin{aligned} \delta B_i &= \sum_{j \neq i} C_{ij} B_j, \\ \delta F_i &= \gamma_5 D_{ii} F_i + \sum_{j \neq i} D_{ij} F_j, \\ \delta \bar{F}_i &= D'_{ii} \bar{F}_i \gamma_5 + \sum_{j \neq i} D'_{ij} \bar{F}_j. \end{aligned} \quad (2.18)$$

⁶Here we consider only the spin-0 and 1/2 particles. We can extend our analysis to the higher spin particles with some complication. In the practical point of view, pion (spin-0) and nucleon (spin-1/2) are enough to be considered as on-shell states, since all the physical process takes place through the interaction of these stable particles as asymptotic states.

The non-zero off-diagonal matrix elements of C , D and D' mean the presence of the chiral multiplet. The explicit construction of these operators for the nucleon can be seen in ref.[13].

To prove eq. (2.16), it is enough to prove $S(\delta A O_A) = 0$ for an arbitrary $A = B_i, F_j$ or $\bar{F}_k$. If the chiral symmetry is not broken,

$$\begin{aligned} \langle O(A) \rangle &= \langle O(A + \delta A) \rangle \\ &= \langle O(A) + \delta A O_A \rangle. \end{aligned} \quad (2.19)$$

This yields

$$\langle \delta A O_A \rangle = 0. \quad (2.20)$$

Here we write the argument A of O explicitly. Therefore $S(\delta A O_A)$ vanishes trivially in this case.

If the chiral symmetry is (explicitly or spontaneously) broken, the masses of the particles in the chiral multiplet become different, i.e. $M_i \neq M_j$ and $m_i \neq m_j$ for $i \neq j$.⁷ Using this fact and eq. (2.13), we obtain

$$S(\delta B_i O_{B_i}) \sim \sum_{i \neq j} C_{ij} \frac{p_i^2 - M_i^2}{p_j^2 - M_j^2} T(B_j O_{B_i}) = 0, \quad (2.21)$$

for boson operators because of the on-shell condition for p_i . For fermions with a positive energy particle,

$$\begin{aligned} S(\delta \bar{F}_i O_{F_i}) &\sim \sum_{i \neq j} D'_{ij} [T(\bar{F}_j O_{F_i}) \frac{\gamma \cdot p_i - m_i}{\gamma \cdot p_i - m_j} u(p_i) \\ &\quad + D'_{ii} [T(O) \frac{1}{\gamma \cdot p_i - m_i} \gamma_5 (\gamma \cdot p_i - m_i)] u(p_i) \\ &= -D'_{ii} T(O) \gamma_5 \frac{\gamma \cdot p_i - m_i}{\gamma \cdot p_i + m_i} u(p_i) = 0, \end{aligned} \quad (2.22)$$

⁷We may construct the effective action where the mass term of the multiplet is chiral symmetric ($M_i = M_j$ or $m_i = m_j$) while the interactions break the $U_A(1)$ invariance. In this case, $S(\delta A O_A) \neq 0$. However the above equality of masses is unstable against the higher order corrections since the interactions break $U_A(1)$ invariance, i.e., $M_i \neq M_j$ and $m_i \neq m_j$ result after the loop corrections. Therefore we exclude the possibility of such accidental degeneracy of the masses in the chiral broken phase.

and

$$\begin{aligned} S(\delta F_i O_{F_i}) &\sim \sum_{i \neq j} D_{ij} \bar{u}(p_i) \left[\frac{\gamma \cdot p_i - m_i}{\gamma \cdot p_i - m_j} T(F_j O_{F_i}) \right] \\ &\quad + D_{ii} \bar{u}(p_i) [(\gamma \cdot p_i - m_i) \gamma_5 \frac{1}{\gamma \cdot p_i - m_i} T(O)] \\ &= -D_{ii} \bar{u}(p_i) \frac{\gamma \cdot p_i - m_i}{\gamma \cdot p_i + m_i} \gamma_5 T(O) = 0, \end{aligned} \quad (2.23)$$

because of the on-shell condition and the Dirac equation which u and $\bar{u}$ satisfy. The same proof holds for anti-particles where the on-shell condition becomes $(\gamma \cdot p_i + m_i) v(p_i) = 0$.

Thus, $S(\delta A O_A) = 0$ for all A irrespective of the realization of the chiral symmetry, which completes the proof of $S(\delta O) = 0$. The essential point is that, by the infinitesimal chiral transformation, the original operator turns into the one creating a particle with different mass or into the same particle with extra γ_5 . The on-shell condition for the original particle kills both.

3 Electric Dipole Moment of Neutron

The measurement of the neutron electric dipole moment (NEDM) gives the most stringent bound on the value of the θ -parameter at present. To extract a bound on θ from experiment, we need to calculate the following matrix element in a reliable way:

$$i \int d^4x \langle n(p') | T[J_\mu(0) L_{CP}(x)] | n(p) \rangle = -D_n \bar{u}(p') \sigma_{\mu\nu} \gamma_5 k^\nu u(p), \quad (3.1)$$

where $L_{CP}(x) = -\theta q(x)$ with $q(x)$ being the topological charge density. (We have used the first-order perturbation for θ since it is known to be small.) Various authors have calculated the l.h.s. of eq.(3.1) using the low energy effective theories of QCD (see the reviews, [2,3]) or the numerical simulation on the lattice [7]. As we discussed in section 2, however, one should keep in mind that only the disconnected part has to be properly taken into account in the calculation when one uses the effective operator L_{CP}^{eff} in place of $-\theta q$.

In the following, we will give a direct estimate of the l.h.s. of eq.(3.1), i.e. the matrix element of $-\theta Q$ using an effective meson-baryon Lagrangian. We will first calculate it in the 2-flavor case to clarify the inconsistency in the previous calculations. Then we examine the 3-flavor case and show that there exists a new tree-level contribution to NEDM which has not been addressed so far. Essential constraints from the WT identity discussed in sec.2 are (i) the vanishing of the matrix element in the chiral limit and/or in the absence of the axial anomaly and (ii) the independence of the result under the change of variables.

The meson part of the low energy effective action incorporating the axial anomaly is well known [15]

$$\mathcal{L}_M(U, \theta) = \frac{f_\pi^2}{4} \text{Tr}(\partial^\mu U \partial_\mu U^\dagger) + \frac{f_\pi^2}{4} \text{Tr}(MU + M^\dagger U^\dagger) + \frac{i}{2} q(x) \text{Tr}(\log U - \log U^\dagger) + \frac{1}{af_\pi^2} q^2(x) - \theta q(x), \quad (3.2)$$

where $f_\pi=93\text{MeV}$, M is the explicit chiral symmetry breaking ($M=\text{diag}(m_\pi^2, m_\pi^2)$ for flavor $SU(2)$ and $M=\text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$ for $SU(3)$), a is a constant which controls the anomaly and U is a chiral field defined by

$$U = \exp(i\phi^a \lambda^a / f_\pi), \quad (3.3)$$

with λ^a ($a=0 \sim N_f$) being the flavor matrices normalized as $\text{Tr}(\lambda^a \lambda^b) = 2\delta_{ab}$. In the $N_f = 2$ case ($\phi^{1,2,3} = \pi^{1,2,3}$, $\phi^0 = \eta$), the equation of motion at the tree level⁸ yields the $PCU_1 C$ relation [16]

$$q(x) = af_\pi \eta(x), \quad (3.4)$$

and the parameter a is written as

$$a = \frac{1}{2}(m_\eta^2 - m_\pi^2), \quad (3.5)$$

which is $O(1/N_c)$ and vanishes in the absence of anomaly.

Before considering the baryon sector, we will examine an implication of the anomalous WT identity in this effective action. By changing the

⁸This is exact since the $q(x)$ integration is gaussian.

variables in the flavor singlet meson such that $\eta \rightarrow \eta - f_\pi \theta / 2$, we can eliminate $-\theta q(x)$ in (3.2) and get

$$\mathcal{L}_M(U e^{-i\theta \lambda^0 / 2}, 0) = \frac{f_\pi^2}{4} \text{Tr}(\partial^\mu U \partial_\mu U^\dagger) + \frac{f_\pi^2}{4} \text{Tr}(MU e^{-i\theta \lambda^0 / 2} + M^\dagger e^{i\theta \lambda^0 / 2} U^\dagger) + \frac{i}{2} q(x) \text{Tr}(\log U - \log U^\dagger) + \frac{1}{af_\pi^2} q^2(x), \quad (3.6)$$

where θ appears only in the explicit symmetry breaking term corresponding to the operator $m\bar{\psi}e^{-i\theta \lambda^0 / 2}\psi$ in the quark level. Both effective actions $\mathcal{L}_M(U e^{-i\theta \lambda^0 / 2}, 0)$ and $\mathcal{L}_M(U, \theta)$ should give the same result since the change of variables does not affect the physical quantities.[11] Therefore we can conclude immediately that the CP violation in the physical quantities should vanish in the absence of anomaly ($a = 0$) and/or in the chiral limit ($m_\pi^2 = 0$).

Let us demonstrate this explicitly in the case of the $\eta \rightarrow 2\pi$ decay. From (3.2) and (3.4), the matrix element for this process is given by the zero momentum insertion of $-\theta q(x) = -\theta f_\pi \eta(x)$,

$$-i\theta f_\pi a \int d^4x \langle \eta | \eta(x) | \pi^+ \pi^- \rangle, \quad (3.7)$$

at the tree level, while from (3.6), we obtain

$$i\theta \frac{f_\pi}{2} m_\pi^2 \times \int d^4x \langle (\eta | \eta(x) | \pi^+ \pi^-) - \frac{1}{f_\pi^2} \langle \eta | \eta \pi^+ \pi^- (x) | \pi^+ \pi^- \rangle \rangle. \quad (3.8)$$

Since at the tree level

$$\int d^4x \langle \eta | \eta | \pi^+ \pi^- \rangle = \frac{m_\pi^2}{f_\pi^2 m_\eta^2} \quad \text{and} \quad \int d^4x \langle \eta | \eta \pi^+ \pi^- | \pi^+ \pi^- \rangle = 1,$$

both (3.7) and (3.8) give the same result:

$$i\theta \frac{a}{f_\pi} \frac{m_\pi^2}{m_\eta^2}, \quad (3.9)$$

which has the correct property mentioned above. If one adopts the first-order chiral perturbation however, (3.9) reduces to $i\theta m_\pi^2 / 2f_\pi$ ([17]) which

does not vanish in the absence of anomaly ($a \rightarrow 0$). (Numerically, the perturbation is not a bad approximation though.) If there exists the small isospin breaking term,

$$\frac{f_\pi^2}{4} \delta m^2 \text{Tr}[\lambda^3(U + U^\dagger)],$$

it causes the mixing between η and the neutral pion (π^0). Then the matrix element is modified as

$$-i\theta f_\pi a \times \int d^4x [\cos \phi \langle \eta | \eta | \pi^+ \pi^- \rangle - \sin \phi \langle \eta | \pi^0 | \pi^+ \pi^- \rangle], \quad (3.10)$$

where

$$\int d^4x \langle \eta | \eta | \pi^+ \pi^- \rangle = \frac{[(m_\eta^2 - m_\pi^2) 2 \sin^2 \phi + m_\pi^2 (1 + 2 \cos^2 \phi) + \delta m^2 \sin 2\phi]}{3f_\pi^2 m_\eta^2},$$

and

$$\int d^4x \langle \eta | \pi^0 | \pi^+ \pi^- \rangle = \frac{[(m_\eta^2 - m_\pi^2) \sin 2\phi - m_\pi^2 \sin 2\phi + \delta m^2 \cos 2\phi]}{3f_\pi^2 m_\pi^2}.$$

Here m_π is the charged pion mass and the mixing angle ϕ satisfies $\tan 2\phi = \delta m^2/a$ with $m_\eta^2 = m_\pi^2 + a + \sqrt{a^2 + (\delta m^2)^2}$ and $m_{\pi^0}^2 = m_\pi^2 + a - \sqrt{a^2 + (\delta m^2)^2}$. (Therefore the quantity a is expressed in terms of the meson masses: $a = (m_\eta^2 + m_{\pi^0}^2 - 2m_\pi^2)/2$.) It is easy to check that both (3.2) and (3.6) also give the same result in this case.

The baryon sector in $N_f=2$ is written in terms of the baryon field transforming as a fundamental representation of $SU(2)$,

$$\mathcal{L}_B(B, U) = \bar{B}(i\gamma \cdot \partial - \alpha U_3^1)B - \gamma \bar{B}MB + \bar{B}\mu\sigma_{\mu\nu}U_3^1 B(\partial^\nu A^\mu). \quad (3.11)$$

Here the first term is manifestly chiral invariant since $U_3 = \exp(i\gamma_5 \phi^a \tau^a / f_\pi)$ with $\tau^a = (1, \vec{\tau})$; the second term breaks $U(2)$ in the same way with the meson sector and the last term denotes the phenomenological (anomalous) magnetic moment of the baryon with μ being a 2×2 matrix and A_μ being the photon field. A few comments are in order. First, we constructed this effective action to make $\mathcal{L}_B U_A(1)$ invariant (see U_3^1 in the last term. It can

be just $U_3^1 = \exp(-i\gamma_5 \phi^0 / f_\pi)$, but the difference does not matter in the tree-level calculation of NEDM.). This is because the electromagnetism does not break $U_A(1)$ although it breaks $SU(2)$.⁹ Secondly, we did not introduce the usual vector coupling $A_\mu [\bar{B}\gamma_\mu(\tau_3 + 1)B]/2$ since it does not contribute to NEDM at the tree level because of its vector nature.

Now let us evaluate the matrix element (3.1) in two different ways which will clarify how the general discussion in section 2 is realized and also what is wrong in some of the previous calculations. Here we use (3.2) as an effective action for the mesons. If we expand $\mathcal{L}_B$ up to the first-order of the meson fields, we get

$$\begin{aligned} \mathcal{L}_B^I &= \bar{B}(i\gamma \cdot \partial - m_N)B + \bar{B}\mu\sigma_{\mu\nu}B(\partial^\nu A^\mu) \\ &\quad + g_s \bar{B}i\gamma_5 \tau^a B \phi^a - \bar{B}\mu\sigma_{\mu\nu}i\gamma_5 \tau^a B(\phi^a / f_\pi)(\partial^\nu A^\mu), \end{aligned} \quad (3.12)$$

where $g_s = \alpha / f_\pi$ and $m_N = \alpha + \gamma m_\pi^2$ are the pion-nucleon coupling constant and the physical nucleon mass respectively. Another form of $\mathcal{L}_B$ is obtained by defining the chiral invariant nucleon $N = \sqrt{U_3^1} B$;

$$\begin{aligned} \mathcal{L}_B^{II} &= \bar{N}(i\gamma \cdot \partial - m_N)N + \bar{N}(\sqrt{U_3^1} i\gamma_\mu \partial^\mu \sqrt{U_3})N \\ &\quad - g_w \bar{N} \tau^a i\gamma_5 N \phi^a + \bar{N}\mu\sigma_{\mu\nu}N(\partial^\nu A^\mu), \end{aligned} \quad (3.13)$$

where $g_w = \gamma m_\pi^2 / f_\pi$. One should note that this change of variables does not alter the final result.[11] If one uses (3.12) (case I), we have a pseudoscalar (ps) nucleon-meson coupling and an additional contact photon-meson-baryon coupling. On the other hand, in (3.13) (case II), we have a ps-coupling from the explicit symmetry breaking and a pseudovector (pv) coupling from the invariant term.

Our CP-violating operator $L_{CP} = -\theta q$ is written by the gluon field carrying zero momentum. Therefore the pv-coupling does not contribute at the tree level and only two diagrams shown in Fig.1 are relevant in case II. The explicit evaluation of them at the tree level gives

$$D_n = \theta(a f_\pi) \frac{1}{m_\eta^2} \frac{1}{m_N} g_w \mu_n. \quad (3.14)$$

⁹In principle, one can add terms which break $U_A(1)$ but vanish in the chiral limit. However, it is quite difficult to fix the coefficients phenomenologically, so we have taken the simplest possible form.

The meaning of each term in D_n is self-evident. af_π just denotes the coupling strength of $q(x)$ to the physical η meson. Rewriting it by the physical quantities, we get

$$D_n = \theta \frac{m_\eta^2 - m_\pi^2}{2m_N^2} \frac{m_\pi^2}{m_\eta^2} \tilde{\gamma} \mu_n, \quad (3.15)$$

where we have defined a dimensionless number $\tilde{\gamma} = \gamma m_N$ characterizing the strength of the explicit symmetry breaking. (3.15) shows that D_n vanishes in the absence of anomaly ($m_\eta \rightarrow m_\pi$) and/or in the chiral limit ($m_\pi \rightarrow 0$), which is consistent with the general constraint. The former aspect ensures that we took into account only the disconnected piece correctly and the latter is related to the fact $\langle \delta O \rangle|_{m_\pi \rightarrow 0} = 0$. One should note that if one adopts the first order of the chiral perturbation, a disappears from the formula: $D_n \rightarrow \theta(m_\pi^2/2m_N^2) \tilde{\gamma} \mu_n$. Although it is not a bad approximation numerically, it is in principle inconsistent since it does not vanish in the absence of anomaly.

If we perform the same calculation using $\mathcal{L}_B^I$, we have three diagrams at the tree level (Fig.2). The sum of the first two gives

$$D_n^{12} = -\theta(a f_\pi) \frac{1}{m_\eta^2} \frac{1}{m_N^2} g_\pi \mu_n = -\theta a \frac{1}{m_\eta^2} \frac{a}{f_\pi m_N} \mu_n, \quad (3.16)$$

which does not vanish in the chiral limit. In other words, if one were not to make $\mathcal{L}_B$ $U_A(1)$ invariant, it gives a wrong result. The remedy comes from the third term related to the $U_A(1)$ invariant magnetic moment which gives

$$D_n^3 = \theta a \frac{1}{m_\eta^2} \mu_n. \quad (3.17)$$

This term cancels a part of D_n^{12} and recovers the desired result, eq.(3.15).

Since it becomes now clear how to do the consistent evaluation of NEDM by the above exercise, let us give a diagnosis of the previous calculations. They are classified in two categories: (i) The direct evaluation of $-\theta Q$ using the effective Lagrangians with an explicit gluon field [18,19] or with

the instanton induced vertex [20], and (ii) calculations using $L_{CP}^{eff} \sim \theta m \bar{\psi} \gamma_5 \psi$ in place of $-\theta Q$. [5,12,14,21,22,23,24]

In the former approach, there arises no contradiction to the general principle since only the disconnected part is treated from the beginning. In ref.[18], the photoproduction of η' off the nucleon is adopted to estimate D_n . A problem is that they have several assumptions for the amplitude which makes it almost impossible to estimate how reliable the result is. In ref.[19], the pion loop contribution, which is a dominant term in the chiral limit due to $\log(m_N^2/m_\pi^2)$, is adopted following ref.[21]. We will show, however, there is a tree-level contribution which is comparable to, or larger in size than, this one-loop result in the last part of this section. In ref.[20], the instanton-induced 't Hooft type vertex [25] is used to calculate NEDM at the one loop. The vertex has a correct structure consistent with our constraint. Unfortunately, the structure of the 't Hooft vertex depends on the number of flavors and the calculation of NEDM is given only in the simplest $N_f = 2$ case in the paper.

In approach (ii), one has to make sure that the direct insertion defined in section 2 is properly eliminated. For example, the direct insertion of L_{CP}^{eff} in the quark field gives a finite result in the approaches using the bag models[5,12] and using the lattice QCD [7], which cannot be true. It is just a calculation of the matrix element $\langle J_\mu \delta N \bar{N} \rangle$ which has to be zero when the proper on-shell condition is imposed. In ref.[14], it is claimed that a rather large $|D_n|$ is obtained at the tree level. However, there are serious problems in the result. First, the contamination from the direct insertion is unavoidable since the matrix element $\langle 0 | \bar{q} \gamma_5 q | \eta' \rangle$, which has both a connected and a disconnected piece, is evaluated (note that η' has both quark and gluon contents due to the axial anomaly). Second, $U_A(1)$ non-invariant magnetic moment of neutron is introduced, which gives an erroneous result being non-vanishing in the absence of anomaly as we already remarked in eq.(3.16).¹⁰ In ref.[21], the current algebra and the chiral per-

¹⁰Once one introduces the $U_A(1)$ invariant magnetic moment in their approach and uses the $\eta' - N$ coupling from the quark model, NEDM vanishes due to the contact photon- η' -nucleon coupling. We would like to thank Prof. A.I. Vainshtein for discussions on this point.

turbation is used to evaluate the CP-violating pion-nucleon coupling $\bar{g}_{\pi N}$ induced by L_{CP}^{eff} . Their result does not vanish in the absence of anomaly ($\alpha \rightarrow 0$) because of the same reason that we mentioned below eq.(3.9). $\bar{g}_{\pi N}$ is also calculated by using the chiral quark model [22], the chiral bag model [23,12] and the Skyrme model [24]. These calculations thus suffer the same criticism.

It is instructive to consider the relation between (i) and (ii) within our effective Lagrangian approach. In approach (i) we have to use the following total action:

$$\mathcal{L}_M(U, \theta) + \mathcal{L}_B(B, U), \quad (3.18)$$

while in approach (ii) there are several choices for $\mathcal{L}_B$ with the mesonic part given by (3.6). The most naive choice is

$$\mathcal{L}_M(U e^{-i\theta\pi^0/2}, 0) + \mathcal{L}_B(B, U e^{-\theta\pi^0/2}), \quad (3.19)$$

where

$$\begin{aligned} \mathcal{L}_B(B, U e^{-\theta\pi^0/2}) &= \bar{B}(i\gamma \cdot \partial - \alpha U_3^\dagger e^{i\theta\pi^0/2} B) \\ &\quad - \gamma \bar{B} M B + \bar{B} \mu \sigma_{\mu\nu} U_3^\dagger e^{i\theta\pi^0/2} (\partial^\nu A^\mu). \end{aligned} \quad (3.20)$$

At the quark level, this choice is rather strange since the quarks in the baryon are unchanged while the quarks in the meson are transformed under the $U_A(1)$ rotation. Furthermore, the CP violating effect appears in several places; the complex baryon mass, the CP violating baryon-meson interaction, the explicit electric dipole moment of the baryon and so on. Each term does not have a definite physical meaning; only the sum of all effects is physically relevant. A more reasonable and simpler choice is

$$\begin{aligned} \mathcal{L}_B(e^{-i\theta\pi^0/2} B, U e^{-i\theta\pi^0/2}) &= \bar{B}(i\gamma \cdot \partial - \alpha \bar{B} U_3^\dagger) B \\ &\quad - \gamma \bar{B} M e^{-i\theta\pi^0/2} B + \bar{B} \mu \sigma_{\mu\nu} U_3^\dagger (\partial^\nu A^\mu), \end{aligned} \quad (3.21)$$

where the CP violating term appears only in the explicit symmetry breaking term. Another choice adopted in ref.[19] is a little complicated:

$$\mathcal{L}_M(U e^{-i\theta\pi^0/2}, 0) + \mathcal{L}_B(e^{i\theta\pi^0/2} B, U e^{-i\theta\pi^0/2}) \quad (3.22)$$

where $b = \gamma m_\pi^2/m_N$. This is chosen so as to keep the baryon mass γ_5 free.¹¹ In this choice of variables, the CP violating coupling $B\pi^0 B\pi^0$ appears from the original CP preserving interaction $\bar{B}\pi^0 B\pi^0$ by the replacement $\eta \rightarrow \eta - b\theta f_\pi/2$ and only this CP violating interaction was used to calculate the NEDM [19,21]. However the CP violating term in the meson sector (3.6) also produces the same effect with the factor $-m_\pi^2/m_\eta^2$. By combining both we obtain the correct factor $(m_\eta^2 - m_\pi^2)/m_\eta^2$, which becomes 1 in the large a limit.

Even within the manifestly consistent approach (i), there are some varieties depending on the approximations, as we mentioned. The meson loop effects are essential in ref.[19], since there is no tree-level contribution to NEDM in his effective Lagrangian. On the other hand, we have the tree-level contribution originating from the $U_A(1)$ invariant magnetic moment as we demonstrated in the $N_f = 2$ case. In the following, we present a calculation in $N_f=3$ and show that the result is comparable to, or larger in magnitude than, that of one loop.

Eq.(3.2) is our meson Lagrangian with

$$\begin{aligned} q(x) &= \alpha f_\pi \eta^0, & a &= \frac{1}{3}(m_{\eta'}^2 + m_\eta^2 - 2m_K^2), \\ \text{and} & & M &= \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2), \end{aligned} \quad (3.23)$$

where we have neglected the small isospin breaking. We introduce the baryon field B as a $(3, 3^*)$ representation of the $SU(3)$ flavor. Then $\mathcal{L}_B$ is written as

$$\begin{aligned} \mathcal{L}_B &= \text{Tr}[\bar{B} i\gamma \cdot \partial B] - \alpha \text{Tr}[\bar{B}_L U B_R U + \bar{B}_R U^\dagger B_L U^\dagger] \\ &\quad + \delta \text{Tr}[\bar{B}_L U B_R M + \bar{B}_R U^\dagger B_L M^\dagger] + \gamma \text{Tr}[\bar{B}_L M B_R U + \bar{B}_R M^\dagger B_L U^\dagger] \\ &\quad + (\text{magnetic moment}), \end{aligned} \quad (3.24)$$

where B_L (B_R) are the left (right) projections of B . The 3rd and 4th term denote the $U(3)$ breaking and the last term denotes the magnetic moment of the octet baryons, the precise form of which is irrelevant here except that it is $U_A(1)$ invariant. The parameters α , δ and γ are determined by

¹¹ Here we consider the $N_f = 2$ case for simplicity, though $N_f = 3$ in ref.[19,21].

the baryon spectrum as in ref.[19].¹² To evaluate NEDM, it is convenient to define the chiral invariant nucleon field by

$$B_L = \sqrt{U} N_L \sqrt{U} \quad \text{and} \quad B_R = \sqrt{U}^\dagger N_R \sqrt{U}^\dagger, \quad (3.25)$$

Using this, expanding U up to the first order of the meson fields and retaining only the part relevant for the neutron (n), we get

$$\begin{aligned} \mathcal{L}_B \rightarrow & \bar{n}(i\gamma \cdot \partial - m_N)n + (\text{pv-coupling}) \\ & - (\eta' g_\eta + \eta g_\eta) \bar{n} i \gamma_5 n + \mu_n \bar{n} \sigma_{\mu\nu} n (\partial^\mu A^\nu), \end{aligned} \quad (3.26)$$

with

$$\begin{aligned} g_\eta &= \frac{1}{f_\pi} [\sqrt{2} \delta (2m_K^2 - m_\pi^2) \cos(\phi + \phi_0) + \gamma m_\pi^2 \sin(\phi + \phi_0)], \\ g_\eta &= \frac{1}{f_\pi} [-\sqrt{2} \delta (2m_K^2 - m_\pi^2) \sin(\phi + \phi_0) + \gamma m_\pi^2 \cos(\phi + \phi_0)]. \end{aligned} \quad (3.27)$$

Here $m_N = \alpha - \delta(2m_K^2 - m_\pi^2) - \gamma m_\pi^2$, μ_n is the neutron magnetic moment and we have used a relation between the flavor eigenstates (η^0 and η^8) and the mass eigenstates (η and η');

$$\begin{aligned} \eta^0 &= \eta' \cos \phi - \eta \sin \phi, \\ \eta^8 &= \eta \cos \phi + \eta' \sin \phi, \end{aligned} \quad (3.28)$$

with the empirical mixing angle $\phi \simeq -11^\circ \sim -20^\circ$ [26] and $\phi_0 \simeq 54.73^\circ$ ($\tan \phi_0 = \sqrt{2}$). Owing to the fact that we introduce the magnetic moment as an $U_A(1)$ invariant form, the contact photon- η^0 -neutron coupling does not arise in the choice of the variables here (as in case II in our previous example). Using $PCU_1 C$, the problem is reduced to evaluating the matrix element

$$-i\theta(a f_\pi) \int d^4x \langle n(p') | T[\eta'(x) \cos \phi - \eta(x) \sin \phi] J_\mu(0) | n(p) \rangle, \quad (3.29)$$

¹²Note that our parameters are $f_\pi/\sqrt{2}$ times α , δ and γ in ref.[19].

which is given by the diagrams similar to those in Fig.1. After a straightforward calculation, we get¹³

$$D_n = \theta \frac{m_\pi^2 + m_\eta^2 - 2m_K^2}{3} \left(\frac{g_\eta'}{m_\eta} \right) \left(\frac{f_\pi}{m_N} \right) (\cos \phi - \frac{g_\eta}{m_\eta^2} \sin \phi) \mu_n, \quad (3.30)$$

which is proportional to “ a ” and vanishes in the chiral limit since $g_\eta(n)$ is proportional to the current masses. Taking the experimental numbers, we get

$$\begin{aligned} D_n &= -5.9 \cdot 10^{-16} \theta e \cdot cm \quad \text{for } \phi = -11^\circ, \\ &= -3.2 \cdot 10^{-16} \theta e \cdot cm \quad \text{for } \phi = -20^\circ, \end{aligned} \quad (3.31)$$

which is different in sign and comparable with, or larger in magnitude than the one-loop result ($3.6 \cdot 10^{-16} \theta$) in ref.[19]¹⁴. Taking the experimental bound on NEDM ($|D_n| \leq 5 \cdot 10^{-25} e \cdot cm$), we obtain the upper bound from the tree graph $|\theta| < (0.8 - 1.6) \cdot 10^{-9}$. On the other hand, if we combine the one-loop result with our result at the tree-level, the bound becomes 2 to 8 times worse since the two contribution have opposite sign with each other.

Here we should mention briefly the same kind of calculation for the CP violating condensation parameter or the topological susceptibility K which is defined by

$$\theta K = \langle -q(x) \rangle_\theta \simeq \theta \int d^4y \langle T q(x) q(y) \rangle. \quad (3.32)$$

If only the disconnected part is properly taken into account in the calculation, K has to be proportional to both a and the current quark mass. The evaluation using the current algebra and the chiral perturbation, however, gives only the latter dependence, as can be seen in ref.[17]. Although K is

¹³A more general form of the $U_L(3) \times U_R(3)$ invariant part of the effective Lagrangian has D and F terms for the pv-coupling[27] In our case, however, such details are irrelevant since pv-couplings do not contribute. On the other hand, in the approaches calculating the one loop diagram, they are important to get the observed $g_{\pi N}$ at the tree level i.e., the Goldberger-Treiman relation $g_{\pi N} = (F + D)m_N/f_\pi$.

¹⁴We have introduced $-\theta q(x)$ in exactly the same way as in ref.[19,21] including the sign. Note also that the sign of D_n from the pion-loop contribution is positive. (see ref.[12].)

an off-shell quantity, (3.2) and (3.6) give the same result, since q is a purely gluonic operator being chiral invariant:

$$K = \frac{f_\pi^2}{2} a \left[1 - 2a \left(\frac{\cos^2 \phi}{m_\pi^2} + \frac{\sin^2 \phi}{m_\pi^2} \right) \right], \quad (3.33)$$

in $N_f = 2$. Here we took into account the isospin violation. In the large “ a ” or small current quark mass limit, (3.34) reduces to the result of ref.[17],

$$K = \frac{f_\pi^2}{4} m_\pi^2. \quad (3.34)$$

As can be seen in all the above examples, the current algebra with chiral perturbation is always missing the factor “ a ”. The reason is easy to understand: Schematically every tree amplitude is proportional to the combination

$$N_f \frac{a}{m_\pi^2}, \quad (3.35)$$

in the flavor symmetric case. Here m_π is the mass of the flavor singlet pseudoscalar meson. Once one adopts the lowest order of the chiral perturbation, it reduces to unity. This tells us that we can improve the result of the current algebra with chiral perturbation by simply multiplying $N_f a / m_\pi^2$.

4 Conclusion

In this paper, we have extensively examined the consistency of the calculation of the low energy matrix element of $\theta F\tilde{F}$.

The anomalous WT identity tells us that the insertion of $\theta F\tilde{F}$ in the hadronic process is related to the *disconnected* insertion of L_{CP}^{eff} while the connected insertion of L_{CP}^{eff} is merely the change of variables $\delta\mathcal{O}$ which vanishes for the on-shell amplitude.

This simple observation gives a constraint on the use of the effective theory to calculate NEDM and also makes it possible to diagnose the previous

calculations. In fact, one cannot construct the effective theory without $U_A(1)$ invariance in the chiral limit. That also implies all the CP violating matrix elements have to be proportional to the current quark masses and the strength of the anomaly. We pointed out, in section 3, the problematic points of the previous calculations of NEDM. Some of them break the condition $\langle \delta\mathcal{O} \rangle|_{m \rightarrow 0, a \rightarrow 0} = 0$, and some of them do not have the correct limiting behavior under $m \rightarrow 0$ and $a \rightarrow 0$. Then we gave a new calculation of NEDM which is consistent with the general constraints above and does not vanish at the tree level. The explicit inclusion of the $U_A(1)$ invariant magnetic moment of the neutron is the essential ingredient of the calculation.

Finally we should mention that it is desirable to calculate NEDM from the first principle. One possibility is a lattice QCD calculation, although there are several problems to be overcome as the recent analysis shows [8]. Another interesting problem related to the operator $F\tilde{F}$ is the spin content of the nucleon [28] where the nucleon matrix element of $F\tilde{F}$ plays a crucial role.[29] Extensive studies using the effective Lagrangians and the lattice QCD consistent with the WT identity are called for on this problem.

Acknowledgements

We would like to thank A. Gocksch and A. I. Vainshtein for useful discussions. This work was supported by the NSF grant NSF-PHY-89-08495 and the US Department of Energy under Grant No. DE-FG02-88ER40388.

References

- [1] R. D. Pececi and H. R. Quinn, *Phys. Rev. Lett.* **36** (1977) 1440; *Phys. Rev. D* **16** (1977) 1791.
- [2] H.-Y. Cheng, *Phys. Rep.* **158** (1988) 1.
- [3] J. E. Kim, *Phys. Rep.* **150** (1987) 1.
- [4] N. F. Ramsey, *Ann. Rev. Nucl. Part. Sci.* **32** (1982) 211.
- [5] V. Baluni, *Phys. Rev. D* **19** (1979) 2227.
- [6] E. P. Shabalin, *Sov. Phys. Usp.* **26** (1983) 297.
- [7] S. Aoki and A. Gocksch, *Phys. Rev. Lett.* **63** (1989) 1125.
- [8] S. Aoki, A. Gocksch, A. V. Manohar and S. R. Sharpe, "Calculating the Neutron Electric Dipole Moment on the lattice", ITP-SB-90-53, BNL-44742, UCSD/PTH 90-7, 40423-08 P90 (1990), to be published in *Phys. Rev. Lett.*
- [9] S. L. Adler, *Phys. Rev.* **177** (1969) 2426; J. S. Bell and R. Jackiw, *Nuovo Cim.* **60A** (1969) 47.
- [10] S. L. Adler and W. A. Bardeen, *Phys. Rev.* **182** (1969) 1517.
- [11] S. Kamefuchi, L. O'Raifeartaigh and A. Salam, *Nucl. Phys.* **28** (1961) 529; M. Bando, T. Kugo and K. Yamawaki, *Phys. Rep.* **164** (1988) 217.
- [12] M. Morgan and G. A. Miller, *Phys. Lett.* **179B** (1986) 379.
- [13] G. A. Christos, *Z. Phys. C* **21** (1983) 83.
- [14] P. Cea and G. Nardulli, *Phys. Lett.* **144B** (1984) 115.
- [15] P. Di Vecchia and G. Veneziano, *Nucl. Phys.* **B171** (1980) 253; P. Nath and R. Arnowitt, *Phys. Rev. D* **23** (1981) 473; C. Rosenzweig, J. Schechter and G. Trahern, *Phys. Rev. D* **21** (1980) 3388; E. Witten, *Ann. of Phys. (N.Y.)* **128** (1980) 363.
- [16] K. A. Milton, W. F. Palmer and S. S. Pinsky, *Phys. Rev. D* **22** (1980) 1124, 1647; G. Veneziano, *Nucl. Phys.* **B159** (1979) 213.
- [17] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys.* **B166** (1980) 493.
- [18] K. Kawarabayashi and N. Ohta, *Nucl. Phys.* **B175** (1980) 477.
- [19] P. Di Vecchia, *Acta Phys. Austriaca Suppl.* **22** (1980) 341.
- [20] E. P. Shabalin, *Yad. Fiz.* **36** (1982) 981 (*Sov. J. Nucl. Phys.* **36** (1982) 575.)
- [21] R. J. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, *Phys. Lett.* **88B** (1979) 123; **91B** (1980) 487 (E).
- [22] K. Kanaya and M. Kobayashi, *Prog. Theor. Phys.* **66** (1981) 2173.
- [23] M. M. Musakhanov and Z. Z. Israilov, *Phys. Lett.* **137B** (1984) 419.
- [24] H. Schnitzer, *Phys. Lett.* **139B** (1984) 217.
- [25] G. 't Hooft, *Phys. Rep.* **142** (1986) 357.
- [26] A. Bramon and M. D. Scadron, *Phys. Lett.* **234B** (1990) 346.
- [27] J. Bijnens, H. Sonoda and M. B. Wise, *Nucl. Phys.* **B261** (1985) 185.
- [28] J. Ashman et. al., *Nucl. Phys.* **B328** (1990) 1.
- [29] T. Hatsuda, *Nucl. Phys.* **B329** (1990) 376; G. Veneziano, *Mod. Phys. Lett.* **A4** (1989) 1605.

Figure Captions

Fig. 1 : The tree-level contributions to NEDM in the effective Lagrangian in case II. There is no contact photon-meson-nucleon coupling in this case.

Fig. 2 : The tree-level contributions to NEDM in the effective Lagrangian in case I. There is a contact photon-meson-nucleon coupling from the $U_A(1)$ invariant magnetic moment of the neutron.

Fig.1

Fig.2