CERN-TH.5839/90

NUB-3001

DILATON, GAUGINO CONDENSATION AND SUPERSYMMETRY BREAKING

T.R. Taylor

CERN, CH-1211 Geneva 23, Switzerland

and

*Department of Physics, Northeastern University
Boston, MA 02115, U.S.A.**

Abstract

Non-perturbative condensation of gauginos belonging to a gauge group of the form $G_1 \otimes G_2 \otimes \cdots \otimes G_p$, where G_n are non-Abelian subgroups, is examined in superstring theory by utilizing the effective Lagrangian techniques. A supersymmetric vacuum with stable expectation values of the dilaton and moduli fields is shown to exist provided that the gauge group satisfies one non-trivial constraint.

*Permanent address

CERN-TH.5839/90

NUB-3001

July 1990

The fundamental constants of superstring theory are related to the vacuum expectation values (VEVs) of the dilaton and moduli fields. In the present letter, the problem of determination of these VEVs and the related problem of mass generation for the dilaton and moduli are addressed in the context of gaugino condensation in the "hidden" sector of superstring theory.

Non-perturbative gaugino condensation [1] has been previously considered as a promising mechanism for supersymmetry breaking in superstring theory. It has been recently pointed out [2,3] that the dependence of the corresponding effective action on the moduli fields is highly constrained by the duality symmetry [4]. For instance, in the simplest case of one modulus T , the invariance under the $SL(2, Z)$ modular group, generated by the duality transformations $T \rightarrow 1/T$ and $T \rightarrow T + i$ [5], determines the form of the effective potential uniquely. This form is consistent with the results of explicit string computations of the so-called threshold effects [6]. Moreover, it has been shown that duality-preserving vacua at $\langle T \rangle = 1$, with broken supersymmetry, do generically occur [2]. The effective potential is however unstable with respect to the fluctuations of the dilaton field S whose VEV determines the string (and gauge) coupling constant.

The following mechanism for fixing the coupling constant has been suggested by Krasnikov [7], and recently discussed in more detail in [8]. Suppose that the gauge group G is a tensor product of several non-Abelian groups G_n , $n = 1, 2, \dots, p$: $G = G_1 \otimes G_2 \otimes \dots \otimes G_p$. The condensation of p gauginos is naively expected to give rise to the effective potential:

$$V \propto M_p^{-2} \left| \sum_{n=1}^{n=p} k_n \langle \lambda \lambda \rangle_n \right|^2, \quad (1)$$

where M_p is the Planck mass and the positive integer k_n is the level of Kač-Moody algebra that generates G_n .¹ In the case of pure SYM theory under consideration, the condensates depend on the coupling constants g_n and the theta angles θ_n in the following

¹The effects of matter fields have been neglected in eq.(1).

way [9]:

$$\langle \lambda \lambda \rangle_n \propto \mu^3 g_n^{-2} \exp \left(\frac{-8\pi^2}{c_n g_n^2} + i \frac{\theta_n}{c_n} \right), \quad (2)$$

where c_n is the Casimir operator in the adjoint representation of G_n . The parameter μ corresponds to the mass scale below which the running of the coupling constants takes place [10,11], and depends on the VEVs of the moduli fields [11,6,2]. One expects these scales to be of the order of the string scale $(\alpha')^{-1/2}$, corresponding to the fixed points of the modular (duality) group. On the other hand, the coupling constants and theta angles can be expressed in terms of the VEV of the dilaton field [12]:

$$\frac{4\pi}{g_n^2} + i \frac{\theta_n}{2\pi} = 16\pi k_n(S). \quad (3)$$

The potential, eq.(1), has one zero-value minimum at $\langle \lambda \lambda \rangle_n = 0$, $n = 1, 2, \dots, p$, which corresponds to the infinite VEV of the dilaton field. The crucial observation [7] is that, if $p > 1$, the potential can have another zero-value minimum, provided that the real and imaginary parts of the dilaton VEV adjust in such a way as to render the coherent sum $\sum_{n=1}^{n=p} \langle \lambda \lambda \rangle_n = 0$. In addition to fixing the string coupling constant $g^2 = (4\text{Re}S)^{-1}$ to a value that corresponds to a weakly coupled string, the above mechanism can generate a hierarchy of mass scales. It was not clear, however, whether supersymmetry is broken or not in such a vacuum. The aim of this work is to re-examine this multi-gaugino mechanism by utilizing the effective Lagrangian techniques which allow systematic derivation of the effective potential, thus going beyond the naïve approximation of eq.(1).

In order to derive the effective action for a strongly-coupled $G = G_1 \otimes G_2 \otimes \dots \otimes G_p$ SYM coupled to supergravity [13], one follows [14,15,16,2] and introduces the composite superfields Y_n :

$$Y_n^3 = (\delta_{ab} W_a^\alpha \epsilon^{\beta\gamma} W_\beta^\dagger)_{\gamma\alpha} / S_0^3, \quad (4)$$

whose scalar components are proportional to $\langle \lambda \lambda \rangle_n$. The chiral compensator S_0 [13,17] is included in the definition of Y_n , eq.(4), in such a way as to fix the conformal weight of Y_n equal to the canonical weight 0 [2]. The choice of S_0 determines the normalization of the gravitational action: $\mathcal{L}_{\text{grav}} \propto [e^{-K/3} S_0 \bar{S}_0]_{\theta=\bar{\theta}=0} R$, where K is the Kähler potential.

In the effective theory of gaugino condensation, the action is specified by the Kähler potential

$$K = -3 \log[(T + \bar{T})(S + \bar{S})^{1/3} - \sum_{n=1}^{n=p} |Y_n|^2] \quad (5)$$

and the superpotential

$$w = \frac{1}{32\pi^2} \sum_{n=1}^{n=p} c_n Y_n^3 \log[Y_n^3 f_n(S) h_n(T)]. \quad (6)$$

The choice of the Y_n -dependence of the Kähler potential [2], eq.(5), is such as to reproduce the kinetic terms $-\{e^{-K/3} S_0 \bar{S}_0\}_D = -\sum_{n=1}^{n=p} D_\mu(\lambda\lambda)_n^{1/3} D^\mu(\bar{\lambda}\bar{\lambda})_n^{1/3} + \dots$ for the gaugino composites $(\lambda\lambda)_n$ [14,15]. The form of the superpotential, eq.(6), is determined by the usual arguments based on the fulfilment of anomalous Ward identities [14,15] which dictate $f_n(S) = \exp(32\pi^2 k_n S/c_n)$. It is consistent with the invariance under the $SL(2, Z)$ modular group of duality transformations of T , provided that Y_n transform as modular forms of weight -1 , and the functions $h_n(T)$ are modular forms of weight 3 . The requirement of the absence of singularities in the effective action for any value of T dictates $h_n(T) \propto \eta^6(T)$ [2]. In the case of a larger number of moduli, which will not be discussed here, the moduli-dependence could be determined either by similar symmetry arguments, or by direct computation of the threshold effects [6].

The scalar potential that will be written below corresponds to the chiral compensator satisfying the condition $[e^{-K/3} S_0 \bar{S}_0]_{\theta=\bar{\theta}=0} = 1$, so that minimizations with respect to S and T can be performed before considering the gravitational field equations. The cosmological constant is given directly by the value of this potential at the minimum. The canonical normalization of the gravitational action $\mathcal{L}_{grav} = (16\pi)^{-1} M_P^2 R$ can be restored, after minimizing the potential, by the appropriate Weyl rescalings with $M_P^2 = 32\pi(\text{Re}S)/\alpha'$. In particular, the gauginos scale as $\lambda'_n \propto M_P^{3/2} \lambda_n$. The scalar potential [13,17] corresponding to the Kähler potential and superpotential of eqs.(5) and (6), written in terms of the variables S , T , and $Z_n = (S + \bar{S})^{-1/8} (T + \bar{T})^{-1/2} Y_n$, is:

$$V = [32\pi^2 (1 - \sum_{n=1}^{n=p} |Z_n|^2)]^{-2} \left\{ V_Z + V_S + V_T - 3 \left| \sum_{n=1}^{n=p} c_n Z_n^3 \right|^2 \right\}, \quad (7)$$

where

$$V_Z = 3 \sum_{n=1}^{n=p} c_n^2 |Z_n|^4 \left| 1 + \log[(S + \bar{S})^{1/2} (T + \bar{T})^{3/2} f_n h_n Z_n^3] \right|^2, \quad (8)$$

$$V_S = \left| \sum_{n=1}^{n=p} c_n Z_n^3 \{ 1 + (S + \bar{S}) f'_n / f_n \} \right|^2, \quad (9)$$

$$V_T = \frac{1}{3} \left| \sum_{n=1}^{n=p} c_n Z_n^3 [3 + (T + \bar{T}) h'_n / h_n] \right|^2. \quad (10)$$

Notice that the modulus T and the dilaton S appear in a very symmetric way in the scalar potential. In fact, in addition to the invariance under $SL(2, Z)$ transformations of T , the effective theory exhibits "approximate" symmetry under the analogous transformations² of S , with Y_n transforming as modular forms of weight $-1/3$. This symmetry would be exact if the functions f_n transformed as modular forms of weight 1 , which is not the case, since $f_n(S) = \exp(32\pi^2 k_n S/c_n)$, as required by anomalous Ward identities.

In the weak coupling limit, i.e. for a fixed, large dilaton VEV $\langle \text{Re}S \rangle = (4g^2)^{-1}$, the dominant contribution to the potential is:

$$V_S \propto \langle \text{Re}S \rangle^2 \left| \sum_{n=1}^{n=p} k_n Z_n^3 \right|^2 \propto M_P^4 \left| \sum_{n=1}^{n=p} k_n \langle \lambda\lambda \rangle_n \right|^2. \quad (11)$$

In terms of the Weyl-rescaled gaugino fields, the leading small g behaviour of the scalar potential, eq.(11), coincides with the naïve expression of eq.(1). The full effective potential is given, however, by a more complicated expression, eqs.(7)-(10), which will be analysed below.

The scalar potential does indeed have a stationary point of the type discussed by Krasnikov [7]. It corresponds to the zero-energy field configuration satisfying the

²Such a strong-weak coupling duality has been conjectured on various grounds in [18].

to a local minimum. This can be seen in the following way. If S is sufficiently large, so that $Z_n = (S + \bar{S})^{-1/2} (T + \bar{T})^{-1/2} Y_n \ll 1$, see eq.(15), then the mass matrix of scalar fluctuations is dominated by the second derivatives of V_Z , eq.(8). Since the point under consideration corresponds to the global minimum of V_Z , this matrix is positive definite in all directions except for the zero modes in the directions constrained by the condition $V_Z = 0$, eq.(15). The remaining part of the potential, which is dominated by the term proportional to $\text{Re} S$ in V_S , eq.(9), does not destabilize any of these modes. Only the modulus T (more precisely, one combination of T and Y_n 's) remains massless, as expected from the fact that the extremum condition [eqs. (15) and (16)] leaves its VEV undetermined. This is quite different from the one-gaugino case [2], in which $\langle T \rangle = 1$ and the modulus acquires a non-zero mass.

Since $V = w = 0$ at the minimum, see eqs.(6) and (12)-(14), the corresponding meta-stable vacuum is supersymmetric [13]. Nevertheless, it is quite remarkable that a weakly-coupled vacuum, with a stable dilaton VEV, exists at all. This is, of course, provided that the gauge group satisfies the necessary discriminant condition. In addition, the integers k_n and c_n should be such as to ensure the existence of small z (weak coupling) solutions of eq.(16). In practice, it is rather difficult to find such gauge groups, especially since k_n and c_n are constrained by the inequality $C_G = \sum_{n=1}^{n=p} k_n (k_n + c_n)^{-1} \dim G_n < 18$ for the hidden sector contribution to the Virasoro central charge. Nevertheless, some simple solutions do really exist, for example, $z^{1/2} = -1/2$ for $SU(2)_{k=1} \otimes SU(2)_{k=2}^4 \otimes SU(2)_{k=3}^4$, with $C_G = 71/5$, satisfies eq.(16). The problem of finding all possible solutions deserves a separate study.

The dilaton VEV and supersymmetry breaking represent two major problems for superstring phenomenology.³ The effective Lagrangian analysis shows that non-perturbative condensation of gauginos belonging to a gauge group of the form $G_1 \otimes G_2 \otimes \dots \otimes G_p$, $p \geq 3$, can indeed stabilize the dilaton VEV in the physically interesting weak-coupling regime. Supersymmetry remains however unbroken in such a vacuum. It remains to

³See, for example, [19] and references therein.

$$\log[Y_n^3 f_n(S) h_n(T)] = -1, \quad (12)$$

$$\sum_{n=1}^{n=p} k_n Y_n^3 = 0, \quad (13)$$

$$\sum_{n=1}^{n=p} c_n Y_n^3 = 0. \quad (14)$$

The first equation is equivalent to

$$Y_n^3 \propto \eta^{-6}(T) \exp(-32\pi^2 k_n S / c_n). \quad (15)$$

The gaugino condensates can be expressed, by using this relation, in terms of the VEV's of the dilaton and modulus fields. After the appropriate Weyl rescalings of the condensates, the result is identical to eq.(2), with a T -dependent running scale μ [2]. Equations (13) and (14) can now be rewritten as:

$$P(z) = P'(z) = 0, \quad (16)$$

where

$$z = \exp(-32\pi^2 S), \quad P(z) = \sum_{n=1}^{n=p} c_n z^{k_n/c_n}. \quad (17)$$

The dilaton VEV is determined by the solution of eq.(16). Notice that there are two equations to be satisfied by one complex variable z . Since k_n and c_n are positive integers, these equations are equivalent to two polynomial equations. Only the roots of order 2 and higher of the equation $P(z) = 0$ satisfy both equations simultaneously. Such roots exist if and only if the corresponding polynomial has a vanishing discriminant. The existence of the stationary point under consideration is therefore possible only for some special gauge groups, such that the corresponding k_n and c_n satisfy this discriminant condition. In particular, there will be no solutions if the gauge group contains $p < 3$ factors.

The scalar potential of eq.(7) is not positive-definite. The zero-value stationary point under consideration corresponds, however, in the weak coupling limit (i.e. for large S),

be seen whether a realistic supersymmetry breaking scenario can be built on the basis of multi-gaugino condensation. In particular, the effects of: 1) matter fields, 2) hidden sectors more complicated than pure SYM, 3) a larger number of moduli, should be explored as possible sources of supersymmetry breaking.

Helpful conversations with L. Alvarez-Gaumé, J.-P. Derendinger, S. Ferrara, V. Kaplunovsky, A.N. Schellekens, R. Stora and G. Veneziano are gratefully acknowledged.

References

- [1] S. Ferrara, L. Girardello and H.P. Nilles, *Phys. Lett. B* 125 (1983) 457;
J.-P. Derendinger, L.E. Ibáñez and H.P. Nilles, *Phys. Lett. B* 155 (1985) 65;
M. Dine, R. Rohm, N. Seiberg and E. Witten, *Phys. Lett. B* 156 (1985) 55.
- [2] S. Ferrara, N. Magnoli, T.R. Taylor and G. Veneziano, preprint CERN-TH.5744/90 (1990).
- [3] A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, preprint CERN-TH.5726/90 (1990);
P. Binétruy and M.K. Gaillard, preprint CERN-TH.5727/90 (1990);
H.P. Nilles and M. Olechowski, preprint MPI-PAE/PTh 25/90 (1990).
- [4] K. Kikkawa and M. Yamasaki, *Phys. Lett. B* 149 (1984) 357;
N. Sakai and I. Senda, *Prog. Theor. Phys.* 75 (1986) 692.
- [5] S. Ferrara, D. Lüst, A. Shapere and S. Theisen, *Phys. Lett. B* 225 (1989) 363;
S. Ferrara, D. Lüst and S. Theisen, *Phys. Lett. B* 233 (1989) 147.
- [6] L.J. Dixon, V.S. Kaplunovsky and J. Louis, preprint SLAC-PUB 5138 (1990).
- [7] N.V. Krasnikov, *Phys. Lett. B* 193 (1987) 37.
- [8] L.J. Dixon, preprint SLAC-PUB 5229 (1990);
L. Dixon, V. Kaplunovsky, J. Louis and M. Peskin, preprint SLAC-PUB 5256 (1990);
J.A. Casas, Z. Lalak, C. Muñoz and G.G. Ross, preprint OUTP-90-07P (1990).
- [9] D. Amati, K. Konishi, Y. Meurice, G.C. Rossi and G. Veneziano, *Phys. Rep.* 162 (1988) 169.
- [10] G. Veneziano, in: *DST Workshop on Superstring Theory*, H.S. Mani and R. Ramachandran, eds. (World Scientific Pub. Co., 1988) p.1;
J. Minahan, *Nucl. Phys. B* 298 (1988) 36;

- V. Kaplunovsky, Nucl. Phys. B 307 (1988) 145;
Z. Bern and D.A. Kosower, Phys. Rev. D 38 (1988) 1888.
- [11] T.R. Taylor and G. Veneziano, Phys. Lett. B 212 (1988) 147
- [12] E. Witten, Phys. Lett. B 149 (1984) 351; Phys. Lett. B 155 (1985) 151;
P. Ginsparg, Phys. Lett. B 197 (1987) 139.
- [13] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B 212 (1983) 413.
- [14] G. Veneziano and S. Yankielowicz, Phys. Lett. B 113 (1982) 231.
- [15] T.R. Taylor, Phys. Lett. B 164 (1985) 43.
- [16] P. Binétruy and M.K. Gaillard, Phys. Lett. B 232 (1989) 83;
G.G. Ross, Phys. Lett. B 211 (1988) 315.
- [17] T. Kugo and S. Uehara, Nucl. Phys. B 222 (1983) 125.
- [18] S.J. Rey, preprint UCSB-TH-89/23 (1989);
A. Strominger, preprint UCSB-TH-90/16 (1990);
T.R. Taylor, unpublished (1990);
A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, preprint CERN-TH.5790/90 (1990).
- [19] T.R. Taylor and G. Veneziano, Phys. Lett. B 213 (1988) 450.