N=1 SUPERSYMMETRY IS REALLY BROKEN TWISTED N=2

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ABSTRACT

A new classical gauge action for the superparticle is suggested, in which $N=1$ space-time supersymmetry arises as a broken infinite dimensional twisted $N=2$ supersymmetry. Besides κ -symmetry, this action has a local ξ -symmetry, which is a special combination of twisted $N=2$ supersymmetries. The light-cone and covariant gauges of this action coincide with the corresponding gauges of the Brink-Schwarz superparticle. The BRST operator breaks the set of twisted $N=2$ supersymmetries of the covariant gauge-fixed action down to one $N=1$ supersymmetry and gives the correct cohomology. In this sense the standard $d=10$ $N=1$ supersymmetry appears as a result of symmetry breaking in the underlying $N=2$ twisted supersymmetric theory.

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1 Introduction

The covariant quantization of the heterotic superstring with the Green-Schwarz action [1] and of the superparticle with the Brink-Schwarz action [2] is a longstanding problem. Both actions have a manifest space-time $N=1$ supersymmetry and, as a payment for that, an extremely complicated gauge symmetry, the so-called Siegel κ -symmetry. Both actions do not have a simple kinetic term for space-time spinors. This is the origin of all sorts of problems in quantizations. Spinors enter the Lagrangian as follows

$$\theta \not{p} d\theta. \quad (1)$$

The reason for that is the manifest $N=1$ space-time supersymmetry, realized in terms of $N=1$ differential forms

$$P_\mu(dX^\mu - \theta \gamma^\mu d\theta). \quad (2)$$

In fact this is the only way to keep $N=1$ supersymmetry manifest if we have only one $N=1$ supersymmetry. If, however, we want to have a normal kinetic term for spinors of the form

$$\lambda d\theta, \quad (3)$$

we are immediately led to a twisted $N=2$ supersymmetry for the kinetic part of the action,

$$P_\mu dX^\mu + \lambda d\theta \quad (4)$$

The problem then arises, whether it is possible to construct the correct $N=1$ supersymmetric theory as a broken twisted $N=2$ supersymmetric theory. This is an important problem also because the covariant gauge-fixed actions of superparticle and of the heterotic superstring do have an infinite number of twisted $N=2$ supersymmetries, as it was pointed out in [3].

An important step in this direction has been undertaken when Warren Siegel has introduced his modifications to superparticle and superstring actions [4]. The Siegel modified superparticle action has the twisted $N=2$ global supersymmetry, which has been described by Green and Hull [5]. Different investigations of the possibilities to perform the quantization of this theory have been performed [6],[7], [8],[5].

Meanwhile, the problem of covariant quantization of BS-superparticle and GS-superstring action was investigated most intensive. It was pointed out by Bengtsson and Oederwal [9] that the main problem in covariant quantization of manifestly supersymmetric superstring and superparticle theories is related to the existence of the second-class constraints, mixed with the first-class ones. Indeed, this was the cause of problems in searching the appropriate BRST operator. However after understanding the issue of the infinite reducibility [10], covariant gauge-fixed actions have been found, both for the superparticle [11] - [12] and for the superstring [13]. The counting of propagating degrees of freedom in these theories in covariant gauges coincided with that in the light-cone gauge. For the covariant superstring action it was shown that the conformal anomaly is vanishing [13]. This was the consequence

of the equivalence between the spinorial representation of the $OSp(10/4)$ supergroup and the one of the $SO(6)$ group [14]. It has been also noted [15] that the covariant action for the superstring can be reproduced from the requirement of the equivalence of the corresponding vacuum functional to the one in the light-cone one, simultaneously with the requirement of vanishing conformal anomaly.

However, the solution of the quantization problem proposed in [11] - [13] was not complete, since despite the discovery of a nice gauge fixed action for each of these theories, it was impossible to construct the corresponding BRST operators with the correct cohomology by any known methods of quantization.

Recently such a BRST operator was found for the superparticle theory [3]. It has been constructed from all fields entering the gauge-fixed action in the free covariant gauge. The existence of such an operator was indicated by the fact that the counting of the propagating degrees of freedom in the gauge-fixed action was correct. However, the procedure of the construction of this operator in [3], starting from the classical action for the superparticle given by Brink and Schwarz [2], was a kind of art rather than a regular procedure.

The purpose of this paper is to suggest a new action which gives a correct description of the superparticle and has a normal kinetic term for spinors. As a result, the classical action contains a part which has a set of global twisted $N=2$ supersymmetries and a part which breaks these symmetries down to one $N=1$ supersymmetry. Besides that in addition to the local κ -symmetry this action has a special combination of local twisted $N=2$ supersymmetries, which will be called ξ -symmetries.

This action has only first-class constraints which are infinite reducible. After a partial gauge-fixing one gets the Brink-Schwarz action with the mixed first- and second-class constraints.

The new action can be quantized in a regular way. By a regular way we mean the possibility to use the full power of the Batalin-Fradkin-Vilkovisky Lagrangian and/or Hamiltonian quantization of gauge theories with finite reducibility [16], [17] supplemented with our recent experience of dealing with infinite reducibility in the case that the corresponding set of fields forms a representation of a supergroup [18], [14].

2 The action

Consider the following classical superparticle action

$$L_d = L_d^{sym} + L_d^{br}, \quad (5)$$

where

$$L_d^{sym} = P_\mu \dot{X}^\mu - \frac{1}{2} P^2 g + \sum_{p=0}^{\infty} \{ \lambda^p \dot{\theta}_p - K^p \zeta_p \}, \quad (6)$$

and

$$L_d^{br} = - \sum_{p=0}^{\infty} F^p \eta_p \quad (7)$$

We are using the following notations ²:

$$\begin{aligned} K^p &= \lambda^p \not{p} , \\ F^p &= \dot{Q}^p + Q^{p+1} , \end{aligned} \quad (8)$$

where Q^p and $\dot{Q}^p$

$$\begin{aligned} Q^p &= \lambda^p - \not{p} \theta_p , \\ \dot{Q}^p &= \lambda^p + \not{p} \theta_p . \end{aligned} \quad (9)$$

are the charges of a twisted N=2 supersymmetry:

$$\begin{aligned} \{Q^p, Q^q\} &= -2 \not{p} \delta^{p,q} , \\ \{\dot{Q}^p, \dot{Q}^q\} &= +2 \not{p} \delta^{p,q} . \end{aligned} \quad (10)$$

The independent fields in this action are the usual fields of the bosonic particle action (P_μ, X^μ, g) and also the anti-commuting ten-dimensional spinors of positive chirality, $(\theta_p)^\alpha$, $(\eta_p)^\alpha$ and those of negative chirality, $(\lambda^p)_\alpha$, $(\zeta_p)_\alpha$ and p starts from zero and goes to infinity.

The full classical action can be presented in the following form

$$L_d = P_\mu \dot{X}^\mu - \frac{1}{2} P^2 g + \sum_{p=0}^{\infty} \{ \lambda^p \dot{\theta}_p - \lambda^p \not{p} \zeta_p - (\lambda^p + \theta_p \not{p} + \lambda^{p+1} - \theta_{p+1} \not{p}) \eta_p \} . \quad (11)$$

The symmetric part of the Lagrangian L_d^{sym} has an infinite set of global twisted $N = 2$ supersymmetries:

$$\begin{aligned} \delta_{N_p=2} X^\mu &= \pm \epsilon^{\pm p} \gamma^\mu \theta_p , \\ \delta_{N_p=2} \theta_p &= \epsilon^{\pm p} , \\ \delta_{N_p=2} \lambda^p &= \mp \epsilon^{\pm p} \not{p} , \\ \delta_{N_p=2} g &= \pm 2 \epsilon^{\pm p} \zeta_p . \end{aligned} \quad (12)$$

The role of L_d^{br} is two-fold:

²The action in the Appendix of [8] can be obtained from our action if one takes $K^p = 0, p = 1, 2, 3, \dots, \infty$. This leads to a crucial difference both for the BRST-operator, given in eq. (38) and for the gauge-fixed action, given in eq. (64).

i) It breaks the set of global twisted N=2 supersymmetries given in eq. (12) so that the full action has only one global N=1 supersymmetry, realized as follows:

$$\begin{aligned} \delta_{N_0=1} \theta_0 &= \epsilon , \\ \delta_{N_0=1} \lambda^0 &= -\epsilon \not{p} , \\ \delta_{N_0=1} X^\mu &= \epsilon \gamma^\mu \theta_0 , \\ \delta_{N_0=1} g &= 2 \zeta_0 \epsilon . \end{aligned} \quad (13)$$

ii) It allows to promote a special combination of global N=2 supersymmetries given in eq. (12) with

$$\epsilon^{-p} = \epsilon^{+(p+1)} \quad (14)$$

to a local ξ -symmetry, to be described later.

The global $N = 2$ supersymmetries will, in fact, survive in the covariant gauge-fixed action and the physical states will form the representations of one simple N=1 supersymmetry due to the special properties of the BRST-operator.

The classical Hamiltonian

$$H_d = \frac{1}{2} P^2 g + K^p \zeta_p + F^p \eta_p \quad (15)$$

is given by the set of first-class constraints

$$T_a = \left\{ \frac{1}{2} P^2 , \quad K^p , \quad F^p \right\} \quad (16)$$

times the Lagrange multipliers

$$\psi^a = \{ g , \quad \zeta_p , \quad \eta_p \} . \quad (17)$$

The only nontrivial part of the superalgebra

$$\{ T_a , \quad T_b \} = f_{ab}^c T_c \quad (18)$$

is the anticommutator

$$\{ K^q , \quad F^p \} = -P^2 (\delta^{q,p+1} - \delta^{q,p}) . \quad (19)$$

Thus our action has a very simple form of the type

$$L_d = p_i \dot{q}^i - T_a(p, q) \psi^a = p_i \dot{q}^i - H_d . \quad (20)$$

In (20) we denoted the set of coordinates $\{ X^\mu , \theta_p \}$ by q^i and the set of canonical conjugate momenta $\{ P^\mu , \lambda^p \}$ by p_i .

The classical action (11) is much more complicated in the second order form, when the equation of motion for P_μ is solved. It is, however, instructive to present it here with the purpose to have some insight into the possibilities to make a generalization to the superstring theory:

$$L_{cl} = \frac{1}{2} g^{-1} (\dot{X}^\mu - \sum_{p=0}^{\infty} \{ \lambda^p \gamma^\mu \zeta_p - (\theta_p - \theta_{p+1}) \gamma^\mu \eta_p \})^2 + \sum_{p=0}^{\infty} \{ \lambda^p \dot{\theta}_p - (\lambda^p + \lambda^{p+1}) \eta_p \} \quad (21)$$

3 Gauge symmetries of the action

The gauge action of the type of (20) has a set of gauge symmetries, defined by the first-class constraints T_a , their algebra and their zero modes. The set of gauge transformations, generated by the first-class constraints is given by the following equations:

$$\begin{aligned} \delta p_i &= \{p_i, T_a\} \xi^a \\ \delta q^i &= \{q^i, T_a\} \xi^a \\ \delta \psi^a &= (\delta_0^a d_t + f_{bc}^a \psi^b) \xi^c \end{aligned} \quad (22)$$

These symmetries are infinite reducible in the following sense. The first-class constraints have zero modes

$$T_a Z_{a_1}^a = 0, \quad (23)$$

the n -stage zero modes have their own zero modes for each n ,

$$Z_{a_n}^{a_{n-1}} Z_{a_{n+1}}^{a_n} + f_{a_{n+1}}^{a_n b} T_b = 0, \quad (24)$$

and the procedure of finding all zero modes never stops. The existence of the first stage zero modes given by (23) means that the action has additional gauge symmetries, as explained in [6], generated by $Z_{a_1}^a$:

$$\delta p_i = 0, \quad \delta q^i = 0, \quad \delta \psi^a = Z_{a_1}^a \xi^{a_1}. \quad (25)$$

In our particular case the gauge symmetries of the action generated by the first class type constraints are

$$\begin{aligned} \delta P^\mu &= 0, \\ \delta X^\mu &= \lambda^p \gamma^\mu \kappa_p + \theta_p \gamma^\mu (\zeta_p - \xi_{p-1}) + \xi P^\mu, \\ \delta \lambda^p &= (\xi_p - \xi_{p-1}) \not{P}, \\ \delta \theta_p &= \not{P} \kappa_p + (\xi_p + \xi_{p-1}), \\ \delta g &= \dot{\xi} + 2\eta_p (\kappa_p - \kappa_{p+1}) + 2\zeta_p (\zeta_p - \xi_{p-1}), \\ \delta \zeta_p &= \dot{\kappa}_p, \\ \delta \eta_p &= \dot{\xi}. \end{aligned} \quad (26)$$

The additional symmetries generated by the first stage zero modes are

$$\begin{aligned} \delta \zeta_p &= \not{P} \omega_p - (\rho_p + \rho_{p-1}), \\ \delta \eta_p &= \not{P} \rho_p, \\ \delta g &= 2\rho_p (\theta_p - \theta_{p+1}) + 2\omega_p \lambda^p. \end{aligned} \quad (27)$$

These transformations have their own zero modes, etc.

4 Quantization

The classical Lagrangian is given by eq. (11) and is rewritten in (20) in the form adjusted to the use of the Batalin-Fradkin effective canonical action [16]. This action in a very general class of gauges is given by the following expression:

$$S_{g,f} = \int dt \left(\sum \{ P_{\min} \dot{Q}_{\min} + P_{\text{nonmin}} \dot{Q}_{\text{nonmin}} \} - H_\Psi \right). \quad (28)$$

The first term in (28) presents a set of minimal fields, including the original classical pairs (p, q) and the set of ghosts $(\bar{P}, C)$, which serve to realize the complete algebra of the first-class constraints and their zero modes. The second term in (28) presents a set of nonminimal pairs, which consists of Lagrange multipliers (π, ψ) and pairs of additional ghosts (anti-ghosts) $(\bar{C}, \bar{P})$. The unitarizing Hamiltonian is given by

$$H_\Psi = H + \{\Psi, \Omega\}, \quad (29)$$

where Ψ is the gauge fermion defining the gauge condition and Ω is the nilpotent BRST operator,

$$\Omega = \Omega_{\min}(p, q; \bar{P}, C) + \Omega_{\text{nonmin}}. \quad (30)$$

The complete information about the first-class constraints T_a , all their zero modes $Z_{a_{n-1}}^{a_n}$ and all structure functions of the algebra of the type $f_{a_{n+1}}^{a_n a}$ is codified in the minimal part of the BRST operator.

The nonminimal part of the BRST operator is universal and for any theory is given by the following equation:

$$\Omega_{\text{nonmin}} = \sum \pi \mathcal{P}, \quad (31)$$

The heart of the theory is $\Omega_{\min}$. For our particular theory given in (11) and in (20) we are able to present the exact result in a closed form, despite the theory is infinite reducible. First of all, we are going to specify the notations. It turns out that the minimal set of fields for the theory under consideration coincides exactly with the full system of classical fields

and ghosts introduced in the covariant quantization of the Brink-Schwarz superparticle [11] - [12]. We will use the notations from [14].

The set of ten-dimensional spinors of alternating Grassmann parity and chirality forms two conjugate spinorial representations of the orthosymplectic supergroup $OSp(10/4)$,

$$\theta(j, m), \quad \lambda(j, -m), \quad (32)$$

where

$$0 \leq j \leq \infty, \quad |m| \leq j. \quad (33)$$

This is the $SO(10) \times SU(2)$ content of the metaplectic representations of $OSp(10/4)$ [14]. The states on the horizontal lines of the triangles, given in the Figs. to ref. [14] fit into $SU(2)$ multiplets $[j, m >, < j, -m]$. When $2j$ is even the field θ is a fermion of a positive chirality, while when $2j$ is odd it is a boson of opposite chirality. For λ -fields fermions with even $2j$ have negative chirality, bosons with odd $2j$ have positive chirality.

The precise relation of new notations with our previous ones in refs. [12], [3] with $0 \leq q \leq p$ is the following:

$$\begin{aligned} \theta(j, m) &= \begin{cases} \theta_{p,q} & = \theta(j = (p+q)/2, m = (p-q)/2), \\ \bar{\theta}_{p+1,q+1} & = \theta(j = (p+q+1)/2, m = (q-p-1)/2) \end{cases} \\ \lambda(j, -m) &= \begin{cases} \bar{\lambda}_{p,q} & = \lambda(j = (p+q)/2, -m = (p-q)/2), \\ \lambda_{p+1,q+1} & = \lambda(j = (p+q+1)/2, -m = (q-p-1)/2) \end{cases} \end{aligned} \quad (34)$$

Using the (j, m) labelling of the fields, which corresponds to an assignment of the fields to a representation of the supergroup $OSp(10/4)$, we are able to present the BRST operator of the theory in a relatively simple form.

We will use the following notations:

$$\begin{aligned} \varepsilon_m &= +1/2 \quad \text{for } m \geq 0 \\ \varepsilon_m &= -1/2 \quad \text{for } m < 0 \\ \mathcal{Z}_m &= 2\varepsilon_m(-1)^{m(2m-1)} \end{aligned} \quad (35)$$

The function $\mathcal{Z}_m$, defined above, has the following property:

$$\mathcal{Z}_m = \mathcal{Z}_{-m-1/2} \quad (36)$$

The dependence of the BRST-operator on spinorial fields will enter through the special combinations:

$$\begin{aligned} s(j, m) &= \not{P}\theta(j, m) - 2\theta(j + \varepsilon_m, m + 1/2)b \\ s_\theta(j, m) &= s(j + \varepsilon_m, m + 1/2) - \delta(m + 1/2)s(j + 1/2, 0) + \delta(m + 1)2\theta(j, 0)b \end{aligned} \quad (37)$$

Now everything is prepared to write down the BRST operator $\Omega_{\min}$ in terms of all spinors, furnishing 2 conjugate representations of $OSp(10/4)$. The procedure behind this is pretty regular, one just uses the Batalin-Fradkin prescription [16]. The result was, in fact, deduced in [3], just because it was expected to exist due to the group theory equivalence between the $OSp(10/4)$ and $SO(6)$ representations. Still its origin from the point of view of quantization was mysterious. But now, taking into account all first type class constraints and their zero modes, one gets

$$\Omega_{\min} = \frac{1}{2}cP^2 + \sum_{j=0}^{\infty} \sum_{m=-j}^m \lambda(j, -m)s''(j, m), \quad (38)$$

where

$$s''(j, m) = s_\theta(j, m) + \lambda(j + 1/2, m + 1/2)\mathcal{Z}_m \quad (39)$$

The minimal BRST-transformations of the θ -fields are very simple in these notations. Taking into account that

$$[\theta(j, m), \lambda(j', m')] = \delta(j, j')\delta(m, -m'), \quad (40)$$

we get for

$$\delta_{BRST}\theta(j, m) = [\theta(j, m), \Omega_{\min}] \quad (41)$$

the following expression:

$$\delta_{BRST}\theta(j, m) = s'(j, m) \quad (42)$$

where

$$s'(j, m) = s_\theta(j, m) + \bar{\lambda}(j, m) \quad (43)$$

and

$$\bar{\lambda}(j, m) = [\lambda(j + 1/2, m + 1/2) + \lambda(j - 1/2, m + 1/2)]\mathcal{Z}_m \quad (44)$$

The operator $\Omega_{\min}$ (38) can be also rewritten in the following form

$$\Omega_{\min} = \frac{1}{2}cP^2 + \sum_{j=0}^{\infty} \sum_{m=-j}^m \lambda(j, -m)(s_\theta(j, m) + \frac{1}{2}\bar{\lambda}(j, m)) \quad (45)$$

It is nilpotent and the solution to its cohomology is the part of the $N=1$ supersymmetry charge

$$q^0 = \lambda^0 - s(0, 0), \quad (46)$$

which cannot be represented through $\not{P}q^0$. This is explained in detail in [3].

5 Different gauges

5.1 Brink-Schwarz action as a partially gauge-fixed new action

One can take the classical action given in (11) and make a partial gauge fixing,

$$L_{p,g,f} = L_{cl} + \pi_p(\hat{Q}^p - Q^{p+1}) \quad (47)$$

The operator

$$\tilde{F}^p = \tilde{Q}^p - Q^{p+1} \quad (48)$$

has the following commutation relations with the constraint F^p :

$$\{F^p, \tilde{F}^q\} = 4 \, p \delta^{p,q} \quad (49)$$

This means that adding the gauge-fixing function as in (47) breaks half of the gauge symmetries generated by F^p , since only a half of the terms in the commutator of F^p with $\tilde{F}^q$ produce the invertible right hand-side. Namely,

$$\{\gamma^+ F^p, \gamma^+ \tilde{F}^q\} = 8\gamma^+ P^+ \quad (50)$$

In this way also the mixing of the first- and second-class constraints is introduced and the structure of the zero modes of the classical symmetries is changed drastically.

To proceed, one integrates over η_p and π_p and gets

$$L_{p,g,f} = P_\mu \dot{X}^\mu - \frac{1}{2} g P^2 + \sum_{p=0}^{\infty} \{ \lambda^p \dot{\theta}_p - \zeta_p P \lambda^p \} \quad (51)$$

under the conditions

$$\tilde{Q}^p + Q^{p+1} = 0, \quad \tilde{Q}^p - Q^{p+1} = 0, \quad (52)$$

which are equivalent to the following ones

$$\tilde{Q}^p = 0, \quad Q^{p+1} = 0, \quad p \geq 0 \quad (53)$$

Inserting these conditions into the remaining part of the partially gauge-fixed action we get

$$L = P_\mu (\dot{X}^\mu - \theta_0 \gamma^\mu \dot{\theta}_0) - \frac{1}{2} (g - 2\theta_0 \zeta_0) P^2 \quad (54)$$

Performing the shift in metric $\delta g = -2\theta_0 \zeta_0$, we get the Brink-Schwarz action [2] for the superparticle,

$$L_{BS} = P_\mu (\dot{X}^\mu - \theta_0 \gamma^\mu \dot{\theta}_0) - \frac{1}{2} g P^2 \quad (55)$$

Thus we end up without ξ -symmetries and only with the κ -symmetry, acting on the fields remaining in the action.

5.2 Semi-light-cone gauge

One can start again with the classical action given in (11) and perform this time a complete gauge fixing,³

$$L_{s,l,c} = L_d + \pi_p (\tilde{Q}^p - Q^{p+1}) + b\dot{c} + \pi(g-1) + \rho \gamma^+ \theta_0 \quad (56)$$

After integrating over η_p , π_p and ρ one gets the semi-light-cone gauge action [19],

$$L_{s,l,c} = P_\mu \dot{X}^\mu - \frac{1}{2} P^2 - \theta_0 \gamma^- P^+ \theta_0 + b\dot{c} \quad (57)$$

³Since after the partial gauge fixing, presented in the previous subsection, we get the BS action containing the second-class constraints, one should include into the path integral the measure of integration $(d\epsilon! P^+)^{-1}$.

5.3 Covariant gauge

To get the covariant gauge starting from the classical action (11) one can use eqs. (28) and equation (29). In our case, as it was already explained, the set of minimal spinorial fields is given by two conjugate $OSp(10/4)$ multiplets $\lambda(j, m)$ and $\theta(j, -m)$. The set of non-minimal fields consists of two sets of fields of opposite statistics, namely the Lagrange multipliers (π, ψ) and anti-ghosts $(\bar{C}, \bar{\mathcal{P}})$ for all stages of reducibility.

The unitarizing Hamiltonian in our case is given simply by the equation (29) with $H = 0$. Our main purpose now is to find a gauge-fixing function in such a way that *all non-minimal fields become non-propagating*. We take the gauge-fixing function in the form

$$\Psi = \sum \{ \bar{C}(\chi(\psi) + \dot{\psi}) + \bar{\mathcal{P}}\psi \} \quad (58)$$

The sum in (58) means the sum over the full set of anti-ghosts and Lagrange multipliers. For the unitarizing Hamiltonian one gets, using equations (29),(30) and (58),

$$H_\Psi = \sum P_{nonmin} \dot{Q}_{nonmin} + T(p, q; \bar{\mathcal{P}}, C) \psi + \pi \chi(\psi) - \mathcal{P}(\bar{C} \frac{\delta \chi}{\delta \psi} + \bar{\mathcal{P}}), \quad (59)$$

where the full set of all reducible constraints is denoted by T ,

$$T = [\bar{\mathcal{P}}, \Omega] \quad (60)$$

Thus H_Ψ indeed contains the $P\dot{Q}$ -term for all non-minimal fields, which is necessary to make them non-propagating after substituting H_Ψ given in equation (59), into equation (28). As a result one gets the gauge-fixed action for the given class of gauges, where only minimal fields are propagating.

$$S_{g,f} = \int dt \sum \{ P_{min} \dot{Q}_{min} - T\psi - \pi \chi(\psi) + \mathcal{P}(\bar{C} \frac{\delta \chi}{\delta \psi} + \bar{\mathcal{P}}) \} \quad (61)$$

For all spinorial Lagrange multipliers we choose the gauge-fixing function to be

$$\chi_{spin}(\psi_{spin}) = \psi_{spin}, \quad (62)$$

and the gauge-fixing function for the multiplier to the reparametrization constraint to be

$$\chi_{rep}(\psi_{rep} = g) = g - 1 \quad (63)$$

Integrating over the Lagrange multipliers to gauges π and over $\mathcal{P}$ one gets the expected free covariant gauge-fixed action, and all non-minimal fields drop from it.

$$S_{g,f} = \int dt \{ P_\mu \dot{X}^\mu + b\dot{c} - \frac{1}{2} P^2 + \sum_{j=0}^{\infty} \sum_{m=-j}^{m=j} \lambda(j, -m) \dot{\theta}(j, m) \} \quad (64)$$

In eq. (64) for our particular theory we are left besides classical fields with ghosts $C = \{c, \theta(j, m)\}$ and their canonical momenta $\mathcal{P} = \{b, \lambda(j, -m)\}$. Note that due to functional integration over $\mathcal{P}$ and the form of the gauge-fixing functions, given in (62), (63), the original canonical momenta to the ghosts $\bar{\mathcal{P}}$ become equal to anti-ghosts $-\bar{C}$.

This action was proposed before in [11] and [12], starting from the Brink-Schwarz superparticle action.

The BRST-operator in this gauge is exactly the Ω_{min} , given in eq. (38).

6 Conclusion

Thus we have found the covariantly quantizable classical action for the superparticle. Its quantization reproduces in a regular way the covariant gauge-fixed action [11], [12] and the BRST operator [3] coinciding with those of the Brink-Schwarz superparticle.

However, as distinct from the previous direct attempts to obtain the BRST operator with the correct cohomology, this new action leads quite straightforwardly to the fully quantized theory. In arbitrary gauge the BRST operator defines the gauge-fixed action, in a simplest covariant gauge all fields are free and spinors package into two conjugate representations of the $OSp(10/4)$ supergroup.

The new action, described in this paper, has *two types of gauge symmetries*: local κ -symmetry, which is $N=1$ space-time supersymmetry transformation with a special parameter $\epsilon = \not{P}\kappa$, promoted to a local symmetry, and local ξ -symmetry, which is the twisted $N=2$ space-time supersymmetry transformation with special parameters $\epsilon^{-p} = \epsilon^{+(p+1)} = \xi_p$, promoted to a local symmetry of the world-sheet.

This set of local symmetries corresponds to the first-class infinite-reducible constraints only, and therefore its covariant quantization is simple and confirms the correctness of the covariant gauge-fixed action [11] - [12] and of the BRST operator obtained in [3].

The main property of the new theory is the existence of an infinite number of global twisted $N=2$ supersymmetries both in the part of the classical action L_d^{sym} , given in eq. (6), and in the covariant gauge-fixed action, presented in equation (64). In the classical theory the part of the action L_d^{br} , given in eq. (7), breaks down all extra supersymmetries so that the global symmetry of the full classical action is reduced to one simple $N=1$ supersymmetry. In the gauge-fixed action all global twisted $N=2$ supersymmetries are unbroken, but the physical states have one simple $N=1$ supersymmetry only, because the BRST-operator does not commute with all extra supersymmetries.

The present investigation of the superparticle quantization leads to a structure of the off-shell superspace which is very much different from the standard one, which is based on the

$N=1$ supersymmetry. One should start off-shell with a set of twisted $N=2$ supersymmetries and only on-shell the standard $N=1$ supersymmetry will arise.

It would be most important to find a generalization of the superparticle classical action given in eq. (11) for the manifestly space-time supersymmetric string theory.

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