Strangeness, Glue and Quark Matter

Content of Neutron Stars

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Abstract

We show that uncertainties in the strength of interactions of hyperons among themselves and with nucleons lead to a large uncertainty in the maximum allowed neutron star mass, even if the properties of nuclear and neutron matter are known with infinite precision around normal nuclear matter density and below. The presence of hyperons in the neutron star will generate a ϕ -meson condensate, however, and this reduces the sensitivity to the strengths of the couplings. The possibility that nucleons have a high strangeness content is explored, but it turns out to have negligible influence on neutron star structure. We consider a novel mechanism for nuclear attraction, a density-dependent glueball condensate. Finally, we determine which of these nuclear equations of state lead to a stable quark matter core in the star, via a first or second order phase transition.

1 Introduction

There has been tremendous interest and activity for three decades in the relation between the nuclear matter equation of state and the properties of neutron stars. This interest has intensified in the last decade with the theoretical activity in the simulation of supernova explosions [1], and with the activity surrounding heavy ion collisions and efforts to extract nuclear matter properties from them [2]. Much recent emphasis has focused on the stiffness of nuclear matter, particularly in the density range $\frac{1}{2} \leq n/n_0 \leq 2$ ($n_0 = 0.153 \text{ fm}^{-3}$ = nuclear matter density at saturation) [3,4]. The compressibility K is of course density-dependent, and this leaves open the possibility that any extrapolation to high density is not unique. In fact, at higher densities (typically $n/n_0 > 2$) hyperons appear due to strangeness changing weak interactions. Although hyperons have been included in many (but not all) of the commonly used equations of state, [5,6] a systematic study has been lacking, as emphasized recently [4]. In a recent letter [7] two of us pointed out a surprising sensitivity of neutron star masses to the uncertainties in the strength of hyperon interactions among themselves and with nucleons. It was also pointed out that care must be taken to insure that consistency between nuclear matter and quark matter is maintained. An otherwise innocuous looking nuclear equation of state may be incompatible with a (physically sensible) transition to quark matter at high density.

It is the purpose of this paper to elaborate on, and extend, the study begun in ref. [7]. In section 2 we discuss the theoretical framework we use to describe hadronic matter, both nuclear and hyperonic. Since the influence of variations in the properties of nuclear matter around the saturation density has been extensively studied, we fix the values of the compressibility, effective mass, etc. at saturation density and focus on the high density equation of state. Reasonable values of the hyperon coupling constants and of the strangeness content of the nucleon are inferred from observed hypernuclei energy levels, from nucleon-nucleon scattering, and from the electromagnetic form factor of the proton. In section 3 we compute neutron star masses based on these relativistic equations of state. We find that a ϕ -meson condensate, not considered in any previous studies known to us, is important at high

densities. In section 4 we allow for either first or second order phase transitions to quark matter. Some hadronic equations of state are incompatible with a high density quark phase, however, and in our opinion these should be discarded as physically unacceptable. The likelihood that quark matter can exist in the cores of the most massive stars is small but not zero. In section 5 we consider a novel mechanism for nuclear attraction: a glueball condensate. Although one might have expected that the value of this condensate would go to zero at high density, indicating a transition to quark matter, this turns out not to occur. In section 6 we estimate the maximum rotation frequency for all the equations of state studied. Only for parameter values we consider physically unlikely can a rotation frequency greater than 10,000 rad/sec be achieved. In section 7 we summarize our conclusions.

2 Relativistic Hadronic Equation of State

We use the theoretical framework of relativistic nuclear mean field theory [8]. Other theoretical frameworks are possible. The advantage of this one is that the resulting equation of state is automatically consistent with special relativity and it readily incorporates hyperons and their interactions. In our opinion it is best to view relativistic nuclear mean field theory as a convenient way of parametrizing the equation of state. Therefore the parameters involved may differ somewhat from their values determined from some other considerations, such as phase shift analyses of nucleon-nucleon scattering.

2.1 Lagrangian and Mean Field Theory

We will base our relativistic mean field theory on the Lagrangian

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_i(i\gamma^\mu\partial_\mu - m_i + g_{\sigma i}\sigma - g_{\omega i}\omega_\mu\gamma^\mu - g_{\phi i}\phi_\mu\gamma^\mu \\ & - g_{\rho i}\rho_\mu^a\gamma^\mu T_a)\psi_i - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu \\ & - \frac{1}{4}\rho_{\mu\nu}^a\rho_a^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho_a^\mu - \frac{1}{3}bm_N(g_{\sigma N}\sigma)^3 - \frac{1}{4}c(g_{\sigma N}\sigma)^4. \end{aligned} \quad (1)$$

Here i represents one of the spin- $\frac{1}{2}$ baryons in the lowest octet ($i =$

$p, n, \Lambda, \Sigma^+, \Sigma^-, \Sigma^0, \Xi^-, \Xi^0$); a sum over i is implied. The vector meson fields are, in obvious notation, ω_μ, ϕ_μ and ρ_μ^a ; and $\omega_{\mu\nu}, \phi_{\mu\nu}$ and $\rho_{\mu\nu}^a$ are the corresponding field strengths. T_a is the isospin generator. No pions are used directly, but the scalar field σ is employed to simulate attractive two-pion exchange, in the usual way. We use the metric and other conventions of Bjorken and Drell [9].

In the relativistic mean field approximation we allow the meson fields to acquire density-dependent average values; the nonzero ones are $\bar{\sigma}, \bar{\omega}_0, \bar{\phi}_0$ and $\bar{\rho}_0^3$. These are driven by the finite density of particle number ($\bar{\sigma}$), baryon number ($\bar{\omega}_0$), strangeness ($\bar{\phi}_0$) and isospin asymmetry ($\bar{\rho}_0^3$). From the Lagrangian (1) one can read off the effective masses ($\bar{m}_i$) and chemical potentials ($\bar{\mu}_i$) of the baryons. These are

$$\begin{aligned}\bar{m}_p &= m_p - g_{\sigma N} \bar{\sigma} \\ \bar{m}_n &= m_n - g_{\sigma N} \bar{\sigma} \\ \bar{m}_\Lambda &= m_\Lambda - g_{\sigma \Lambda} \bar{\sigma} \\ \bar{m}_{\Sigma^-} &= m_{\Sigma^-} - g_{\sigma \Sigma} \bar{\sigma} \\ \bar{m}_{\Xi^-} &= m_{\Xi^-} - g_{\sigma \Xi} \bar{\sigma}\end{aligned}\tag{2}$$

and

$$\begin{aligned}\bar{\mu}_p &= \mu_p - g_{\omega N} \bar{\omega}_0 - \frac{1}{2} g_{\rho N} \bar{\rho}_0^3 - g_{\phi N} \bar{\phi}_0 \\ \bar{\mu}_n &= \mu_n - g_{\omega N} \bar{\omega}_0 + \frac{1}{2} g_{\rho N} \bar{\rho}_0^3 - g_{\phi N} \bar{\phi}_0 \\ \bar{\mu}_\Lambda &= \mu_\Lambda - g_{\omega \Lambda} \bar{\omega}_0 - g_{\phi \Lambda} \bar{\phi}_0 \\ \bar{\mu}_{\Sigma^-} &= \mu_{\Sigma^-} - g_{\omega \Sigma} \bar{\omega}_0 + g_{\rho \Sigma} \bar{\rho}_0^3 - g_{\phi \Sigma} \bar{\phi}_0 \\ \bar{\mu}_{\Xi^-} &= \mu_{\Xi^-} - g_{\omega \Xi} \bar{\omega}_0 + \frac{1}{2} g_{\rho \Xi} \bar{\rho}_0^3 - g_{\phi \Xi} \bar{\phi}_0.\end{aligned}\tag{3}$$

The other three baryons are unimportant at zero temperature. We neglect fluctuations in the meson fields about these mean values.

The particle densities are given in terms of the Fermi momenta via

$$n_i = k_i^3 / 3\pi^2.\tag{4}$$

The Fermi momenta, in turn, are related to the effective chemical potentials via

$$\bar{\mu}_i = (\bar{m}_i^2 + k_i^2)^{\frac{1}{2}}. \quad (5)$$

In a neutron star the matter is locally electrically neutral and in equilibrium under the strong, electromagnetic and weak interactions. Chemical equilibrium among the five baryons listed above as well as electrons and muons implies the relations

$$\begin{aligned} \mu_p &= \mu_n - \mu_e \\ \mu_\Lambda &= \mu_n \\ \mu_{\Sigma^-} &= \mu_{\Xi^-} = \mu_n + \mu_e \\ \mu_\mu &= \mu_e \end{aligned} \quad (6)$$

where $\mu_e = (m_e^2 + k_e^2)^{\frac{1}{2}}$, $n_e = k_e^3/3\pi^2$ and similarly for muons. Electrical neutrality requires

$$n_p = n_e + n_\mu + n_{\Sigma^-} + n_{\Xi^-}. \quad (7)$$

The total pressure and energy density are succinctly expressed in terms of the effective masses and chemical potentials.

$$\begin{aligned} P &= \frac{1}{2}m_\omega^2\bar{\omega}_o^2 + \frac{1}{2}m_\phi^2\bar{\phi}_o^2 + \frac{1}{2}m_\rho^2(\bar{\rho}_o^3)^2 - \frac{1}{2}m_\sigma^2\bar{\sigma}^2 \\ &\quad - \frac{1}{3}bm_N(g_{\sigma N}\bar{\sigma})^3 - \frac{1}{4}c(g_{\sigma N}\bar{\sigma})^4 \\ &\quad + \sum_i P_{FG}(\bar{m}_i, \bar{\mu}_i) + P_{FG}(m_e, \mu_e) + P_{FG}(m_\mu, \mu_\mu), \end{aligned} \quad (8)$$

$$\begin{aligned} \rho &= \frac{1}{2}m_\omega^2\bar{\omega}_o^2 + \frac{1}{2}m_\phi^2\bar{\phi}_o^2 + \frac{1}{2}m_\rho^2(\bar{\rho}_o^3)^2 + \frac{1}{2}m_\sigma^2\bar{\sigma}^2 \\ &\quad + \frac{1}{3}bm_N(g_{\sigma N}\bar{\sigma})^3 + \frac{1}{4}c(g_{\sigma N}\bar{\sigma})^4 + \sum_i \rho_{FG}(\bar{m}_i, \bar{\mu}_i) \\ &\quad + \rho_{FG}(m_e, \mu_e) + \rho_{FG}(m_\mu, \mu_\mu). \end{aligned} \quad (9)$$

In these equations $P_{FG}(x, y)$ and $\rho_{FG}(x, y)$ are the relativistic, noninteracting expressions for the pressure and energy density of fermions with mass x , chemical potential y and Fermi momentum $(y^2 - x^2)^{\frac{1}{2}}$. See ref. [8] or [10] for analytic formulas. Equations (6) and (7) determine the relative abundances of all the particles if one is given the Fermi momentum of one of them, say k_n . This leaves as unknowns the values of the mean fields $\bar{\sigma}, \bar{\omega}_0, \bar{\rho}_0^3$ and $\bar{\phi}_0$. These mean fields are determined by maximizing the pressure at fixed baryon chemical potential $\mu_B = \mu_n$, or by minimizing the energy at fixed baryon density $n_B = n_n + n_p + n_\Lambda + n_{\Sigma^-} + n_{\Xi^-}$. One then derives the conditions

$$m_\omega^2 \bar{\omega}_0 = \sum_i g_{\omega i} n_i \quad (10)$$

$$m_\rho^2 \bar{\rho}_0^3 = \sum_i I_{3i} g_{\rho i} n_i \quad (11)$$

$$m_\phi^2 \bar{\phi}_0 = \sum_i g_{\phi i} n_i \quad (12)$$

where I_{3i} is the third component of isospin for the i 'th baryon, and

$$m_\sigma^2 \bar{\sigma} + b m_N g_{\sigma N}^3 \bar{\sigma}^2 + c g_{\sigma N}^4 \bar{\sigma}^3 = \sum_i g_{\sigma i} n_i^s. \quad (13)$$

In (13) n_i^s is the scalar density

$$n_i^s = 2 \int \frac{d^3 p}{(2\pi)^3} \frac{\bar{m}_i}{(p^2 + \bar{m}_i^2)^{\frac{1}{2}}} \theta(k_i - p). \quad (14)$$

and again, an analytic expression may be found in ref. [8] or [10]. In general this set of self-consistent equations must be solved numerically.

2.2 Nuclear Matter

There are a large number of parameters to determine. In this subsection we focus on the parameters dealing with the neutron and proton sector. In the next subsection we consider the hyperons.

For the nucleons only, there are 6 parameters: $g_{\sigma N}/m_\sigma$, $g_{\omega N}/m_\omega$, $g_{\rho N}/m_\rho$, $g_{\phi N}/m_\phi$, b and c . Leaving aside the ϕ -coupling, we impose the following constraints for saturated isospin-symmetric nuclear matter.

Saturation density [11]

$$n = n_p + n_n = \frac{2k_F^3}{3\pi^2} = 0.153 fm^{-3} \quad (15)$$

Binding energy [11]

$$\rho/n - m_N = -16.3 MeV \quad (16)$$

Landau mass [12]

$$m_L = (\bar{m}_N^2 + k_F^2)^{\frac{1}{2}} = 0.83m_N \quad (17)$$

Charge symmetry energy [11]

$$a_{sym} = \left(\frac{g_{\rho N}}{m_\rho}\right)^2 \frac{k_F^3}{12\pi^2} + \frac{k_F^2}{6m_L} = 32.5 MeV \quad (18)$$

Compressibility [11,13]

$$K = k_F^2 \frac{d^2}{dk_F^2} \left(\frac{\rho}{n}\right) = 300 MeV \quad (19)$$

Of these, the compressibility has the greatest uncertainty. One can find values in the literature ranging from 150 to 400 MeV. As we said before, the influence of these numbers on neutron star masses has been extensively studied recently, so we simply take these as given. One deduces

$$\begin{aligned} g_{\sigma N}/m_\sigma &= 1.52502 \times 10^2 MeV^{-1} \\ g_{\omega N}/m_\omega &= 8.63433/782 MeV \\ g_{\rho N}/m_\rho &= 8.55451/770 MeV \\ b &= 3.41799 \times 10^{-3} \\ c &= 1.46 \times 10^{-2} \end{aligned} \quad (20)$$

The SU(3) quark model predicts $g_{\phi N} = 0$ since, obviously, the nucleons have no valence s -quarks. However, a real-life nucleon could possibly couple to the ϕ because of $s\bar{s}$ pairs in its wave function¹. This possibility has recently been studied quantitatively by Jaffe [15]. He infers $g_{\phi N}/g_{\omega N} = 0.31$ on the basis of a pole model for the electromagnetic form factor of the nucleon. Some analyses of low energy nucleon-nucleon and hyperon-nucleon scattering extract a nonzero value for $g_{\phi N}$. For example, Maessen, Rijken and de Swart [16] infer $g_{\phi N}/g_{\omega N} = 0.1$. Therefore we are compelled to at least consider the possibility that $g_{\phi N} \neq 0$.

If $g_{\phi N} \neq 0$, how are the previously determined parameters for nuclear matter changed? It is clear that if we allow for a ϕ -condensate then the repulsion provided by the ω must be reduced. From Eqs. (3), (8), (9), (10) and (12) it is apparent that $\bar{\phi}_0$ and $\bar{\omega}_0$ contribute in exactly the same way, at least concerning the nucleons. If $g_{\phi N} \neq 0$, we would obtain the same equation of state if $g_{\omega N}$ is replaced by a smaller $g'_{\omega N}$ determined by

$$\frac{g_{\omega N}^2}{m_{\omega}^2} = \frac{g_{\omega N}'^2}{m_{\omega}^2} + \frac{g_{\omega N}^2}{m_{\phi}^2} \quad (21)$$

where $g_{\omega N}/m_{\omega}$ is given by Eq. (20). The particular values of $g_{\phi N}$ we consider in this paper will be given in the next subsection.

2.3 Hyperons

In our first report [7] we considered the cases shown in Table 1. We did not allow for the possibility of a ϕ -condensate, nor did we allow for the appearance of Ξ^- at high density. The first row in the table corresponds to a “universality” assumption among all baryon-meson couplings. The last row corresponds to the introduction of the hyperons as noninteracting particles: they participate only in the sense of satisfying the equations of chemical equilibrium (6) and the equation of electrical neutrality (7). The middle row corresponds to some sort of simple quark counting and gives a reasonable first description of Λ -hypernuclear energy levels in the framework of relativistic nuclear mean field theory [17]. The fourth

¹There is evidence of $s\bar{s}$ pairs in the nucleon from analysis of the πN σ -term and polarized electro- production [14]

row corresponds to a least-squares fit to Λ -hypernuclei with the constraint that $g_{\sigma\Lambda} = g_{\omega\Lambda}$. The value obtained [18] was 0.21 ± 0.02 . The second row corresponds to a similar fit [18] but with $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$ treated as independent parameters. The uncertainties in the ratios shown in the second row are ± 0.48 !

In the absence of hyperons the maximum neutron star mass found in ref. [7] was $2.09 M_{\odot}$. As one goes down the rows in the table, the maximum neutron star mass decreases, to 1.93, 1.62, 1.32, 1.20 and $1.16 M_{\odot}$. This is easy to understand. By decreasing the strength of the hyperon couplings, it is energetically favorable to convert nucleons into hyperons, because the hyperons do not feel the predominantly repulsive forces. The equation of state is softened, and the maximum star mass is reduced.

The only parameter modification we would make to ref. [7] is to fix $g_{\rho\Sigma} = g_{\rho N}$. This is motivated by both the SU(3) quark model and the vector meson dominance model. We have checked that this parameter change has a negligible influence on the equation of state and on the neutron star mass curve.

Here we would like to extend our considerations to ϕ -condensates and to allow for the Ξ , the last member of the lowest octet. The parameters in the equations of state we study are listed in Table II. The nomenclature is now explained. The first number is the ratio $g_{\sigma\Lambda}/g_{\sigma N}$. The value 0.48, as explained before, corresponds to the best fit to Λ -hypernuclei. The symbol ϕ after 0.48 indicates that a nonzero ϕ -condensate is present. The symbol J indicates that the nucleon couples to the ϕ with a strength approximately given by Jaffe [15]. The set of couplings 0.58 ϕ JJ is inferred from Maessen et al. [16]. We are also curious to know what happens when all the mass shifts are identical ($g_{\sigma i} = g_{\sigma N}$ for all i), or when the mass ratios are identical ($g_{\sigma i} = m_i g_{\sigma N}/m_N$ for all i). Finally, an x indicates that the appearance of Ξ^- is allowed for. In all cases $g_{\rho\Sigma} = g_{\rho\Xi} = g_{\rho N}$, $g_{\omega\Lambda} = g_{\omega\Sigma}$ and $g_{\phi\Lambda} = g_{\phi\Sigma}$.

3 Stars Made of Nucleons and Hyperons

To determine the mass and structure of cold, non-rotating, spherically symmetric stars, we use the Tolman-Oppenheimer-Volkoff equation

$$\begin{aligned}
-r^2 \frac{dP}{dr} &= G(\rho + P)(M + 4\pi r^3 P) \left(1 - \frac{2GM}{r}\right)^{-1} \\
M(r) &= 4\pi \int \rho r^2 dr
\end{aligned} \tag{22}$$

The equations of state $P(\rho)$ were computed for each of the models listed in Table 2. Because of the large number of models, it is not possible here to display graphically results from all of them. Rather, we will focus on the trends followed by an increase or decrease of the strength of the hyperon couplings, on the ϕ -meson condensate and on the influence of the heavier hyperon Ξ^- . Some of these trends have also been reported in ref. [6].

In Fig. 1 we plot the pressure (P) vs. energy density (ρ) for the EOS{.48} and the EOS{.48 ϕ x}. The EOS { .48 ϕ x } is slightly stiffer at high density because of the ϕ -meson condensate. In its absence, the EOS is softened when nucleons convert to hyperons, partly because new degrees of freedom always soften an EOS, and partly because the hyperons couple more weakly than the nucleons to the repulsive vector field. With the ϕ -meson condensate the overall vector field repulsion is slightly increased. The models EOS { .48 ϕ }, EOS { .48 ϕ J }, EOS { .58 ϕ JJ }, and EOS { .58 ϕ JJx } are all nearly identical to the EOS { .48 ϕ x } model, i.e., the ϕ -hyperon coupling is more important than the ϕ -N coupling or the slight variation in the σ -couplings.

In Figure 2 we compare the particle abundances for EOS { .48 } and EOS { .48 ϕ x }. At low density neutrons are the dominant component. As the density rises, protons and hyperons become important. Above 1 to 1.5 baryons/ fm^3 , lambdas are the dominant baryon species.

In Figure 3 we display the star mass curves for EOS { .48 }, EOS { .48 ϕ } and for the EOS without any hyperons or ϕ -meson condensate, namely row 1 of Table 1. Adding the hyperons softens the EOS and reduces the maximum star mass, as stated earlier. Further addition of the ϕ -meson condensate stiffens the EOS somewhat and brings the mass curve closer to the case of no hyperons. Because the EOS { .48 ϕ }, EOS{ .48 ϕ x }, EOS { .48 ϕ J }, EOS{ .58 ϕ JJ }, and EOS{ .58 ϕ JJx } are all nearly identical on a P vs. ρ graph, they all give nearly the same star mass

curve. The particle abundances are somewhat different at high density, but this does not influence P vs. ρ and hence does not influence the star mass.

In Figure 4 we plot P vs. ρ for EOS{1}, EOS{1 ϕ } and EOS{1 ϕ J}. Recall that the 1 refers to the ratio $g_{\sigma i}/g_{\sigma N}$. Compared to the 0.48 class of EOS the effective hyperon masses are smaller. Thus the threshold for hyperon production is reduced, and this softens the EOS, as seen by comparing Figures 1 and 4. The EOS {1} is the softest. By introducing the ϕ -meson condensate, and then the coupling of nucleons to the ϕ , the EOS is stiffened. The star mass curves corresponding to these EOS are shown in Figure 5. The EOS {1} is so softened that it displays a strong first order phase transition, as is evidenced by the horizontal portion of Figs. 4 and 5 for $\rho \simeq 200 - 300 \text{ MeV}/fm^3 (3.5 - 5 \times 10^{14} \text{ g/cm}^3)$. The transition involves a change from predominantly normal nuclear matter to a state which is dominated by hyperonic (Λ) matter. The transition is removed when the ϕ condensate is introduced. Central densities are greater than in the absence of strange baryons and mesons by a factor of two to four. This has implications for the maximum Keplerian rotation frequency, to be discussed in section 6. As before EOS {1 ϕ x} is nearly identical to EOS {1 ϕ }.

In ref. [7] we found that decreasing the hyperon coupling constants substantially softened the EOS and decreased the maximum star mass. Converting nucleons to hyperons decreased the energy stored in the repulsive vector fields. This more than compensated the larger effective hyperon masses $\bar{m}_i = m_i - g_{\sigma i} \bar{\sigma}$ caused by the smaller $g_{\sigma i}$. Once ϕ -meson condensation is allowed, however, the hyperons can no longer hide from all repulsive vector fields. It is interesting to see what happens when the strength of the hyperon interactions is varied in the presence of a ϕ -meson condensate. The phase transition in Fig. 5 for EOS {1} is also found in EOS {1.187} and in fact is much stronger there. However the transition remains in this case even in the presence of the ϕ condensate. Figure 6 shows EOS {.48 ϕ }, EOS {1 ϕ } and EOS {1.187 ϕ }. The EOS softens, as the hyperon couplings are increased, at low density. The attractive energy expressed through the σ -meson condensate and through the reduction in effective masses beats the repulsive vector energies at low density. At high density the repulsive vector fields win, and the pressure is greater for the larger hyperon couplings. This behavior manifests itself in the star

mass curves, too. The EOS $\{1\phi\}$ and EOS $\{1.187\phi\}$ have lower maximum masses than EOS $\{.48\phi\}$, but they are achieved at higher central density. See Figure 7.

In concluding this section we remark that, in our opinion, the “best” EOS based on present knowledge is EOS $\{.48\phi x\}$. Some others, namely EOS $\{.48\phi\}$, EOS $\{.48\phi J\}$, EOS $\{.58\phi JJ\}$ and EOS $\{.58\phi JJx\}$ are so similar as to be nearly indistinguishable from it.

4 Quark Matter Cores

At high density we expect a transition to quark matter because hadrons are composites of quarks and gluons. The prevailing opinion is that the transition is first order. This is supported by lattice gauge simulations at finite temperature but zero chemical potential [19]. The possibility that quark matter may exist in the cores of the most dense stars, or that whole stars made of quark matter may exist, has been studied seriously by many people for about fifteen years [20-28].

Our purpose here is three-fold. First, we would like to know whether our favorite EOS $\{.48\phi x\}$, or variations of it, allows for a quark matter core in the most massive stars. Second, we would like to study the effects on a star if the quark transition is second order. To our knowledge this has not been studied before. Third, we want to reinforce the statement made in our first report [7] that some EOS may actually be inconsistent with a transition to quark matter at high density. We would consider any such hadronic EOS to be physically unacceptable.

Conclusions, such as the stability of neutron stars with quark interiors, are quite model dependent as one would expect. Depending on the specific equation of state for both the hadronic and the quark matter phases, one may find that quark-core stars may [24,27,28] or may not [21,22,25,26,28] be stable. Our approach here most closely resembles that of Serot and Uechi [28]. It differs primarily in that we use a nonlinear σ -potential which allows us to fix K and m_L to “experimental” values, and we include hyperons and the ϕ meson. As we stated above, we use the transition to test the consistency of the equation of state.

Another possibility discussed in the literature, is that of self-bound or strange

matter stars [29, 27, 30]. These objects may exist if it is assumed that quark matter is the true ground state of nuclear matter and as such persists all the way to the surface of the star, i.e., there is not a significant nuclear shell. The star is assumed to end at a density discontinuity when $P = 0$. We do not feel that this approach is thermodynamically justified as was argued in ref. [31], where a low pressure phase of “nugglets” is assumed to be present. In ref. [30] earlier it was argued that strange matter at low density should resemble a collection of closely packed Λ particles. In either case the structure of the star resembles that of the “hybrid” stars we describe. We will only comment on strange stars in our discussion of rotation.

Usually the quark matter EOS is described by perturbative QCD [10,20,21,23,27,28], and we subscribe to that approach here. It turns out that muons almost never appear in quark matter. We leave them out of our discussion, although they were included in our numerical results. We set the electron, up and down masses to zero. The strange quark mass is taken to be 180 MeV. In chemical equilibrium $\mu_d = \mu_s = \mu_u + \mu_e$. In terms of the baryon and electric charge chemical potentials, μ_B and μ_E :

$$\begin{aligned}\mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_E \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_E \\ \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_E\end{aligned}\tag{23}$$

These will be useful later. The pressure contributed by the quarks is computed to order $\alpha_s = \frac{g^2}{4\pi}$ where g is the QCD coupling constant. Since the range of chemical potentials considered here is not very great, we neglect the logarithmic running of α_s on μ . Confinement is simulated by a bag constant B . Thus

$$P = P_e + P_\mu + P_d + P_s - B\tag{24}$$

where

$$P_e = \frac{\mu_e^4}{12\pi^2},$$

$$\begin{aligned}
P_u &= \frac{\mu_u^4}{4\pi^2} \left(1 - \frac{2}{\pi} \alpha_s\right), \\
P_d &= \frac{\mu_d^4}{4\pi^2} \left(1 - \frac{2}{\pi} \alpha_s\right), \\
P_s &= \frac{1}{4\pi^2} \left[\mu_s k_s (\mu_s^2 - \frac{5}{2} m_s^2) + \frac{3}{2} m_s^4 \ell n \left(\frac{\mu_s + k_s}{m_s} \right) \right] \\
&\quad - \frac{\alpha_s}{\pi} \left[\frac{3}{2} (\mu_s k_s - m_s^2 \ell n \left(\frac{\mu_s + k_s}{m_s} \right))^2 - k_s^4 \right].
\end{aligned} \tag{25}$$

Here $k_s \equiv (\mu_s^2 - m_s^2)^{\frac{1}{2}}$ but this is not the s-quark Fermi momentum unless $\alpha_s = 0$.

There are only the two independent chemical potentials μ_B and μ_E . μ_E is adjusted so that the matter is electrically neutral, $\partial P / \partial \mu_E = 0$. The baryon density is given by $n = \partial P / \partial \mu_B$. Quark matter and hadron matter are in equilibrium, and undergo the phase transition, when the pressures and baryon chemical potentials of the two phases are equal. The stable, thermodynamically preferred, phase is the one with the greater pressure at a given baryon chemical potential.

To illustrate the nontriviality of the Gibbs matching conditions at the phase transition we provide Figures 8 and 9. Figure 8 displays P vs. μ_B for EOS $\{.48\}$ and EOS $\{.48\phi\}$, and Figure 9 displays P vs. μ_B for EOS $\{1\phi\}$. In addition, the quark pressure is plotted for $\alpha_s = 0, 0.2$ and 0.4 , but neglecting B . To obtain the full pressure in the quark phase B must be subtracted from the curves. For a given value of B , the quark and hadron pressure curves may cross once, twice, or not at all. If they never cross then, generally speaking, and for $B = 0$ or B very small, the quark pressure will always exceed the hadron pressure. Then hadron matter cannot exist thermodynamically! The preferred case is when the curves cross once, and when the quark pressure exceeds the hadron pressure at large density. If the curves cross twice, the lower crossing must be ignored as unphysical, otherwise quark matter would be stable at low and high density and hadron matter would be stable at medium density! This even places some constraint on the possible location of the phase transition. For example, in Fig. 9, if $\alpha_s = 0.4$ then the chemical potential at the transition must equal or exceed about 1600 MeV. This corresponds to a density $n_B \geq 9n_0$. However at these densities we do not find stable configurations. We further note that the absence of a ϕ condensate ($EOS\{1\}$) is not sensible even with $\alpha_s = 0.0$.

Now we investigate whether a transition to quark matter is possible in the star's core. In Figure 10(a) we fix $\alpha_s = 0$ and adjust B so that the EOS $\{.48\phi\}$ undergoes a first order transition at $n/n_0 = 1.1, 2, 3$ or 4 (density quoted refers to the hadron side, the density on the quark side is greater). In Figure 10(b) we choose $\alpha_s = 0.4$ instead. A larger value of α_s allows for a greater range of transition densities leading to a stable star with a quark core. It would seem physically sensible, and consistent with the heavy ion experiments [2], that the hadron transition density is at least $4n_0$. Then if the transition is above $4n_0$, EOS $\{.48\phi\}$ does not allow for a quark core.

The EOS $\{1\phi\}$ allows for stars with greater central densities than EOS $\{.48\phi\}$. Thus one might suspect that it is more conducive to quark matter formation. The case $\alpha_s = 0$ is shown in Figure 11. The hadron transition density here could be as high as $\sim 6n_0$ before the star becomes gravitationally unstable. Thus a quark core in stars described by EOS $\{1\phi\}$ is possible, but marginal.

How can we obtain a second order phase transition to quark matter? This is more delicate and more difficult to achieve than a first order transition, because in a second order transition not only is P continuous but so is $n_B = \partial P / \partial \mu_B$. Apart from accidental or exceptional circumstances, this will require another parameter. In ref. [32] lattice gauge simulations at finite temperature were modeled with an additional, phenomenological term in the pressure linear in T . We consider, by analogy, a phenomenological μ^2 term in the pressure of quark matter. The simplest implementation of this idea is to have massless u, d and s quarks. Then electric charge neutrality is achieved without any leptons. We parametrize quark matter as

$$P = a\mu_B^4 + b\mu_B^2 - B \quad (26)$$

where $a = \frac{1}{108\pi^2}(1 - \frac{2}{\pi}\alpha_s)$ is the same coefficient for quark matter with lowest order in α_s , QCD corrections, and b is a new phenomenological parameter. We adjust b and B to produce a second order transition to a specified hadron EOS. The pressure-energy density relation is

$$P = \frac{1}{3}(\rho - 4B) + \frac{b}{9a}\{[b^2 + 12a(\rho - B)]^{\frac{1}{2}} - b\}. \quad (27)$$

In Fig. 12 we compare first and second order transitions from EOS $\{.48\phi\}$ to quark matter with $\alpha_s = 0$. (The EOS $\{.48\phi\}$ was not consistent with a second order transition and $\alpha_s = 0.4$). Figure 12(a) has the transition out of hadron matter at $2n_0$ while Figure 12(b) has it at $4n_0$. A second order transition produces a star mass curve which is remarkably similar to the mass curve in the absence of a transition. A second order transition just softens the EOS a little and reduces the maximum mass by a small amount. A second order transition would greatly increase the likelihood of a quark matter core.

5 Gluon Condensate

The nagging problem with the approach to the quark phase transition discussed in section 4 is that two independent models are being used: a low density hadron model and high density perturbative QCD. In principle the fundamental parameters of QCD determine the parameters used in the low energy effective hadron model. In practice we do not know how to make the connection.

Here we would like to exploit the concepts of broken chiral symmetry and the anomalous trace of the energy-momentum tensor in QCD, as embodied in a low energy, effective Lagrangian [33]. In pure gluodynamics this Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - B\{4(\frac{\chi}{\chi_0})^4 \ln(\frac{\chi}{\chi_0}) - (\frac{\chi}{\chi_0})^4 + 1\} \quad (28)$$

where χ represents scalar gluonium, or the lowest mass 0^{++} glueball state. In the vacuum the lowest energy state is obtained by minimizing the potential in Eq. (28), which results in $\bar{\chi} = \chi_0$. A nonzero value of $\bar{\chi}$ corresponds to the existence of a gluon condensate. Fluctuations about this average gluon condensate are massive glueballs with mass related to B . At high temperature or density we would expect the glueball condensate to go to zero, resulting in quark-gluon matter. When $\bar{\chi} = 0$ the potential in Eq. (28) is B . This is the shift in the energy of the “perturbative” vacuum compared to “nonperturbative” vacuum. Hence B may be identified with the bag constant.

In the confined phase all hadrons are given a mass proportional, in the chiral

symmetry limit, to the gluon condensate $\bar{\chi}$ as a result of the breaking of scale invariance.

$$\bar{m}_i = m_i \left(\frac{\bar{\chi}}{\chi_0} \right) \quad (29)$$

Being a scalar field χ will provide for attraction among baryons at finite density. We make the novel and extreme assumption that all nuclear attraction is provided by the glueball condensate. Then the pressure is the same as Eq. (8) except that all masses are scaled by $\bar{\chi}/\chi_0$ and the σ -potential is replaced by the χ -potential.

$$\begin{aligned} P = & \frac{1}{2} \bar{m}_\omega^2 \bar{\omega}_0^2 + \frac{1}{2} \bar{m}_\phi^2 \bar{\phi}_0^2 + \frac{1}{2} \bar{m}_\rho^2 (\bar{\rho}_0^3)^2 \\ & - B \left\{ 4 \left(\frac{\bar{\chi}}{\chi_0} \right)^4 \ell n \left(\frac{\bar{\chi}}{\chi_0} \right) - \left(\frac{\bar{\chi}}{\chi_0} \right)^4 + 1 \right\} \\ & + \sum_i P_{FG}(\bar{m}_i, \bar{\mu}_i) + P_{FG}(m_e, \mu_e) + P_{FG}(m_\mu, \mu_\mu). \end{aligned} \quad (30)$$

This involves a reduction of parameters: $g_{\sigma i}$, b and c are gone, replaced by the bag constant B as a new parameter.

We refit all parameters to the properties of nuclear matter as given by Eqs. (15), (16) and (18). We get $B = 47.2 \text{ MeV}/f m^3 = (138 \text{ MeV})^4$. This is a rather small value, probably at the limit of acceptability. In order to make room for additional sources of nuclear attraction, such as $\sigma(\pi\pi)$ exchange, it would seem that B would have to be reduced further.

Because Eq. (30) has fewer parameters it predicts the Landau mass and compressibility at saturation density.

$$\begin{aligned} m_L &= 0.76 m_N \\ K &= 1600 \text{ MeV} \end{aligned} \quad (31)$$

The Landau mass is not too bad. The compressibility is extraordinarily large! However, if we plot the energy per baryon of isospin symmetric nuclear matter, as in Figure 13, we see that the curve is very asymmetric about the minimum. Thus K alone is not a good measure of how stiff nuclear matter is.

As a function of density, $\bar{\chi}/\chi_0$ is one at $n = 0$, it decreases smoothly to a minimum value of about 0.69 at $n = 1.55n_0$, and then increases slowly to about 1.2 at high density. We had expected $\bar{\chi}$ to decrease continuously toward zero.

For electrically neutral hadron matter we use the same hyperon coupling constants as EOS{1 ϕ x} except that χ replaces σ , of course. A numerical calculation yields the P vs. ρ curves in Figure 14. Below $2.5 \text{ GeV}/fm^3$ the glueball condensate gives an EOS which is much stiffer than a σ -meson condensate. Thus it is no surprise that the glueball condensate leads to more massive stars, with a maximum mass of $2.55 M_\odot$. This mass is achieved with a relatively low central density of $1.25 \times 10^{15} g/cm^3$. See Figure 15.

The baryon abundances are qualitatively like those of EOS {48 ϕ x}. The significant difference is that the Λ becomes the dominant species at the relatively low density of 0.25 baryons/ fm^3 .

The most disappointing aspect of the glueball condensate model is that a transition to quark matter does not occur. First of all, $\bar{\chi}$ does not go to zero at high density. Worse yet, if we interpret B in Eq. (30) as the bag constant, the same one as used in Eq. (23), then the quark and hadron pressure vs. μ_B curves never cross! The quark pressure is greater than the hadron pressure, so quark matter is stable but hadron matter is not. This is true for the values of α_s we have been using: 0, 0.2 and 0.4. If we crank-up α_s to a value near $\pi/2$ (see Eq. (25)) we can bring the quark pressure down and perhaps force a crossing, but then the order α_s correction is not small and we have no confidence in the perturbative QCD EOS.

Perhaps a mixture of σ and χ terms would give a better hadronic EOS. This seems doubtful, because the χ attraction would have to be reduced, meaning B would have to be reduced, and it is already too small. Another possibility to repair this difficulty is to vary the glue coupling to the hyperon states.

In a previous study of the effective Lagrangian approach at finite temperature [33], the gluon condensate was driven to zero by the presence of the very massive hadron states. Indeed when only the low lying meson nonet was included, no phase transition was found. It may be that at finite density, a phase transition is dependent on the presences of the more massive baryon states beyond the octet considered here.

This possibility will be pursued elsewhere [34].

6 Rotation

Finally we would like to address the question of rotation with respect to the models presented here. Our results for the mass and density of stars assume that the stars are not rotating. The exact treatment of rotating neutron stars is significantly more complicated and has been discussed in detail in ref. [35]. For our purposes it is sufficient to note that the maximum Keplerian frequency, Ω_K , is accurately (to $\simeq 10\%$) given by [36,37]

$$\Omega_K = 0.72 \times 10^4 s^{-1} (M/M_\odot)^{\frac{1}{2}} (10 km/R)^{\frac{3}{2}} \quad (32)$$

for a star of maximum mass M and corresponding radius R . It has recently been shown however that stars with $\Omega \geq 0.9\Omega_K$ develop instabilities [38].

The errant observation [39] of a pulsar in the remnant of SN1987 with a rotational frequency, $\Omega \sim 1.24 \times 10^4 s^{-1}$ led to a flurry of theoretical work concerning the viability of various equations of state [37,40-42]. The problem in obtaining a large rotation frequency has led some to speculate that perhaps this pulsar contains a quark core or that normal nuclear matter is not the true ground state but rather that strange-quark-matter is, making a strange-quark-star [27,41-44]. Nevertheless, as we find here, even in these cases $\Omega \leq 10^4 s^{-1}$ for both stars with quark cores as well as self-bound strange stars. (The exception is the paper by Glendenning [27], who finds large values of Ω for the strange stars, but only when the bag constant is chosen at a value larger than the upper limit corresponding to the assumption that quark matter is the ground state).

For completeness, we give our results for the maximum values of Ω using the models in Table 2 and a selection of these including a quark core. These results are shown in Tables 3 and 4 and graphically in Figure 16 for the EOS $\{.48\phi x\}$ (a), with a quark phase transition at $1.1 n_o$ (b), and at $3n_o$ (c) with $\alpha_s = 0.4$. All three cases correspond to a star with $M = 1.65M_0$ (the maximum when $n_c = 3n_o$).

There is a definite trend to be pointed out in Table 3. Recall that in our

Compounding the uncertainty in the nuclear equation of state, is the uncertainty in the transition to quark matter. Interestingly, we find that not all of our “reasonable” equations of state are thermodynamically consistent with such a transition. Assuming a first order transition and a perfect gas equation of state for relativistic quarks, we find that if the transition to quark matter occurs at densities $n_B \gtrsim 4n_0$ it is quite unlikely that stable stars with quark interiors exist. (A possible exception to this was EOS{1 ϕ } where stability may occur up to $\sim 6n_0$). Based on Bevalac experiments [2] it is unlikely that the transition occurs below $3 - 4n_0$. We have also considered the possibility that the transition is second order. To do so, we must include an additional phenomenological parameter. Suprisingly, the stellar mass curves closely resemble the case with no transition.

We have also considered the possibility that the attractive nuclear interaction is due to a gluon condensate. This model predicts the bag constant ($B \simeq 47 \text{ MeV}/f m^3$), the Landau mass ($m_L = .76 m_N$) and the compressibility ($K = 1600 \text{ MeV}$). With the exception of the latter, they are marginally acceptable.

Finally, we considered briefly the maximum allowed Keplerian rotation frequencies for many of our models. With few exceptions did we find rotation frequencies greater than $10^4 s^{-1}$.

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Figure Captions

Fig. 1. A comparison of EOS{.48} and EOS{.48 ϕ x}. Plotted are the curves for P vs. ρ .

Fig. 2. (a) The fraction of the total baryon density n_B is shown for n_n, n_p, n_Λ and n_Σ as a function of n_B for EOS{.48}. (b) The fraction of the total baryon density n_B is shown for $n_n, n_p, n_\Lambda, n_\Sigma$ and n_Ξ as a function of n_B for EOS{.48 ϕ x}.

Fig. 3. Stellar mass as a function of the central energy density $\rho(0)$ for EOS{.48}, EOS{.48 ϕ x} and for the equation of state for nucleons only (N).

Fig. 4. A comparison of EOS{1}, EOS{1 ϕ } and EOS{1 ϕ J}. Plotted are the curves for P vs. ρ . As one can see, EOS{1} exhibits a first order phase transition at $\rho \simeq 200\text{--}300 \text{ MeV fm}^{-3}$.

Fig. 5. Stellar mass as a function of the central energy density $\rho(0)$ for EOS{1}, EOS{1 ϕ }, EOS{1 ϕ J} and for pure nucleons (N).

Fig. 6. A comparison of EOS{.48 ϕ }, EOS{1 ϕ } and EOS{1.187 ϕ }. Plotted are the curves for P vs ρ . Note the strong first order phase transition for EOS{1.187 ϕ }. Unlike the case for EOS{1} this transition is not removed by the presence of the ϕ condensate, nor is it removed when Ξ 's were included in the EOS{1.187 ϕ x}. The transition begins below nuclear density and as such we consider this class of equations of state unphysical.

Fig. 7. Stellar mass as a function of the central energy density $\rho(0)$ for EOS{.48 ϕ }, EOS{1 ϕ }, EOS{1.187 ϕ } and for pure nucleons (N).

Fig. 8. The pressure P vs. the baryon chemical potential μ for the EOS{.48} and EOS{.48 ϕ }, for hadrons and for quark matter (with $B = 0$) for $\alpha_s = 0.0, 0.2$, and 0.4 as shown. It is clear, for example, that with $\alpha_s = 0.4$, certain values of B may be compatible with EOS{.48 ϕ } but not EOS{.48}.

Fig. 9. As in Figure 8, using EOS {1 ϕ } for hadrons.

Fig. 10.(a) Stellar mass as a function of the central energy density $\rho(0)$ for EOS{.48 ϕ } with a first order phase transition to quark matter at $1.1 n_0$ (dotted), $2n_0$ (small dashed), $3n_0$ (large dashed), and at $4n_0$ (dot-dashed) and with no transition at all (solid). The strong fine-structure constant was taken as $\alpha_s = 0.0$. (b) Same

as in (a) with $\alpha_s = 0.4$.

Fig. 11. Stellar mass as a function of the central energy density $\rho(0)$ for EOS $\{1\phi\}$ with a first order phase transition to quark matter at $2n_0$ (dashed), $3n_0$ (dot-dashed), $8n_0$ (dotted), and with no transition at all (solid). $\alpha_s = 0.0$ was chosen.

Fig. 12. (a) Stellar mass as a function of the central energy density $\rho(0)$ for EOS $\{.48\phi\}$. Shown is a comparison between a first order transition (dotted) and a second order transition (dashed) to quark matter both at $n_B = 2n_0$, with $\alpha_s = 0.0$. The solid curve assumes no transition. (b) As in (a) with $n_B = 4n_0$.

Fig. 13. The energy per baryon, E/N , as a function of the baryon density. Attractive interactions are accounted for by $\sigma(\pi\pi)$ exchange in the solid curve and by a gluon condensate χ in the dotted curve. Note that near normal nuclear matter density, the two are very similar, whereas they differ greatly at large n_B .

Fig. 14. A comparison of EOS $\{.48\phi x\}$ and the equation of state with a gluon condensate.

Fig. 15. Stellar mass as a function of the central energy density $\rho(0)$ for EOS $\{.48\phi x\}$ and the equation of state with a gluon condensate.

Fig. 16. (a) The density and mass as a function of the radius of the star for EOS $\{.48\phi x\}$. (b) As in (a) but with a transition to quark matter at $n_B = 1.1n_0$. (c) As in (b) with the transition at $n_B = 3n_0$.

Table 1: Ratios of coupling constants used in ref. [7].

$\frac{g_{\sigma\Lambda}}{g_{\sigma N}}$	$\frac{g_{\omega\Lambda}}{g_{\omega N}}$	$\frac{g_{\rho\Lambda}}{g_{\rho N}}$	$\frac{g_{\sigma\Sigma}}{g_{\sigma N}}$	$\frac{g_{\omega\Sigma}}{g_{\omega N}}$	$\frac{g_{\rho\Sigma}}{g_{\rho N}}$
1	1	0	1	1	1
0.48	0.56	0	0.48	0.56	0.56
1/3	1/3	0	1/3	1/3	1/3
0.21	0.21	0	0.21	0.21	0.21
0	0	0	0	0	0

Table 2: Ratios of coupling constants used in this work.

EOS	$\frac{g_{\sigma\Lambda}}{g_{\sigma N}}$	$\frac{g_{\omega\Lambda}}{g_{\omega N}}$	$\frac{g_{\phi\Lambda}}{g_{\omega\Lambda}}$	$\frac{g_{\sigma\Sigma}}{g_{\sigma N}}$	$\frac{g_{\phi N}}{g_{\phi\Lambda}}$	$\frac{g_{\sigma\Xi}}{g_{\sigma N}}$	$\frac{g_{\omega\Xi}}{g_{\omega N}}$	$\frac{g_{\phi\Xi}}{g_{\phi\Lambda}}$
.48	.48	.56	0	.48	0			
.48 ϕ	.48	.56	1	.48	0			
.48 ϕJ	.48	.56	1	.48	.5			
.58 ϕJJ	.58	.66	.75	.58	.2			
1.0	1.0	.66	0	1	0			
1.0 ϕ	1.0	.66	1	1	0			
1.0 ϕJ	1.0	.66	1	1	.5			
1.187	1.187	.66	0	1.275	0			
1.187 ϕ	1.187	.66	1	1.275	0			
1.187 ϕJ	1.187	.66	1	1.275	.5			
.48 ϕx	.48	.56	1	.48	0	.48	1/3	2
1.0 ϕx	1.0	.66	1	1	0	1	1/3	2
.58 ϕJJx	.58	.66	.75	.58	.2	.58	1/3	2

Table 3: Maximum star mass with corresponding radius and maximum Keplerian frequency for various EOS.

EOS	$M_{\text{max}}(M_{\odot})$	$R(\text{km})$	$\Omega(10^4\text{s}^{-1})$
.48	1.62	11.5	.74
.48 ϕ	1.81	11.6	.78
.48 ϕJ	1.86	11.4	.81
.58 ϕJJ	1.87	11.5	.80
1.0	1.40	7.5	1.3
1.0 ϕ	1.58	8.6	1.1
1.0 ϕJ	1.57	8.7	1.1
1.187 ϕ	1.63	7.9	1.3
1.187 ϕJ	1.63	7.9	1.3
1.0 ϕx	1.53	8.6	1.1

Table 4: Maximum star mass with corresponding radius and maximum Keplerian frequency for various hadronic to quark matter phase transitions, and for glue condensation.

EOS	n_c/n_0	α_s	$M_{\max}(M_{\odot})$	$R(\text{km})$	$\Omega(10^4\text{s}^{-1})$
.48	2	0	1.39	9.0	.99
	3	0	1.49	10.0	.88
	2	0.2	1.59	9.6	.96
	3	0.2	1.56	9.9	.91
.48 ϕ	1.1	0	1.52	8.9	1.1
	1.1	.4	1.68	9.8	.96
	2	0	1.43	9.4	.94
	2	.4	1.61	10.	.91
	2(2nd)	0	1.75	11.6	.76
	3	0.4	1.65	11.4	.76
	4	0	1.74	12.4	.69
	4(2nd)	0	1.79	11.9	.74
1.0 ϕ	2	0	1.48	9.0	1.0
	3	0	1.46	9.3	.97
	4	0	1.46	9.2	.99
	8	0	1.52	9.3	.99
glue			2.55	14.8	.64

Fig. 1

Fig. 2(a)

Fig. 2(b)

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10(a)

Fig. 10(b)

Fig. 11

Fig. 12(a)

Fig. 12(b)

Fig. 13

Fig. 14

Fig. 15

Fig. 16(a)

Fig. 16(b)

Fig. 16(c)