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DARK MASS MOMENTS

A. De Rújula, Ph. Jetzer and E. Massó

CERN, CH-1211 Geneva, Switzerland

Abstract

In a potentially seminal paper Paczyński has demonstrated a way to detect the “dark” constituents of the halo of our galaxy, should their mass lie in the approximate domain $10^{-6} < M/M_{\odot} < 10^{-1}$. (This range snugly and auspiciously overlaps with that of H/He aggregates heavy enough not to evaporate, but too light to ignite or to be visible). The massive halo objects (MHOs), when crossing close to the line of sight of a more distant and “fixed” star, would gravitationally “micro-lense” its image, resulting in a time-dependent achromatic amplification with a predictable shape. Observations now in preparation are to monitor up to one million stars in the Large Magellanic Cloud (LMC) during ~ 1 year. Barring unsuspected backgrounds, these observations will either establish or exclude a halo dark mass constituency in the quoted mass range. In the event of a discovery, the next most crucial question will concern the MHO mass distribution, well concealed in the raw data by convolutions with the distributions of various unobservable quantities, that describe the motion and position of individual MHOs. We discuss the extraction from the data of this MHO mass function and we prove that its moments are explicitly related to the moments of the observed distribution of durations of the micro-lensing events.

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Paczynski¹ has shown how to detect "dark" constituents of our galactic halo, should they weigh between a tenth and a millionth solar masses. Massive halo objects (MHOs), crossing near the direction of a distant star, gravitationally "micro-lense" its image², resulting in an achromatic amplification of predictable temporal shape. If MHOs are discovered, the crucial question becomes their mass distribution. We discuss the measurability of this function and show how its moments are obtainable from those of the observed event durations.

Three of the most challenging problems in fundamental physics are problems of mass. In the particle-physics field, a key question concerns the unknown mechanism endowing the different elementary particles with their specific masses. In the astrophysical domain the conundrum³ lies in the nature of the non-luminous matter of galactic halos. In the cosmological realm, the puzzle⁴ is the mysterious "non-baryonic" form of mass (or energy density) supposedly dominating the universe. We shall be concerned with the second of these problems, progress towards whose solution is quite likely to occur in the near future.

The mass of a galaxy within a centred sphere of radius R can be inferred from its "rotation curve" $v(R)$, the velocity of tenuous "test-particle" gas in orbit around the galaxy, measured via Doppler shifts^{5,6,7}. The Milky Way and scores of other galaxies are observed to be embedded in an "invisible halo" roughly an order of magnitude more extended and more massive than their visible domain, which consists of stars, gas and dust. What the constituents of these halos are, we do not know. Fashions have come and gone, wherein the "most favoured candidates" were neutrinos, primordial black holes, or more exotic objects such as photinos, higgsinos, shadow matter, wimps, champs, cosmions, nuclearites, and newtories, all of whose definitions and alleged virtues we need not recall.

Even though physicists may have unleashed their imagination on weird dark-mass candidates, there is no watertight syllogism why the essence of galactic halos need be out of the ordinary; the final word on arguments favouring a "non-dissipative" (non-atomic) halo constituency⁸ ought to await the development of an undisputed theory of galacto-genesis.

Primordial nucleosynthesis⁹ results only in light elements: mainly ^1H , ^4He ($\sim 25\%$ by mass), and minute amounts of ^2H , ^3He and ^6Li . The universe is thought to be too young for stars to have converted most of this material into heavier atoms. It is therefore unlikely that the galactic dark mass, outweighing the observed H and He by more than a factor of five, consists of heavier elements. The observed low metallicity of stars far from the galactic centre also precludes the possibility of a halo that is predominantly made of "metals" (such as moons, CO_2 , SiO_2 , or even H_2O , in cosmology's less than precise parlance). Our current understanding of cosmogenesis strongly favours⁴ a "critical" universe: one that sits exactly at the borderline between an open and a closed geometry. In a critical universe the calculated abundances of H and He exceed, by a common factor $\sim \mathcal{O}(5)$, the mean abundances inferred from the visible parts of galaxies. The large amounts of unobserved light elements may pervade intergalactic space, or be gathered in galactic halos in a yet undetected form.

Collapsed objects that were once ordinary atomic matter (black holes and neutron stars) are excluded as candidate halo constituents by the observed absence of copious supernova

remnants, except for super-massive black holes ($M_{\text{BH}} > 100M_{\odot}$) that would allegedly ingurgitate the whole parent star in their implosive birth. With this exception, and in conclusion, MHOs, if made of ordinary matter, should be aggregates of the lightest primordial elements: Hydrogen and Helium. In the narrow range $0.1 > M/M_{\odot} > 0.08$ they would be hard-to-spot white dwarfs. Lighter MHOs are below the nuclear-ignition threshold, they would be "brown dwarfs", Jupiters, or bodies akin to even smaller "outer" planets. The possible origin of small hydrogenoid planets, by fragmentation or by some other mechanism, is too ill-understood to enable us to draw useful conclusions. The ambient radiation, or their own body heat, would make sufficiently small objects evaporate rapidly, not unlike light gases escape the Earth's gravitational pull from the top of its atmosphere. To estimate a lower-mass cut-off, consider a condensed body of unspecified origin, average density $\rho \sim 1 \text{ gr/cm}^3$, and surface temperature T_s . The condition that its rate of evaporation be insufficient to halve its mass in a billion years yields $M > 10^{-7}M_{\odot} (T_s/30 \text{ K})^{3/2}$. Notice that, to make this bound safe, we have chosen a rather low reference temperature and a fairly short evanescence time.

Accepting the above arguments, we concentrate our discussion on objects made of ordinary matter. We shall see that the range of MHO observability via micro-lensing auspiciously overlaps with the mass bracket of H/He aggregates that are heavy enough not to evaporate, but too light to ignite or to be visible. But, since micro-lensing signatures do not give away the substance of their cause, the only fully decisive result of their search would be the non-existence of MHOs (in a competing nomenclature⁹ "Astrophysical" and "Compact" are inserted into the MHO acronym, to contrast the resulting abbreviation with that for Weakly Interacting Massive Particles).

It is sometimes stated that an extrapolation of the observed star mass-function (numbers of stars per unit mass) beyond its low-mass observational cut-off does not seem to favour a large abundance of light objects. But we do not understand well enough the genesis of celestial bodies to trust this extrapolation: it is for the experiments we are discussing to tell.

A micro-lensing event occurs when a MHO crosses close to the line of sight of a distant and "fixed" star, as in Figure 1. We neglect the relative motion of the observer and the source; this and other modifications of Paczyński's simplified approach¹ have been shown to have minor impact in all cases of interest here (Griest, K. University of California at Berkeley. Preprint CfPA-TH-90-007, 1990). The deflection of the star's light by the MHO's gravity produces an (unresolvable) multiple image and an observable time-dependent and colour-preserving magnification of the original brightness of the source. This is the lensing signature.

Observations now in preparation are to monitor up to one million stars in the Large Magellanic Cloud (LMC) during ~ 1 year (Ferlet, R. *et al.* Proposal to the C.A.S.E. Saclay, 1990; Moniez, M. Orsay Preprint LAL-90-20, 1990; Alcock, C. *et al.* and Perlmutter, S. *et al.*, second-hand private communications). Barring unsuspected backgrounds (such as non-repetitive star flares with no temperature variation) these observations will either find or exclude a halo dark mass constituency in the range $10^{-6} < M/M_{\odot} < 10^{-1}$. In the event of a discovery, the next most crucial question will concern the MHO mass distribution, well

concealed in the raw data by convolutions with the distributions of various unobservable quantities, that describe the motion and position of individual MHOs. We shall discuss the extraction from the data of this MHO mass function and prove that its moments are explicitly related to the moments of the observed distribution of durations of the micro-lensing events.

Consider, as in Figure 1, the plane perpendicular to the line of sight and located at the MHO's actual position, a plane onto which we shall project all images and velocities. Individual light rays are deflected¹⁰ by an angle $\sim 2R_S/y$, with y their minimum distance to the centre of a deflector, whose Schwarzschild radius is R_S . For a point-like source and perfect alignment, the source's image would be a ring with a projected "Einstein" radius:

$$R_E^2 = \frac{4GMD}{c^2} x(1-x), \quad (1)$$

with M the MHO mass and $D(xD)$ the distance from observer to source (to the MHO). In the only case to be discussed here, $D \sim 55$ kpc will be our distance from the LMC. We assume the projected image of the undisturbed source to have a radius smaller than R_E , effectively excluding only tight binary systems¹¹ and red giants¹, which we discuss in detail elsewhere (CERN Preprint TH-5787, 1990). Let d be the impact parameter of the MHO trajectory to the straight line of sight, let $t = 0$ be the instant of closest approach, and let v_T be the MHO velocity in the transverse plane, all as in Figure 1. It is a matter of simple geometry to compute the magnification A as a function of (MHO) time, with the result:

$$A(t) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}, \quad (2.a)$$

$$u^2 = \frac{d^2 + v_T^2 t^2}{R_E^2}. \quad (2.b)$$

where

In Figure 2, $\Delta m = -2.5 \log A(t)$ is displayed as a function of time. The light curve is a universal function¹ uniquely determined by two parameters: d/R_E and v_T/R_E . For a given micro-lensing event, it is useful to trade these parameters for two directly measurable quantities. We choose these, as in Figure 2, to be the magnification at maximum $A(0) = A_{max}$, and the time duration, T , throughout which the magnitude shift $|\Delta m|$ is greater than half its peak value $|\Delta m_{max}|$ or, in terms of A :

$$\log A(\pm T/2) \equiv \frac{1}{2} \log A_{max}. \quad (3)$$

Equations (2) can be inverted to relate d/R_E and R_E/v_T to the directly observable quantities:

$$v_{min}^2 \equiv \frac{d^2}{R_E^2} = 2[(1 - A_{max}^{-2})^{-1/2} - 1], \quad (4)$$

and

$$T^2 = \frac{8R_E^2}{v_T^2} g(A_{max}), \quad (5.a)$$

with

$$g(A) \equiv \frac{1}{\sqrt{1-1/A}} - \frac{1}{\sqrt{1-1/A^2}}. \quad (5.b)$$

The expected rate of micro-lensing events depends on the velocity and spacial distributions of MHOs within the galaxy. We adopt the model of an "isothermal" spherical halo^{12,8,3} in which the normalized MHO number-distribution as a function of v_T is

$$f(v_T)dv_T = \frac{2}{v_H^3} v_T e^{-v_T^2/v_H^2} dv_T, \quad (6)$$

with $v_H \approx 210$ km/s the velocity dispersion implied by the rotation curve of our galaxy (we have ignored the negligible effects of a cut-off at the galactic escape velocity $\sim 3v_H$). Let $\rho_0 \approx 7.9 \cdot 10^{-3} M_\odot/\text{pc}^3$ be the local dark-mass density, $a \sim 5.6$ kpc be the galactic "core" radius, $R_{GC} = 8.5$ kpc be our distance from the centre of the galaxy, and let α be the angle between the line of sight and the direction of the galactic centre ($\sim 82^\circ$ for the LMC). In this halo model the mass density at a distance xD from an Earth-bound observer is

$$\rho = \rho_0 H(x), \quad (7.a)$$

with

$$H(x) = \frac{a^2 + R_{GC}^2}{a^2 + R_{GC}^2 + D^2 x^2 - 2DR_{GC}x \cos \alpha}. \quad (7.b)$$

Let dn/dM be the MHO number density distribution as a function of mass. Its normalization (should the halo dominantly consist of MHOs) is:

$$\int M \frac{dn(x, M)}{dM} dM = \rho(x). \quad (8)$$

In what follows, we use Jupiter as our unit of mass and define $\mu \equiv M/M_J$, with $M_J = 1.9 \times 10^{27}$ kg $\sim 0.95 \times 10^{-3} M_\odot$. We also normalize our expressions to a "typical" Einstein radius

$$r_E^2 \equiv \frac{4GM_J D}{c^2} \sim (0.67 \text{ a.u.})^2 \sim (10^8 \text{ km})^2. \quad (9)$$

The differential rate $d\Gamma$ at which a single star is micro-lensed by MHOs in the interval $d\mu du_{min} dv_T dx$ is the rate at which (at fixed μ , x and v_T) these objects cross close to the line of sight, such as to produce an image amplification peaking at A_{max} (or u_{min}), to wit:

$$d\Gamma = 2Dr_E v_T f(v_T) [\mu x(1-x)]^{1/2} \frac{dn}{d\mu} d\mu du_{min} dv_T dx, \quad (10)$$

Should the MHO mass distribution not be factorizable into a function of mass times a function of distance from the galactic centre ($\propto \rho(x)$), there would be no way to extricate $dn/d\mu$ from the various convolutions in the observable integrated rate Γ , but to compare data from sources at different distances: the LMC, the SMC, high red-shift supernovas¹³, and various globular clusters, which we discuss elsewhere (CERN Preprint TH-5787, 1990).

To proceed with our discussion of source stars in the LMC, we make the plausible assumption that, but for its normalization, $dn/d\mu$ is everywhere in our halo the same function of μ , that is:

$$\frac{dn}{d\mu} = H(x) \frac{dn_0}{d\mu}, \quad (11)$$

with $dn_0/d\mu$ the local MHO mass distribution, and $H(x)$ as in eq.(7b).

For an experiment that monitors N_* stars during a total observation time t_{obs} , the differential number of micro-lensing events is:

$$dN_{ev} = N_* t_{obs} dT. \quad (12)$$

The total number of events is simply obtained by integration over $d\mu du_{min} dv_T dx$; all the subtleties reside in the limits of integration. We assume the Milky Way halo, as described by eqs.(7), to extend all the way to the LMC: $0 \leq x \leq 1$. The v_T distribution is limited to $0 \leq v_T \leq 3v_H$, but the upper (escape-velocity) limit can be substituted for infinity with insignificant effect on the results. The upper limit of the u_{min} integration is determined by the experimental threshold in magnitude shift, Δm_{TH} : the minimum magnitude variation sufficient for the reconstruction of the light curve to the point of measuring its width, T . We set $0 \leq u_{min} \leq u_{TH}$ with, in terms of the other customary and redundant quantities,

$$u_{TH}^2 = 2[(1 - A_{TH}^{-2})^{-1/2} - 1], \quad (13.a)$$

$$A_{TH} = 10^{-\Delta m_{TH}/2.5}. \quad (13.b)$$

The range of MHO masses in eqs (10,12) is in principle arbitrary, but there is an experimental stricture that affects the observable values of μ in a non-trivial fashion: the limited measurable range in T , the event durations of eq.(3). To maximize sensitivity, the focal plane of a telescope may be equipped with "Charge Coupled Devices" (CCDs), that convert single photons into digital signals. For a medium-sized telescope looking at a star in the LMC, current technology sets a limit $T \geq T_{min} \sim \mathcal{O}(\text{tens of minutes})$. The brevity of human life sets a limit $T \leq T_{max} \sim \mathcal{O}(\text{years})$. Thus, the total number of observable¹ micro-lensing events is

$$N_{ev}^{obs} = \int dN_{ev} \Theta(T - T_{min}) \Theta(T_{max} - T), \quad (14)$$

where the integration is over the full range of $d\mu du_{min} dv_T dx$, that we have defined. From eq.(5.a) we find the relation between T and the integration variables to be:

$$T^2 = \frac{8r_E^2 \mu x(1-x)}{v_T^2} g \left(\frac{u_{min}^2 + 2}{u_{min} \sqrt{u_{min}^2 + 4}} \right). \quad (15)$$

Notice that we are discussing the case in which the observer is granted telescope time and good weather conditions every night of the year. Corrections to this idealization are beyond our mathematical powers, and should be made *a posteriori*.

Eq.(14) allows one to rigorously ascertain the range of MHO masses, to which a particular experiment has access. To do this, define $dN_{ev}^*(\mu)$ as dN_{ev} with the MHO mass

distribution concentrated at a fixed mass $\bar{\mu}$: $dn_0/d\mu = n_0 \delta(\mu - \bar{\mu})$. Introduce the efficiency function

$$\epsilon_0(\bar{\mu}) \equiv \frac{\int dN_{ev}^*(\bar{\mu}) \Theta(T - T_{min}) \Theta(T_{max} - T)}{\int dN_{ev}^*(\bar{\mu})}, \quad (16)$$

that measures the fraction of the total number of observable ($u_{min} < u_{TH}$) MHOs that also meet the event-duration limits, at a fixed MHO mass $M = \bar{\mu} M_J$. A sample ϵ_0 is shown in Figure 3a. At the low-mass (high-mass) end, the efficiency depends strongly on $T_{min}(T_{max})$ and more weakly on Δm_{TH} . In the example of Figure 3a, the efficiency drops below 80% for $\mu < 2 \cdot 10^{-3} (M < 1.9 \cdot 10^{-8} M_\odot)$ and for $\mu > 3.2 \cdot 10^2 (M > 0.30 M_\odot)$. We re-emphasize how cosily this range coincides with the unexplored domain of MHOs that would be aggregates of Hydrogen and Helium.

To an excellent approximation the exact expression eq.(14) can be substituted for

$$N_{ev}^{obs} = \int dN_{ev} \epsilon_0(\mu), \quad (17)$$

the meaning of which is that the number of lensing events observed in a given experiment is a convoluted measure of the product of the true mass distribution $dn/d\mu$ times the efficiency function $\epsilon_0(\mu)$.

In comparing conceivable observational campaigns, it is useful to have an analytical approximation for $\epsilon_0(\mu)$ as a function of the experimental limitations. A fair and square estimate (close to Paczyński's original one¹) is:

$$\epsilon_0(\mu) = \Theta(\mu - \mu_{min}) \Theta(\mu_{max} - \mu), \quad (18)$$

with

$$\mu_{min} = 5.5 \times 10^{-6} \frac{(T_{min}/10 \text{ minutes})^2}{g(A_{TH})}, \quad (19)$$

$$\mu_{max} = 9.4 \times 10^2 \frac{(T_{max}/3 \text{ months})^2}{g(A_{TH})}. \quad (20)$$

Here, the numerical constants refer to sources in the LMC, but the functional form, inspired by eq.(18) and by the fact that events steeply accumulate at threshold¹, is clearly universal. We have numerically determined that a very good approximation to $\epsilon_0(\mu)$ (better than 5% for a broad range of conceivable experimental limits) is:

$$\epsilon_0(\mu) = \frac{1}{1 + (\mu_{min}/\mu)^{1.1}} \frac{1}{1 + (\mu/\mu_{max})^{1.2}}, \quad (21)$$

with the same values of μ_{min} and μ_{max} as above. The exact and approximate forms of ϵ_0 are displayed in Figure 3a.

Now that we have analyzed the sensitivity of an observational program to different MHO masses, we proceed to study the extraction of $dn/d\mu$ from the data, discussing different possibilities in order of their increasing usefulness.

The weakest test concerns the distribution of events in A_{max} : it gives no information whatsoever on $dn/d\mu$, since the lensing probability for a given source star is independent

of MHO mass¹ for a fixed (small) optical depth (fixed integrated MHO mass-density along the line of sight).

The microlensing rate and mean duration are rough measures of the average MHO mass¹. From the normalization requirement eq.(8) and the factorization hypothesis eq.(11) we have that $n_0 = \rho_0/(M_J < \mu >)$, so that $dn_0/d\mu$ must scale as $<\mu>^{-1}$. It then follows from eqs.(10,15,17) that

$$N_{ev} \sim 0.17 D r_E N_* t_{obs} v_H u_{TH} \frac{\rho_0}{M_J \sqrt{<\mu>}} \epsilon_0(<\mu>) \quad (22)$$

$$\sim 19 \frac{N_*}{10^6} \frac{t_{obs}}{4 \text{ months}} u_{TH} \epsilon_0(<\mu>) <\mu>^{-1/2},$$

$$<T> \sim (2.9 \text{ days}) h(A_{TH}) <\mu>^{+1/2}, \quad (23a)$$

$$h(A_{TH}) \equiv \frac{2}{u_{TH}} \int_0^{u_{TH}} \sqrt{g(A)} du, \quad (23b)$$

with the function h varying from $h=1.2$ to $h=0.8$ in the interval $A_{TH}=1.1$ to 1.6 . Though these expressions correctly reflect the scaling law between rate and mean MHO mass, their numerical coefficients (given above for a mass distribution peaking sharply at its average mass) would clearly depend on the assumed mass function.

The experimental distribution with the closest relation to the MHO mass function is $d\Gamma/d(T^2)$, since $T^2 \propto \mu$, as in eq.(15). But the necessary averages over x and v_T obliterate a sharp sensitivity to $dn/d\mu$. An interesting way to gauge the incidence of this effect is to compute the probability $P(\mu, T, A_{max})$ that an event of duration T and maximum amplification A_{max} be produced by a MHO of mass μ . It is convenient at this point to introduce the variable

$$\tau \equiv \frac{v_H}{r_E} \frac{T}{\sqrt{8g(A_{max})}}, \quad (24)$$

since the function $P(\mu/\tau^2)$ is universal (independent of A_{max}). The factor v_H/r_E occurs in the definition of τ to make it dimensionless; this will simplify the algebra considerably as we move along. At fixed τ :

$$P(\mu, \tau, A_{max}) \propto \int_0^{v_{TH}} du'_{min} \int \delta(\tau' - \tau) d\tau' \delta(A_{max} - A') R_E(x) H(x) dx f(v_T) dv_T. \quad (25)$$

An example with $\tau = 0.4$ (or $T = 4$ days for $A_{max} = 1.34$) and $u_{TH} = 1.67$ ($A_{TH} = 1.1$) is given in Figure 4, showing one Jovian mass ($\mu = 1$) to be the most probable estimate. The value of μ for which $P(\mu)$ is maximal, as well as the whole curve, scale as $\mu = (\tau/0.4)^2$. From Figure 4 one can read that the 50% (90%) confidence interval on μ embraces one (two) orders of magnitude. Clearly, one would like to do better than this in extracting information on MHO masses. This we proceed to do.

To illustrate the sensitivity of $d\Gamma/d\tau$ to the MHO mass distribution consider, for the sake of discussion, a power law form $dn/d\mu \propto \mu^p$, extending from μ_{down} to μ_{up} . Then,

$$\frac{d\Gamma}{d\tau} \propto \frac{1}{\tau^4} \int_{\mu_{down}}^{\mu_{up}} \epsilon_0(\mu) \mu^{2+p} d\mu \int_0^1 [x(1-x)]^2 H(x) e^{-[\mu x(1-x)/\tau^2]} dx. \quad (26)$$

Examples of $d\Gamma/d\tau$ for $-2 \leq p \leq 1$ are given in Figure 5. If $p > -1$, $d\Gamma/d\tau$ peaks towards μ_{up} . Equation (22) indicates that, for p sufficiently larger than -1 and $\mu_{up} \sim \mathcal{O}(\mu_{max})$ the number of events may become so meagre that one could do no better than to estimate on a probabilistic basis the individual masses of the MHOs responsible for them, as we did before. In all other cases the events would have relatively brief T 's, as shown by eq.(23), and the micro-lensing rate of eq.(22) would be sufficient to do more than rough mass estimates. Ideally one may have enough data to attempt to reconstruct $dn/d\mu$ from $d\Gamma/d\tau$ as in Figure 5. More realistically, the data may be sufficient to reconstruct a few moments of the MHO mass distribution, as we presently discuss.

Should MHOs be found, a higher statistics "continuous" survey of the LMC would be justified, to complement the "sporadic" observations that suffice for a first discovery effort. It is on this optimistic but not over-idealized footing of relatively abundant data that we place ourselves in the rest of this paper.

Consider an experiment that observes N_* stars during a time t_{obs} , that has thresholds T_{min} , T_{max} and Δm_{TH} , and has recorded a set of micro-lensing events and measured their individual widths T and peak magnifications A_{max} . From this data we compute the value of the variable τ of eq.(24) on an event-by-event basis.

The cumulative n^{th} moment of τ is constructed from the observations as

$$<\tau^n> = \sum_{\text{events}} \tau^n, \quad (27)$$

These moments are essentially those of the distribution of observed time durations, slightly doctored for convenience by the factors $\sqrt{g}$ and v_H/r_E in eq.(24). As a function of the quantities μ , v_T , x and $u_{min} = d/R_E$, that describe a given MHO and its motion, the variable τ is given by:

$$\tau = \frac{v_H}{v_T} [\mu x(1-x)]^{1/2}. \quad (28)$$

Looking back to eq.(10), one realizes that the theoretical expression, $\int \tau^n d\Gamma$, for the n th τ -moment factorizes into a product of moments of "known" distributions ($f(v_T)$ and $\sqrt{x(1-x)}H(x)$) times a moment of the MHO mass distribution. Thus, one can extract the mass function of interest, either in the sense of its moments or by the inverse Mellin transform. This is the gist of our argument on dark mass moments. Once again, because of the experimental limitation to a finite range of event durations ($T_{min} < T < T_{max}$), reality is a little harder. We proceed to describe it.

Eqs.(14,15) show that an upper or lower limit on T constrains the variables μ , v_T , x and u_{min} in an interwoven fashion. As a consequence, the theoretical expression for the observable τ -moments

$$<\tau^n> = \int dN_{ev} \tau^n \Theta(T - T_{min}) \Theta(T_{max} - T), \quad (29)$$

does not exactly factorize: the integration limits correlate the different functions on which dN_{ev} depends. We shall find the incidence of this effect to be modest.

density in MHOs is $<\mu^1 >$ Jupiter masses per cubic parsec. The mean MHO mass is $<\mu^1 > / <\mu^0 >$ Jovian masses. The ratio

$$f \equiv \frac{M_J}{\rho_0} <\mu^1 > \sim 0.12 \text{ pc}^3 <\mu^1 > \quad (37)$$

is the fraction of the local dark mass density detected in the form of MHOs. A result $f < 1$ implies either that MHOs do not constitute all of the galactic dark matter and/or that some MHOs have escaped detection because of the experimental limitations: the $T_{\min}$ and $T_{\max}$ thresholds, but not the threshold in magnitude shift Δm_{TH} , that mainly affects the overall statistics. The unlikely result that $f > 1$ could mean one or more of the following: that the hypothesis eq.(11) is to be substituted for a non-factorized form contrived to optimize micro-lensing, that the halo of the Milky Way departs significantly from a spherical shape (having, for instance, a dense “finger” towards the LMC), or that we have not understood long-distance gravitation.

The dark mass moments can be used, not only to construct mean values, but also to ascertain the spread of the function $dn_0/d\mu$. The statistical dispersion in $\mu^{m/2}$, is:

$$\frac{\Delta \mu^{m/2}}{<\mu^{m/2}>} = \frac{\sqrt{<\mu^m> - <\mu^{m/2}>^2}}{<\mu^{m/2}>}, \quad (38)$$

and “wider” measures of the mass spread can also be considered. Given sufficiently many well-determined moments, one may dare use their Mellin transform to reconstruct $dn/d\mu$.

We have seen that, should the halo of our galaxy have an MHO component, various possibilities are to be contemplated:

- If the MHO mass function peaks close to the upper edge of detectability ($M \sim 0.1 M_\odot$), one expects no more than a handful of long-duration events per year, and the MHO masses could only be estimated on a likelihood basis. In this occurrence, the case for a search for white and brown dwarfs, via visible or infra-red satellite astronomy, will be greatly reinforced.
- If the MHO masses narrowly bracket the currently contemplated lowest detectable value $\sim 10^{-6} M_\odot$, there will be data galore. Some MHOs will go undetected, and the measured MHO mass density should fail to saturate the upper bound imposed by the halo density, as deduced from our galaxy’s rotation curve: $f < 1$ in eq.(37). In this case, larger ground-based telescopes ought to be equipped with fast light-collecting sensors, and stars in globular clusters may become targets of choice.
- In cases intermediate between the two above extremes, abundant data may be gathered to provide precise information on the MHO mass distribution, or at least on a few of its moments. If $f > 1$ the situation will be puzzling indeed. If $f = 1$ the mystery of the galactic dark mass will have been solved: the temptation to conclude that MHOs are good old “planets” would be irresistible. If f turns out to be significantly below unity, or if it vanishes, the arguments we have reviewed in the introductory paragraphs can be run backwards, strengthening the case for a halo constituency that is not just ordinary matter.

Define, in analogy to eq.(16):

$$\epsilon_n(\mu) \equiv \frac{\int dN_{ev}^*(\mu) \Theta(T - T_{\min}) \Theta(T_{\max} - T) \tau^n}{\int dN_{ev}^*(\mu) \tau^n}. \quad (30)$$

To an excellent approximation eq.(27) may be substituted by

$$<\tau^n> = \int dN_{ev} \epsilon_n(\mu) \tau^n. \quad (31)$$

This expression *does* factorize as:

$$<\tau^n> = V v_{TH} \Gamma(2-m) \widehat{H}(m) <\mu^m>, \quad (32)$$

with $m \equiv (n+1)/2$ and

$$V \equiv 2N_* t_{obs} D \tau_E v_H = 26 \text{ pc}^3 \frac{N_*}{10^6} \frac{t_E}{4 \text{ months}}, \quad (33)$$

$$\Gamma(2-m) \equiv \int_0^\infty \left[\frac{v_T}{v_H} \right]^{1-n} f(v_T) dv_T, \quad (34)$$

$$\widehat{H}(m) \equiv \int_0^1 [x(1-x)]^m H(x) dx, \quad (35)$$

$$<\mu^m> \equiv \int d\mu \epsilon_n(\mu) \frac{dn_0}{d\mu} \mu^m. \quad (36)$$

Notice that the convergence of the integrals in v_T and x restricts the moments to the range $-3 < n < 3$ ($-1 < m < 2$). The function $\Gamma(2-m) \widehat{H}(m)$ is plotted in Figure 6, for target stars in the LMC.

Comparison of eqs.(32) to (36) with the experimental moments of eq.(27) allows one to extract the MHO mass moments, $<\mu^m>$, from the data. But the experimental limitation to a finite range of measurable micro-lensing durations is reflected in a drawback of the above expressions: the mass moments of eq.(36) do not weigh $dn_0/d\mu$, but the different functions $\epsilon_n(\mu) dn_0/d\mu$. For different values of n the functions $\epsilon_n(\mu)$ are very similar, but not identical. An example is given in Figure 3b, where $\epsilon_1(\mu)$ is compared with $\epsilon_{-1}(\mu)$. In practice the difference between these various efficiencies is not a serious problem. If the function $dn_0/d\mu$ peaks in the large region of μ values wherein $\epsilon_n \sim 1$, $\forall n(-3, 3)$, the moments of eq.(36) are the moments of $dn_0/d\mu$. If it peaks at the high mass end of micro-lensing observability, $\mu \sim \mu_{\max}$, there will anyhow be too few events to attempt to reconstruct a mass distribution in much detail. If $dn_0/d\mu$ is concentrated at $\mu_{\min}$, a rough reconstruction of the mean MHO mass will inform us that this is the case. In this situation, there is simply no way to fully access the mass distribution, except to perform an experiment with a lower $T_{\min}$. But trial functional shapes for $dn_0/d\mu$ can still be used to fit the data on dN/dr , the price to pay is one of elegance. In what follows we discuss in more detail the favourable case in which we are not cornered against the limits of observability.

Some moments are of particular interest. The number of events is $N_{ev}^{obs} = <\tau^0>$. The mean local density of MHOs (number per cubic parsec) is $<\mu^0>$. The average local mass

It is difficult to decide which of the possible outcomes of a dedicated micro-lensing search, including total failure, would be the most fascinating.

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Figure Captions

Fig. 1: Definition of various quantities describing a micro-lensing event. The observer is O, the source is S and M is the MHO.

Fig. 2: Plot of $-\Delta m = 2.5 \log A(t)$ as a function of time (in units of 10^5 sec) for a pointlike source at the distance of the LMC. The MHO is at a distance of 25 kpc and has a transverse velocity of 200 km/s and a mass of $10^{-3} M_\odot$. The lighter curve, that serves to define the event duration T , corresponds to $d = 1/2 R_E$, whereas the lower one has $d = R_E$.

Fig. 3: (a) The efficiency $\epsilon_0(\mu)$ and its approximations as functions of $\log(\mu)$. The continuous and dotted lines are eqs.(16) and (18), respectively. The (almost invisible) dashed line is Eq.(21) with $\log \mu_{min} = -5.3$, $\log \mu_{max} = 3.1$. The experimental limitations are taken to be $T_{min} = 10$ minutes, $T_{max} = 0.3$ years and $|\Delta m_{TH}| = 0.1$. (b) The solid line is $\epsilon_{-1}(\mu)$ and the dotted line is $\epsilon_1(\mu)$, defined in eq.(30), with the same experimental limitations.

Fig. 4: The probability distribution $P(\mu)$ of eq.(25) for $\tau = 0.4$ (or $T = 4$ days for $A_{max} = 1.34$) and $u_{TH} = 1.67 (A_{TH} = 1.1)$.

Fig. 5: Number of events as a function of the variable τ , defined in eq.(24), for a mass distribution $dn/d\mu \propto \mu^p$, extending from $\mu_{down} = 10^{-2}$ to $\mu_{up} = 10$, for p varying from -2 to $+1$ in half-integer steps. The normalizations are arbitrary, and the event durations obey 10 minutes $< T < 4$ months.

Fig. 6: The natural logarithm of $\tilde{H}(m) \times \Gamma(2-m)$, defined by eqs.(34,35), as a function of m , for the LMC.

FIGURE 1

FIGURE 2

FIGURE 3a

FIGURE 3b

FIGURE 4

FIGURE 5

FIGURE 6