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LEFT-HANDED CURRENTS AND CP VIOLATION

John Ellis, Mary K. Gaillard ^{*)} and D.V. Nanopoulos ⁺⁾

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A B S T R A C T

An analysis is presented of a model for CP violation due to Kobayashi and Maskawa, which has six quarks and purely left-handed currents. The model can reproduce approximately results of the Glashow, Iliopoulos and Maiani four-quark model in CP conserving processes. It gives a close approximation to superweak predictions for CP violation in common K decays, whatever the masses of the fifth and sixth quarks. In principle there are substantial deviations from superweak predictions in rare K decays, but these seem difficult to observe in practice. There are also deviations from superweak results for CP violation in charmed particle decays. The neutron electric dipole moment is very sensitive to the masses of the b and t quarks, lying in a range containing the superweak prediction.

^{*)} And Laboratoire de Physique Théorique et Particules Élémentaires, laboratoire associé au C.N.R.S.

⁺⁾ Present address : Service de Physique Théorique, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05.

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1. INTRODUCTION

Shortly after the discovery of the new particles, several authors^{1,2)} discussed models with three charge $\frac{2}{3}$ quarks and three charge $-\frac{1}{3}$ quarks, and proposed that more than one of these new quarks might be jointly responsible for the happenings in e^+e^- annihilation and elsewhere. More recently, phenomenological analyses have tended to favour the excitation of just one new quark degree of freedom in present data³⁾. However, interest in six quark models has been maintained by the possibility that the $e^+\mu^+$ events discovered⁴⁾ at SPEAR may come from the decays of a new heavy lepton with its own new neutrino. If this were true, it would be natural to restore some lepton-hadron symmetry by increasing the number of (coloured) quarks also to six. This idea has a calculational basis, in that unless the new leptons and neutrinos (surprisingly) have pure V or pure A couplings, then the 4-quark model's delicate cancellation⁵⁾ of triangle anomalies will be destroyed. With it would go the renormalizability of the theory. The anomaly cancellation is restored in the usual Weinberg-Salam⁶⁾ $SU(2) \times U(1)$ model if there are two more quarks t and b with charges $\frac{2}{3}$ and $-\frac{1}{3}$ respectively. They and the three conventional quarks u, d and s and the charmed quark c would together form three doublets under the weak $SU(2)$ group. The anomaly cancellation is probably only a shallow reflection of a more profound reason for lepton-quark symmetry. But why so many quarks or leptons in the first place?

A possible reason to introduce six quarks was pointed out some time ago by Kobayashi and Maskawa⁷⁾. They observed that in such a model the unitary matrix representing charged weak currents has one phase parameter in addition to the familiar real angles, such as the Cabibbo angle, that describe quark mixing. This extra phase parameter introduces CP violating effects, and links the problems of the origin of the Cabibbo angle and of CP violation.

The Kobayashi-Maskawa model has recently been revived by Pakvasa and Sugawara⁸⁾, and independently by Maiani⁹⁾. Both papers point out that under suitable conditions the model reduces approximately to the superweak¹⁰⁾ form of CP violation for strange particle decays. Pakvasa and Sugawara⁸⁾ also propose a model for the new particles involving all three new quarks, whereas Maiani⁹⁾ assumes no degeneracy between quark masses.

In this paper we discuss systematically the possible patterns of CP violation in the Kobayashi-Maskawa⁷⁾ model. We analyse the validity of the superweak approximation in ordinary K decays, rare K decays and charmed particle decays. We also estimate the neutron electric dipole moment, finding it to be

$$\left| \frac{D}{e} \right|_n \sim \left\{ \begin{array}{l} (10^{-27} - 10^{-28}) \text{ cm} \\ (10^{-30} - 10^{-31}) \text{ cm} \end{array} \right\} \text{ for } m_{t,b} \sim \left\{ \begin{array}{l} m_w \\ 5 \text{ GeV} \end{array} \right.$$

As suggested by Pakvasa and Sugawara, superweak results for common K decays can be approximately recovered if the t and c quarks are almost degenerate, but we disfavour this possibility on phenomenological grounds as mentioned below. We in fact find that approximate superweak results obtain for common K decays independently of the masses of the t and b quarks. The model predicts large deviations from superweak theory in rare K decays, and also has deviations from superweak predictions in charmed particles, but all these seem difficult to observe.

Our calculations are collected in Section 2. The model is introduced in Section 2.1, and applied to common K decays in Sections 2.2, 2.3 and 2.4. Section 2.5 contains a discussion of rare K decays, charmed meson decays are studied in Section 2.6, and the neutron electric dipole moment estimated in Section 2.7. Our conclusions are summarized and discussed in Section 3.

2. CALCULATIONS

2.1 The model

Let us consider models with N quarks of charge $\frac{2}{3}$ and N of charge $-\frac{1}{3}$ *, and suppose their weak and electromagnetic interactions are described by the $SU(2) \times U(1)$ Weinberg-Salam model, with the "left" components of the quark fields collected into N doublets under the weak $SU(2)$ and the "right" components left as singlets. The (V-A) charged weak current is then described by an $N \times N$ unitary matrix U:

$$J_{\mu}^{-} = \bar{q}_p \gamma_{\mu} L U q_n \quad \text{where } L \equiv \frac{1}{2}(1 - \gamma_5) \quad (2.1)$$

Glashow, Iliopoulos and Maiani¹¹⁾ pointed out that in such a model the neutral weak current is purely diagonal in quark fields, being a mixture of the electromagnetic current and

$$J_{\mu}^0 = \bar{q}_p \gamma_{\mu} L U^{\dagger} U q_p + \bar{q}_n \gamma_{\mu} L U^{\dagger} U q_n \quad (2.2)$$

In any such model $\Delta S = 1$ neutral currents and $\Delta S = 2$ transitions are very strongly suppressed. In particular free quark model calculations of diagrams like those of Fig. 1 in the case $N = 2$ (with just X = u, c quarks exchanged) led to

$$\Delta m \equiv m_{K_L} - m_{K_S} \approx \frac{G_F f_K^2}{\sqrt{2}} \frac{\alpha}{4\pi} \left(\frac{m_c}{3.8 \text{ GeV}} \right)^2 \sin^2 \theta_c \cos^2 \theta_c \quad (2.3)$$

*) Each type of quark comes in three colours, but the weak and electromagnetic interactions are presumed to be colour-blind.

The experimental value of Δm implies¹²⁾ $m_c = 0(1.5 \text{ to } 2) \text{ GeV}$, an estimate not greatly altered by asymptotic freedom effects¹³⁾.

The unitary matrix U is generally characterized by N^2 parameters, 1 of which is an unobservable over-all phase, and $2(N-1)$ of which can be removed by suitable choices of phases for the quark fields. The matrix U therefore has $(N-1)^2$ observable parameters. In the case $N = 2$, the $SU(4)$ model of Glashow, Iliopoulos and Maiani¹¹⁾, there is just one free parameter and U can be taken as an orthogonal matrix

$$J_\mu^- = (\bar{u}, \bar{c}) \gamma_\mu L \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad (2.4)$$

with θ_c the Cabibbo angle. In the case $N = 3$, U has 4 observable parameters, only 3 of which can be absorbed into the 3 angles parametrizing 3-dimensional orthogonal matrices^{*)}. As pointed out originally by Kobayashi and Maskawa⁷⁾, there is another parameter which can be taken as a complex phase, and which introduces CP violation. Introducing new "top" t ($Q = 2/3$) and "bottom" b ($Q = -1/3$) quarks, and parametrizing U as done by Kobayashi and Maskawa, we have:

$$J_\mu^- = (\bar{u}, \bar{c}, \bar{t}) \gamma_\mu L \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_3 - s_2 s_3 e^{i\delta} & c_1 c_3 + s_2 s_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2.5)$$

where $c_i(s_i) \equiv \cos \theta_i$ ($\sin \theta_i$), $i = 1, 2, 3$.

The model (2.5) reduces to the Glashow-Iliopoulos-Maiani model (2.4) in the limit $\theta_1 \rightarrow \theta_c$, $\sin \theta_2, \sin \theta_3 \rightarrow 0$, $\delta \rightarrow 0$, and there is nothing to prevent the Kobayashi-Maskawa (KM) model from containing the Glashow Iliopoulos and Maiani (GIM) model as a good approximation to the weak interactions of the (u,d,s,c) sector.

Inspection of the weak current (2.5) suggests that the experimental validity of Cabibbo theory for u, d and s quarks would restrict possible values of the

*) In general an $N \times N$ orthogonal matrix is determined by $[N(N-1)]/2$ real parameters leaving $[(N-1)(N-2)]/2$ complex parameters in the general case. The 6-quark model is unique in that there is only one CP violating parameter and our subsequent analysis is based on this assumption. However, we expect it to be approximately valid even if there are more quarks beyond t and b, just as previous estimates¹²⁾ of m_c are not altered by going to 6 quarks.

new angle parameters. The best estimate of the relative magnitudes of $\Delta S = 0$ and $\Delta S = 1$ couplings seems to come¹⁴⁾ from semileptonic decays of baryons, and yields $\sin \theta_c = 0.230 \pm 0.003$. From a recent compilation of Jarlskog¹⁵⁾ we can estimate $\cos^2 \theta_c$, by comparing nuclear β decay with μ decay. We find $\cos^2 \theta_c = 0.9481 \pm 0.004$, where the dominant errors come from differing estimates of the nuclear quantity Ft , and from uncertainties in the (in principle calculable) higher order weak and electromagnetic corrections at the strong decay vertex. We then find $\cos^2 \theta_c + \sin^2 \theta_c = 1.001 \pm 0.004$ experimentally. Allowing one standard deviation, this implies that in the KM model (2.5) there must be small "leakage" of the weak coupling of the u quark to the b quark: $s_1^2 s_3^2 < 0.003$. Since $s_1^2 \approx \sin^2 \theta_c \approx 0.05$, this means

$$s_3^2 < 0.06 \quad (2.6)$$

so that $|s_3|$ cannot be much bigger than $\sin \theta_c$.

Even in the continued absence of charmed particles, there are some more restrictions on s_2 and c_2 . Consider first the proposal that just one new quark c has a mass 0(1.5 to 2) GeV. The experimental absence of visible tracks of charmed particles suggest a lifetime not much more than 10^{-12} sec. As particles with masses ~ 2 GeV and unit weak couplings to light quarks probably have lifetimes $0(10^{-13})$ sec, and since the $\Delta C = 1$, $\Delta S = 0$ coupling in (2.5) is limited in magnitude to $0(\sin^2 \theta_c) \approx 0.05$, this means charm decays to states containing strange quarks must dominate, so that c_2^2 cannot be too small. Consider now the possibility that both c and t have masses 0(1.5 to 2) GeV: then by the previous lifetime argument both c_2^2 and s_2^2 must be $\geq 1/10$, and strange quarks must dominate the decays both of naked charm and of naked top states. The possibility that b has a low mass 0(1.5 to 2) GeV seems untenable, because its decays to a u quark are suppressed by $s_1^2 s_3^2 < 0.003$, and decays to states containing c or t quarks would be strongly suppressed by phase space, though in principle the couplings could be of magnitude ~ 1 . We cannot rule out a low top mass on the same grounds, but the observed spectroscopy of new states at SLAC and DESY³⁾ disfavors this possibility.

It is clear that CP violation can be introduced *a fortiori* into models¹⁶⁾ with right-handed currents also. We dislike such models purely for aesthetic reasons. First because there are no direct traces of right-handed couplings between u, d and s quarks, and there are theoretical problems¹⁷⁾ with a right-handed coupling between the c and d quarks. Secondly because (e- μ) universality is experimentally very good¹⁸⁾ ($g_e/g_\mu = 1.007 \pm 0.0014$), and this is natural only if any new heavy lepton has a left-handed coupling to its own massless neutrino¹⁹⁾. The most natural way to cancel triangle anomalies and restore renormalizability

is to introduce a third left-handed quark doublet. The Higgs content is more complicated in theories with right-handed currents as well as left. In particular there are several charged and neutral physical Higgs bosons^{*)}, and one is quite bad enough. As we show in the Appendix, CP violation can be incorporated in a left-handed current model using just the original complex doublet of Higgs fields, and hence just one physical Higgs particle.

The KM model has recently been revived by Pakvasa and Sugawara⁸⁾, and independently by Maiani⁹⁾, who presented interesting estimates of CP violation in K decays and of the neutron electric dipole moment. We now proceed to our calculations of these and other CP violating quantities.

2.2 CP violation in the neutral K mass matrix

It is apparent from Fig. 1 that the effective higher order $\Delta S = 2$ transition between K^0 and $\bar{K}^0$ introduces CP violation when $X = c$ or t . Indeed these diagrams give the superweak¹⁰⁾ state mixing which experimentally¹⁴⁾ is the dominant source of CP violation in neutral K decays. The analysis of Appendix F of Ref. 12 tells us immediately that in the six quark model the effective $\Delta S = 2$ operator is proportional to an integral of the form

$$\int \frac{d^4 k}{(2\pi)^4} \left\{ \frac{\gamma^\alpha \not{k} \gamma^\beta L}{k^2 - m_W^2} \right\} \left\{ \frac{\gamma_\beta \not{k} \gamma_\alpha L}{k^2 - m_W^2} \right\} \left\{ -\frac{C_1 S_1 C_3}{k^2 - m_u^2} + \right. \\ \left. + \frac{S_1 C_2 (C_1 C_2 C_3 - S_2 S_3 e^{i\delta})}{k^2 - m_c^2} + \frac{S_1 S_2 (C_1 S_2 C_3 + C_2 S_3 e^{i\delta})}{k^2 - m_t^2} \right\}^2 \quad (2.7)$$

The integral (2.7) is safely convergent even in the absence of the W propagators, so we assume $m_{u,c,t}^2 \ll m_W^2$ and set the W propagators $\approx 1/m_W^2$. We will also assume $m_{c,t}^2 \gg m_u^2$ and set $m_u^2 \approx 0$. The pole terms in the integrand can then be written as

$$\left\{ \frac{S_1 C_1 C_3}{k^2} \left[\frac{C_2^2 m_c^2}{k^2 - m_c^2} + \frac{S_2^2 m_t^2}{k^2 - m_t^2} \right] + \frac{S_1 S_2 C_2 S_3 e^{i\delta} (m_t^2 - m_c^2)}{(k^2 - m_c^2)(k^2 - m_t^2)} \right\}^2 \quad (2.8)$$

Then the ratio $\text{Im } M_{12}/\Delta m$ of the CP violating and CP conserving $\Delta S = 2$ terms in the mass matrix is just the ratio of the integral

$$2 \int d^4 k \left\{ \frac{S_1 S_2 C_2 S_3 \sin \delta (m_t^2 - m_c^2)}{(k^2 - m_c^2)(k^2 - m_t^2)} \right\} \left\{ \frac{S_1 C_1 C_3 C_2^2 m_c^2}{k^2 - m_c^2} + \frac{S_1 C_1 C_3 S_2^2 m_t^2}{k^2 - m_t^2} + \right. \\ \left. + \frac{k^2 S_1 S_2 C_2 S_3 \cos \delta (m_t^2 - m_c^2)}{(k^2 - m_c^2)(k^2 - m_t^2)} \right\} \quad (2.9)$$

*) J. Prentki, private communication.

and the integral

$$\int d^4k \left\{ \frac{S_1^2 C_1^2 C_3^2}{k^2} \left[\frac{C_2^2 m_c^2}{k^2 - m_c^2} + \frac{S_2^2 m_t^2}{k^2 - m_t^2} \right]^2 + \frac{2S_1^2 C_1 S_2 C_2 S_3 C_3 \cos \delta (m_t^2 - m_c^2)}{(k^2 - m_c^2)(k^2 - m_t^2)} \times \right. \\ \left. \times \left[\frac{C_2^2 m_c^2}{k^2 - m_c^2} + \frac{S_2^2 m_t^2}{k^2 - m_t^2} \right] + \frac{k^2 S_1^2 S_2^2 C_2^2 S_3^2 \cos 2\delta (m_t^2 - m_c^2)^2}{(k^2 - m_c^2)^2 (k^2 - m_t^2)^2} \right\} \quad (2.10)$$

Thus we find

$$\left| \frac{\text{Im } M_{12}}{\Delta m} \right| \approx \frac{2S_1^2 C_1 S_2 C_2 S_3 C_3 \sin \delta (m_t^2 - m_c^2) \left\{ C_2^2 m_c^2 \left[\frac{m_t^2}{(m_t^2 - m_c^2)^2} \ln \left(\frac{m_t^2}{m_c^2} \right) - \frac{1}{m_t^2 - m_c^2} \right] + \right. \\ \left. + S_2^2 m_t^2 \left[\frac{m_c^2}{(m_t^2 - m_c^2)^2} \ln \left(\frac{m_c^2}{m_t^2} \right) + \frac{1}{m_t^2 - m_c^2} \right] \right\}}{S_1^2 C_1^2 C_3^2 \left[C_2^4 m_c^2 + S_2^4 m_t^2 + \frac{2S_2^2 C_2^2 m_t^2 m_c^2}{(m_t^2 - m_c^2)} \ln \left(\frac{m_t^2}{m_c^2} \right) \right]} \quad (2.11)$$

where we have dropped from both the numerator and denominator terms which are relatively small if $s_3^2 \ll c_3^2$ as argued in Section 2.1.

We will return to the analysis of (2.11) in Section 2.4: for now we just make two observations:

- i) The observed magnitude of CP violation effects in the neutral K mass matrix clearly implies $|s_2 s_3 \sin \delta| = 0(10^{-3})$. We leave the derivation of this very small number in a natural way as an exercise for ourselves and our readers.
- ii) The formulae for $|\text{Im } m_{12}/\Delta m| \propto (m_t - m_c)$ quoted in references 8 and 9 are good approximations to (2.11) only when $|m_t - m_c| \ll m_c$.

2.3 Direct CP violating decays of K mesons

Lowest order weak decay amplitudes for K^0 mesons will violate CP when they involve charmed or top quarks. It is convenient to introduce the idea of "top-spin" which interchanges t and c quarks. It is easy to see from (2.5) that the effective lowest order four-quark CP violating operator is a top-spin vector:

$$d\ell_{\text{eff}}^{\text{CPX}} = \frac{G_F}{\sqrt{2}} \bar{s} \gamma_\mu L (t\bar{t} - c\bar{c}) \gamma^\mu L d \quad (2.12)$$

and so its matrix elements between topless^{*)} and charmless states would vanish in a top-spin invariant world. In fact, top-spin is broken by the top and charm quark mass difference, and matrix elements of (2.12) are in general non-zero. We analyse the strength of this CP violation in two ways.

Feynman diagrams

In this traditional technique, one inspects all Feynman diagrams (Fig. 2) which take the initial valence quarks to the final ones via $\mathcal{H}_{\text{eff}}^{\text{CPX}}$ including all possible gluon exchanges^{**)}. The CP violating contribution of Fig. 2a in the 't Hooft-Feynman²⁰⁾ gauge is given by

$$\begin{aligned} & S_1 S_2 S_3 C_2 \sin \delta g^2 \int \frac{d^4 p}{(2\pi)^4} \frac{\bar{s}_\mu L (\not{p} + m_x) \gamma^\mu L}{(p^2 - m_x^2)((p-q)^2 - m_w^2)} \\ &= S_1 S_2 S_3 C_2 \sin \delta g^2 \int \frac{d^4 p}{(2\pi)^4} \frac{-2 \not{p} L}{(p^2 - m_x^2)((p-q)^2 - m_w^2)} \end{aligned}$$

Its contribution to CP violation is the difference between c and t exchanges:

$$S_1 S_2 S_3 C_2 \sin \delta \int \frac{d^4 p}{(2\pi)^4} \frac{-2 \not{p} L (m_c^2 - m_t^2)}{((p-q)^2 - m_w^2)(p^2 - m_c^2)(p^2 - m_t^2)}$$

which is finite and proportional to^{***)}

$$\frac{G_F}{\pi^2} S_1 S_2 S_3 C_2 \sin \delta \not{q} L (m_t^2 - m_c^2) \left(1 + O\left(\frac{q^2, m_s^2}{m_w^2}\right) \right) \quad (2.13)$$

The first term in (2.13) is absorbed in the quark wave function renormalization, so that the final contribution of Fig. 2a to the CP violating amplitude is

$$\frac{G_F}{\pi^2} S_1 S_2 S_3 C_2 \sin \delta m_s (m_t^2 - m_c^2) \times O\left(\frac{q^2, m_s^2}{m_w^2}\right) \quad (2.14)$$

*) Ordinary hadrons are assumed to be topless and bottomless, which is presumably why their degrees of freedom are excited at low energies.

**) Graphs with gluons emitted and absorbed by the same fermion line can be discussed similarly.

***) In this and forthcoming formulae, m_s should probably be interpreted as $O(M_K)$.

The Higgs graph of Fig. 2b gives

$$\begin{aligned}
 & S_1 S_2 S_3 C_2 \sin \delta g^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{m_w^2} \frac{(m_d L - m_x R)(\not{p} + m_x)(m_s R - m_x L)}{((p-q)^2 - m_w^2)(p^2 - m_x^2)} \\
 &= S_1 S_2 S_3 C_2 \sin \delta \frac{g^2}{m_w^2} \int \frac{d^4 p}{(2\pi)^4} \times \\
 & \quad \times \frac{\not{p}(m_x^2 L - m_s m_d R) - m_x^2 (m_s R + m_d L)}{((p-q)^2 - m_w^2)(p^2 - m_x^2)} \quad (2.15)
 \end{aligned}$$

After taking the difference of graphs with $X = c, t$, the term in (2.15) proportional to $m_s m_d R$ is finite and gives to the CP violating amplitude a contribution

$$\frac{G_F}{\pi^2} S_1 S_2 S_3 C_2 \sin \delta \times O\left(\frac{m_s^2 m_d (m_t^2 - m_c^2)}{m_w^2}\right) \quad (2.16)$$

The terms in (2.15) proportional to $m_x^2 [\not{p}L - (m_s R + m_d L)]$ have logarithmic divergences which are absorbed into the wave function and mass renormalizations, yielding a finite part of the order of magnitude (2.14). The top half of Fig. 2c is

$$S_1 S_2 S_3 C_2 \sin \delta g^2 g_{\text{strong}} \int \frac{d^4 p}{(2\pi)^4} \frac{\gamma_\mu L (\not{p} + m_x) \gamma_\alpha (\not{p}' + m_x) \gamma^\mu L}{(p^2 - m_x^2)(p'^2 - m_x^2)((p-q)^2 - m_w^2)}$$

The leading part of the integrand is $O(1/p^4)$, which becomes $(m_t^2 - m_c^2)/p^6$ after subtraction, making the integral finite. As in the analysis of Fig. 2a the leading term is absorbed, this time in a coupling constant renormalization, and the contribution of the remainder is again like (2.14).

Turning now to Fig. 2d, we follow intuition based on the $K_L \rightarrow \gamma\gamma$ analysis of Ref. 12. When X is a u quark, the quark mass and the digluon mass are similar, and we expect the graph to be given in order of magnitude by the triangle anomaly. When X is a heavy c or t quark, the anomaly contribution is expected to be cancelled as $\beta \equiv m_X^2/m_{\text{digluon}}^2 \rightarrow \infty$ like $F(\beta)$ in Fig. 8 of Ref. 12. We therefore expect a very small contribution to the amplitude, suppressed relative to the u quark diagram by $O(m_u^2/m_c^2 - m_u^2/m_t^2)^*$. As for Fig. 2e, our intuition suggests a suppression by a factor $\sim 1/m_X^2$ for each heavy quark propagator, so that the sum of such

*) In this formula m_u should probably be interpreted as a typical non-strange meson mass.

diagrams should be suppressed relative to the u quark piece by

$$\left(\frac{m_u^2}{m_c^2, m_t^2} \right)^\alpha$$

with α an integer ≥ 2 . This is consistent with top-spin, which says that the CP violating amplitude should vanish as $m_c \rightarrow m_t$, and suggests a contribution of the graphs of Fig. 2e which is

$$S_1 S_2 S_3 C_2 \sin \delta G_F \times O \left((m_K)^{1+\alpha} \left(\frac{1}{m_c^\alpha} - \frac{1}{m_t^\alpha} \right) \right) : \alpha \text{ integer } \geq 2 \quad (2.17)$$

Finally we turn to the simplest graph of Fig. 2f. As observed by Pakvasa and Sugawara⁸⁾, its contribution to the amplitude is suppressed by Zweig's rule and we may take its ratio to the allowed diagram with a $(\bar{u}u)$ pair in the final state to be^{*)}

$$\propto \left(Z_c^S - Z_t^S \right) \frac{S_2 C_2 t_3 \sin \delta}{C_1} \quad (2.18)$$

where $(Z_{c,t}^S)^2$ is the coupling for an $I = 0, J^P = 0^+ \pi\pi$ state with a mass m_K to a $(\bar{c}c)$, $(\bar{t}t)$ pair. The experimental suppression of the cascade decay $\psi'(3700) \rightarrow J/\psi(3100) + \pi\pi$ relative to the allowed decay $\rho'(1600) \rightarrow \rho(770) + \pi\pi$, where the $\pi\pi$ system has similar isospin, spin and mass, suggests

$$|Z_c^S|^2 = O(10^{-2}) \quad \text{or} \quad Z_c^S = O(10) \quad (2.19)$$

We cannot estimate Z_t^S very well, except to believe that as $m_t > m_c$, probably $Z_c^S > Z_t^S$.

Wilson expansion

More modern²¹⁾ than the Feynman graph analysis just described is to expand the effective CP violating Hamiltonian as a series in local operators of increasing dimension:

*) Hard gluon exchange in Fig. 2f is expected to give a short distance enhancement. This may be less²²⁾ than the similar enhancement for the $(\bar{u}u)$ diagram²³⁾, but the order of magnitude should be similar.

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{CPX}} = G_F \left\{ (\bar{s}d) F_1 + (\bar{s} \not{D} d) F_2 + (\bar{s} D^2 d) F_3 + \right. \\ \left. + \bar{s} \gamma_\mu L (c \bar{e} - t \bar{\tau}) \gamma^\mu L d F_4 \right. \end{aligned} \quad (2.20)$$

+ operators of higher dimension

where $D_\mu \equiv p_\mu + ig B_\mu$, B_μ a gluon field. Dimensional analysis indicates that

$$F_1 = O(m_Q m_W^2), F_2 = O(m_W^2), F_3 = O(m_Q), F_4 = O(1) \quad (2.21)$$

and higher order terms are suppressed by powers of $1/m_W$. The first two terms of (2.20) are absorbed into mass, wave function and coupling constant renormalizations. The third operator is not a top-spin vector, whereas the underlying interaction was. Therefore the coefficient function $F_3 = 0[m_s(m_t^2 - m_c^2)/m_W^2]$. The fourth term is a top-spin vector operator whose matrix elements are given by diagrams like those of Figs. 2c, d, e and f discussed above^{*)}. We expect its matrix element to be proportional to some factor $(X_c - X_t)$ which vanishes as $m_t - m_c \rightarrow 0$, and from our Feynman graph discussion may either behave as

$$\left(\frac{m_K}{m_c} \right)^\alpha - \left(\frac{m_K}{m_t} \right)^\alpha$$

for some $\alpha > 1$, or like the Zweig factors $(Z_c^S - Z_t^S)$ of (2.18), or both.

2.4 CP violation in neutral K decays

We discuss CP violation in the $K^0 - \bar{K}^0$ system using the standard definitions and conventions set out by Kleinknecht¹⁴⁾. There is the mixing parameter

$$\epsilon \equiv \frac{\text{Im } \Gamma_{12}/2 + i \text{Im } M_{12}}{i(\Gamma_S - \Gamma_L)/2 - \Delta m} \quad (2.22)$$

where $\Gamma_{12} = 2\pi \sum_F \rho_F \langle \bar{K}^0 | \mathcal{H}_{wk} | F \rangle \langle F | \mathcal{H}_{wk} | K^0 \rangle$, $\Gamma_S - \Gamma_L$ is the difference between the K_S and K_L lifetimes, and we estimated $\text{Im } M_{12}$ and Δm in Section 2.2. Choosing

*) Note that multigluon operators do not appear in the series (2.20): their coefficients F_i would be suppressed by powers of m_W^{-1} . However, soft gluons in the hadronic wave functions are relevant in taking matrix elements of local operators.

a phase convention so that the $K \rightarrow 2\pi$ ($I = 0$) amplitude is real apart from the strong interaction phase shift δ_0 :

$$\langle \pi\pi (I=0) | K^0 \rangle = e^{i\delta_0} A_0 : A_0 \text{ real} \quad (2.23a)$$

then

$$\langle \pi\pi (I=2) | K^0 \rangle = e^{i\delta_2} A_2 \quad (2.23b)$$

and the standard CP violation parameters η_{+-} and η_{00} obey

$$\left. \begin{aligned} \eta_{+-} &= \epsilon + \epsilon' \\ \eta_{00} &= \epsilon - 2\epsilon' \end{aligned} \right\} \quad (2.24)$$

where

$$\epsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \frac{\text{Im } A_2}{A_0} \quad (2.25)$$

From Eq. (2.22) it follows that

$$\text{Arg } \epsilon = \tan^{-1} \left(\frac{2\Delta m}{\Gamma_S - \Gamma_L} \right) + \phi_D$$

where

$$\phi_D = -\tan^{-1} \left(\frac{\text{Im } \Gamma_{12}}{2 \text{Im } M_{12}} \right) \quad (2.26)$$

In the superweak theory where all CP violation arises from the $K^0 - \bar{K}^0$ mixing matrix we would have $\epsilon' = 0 = \phi_D$. In milliweak theories ϵ' arises from the CP violating phase difference ξ between $\langle \pi\pi (I=0) | K^0 \rangle$ and $\langle \pi\pi (I=2) | K^0 \rangle$, while ϕ_D comes from the CP violating phase difference between $\langle \pi\pi (I=0) | K^0 \rangle$ and the amplitudes for all other ($K^0, \bar{K}^0$) decays.

Examining the KM model, we first observe that the successful estimate¹²⁾ of m_c from Δm in the four quark model¹¹⁾ implies that the denominator of (2.11) $\approx s_1^2 m_c^2$. We therefore have

$$\frac{\text{Im } M_{12}}{\Delta m} \approx 2 c_1 s_2 c_2 s_3 c_3 \sin \delta \left[\cos 2\theta_2 \frac{m_t^2}{m_t^2 - m_c^2} \ln \left(\frac{m_t^2}{m_c^2} \right) + s_2^2 \left(\frac{m_t^2}{m_c^2} \right) - c_2^2 \right] \quad (2.27)$$

By comparison, the CP violating phase difference ξ can be obtained from the estimates of Section 2.3. From Figs. 2a, b, c we use (2.14) and the approximation

$$\langle \pi\pi (\Sigma=0) | K^0 \rangle \approx S_1 C_1 C_2 G_F M_K^3 \quad (2.28)$$

to estimate

$$|\bar{3}| \approx \left| \frac{S_2 C_2 t_3 \sin \delta}{C_1} \right| \frac{(M_t^2 - m_c^2)}{\pi^2 M_W^2} \quad (2.29a)$$

From Figs. 2d, e we get

$$|\bar{3}| \approx \left| \frac{S_2 C_2 t_3 \sin \delta}{C_1} \right| \left(\frac{M_u^2}{m_c^2} - \frac{M_u^2}{M_t^2} \right) \quad (2.29b)$$

where m_u probably ≤ 100 MeV. Finally from Fig. 2f we get

$$|\bar{3}| \approx \left| \frac{S_2 C_2 t_3 \sin \delta}{C_1} \right| (Z_c^S - Z_t^S) \quad (2.29c)$$

where $Z_c^S \sim 1/10 \gtrsim Z_t^S$. Comparing the different estimates (2.29) we see that for $m_t^2 \ll m_W^2$ the largest is (2.29c), while (2.29a) may become competitive only for $m_t^2 \approx m_W^2$. Comparing (2.27) and (2.29c) we see that CP violation in the mass mixing matrix is dominant whatever the value of m_t . For example, for $m_t - m_c \ll m_c$ Eq. (2.27) becomes

$$\left| \frac{\text{Im } M_{12}}{\Delta m} \right| \approx 2 \left| C_1 C_2 C_3 S_2 S_3 \sin \delta \right| \left(\frac{m_t - m_c}{m_c} \right) \quad (2.30)$$

which reproduces the results of Pakvasa and Sugawara⁸⁾ and of Maiani⁹⁾. If we suppose that

$$|Z_c^S - Z_t^S| \approx \frac{1}{10} \left(\frac{m_t - m_c}{m_c} \right) \quad (2.31)$$

for $m_t - m_c \ll m_c$ then we get

$$|\xi| \approx \frac{1}{10} \left| \frac{c_2 s_2 s_3 \sin \delta}{c_1 c_3} \right| \left(\frac{m_t - m_c}{m_c} \right) \quad (2.32)$$

and we know from the Section 2.1 that $|c_1| \approx |c_3| \approx 1$ so that $\left| \frac{\text{Im } M_{12}}{\Delta m} \right| \gg |\xi|$.
For $m_t \gg m_c$,

$$\left| \frac{\text{Im } M_{12}}{\Delta m} \right| \approx 2 \left| c_1 c_2 c_3 s_2 s_3 \sin \delta \right| \left[c_2^2 \left(\ln \left(\frac{m_t^2}{m_c^2} \right) - 1 \right) + s_2^2 \frac{m_t^2}{m_c^2} \right] \quad (2.33)$$

and whether we take

$$\frac{Z_t^S}{Z_c^S} \approx 0 \quad \text{or} \quad \frac{1}{\ln \left(\frac{m_t^2}{m_c^2} \right)} \quad (2.34)$$

which seem to bracket the range of possibilities, it is clear that for $|c_2| = 0(1)$, again $\left| \frac{\text{Im } M_{12}}{\Delta m} \right| \gg |\xi|$.

The KM model therefore approximates superweak results whatever the value of m_t . In fact we have

$$\begin{aligned} |\epsilon'| &= \frac{1}{\sqrt{2}} \left| \frac{\text{Im } A_2}{A_0} \right| \approx \frac{1}{\sqrt{2}} |\xi| \left| \frac{A_2}{A_0} \right| \\ &\approx \frac{1}{30} |\xi| \end{aligned} \quad (2.35)$$

whereas

$$\begin{aligned} |\phi_D| &\approx \frac{|\xi|}{2} \left[\frac{\Gamma(K^0 \rightarrow 2\pi(I=2)) + \Gamma(K^0 \rightarrow 3\pi)}{\Gamma(K^0 \rightarrow 2\pi(I=0))} \right] / \left| \frac{\text{Im } M_{12}}{\Delta m} \right| \\ &\approx 2 \times 10^{-3} |\xi| / \left| \frac{\text{Im } M_{12}}{\Delta m} \right| \end{aligned} \quad (2.36)$$

and

$$|\epsilon| \approx \frac{1}{\sqrt{2}} \left| \frac{\text{Im } M_{12}}{\Delta m} \right| \quad (2.37)$$

From Eqs. (2.30), (2.32), (2.33) (2.34) and (2.37) we get $|\xi|/|\epsilon| \lesssim 1/15$ which gives, using Eq. (2.35)

$$|\epsilon'| \lesssim \frac{1}{450} |\epsilon| \quad (2.38)$$

and from Eq. (2.36)

$$|\phi| \lesssim 10^{-4} \quad (2.39)$$

By comparison, the present experimental limits¹⁴⁾ are

$$|\epsilon'| \lesssim \frac{1}{50} |\epsilon|, \quad |\phi| \lesssim \frac{1}{20}$$

The amplitude for $K \rightarrow 3\pi$ is completely determined in terms of the $K \rightarrow 2\pi$ amplitude in the limit of exact chiral symmetry and in the approximation that the matrix element is linear in energy. Thus the CP violating phase of the dominant $\Delta I = \frac{1}{2}$ transition is zero by definition [c.f. Eq. (2.23a)] in this approximation which is probably good to within 10%. Thus we expect

$$|\epsilon'_{3\pi(\Delta I = \frac{1}{2})}| = \left| \frac{3}{10} \right| \lesssim \frac{|\epsilon|}{150}$$

and

$$|\epsilon'_{3\pi(\Delta I = \frac{3}{2})}| \approx |\epsilon'| \approx \left| \frac{3}{20} \right| \lesssim \frac{|\epsilon|}{300}$$

For $K^\pm$ -decay, the only observable effects are differences in rates and spectra which are further suppressed by the lack of final state interactions. For semi-leptonic decays, the model predicts no CP violation to lowest order in the weak interaction. We conclude that the KM model is completely consistent with existing data on CP violation in common neutral K decays, and likely to remain so for some time.

2.5 CP violation in rare K decays

We saw in the previous section that the KM model predicts very small deviations from superweak results for the dominant neutral K decays. Rare K decays such as $K \rightarrow 2\gamma$, $\mu^+\mu^-$, πe^+e^- and $\pi\nu\bar{\nu}$ occur only in higher order, and at rates generally governed by the GIM mechanism involving cancellations between light and heavy quarks. Therefore, such decays might be expected to show larger deviations from superweak predictions, and we now analyse them.

K → 2γ

There are two possible states for the photon pair: $2\gamma(+) \equiv F_{\mu\nu} F^{\mu\nu}$ with $CP = +1$, $2\gamma(-) \equiv \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$ with $CP = -1$. In a CP invariant world $K_1 \rightarrow 2\gamma(+)$ and $K_2 \rightarrow 2\gamma(-)$. The analysis of Gaillard and Lee¹²⁾ suggests that the dominant contribution to $K_1 \rightarrow 2\gamma(+)$ comes from the usual long distance mechanism of the 2π intermediate state shown in Fig. 3. The short distance contribution of Fig. 4 is expected to be relatively suppressed by $O(m_q^2/m_W^2)$. On the other hand the $K_2 \rightarrow 2\gamma(-)$ decay should go via the diagram of Fig. 4, with the usual anomalous triangle diagram. However, as discussed in Gaillard and Lee¹²⁾, and in Section 2.3 of this paper, the triangle diagrams with heavy quarks are suppressed for small $m(\gamma\gamma)$, ostensibly by $O(m_u^2/m_c^2 - m_u^2/m_t^2)$. A more phenomenological way of estimating the contribution of the triangle diagram of Fig. 4 is to approximate it by pseudoscalar meson poles¹²⁾ as indicated in Fig. 5. The dominant contributions then come from the η and η' intermediate states, and the contributions of charmed (top) quarks are suppressed by a Zweig factor Z_c^P . The observed rate of the decay $\psi'(3700) \rightarrow J/\psi(3100) + \eta$ enables us to estimate

$$|Z_c^P| \leq \frac{1}{10} \quad (2.40)$$

and again presumably $|Z_t^P| \leq |Z_c^P|$. The relative strength of the η_c contribution is $O(m_\eta^2, m_{\eta'}^2/m_{\eta_c}^2)$ and may be comparable.

In the presence of CP violation, since the dominant mechanism for producing the $\gamma\gamma(+)$ state is the 2π intermediate state, the pattern of $K_{S,L} \rightarrow \gamma\gamma(+)$ decays will be the same as that for $\pi\pi$ final states discussed in Section 2.4. Superweak theory will be approximately valid, with deviations characterized by ϵ' : $|\epsilon'|/|\epsilon| \leq \frac{1}{450}$ from Eq. (2.38), and ϕ_D as always given by Eq. (2.39). In the case of the $\gamma\gamma(-)$ state,

$$\mathcal{A}(K_S \rightarrow \gamma\gamma(-)) = \epsilon \mathcal{A}(K_2 \rightarrow \gamma\gamma(-)) + \mathcal{A}(K_1 \rightarrow \gamma\gamma(-)) \quad (2.41)$$

with the CP violating amplitude $\mathcal{A}[K_1 \rightarrow \gamma\gamma(-)]$ given by

$$\epsilon'_{\gamma\gamma} \equiv \frac{i \mathcal{A}(K_1 \rightarrow \gamma\gamma(-))}{\mathcal{A}(K_2 \rightarrow \gamma\gamma(-))} \approx (Z_c^P - Z_t^P) \frac{s_2 c_2 t_3 \sin \delta}{c_1} \quad (2.42)$$

so that [cf. Eq. (2.29)]

$$\frac{|\epsilon'_{\gamma\gamma}|}{|\epsilon|} \approx \frac{|3|}{|\epsilon|} \leq \frac{1}{15} \quad (2.43)$$

if we follow the analysis of Section 2.4. In the case of $\gamma\gamma(-)$ decays the deviations from superweak are larger than for $\pi\pi$ or $\gamma\gamma(+)$, but since even $K_S \rightarrow \gamma\gamma$ has not yet been seen, the prospects for experiments able to see $\epsilon'_{\gamma\gamma}$ as given by Eq. (2.43) are hardly immediate.

$K \rightarrow 2\mu$

Again there are two possible states for the μ pair: $2\mu(+) \equiv \bar{\mu}\mu$ with $CP = +1$, $2\mu(-) \equiv \bar{\mu}\gamma_5\mu$ with $CP = -1$. The analysis of Gaillard and Lee¹²⁾ suggests that the dominant contribution to $2\mu(+)$ comes from the $\gamma\gamma(+)$ intermediate state, and hence via the $\pi\pi$ intermediate state as in Fig. 6. Hence as in $K_{S,L} \rightarrow 2\gamma(+)$, the pattern of CP violation in $K_{S,L} \rightarrow 2\mu(+)$ should be the same as in $K_{S,L} \rightarrow \pi\pi$. Gaillard and Lee found that for $2\mu(-)$ there were three relevant diagrams, those shown in Fig. 7, with the long distance diagram of Fig. 7a dominating the short distance diagrams of Fig. 7b, c by a factor of (5 to 10). We then find

$$\frac{\langle 2\mu(-) | K_S \rangle}{\langle 2\mu(-) | K_L \rangle} \approx \epsilon + \epsilon' + \left(\frac{1}{5} \leftrightarrow \frac{1}{10} \right) \epsilon'_{\mu\mu} \quad (2.43)$$

where $\epsilon'_{\mu\mu}$ is the ratio of the CP violating to CP conserving parts of the Figs. 7b, c. Taking the Gaillard and Lee calculations of these graphs and extending them to top quarks we find

$$|\epsilon'_{\mu\mu}| \approx \left(\frac{m_t^2 - m_c^2}{m_c^2} \right) \frac{s_2 c_2 t_3 \sin \delta}{c_1} \quad (2.44)$$

and hence

$$\frac{|\epsilon'_{\mu\mu}|}{|\epsilon|} \approx \frac{1}{\sqrt{2} c_1^2 c_3^2} \frac{\left(\frac{m_t^2 - m_c^2}{m_c^2} \right)}{\left[c_2^2 \left(\ln \left(\frac{m_t^2}{m_c^2} \right) - 1 \right) + s_2^2 \left(\frac{m_t^2}{m_c^2} \right) \right]} \quad (2.45)$$

in the case $m_t \gg m_c$. If we assume $s_2^2 \ll c_2^2 \approx 1 \approx c_1^2 \approx c_3^2$, and that the top degree of freedom has not yet been excited at SPEAR II or DORIS, so that $m_t \geq 4$ GeV, then we get

$$\frac{|\epsilon'_{\mu\mu}|}{|\epsilon|} \geq 4 \quad (2.46)$$

Hence the deviations from superweak theory in $K_S \rightarrow 2\mu(-)$ are in principle substantial: but they are presumably unobservable.

$$K_L \rightarrow \pi^0 e^+ e^-$$

As pointed out by Gaillard and Lee, the decay $K_L \rightarrow \pi^0 e^+ e^-$ is forbidden by CP to proceed via γ or Z exchange, and the $W^+ W^-$ contribution is expected to be strongly suppressed. Therefore the decay $K_L \rightarrow \pi^0 e^+ e^-$ can proceed via superweak state mixing or via CP violation in the $K^0 \rightarrow \pi^0 e^+ e^-$ amplitude itself. Extending the calculations of Appendix D of Gaillard and Lee gives

$$\frac{\langle \pi^0 e^+ e^- | K_L \rangle}{\langle \pi^0 e^+ e^- | K_S \rangle} \approx \epsilon + \epsilon'_{\pi ee}$$

with

$$|\epsilon'_{\pi ee}| \approx \frac{\ln\left(\frac{m_K^2}{m_c^2}\right)}{\ln\left(\frac{m_c^2}{m_h^2}\right)} \left| \frac{s_2 c_2 t_3 \sin \delta}{c_1} \right| \quad (2.47)$$

where m_h is a typical charmless and topless hadronic mass, probably $\sim m_\rho$. It is then easy to get

$$\frac{|\epsilon'_{\pi ee}|}{|\epsilon|} \gtrsim 1 \quad (2.48)$$

with $m_t \gtrsim 4$ GeV. Again a substantial deviation from superweak theory in principle, but even so

$$\frac{\Gamma(K_L \rightarrow \pi^0 e^+ e^-)}{\Gamma(K_L \rightarrow \text{all})} \approx 5 \times 10^{-12} \quad (2.49)$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

The decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is also strongly suppressed in a CP invariant world. In the presence of CP violation we have

$$\frac{\langle \pi^0 \nu \bar{\nu} | K_L \rangle}{\langle \pi^0 \nu \bar{\nu} | K_S \rangle} \approx \epsilon + \epsilon'_{\pi \nu \bar{\nu}}$$

with $\epsilon'_{\pi \nu \bar{\nu}}$ estimated by reference to Section 2 of Gaillard and Lee:

$$|\epsilon'_{\pi\nu\bar{\nu}}| \approx \frac{m_t^2 \ln\left(\frac{m_W^2}{m_t^2}\right) - m_c^2 \ln\left(\frac{m_W^2}{m_c^2}\right)}{m_c^2 \ln\left(\frac{m_W^2}{m_c^2}\right)} \times \left| \frac{s_2 c_2 t_3 \sin \delta}{c_1} \right| \quad (2.50)$$

and with $m_t \gtrsim 4$ GeV we can get

$$\frac{|\epsilon'_{\pi\nu\bar{\nu}}|}{|\epsilon|} \gtrsim 4 \quad (2.51)$$

but still

$$\frac{\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\Gamma(K_L \rightarrow \text{all})} \approx 10^{-13} \quad (2.52)$$

2.6 CP violation in charmed particle decays

The discussion here parallels that for K decays, but with differences because the lifetimes of charmed particles are presumably very short, and because the difference between the lifetimes of the two components of the D^0 - $\bar{D}^0$ system is relatively small. As in the case of strange decays, the KM model produces no detectable CP violating effects in amplitudes for semileptonic decays, and CP violating effects in $D^\pm$ and $F^\pm$ decays are expected at the same level as in D^0 , $\bar{D}^0$ decays, which we now estimate.

Following Pais and Treiman²⁴⁾, we introduce two components D_S and D_L of the $(D^0, \bar{D}^0)$ system which have definite lifetimes:

$$D_{S,L} = \frac{1}{\sqrt{2(1+|\epsilon_c|^2)}} \left[(1+\epsilon_c) D^0 \pm (1-\epsilon_c) \bar{D}^0 \right] \quad (2.53)$$

where ϵ_c is a CP violating mixing parameter, defined analogously to ϵ for the K system:

$$\epsilon_c \equiv \frac{\text{Im } \Gamma_{12}^c / 2 + i \text{Im } M_{12}}{i(\Gamma_S^c - \Gamma_L^c) / 2 - (m_S^c - m_L^c)} \quad (2.54)$$

where all quantities in Eq. (2.54) are defined by analogy with the K parameters in Eq. (2.22). We define D_S to be the state whose $S = 0$ decay products mainly have

CP = +: it is not obvious that $\Gamma_S^C > \Gamma_L^C$. As argued in Section 2.1 the dominant decays of charmed states in the KM model are still to strange states:

$$\langle S = -1 | D \rangle \propto (c_1 c_2 c_3 - s_2 s_3 e^{i\delta})$$

and decays to non-strange states are still suppressed

$$\langle S = 0 | D \rangle \propto (-s_1 c_2)$$

So that we expect (since $|s_3| \ll 1$)

$$\frac{\Gamma(D \rightarrow S=0)}{\Gamma(D \rightarrow S=-1)} \approx \frac{s_1^2}{c_1^2 c_3^2} \approx \sin^2 \theta_c$$

Moreover in a CP invariant world the $\Delta C = 1$, $\Delta S = 0$ four-quark operator is predominately a U-spin vector (see Eq. (2.60) below). Hence the parameters $\Delta \Gamma^C = \Gamma_S^C - \Gamma_L^C$ and $\Delta m^C = m_S^C - m_L^C$ are the absorptive and dispersive parts, respectively, of the second order weak amplitude $\langle D^0 | \mathcal{H}_w^2 | D^0 \rangle$. Again $\mathcal{H}_w^2 (\Delta C = 2)$ is predominantly an effective U-spin vector, and $D^0, \bar{D}^0$ are singlets, so we expect:

$$\left| \frac{\Delta \Gamma^C}{\Gamma^C} \right| \sim \left| \frac{\Delta m^C}{\Gamma^C} \right| \lesssim \sin^2 \theta_c \left(\frac{m_S^2 - m_d^2}{m_c^2} \right) \sim 10^{-2} \quad (2.55)$$

where $\Gamma^C = \frac{1}{2}(\Gamma_L^C + \Gamma_S^C)$. We also assume for simplicity $|\text{Im } \Gamma_{12}^C| \lesssim |\text{Im } M_{12}^C|$, so that an order of magnitude estimate of $|\epsilon_c|$ can be got from

$$|\epsilon_c| \approx \left| \frac{\text{Im } M_{12}^C}{\Delta m^C} \right| \quad (2.56)$$

The calculation of Eq. (2.56) then follows precisely the analysis of Section 2.2. CP violation arises from imperfect cancellation between loops containing b and s quarks, and would vanish in the limit of exact "bottom-spin" ($m_b = m_s$)^{*}. However this limit is not physically relevant as presumably $m_b \gtrsim 1.5 \text{ GeV}$ ^{**} $\gg m_s$, in which case

$$\left| \frac{\text{Im } M_{12}^C}{\Delta m^C} \right| \approx \frac{c_3 s_2 s_3 \sin \delta \left[\cos 2\theta_3 m_s^2 \ln \left(\frac{m_b^2}{m_s^2} \right) - c_3^2 m_s^2 + s_3^2 m_b^2 \right]}{c_1 c_2 \left[c_3^4 m_s^2 + s_3^4 m_b^2 \right]} \quad (2.57)$$

*) Eventually it may be necessary to introduce a spin exchanging top and bottom quarks. This we term "side-spin": American readers will recognize this as "english".

**) Note that the lifetime considerations of Section 2.1 and the SPEAR limits on narrow resonances may allow the $b\bar{b}$ vector meson mass spectrum to start as low as (6-7) GeV, in which case m_b could be as low as (3-4) GeV.

Comparing Eq. (2.57) with Eq. (2.11) we see that unless $m_b \gg m_t$, then $|\epsilon_c| \approx |\epsilon|$ in order of magnitude, and we expect small CP violation in the $D^0-\bar{D}^0$ mass matrix.

As observed by Pais and Treiman²⁴⁾, the parameters $\Delta m^c/\Gamma^c$, $\Delta\Gamma^c/\Gamma^c$ and $\text{Re } \epsilon_c$ can in principle be measured in semileptonic decays of states prepared as pure D^0 or $\bar{D}^0$:

$$\frac{\Gamma(D^0 \rightarrow \bar{l} + \nu + X)}{\Gamma(D^0 \rightarrow l + \nu + X)} \times \frac{\Gamma(\bar{D}^0 \rightarrow l + \nu + X)}{\Gamma(\bar{D}^0 \rightarrow \bar{l} + \nu + X)} \approx \frac{1}{4} \left[\left(\frac{\Delta m^c}{\Gamma^c} \right)^2 + \frac{1}{4} \left(\frac{\Delta \Gamma^c}{\Gamma^c} \right)^2 \right] \left(1 + \dots \right) \quad (2.58)$$

which measures the (squared) rate of "wrong sign" semileptonic decays. Also:

$$\frac{\Gamma(D^0 \rightarrow \bar{l} + \nu + X)}{\Gamma(D^0 \rightarrow l + \nu + X)} \bigg/ \frac{\Gamma(\bar{D}^0 \rightarrow l + \nu + X)}{\Gamma(\bar{D}^0 \rightarrow \bar{l} + \nu + X)} \approx 1 - 8 \text{Re } \epsilon_c + \dots \quad (2.59)$$

which may differ from 1 by $O(10^{-2})$ if the estimate (2.56) is accepted. Using Eqs. (2.55) and (2.58), the rate of "wrong sign" semileptonic decays may be relatively suppressed by $O(10^{-4})$, so one is looking for small effects in very weak processes!

The most interesting place to look for CP violating effects is presumably in $\Delta S = 0$ hadronic decays. In addition to the superweak mixing effect (2.57) there is direct CP violation in decay amplitudes. From the KM model (2.5) we find (omitting γ matrices and a factor of $G_F/\sqrt{2}$).

$$\begin{aligned} d\mathcal{L}_{\Delta C=-1}^{\Delta S=0} = \frac{(\bar{c}u)}{2} \Big\{ (\bar{d}d - \bar{s}s) \Big[c_1 s_1 c_2 + s_1 c_3^2 c_1 c_2 - s_1 c_3 s_2 s_3 \cos \delta - i s_1 c_3 s_2 s_3 \sin \delta \Big] \\ + (\bar{d}d + \bar{s}s) \Big[c_1 s_1 c_2 s_3^2 + s_1 c_3 s_2 s_3 \cos \delta + i s_1 s_2 s_3 c_3 \sin \delta \Big] \Big\} \quad (2.60) \end{aligned}$$

Introducing $\mathcal{H}_1 \propto (\bar{c}p)(\bar{n}n - \bar{\lambda}\lambda)$, $\mathcal{H}_2 \propto (\bar{c}p)(\bar{n}n + \bar{\lambda}\lambda)$ and extracting the CP violating phases $e^{i\alpha_1}$, $e^{i\alpha_2}$ explicitly, we get

$$\left. \begin{aligned} d\mathcal{L}_{\Delta C=-1}^{\Delta S=0} &= e^{i\alpha_1} d\mathcal{L}_1 + e^{i\alpha_2} d\mathcal{L}_2 \\ d\mathcal{L}_{\Delta C=+1}^{\Delta S=0} &= e^{-i\alpha_1} d\mathcal{L}_1^\dagger + e^{-i\alpha_2} d\mathcal{L}_2^\dagger \end{aligned} \right\} \quad (2.61)$$

where

$$\tan \alpha_1 = - \frac{c_3 s_2 s_3 \sin \delta}{c_1 c_2 + c_3^2 c_1 c_2 - c_3 s_2 s_3 \cos \delta}$$

and

$$\tan \alpha_2 = \frac{s_2 s_3 c_3 \sin \delta}{c_1 c_2 s_3^2 + c_3 s_2 s_3 \cos \delta}$$

(2.62)

It is apparent from Eq. (2.62) that $|\tan \alpha_1|$ is expected to be small, whereas $|\tan \alpha_2|$ is not necessarily small.

We parametrize the matrix elements of $\mathcal{H}$ as follows:

$$\langle f | \mathcal{H}_i | D \rangle = \langle \bar{f} | \mathcal{H}_i^\dagger | \bar{D} \rangle = T_i e^{i(\beta_i + \delta_s)} \quad (i=1,2) \quad (2.63)$$

where δ_s is the elastic phase shift, β_i a residual phase related to inelasticity, and T_i is real. Combining Eqs. (2.60), (2.61) and (2.63) we then get, dropping the common phase δ_s

$$\left. \begin{aligned} \langle f | \mathcal{H}_{\Delta C = -1} | D^0 \rangle &= T_1 e^{i(\alpha_1 + \beta_1)} + T_2 e^{i(\alpha_2 + \beta_2)} \\ \langle \bar{f} | \mathcal{H}_{\Delta C = +1} | \bar{D}^0 \rangle &= T_1 e^{i(\beta_1 - \alpha_1)} + T_2 e^{i(\beta_2 - \alpha_2)} \end{aligned} \right\} \quad (2.64)$$

In order to use the CP violating amplitudes (2.64), we first go back to the decomposition (2.53) of D_S and D_L , from which we find the following expressions for D^0 and $\bar{D}^0$ states:

$$D^0 = \frac{(D_S + D_L)}{\sqrt{2}} \frac{\sqrt{1 + |\epsilon_c|^2}}{(1 + \epsilon_c)}, \quad \bar{D}^0 = \frac{(D_S - D_L)}{\sqrt{2}} \frac{\sqrt{1 + |\epsilon_c|^2}}{(1 - \epsilon_c)}$$

States prepared initially as pure D^0 or $\bar{D}^0$ will then have their D_S , D_L decomposition change in the normal way with time:

$$D_{op}^0(t) \equiv \eta \left[D_S e^{-\Gamma_S t/2} + D_L e^{-\Gamma_L t/2} e^{-i\Delta M^c t} \right]$$

$$\bar{D}_{sp}^0(t) \equiv \tilde{\eta} \left[D_s e^{-\Gamma_s^c t/2} - D_L e^{-\Gamma_L^c t/2} e^{-i\Delta m^c t} \right] \quad (2.65)$$

where

$$\eta \equiv \frac{\sqrt{1+|\epsilon_c|^2}}{\sqrt{2}(1+\epsilon_c)}, \quad \tilde{\eta} \equiv \frac{\sqrt{1+|\epsilon_c|^2}}{\sqrt{2}(1-\epsilon_c)}$$

Substituting Eqs. (2.64) into (2.65) and neglecting various terms second order in the small quantities ϵ_c , $\Delta m^c/\Gamma^c$, $\Delta\Gamma^c/\Gamma^c$, T_2 and $\sin \alpha_1$ we find that for states f with $CP = \pm 1$:

$$|\langle f | \bar{D}_{sp}^0(t) \rangle|^2 = \frac{4T_1^2}{\Gamma_s} \left\{ |\eta|^2 + 2\text{Re} \left[\frac{\epsilon_c + i \frac{T_2}{T_1} e^{i\beta} \sin \alpha_2 + i \sin \alpha_1}{\Gamma^c \pm i\Delta m^c} \right] \right\} + \dots$$

where $\beta \equiv \beta_2 - \beta_1$. From this we get

$$\begin{aligned} \frac{\Gamma(D^0 \rightarrow f)}{\Gamma(\bar{D}^0 \rightarrow f)} &\simeq \frac{|\eta|^2 + 2(\text{Re } \epsilon_c - \frac{T_2}{T_1} \sin \alpha_2 \sin \beta)}{|\tilde{\eta}|^2 - 2(\text{Re } \epsilon_c - \frac{T_2}{T_1} \sin \alpha_2 \sin \beta)} \\ &\simeq 1 - 4 \frac{T_2}{T_1} \sin \alpha_2 \sin \beta + \dots \end{aligned} \quad (2.66)$$

both for states f with $CP = \pm 1$.

We observe that the result (2.66) is non-zero only if the CP violating phase $\sin \alpha_2$ [cf. Eq. (2.62)] is non-zero. This agrees with the result of Pais and Treiman²⁴⁾ in the limit ϵ_c , $\Delta m^c/\Gamma^c$, $\Delta\Gamma^c/\Gamma^c$ small. The phase difference β is not in general small^{*)}: therefore using Eqs. (2.60) and (2.62) we can write

$$\frac{\Gamma(D^0 \rightarrow f)}{\Gamma(\bar{D}^0 \rightarrow f)} \simeq 1 + X S_2 S_3 \sin \delta \quad (2.67)$$

where we have set $|c_i| \simeq 1$ and X is a factor $O(1)$ unless there is some special

*) β need not be small provided that there are several open channels which can communicate strongly with the state f .

reason favouring or disfavouring matrix elements $\langle f | \mathcal{H}_2 | D \rangle$. Since $\mathcal{H}_2$ is a U-spin scalar, whereas $\mathcal{H}_1$ is a U-spin vector, we expect that for transitions to states with definite U-spin

$$\frac{\Gamma(D^0 \rightarrow (U=1))}{\Gamma(\bar{D}^0 \rightarrow (U=1))} \approx 1 + U s_2 s_3 \sin \delta$$

$$\frac{\Gamma(D^0 \rightarrow (U=0))}{\Gamma(\bar{D}^0 \rightarrow (U=0))} \approx 1 + \frac{1}{U} s_2 s_3 \sin \delta$$

where

$$U \approx \frac{m_s^2 - m_d^2}{m_c^2} \quad \left(\approx \frac{1}{10} ? \right)$$

is a measure of U-spin symmetry violation. If $|s_2 s_3 \sin \delta| \approx 10^{-3}$ as suggested by the calculation of $|\varepsilon|$ in Section 2.2, then

$$\left. \begin{aligned} \frac{\Gamma(D^0 \rightarrow (U=1))}{\Gamma(\bar{D}^0 \rightarrow (U=1))} - 1 &= O(10^{-4}) \\ \frac{\Gamma(D^0 \rightarrow (U=0))}{\Gamma(\bar{D}^0 \rightarrow (U=0))} - 1 &= O(10^{-2}) \end{aligned} \right\} \quad (2.68)$$

The estimates (2.68) seem the least undistinctive consequences of the KM model for CP violating effects in charmed particle decays. We re-emphasize that effects of the size (2.68) would not be expected in a "superweak" theory of CP violation in the D^0 - $\bar{D}^0$ system. The simplest example of a possible $U = 0$ final state is $K^0 \bar{K}^0$, and $K^0 \bar{K}^0$ scattering at an invariant energy $s \approx (2 \text{ GeV})^2$ is presumably sufficiently inelastic so that there will be no *a priori* suppression of the CP violating effect ($\sin \beta \sim 1$).

2.7 The electric dipole moment of the neutron

In this section we estimate the dipole moment of the physical neutron in terms of the quark dipole moment. In general²⁵⁾, the effects of weak interactions are expected to induce electromagnetic form factors which violate parity and/or time reversal invariance, via diagrams of the types illustrated in Fig. 8. In particular, an effective $\bar{q}q\gamma$ coupling of the form

$$E_\mu^j(\bar{q}q\gamma) = e f_D(k^2) : \sigma_{\mu\nu} k^\nu \gamma_5 \quad (2.69)$$

violates both P and T, since hermiticity of the electromagnetic current requires that $f_D(k^2)$ be purely imaginary. The strength of the electric dipole moment is given by

$$|D| = e |f_D(0)| \quad (2.70)$$

On general symmetry grounds, the quark dipole moment must vanish in the limits

- a) of vanishing external quark mass, because Eq. (2.69) is a helicity flip operator, and
- b) when any two quarks of the same charge are degenerate in mass⁹).

To estimate the strength of the dipole moment of the quark q_α , we first separate the contributions of the effective $\bar{q}_\alpha q_\alpha \gamma$ vertex into diagrams which are one particle irreducible and reducible, respectively, as illustrated in Fig. 9. The parity violating part of the self-energy insertion is of the form

$$\Sigma^A(p) = [\not{A}(p^2) + m_\alpha B^A(p^2)] \gamma_5 \quad (2.71)$$

and evaluating Fig. 9b with external lines on-shell gives

$$E_\mu^\alpha(9b) = e Q_\alpha A^\alpha \gamma_\mu \gamma_5$$

and there is no contribution to the dipole moment.

Next we consider the one particle irreducible vertex of Fig. 9a. As noted by Maiani⁹), no quark dipole moment arises in lowest order in the weak interaction since the corresponding diagrams (e.g. Fig. 8a) are real:

$$E_\mu(8a) \propto U_{\alpha\beta} U_{\beta\alpha}^\dagger F(m_\beta) = |U_{\alpha\beta}|^2 F(m_\beta) \quad (2.72)$$

where U is the unitary weak coupling matrix defined in Eqs. (2.1) and (2.5). More generally, in order to generate a complex amplitude, at least two charged current transitions not related by complex conjugation must occur. Thus the diagram of Fig. 8b can contribute

$$E_\mu(8b) \propto U_{\alpha\beta} U_{\beta\gamma}^\dagger U_{\gamma\delta} U_{\delta\alpha}^\dagger F(m_\beta, m_\gamma, m_\delta) \quad (2.73)$$

while the graph of Fig. 8c cannot: since Z^0 couplings are real and diagonal, $E_\mu(8c)$ is of the form (2.72). Finally, diagrams involving a closed fermion loop, such as that of Fig. 8d also induce real amplitudes:

$$E_\mu(8d) \propto |U_{d\beta}|^2 |U_{\delta\delta}|^2 F(m_\beta, m_\delta, m_\delta) \quad (2.74)$$

In summary, the dipole moment arises in lowest order from diagrams in which two charged bosons are exchanged along a single quark line. Since the quark charges are such that the two charged boson lines cannot cross, the lowest order contributions to Fig. 9a are those of Fig. 10, where $E_\mu^{\gamma\beta}$ is the effective $\bar{q}_\gamma q_\beta \gamma$ vertex and $\Sigma^{\gamma\beta}$ is the off-diagonal self-energy (Figs. 11-13).

In the 't Hooft-Feynman gauge²⁰⁾ which we adopt hereafter, the diagram of Fig. 10a in fact gives no contribution to the dipole moment. The W-propagator is proportional to $g_{\mu\nu}$, so the contribution of Fig. 10a takes the form

$$E_\mu^{\alpha}(10a) \propto \gamma_\nu L O_\mu \gamma^\nu L = \gamma_\nu L O_\mu R \gamma^\nu \quad (2.75)$$

where $L, R = \frac{1}{2}(1 \mp \gamma_5)$ are the usual helicity projection operators. The electric dipole moment is determined from E_μ by

$$e f_D(k^2) k^\mu = \frac{1}{8} \text{Tr}(\sigma^{\mu\rho} \gamma_5 E_\mu^{\alpha})$$

and vanishes identically for a term of the form (2.75)

$$e f_D^{(10a)}(k^2) k^\mu \sim -\text{Tr}[\gamma_\nu \sigma^{\mu\rho} \gamma^\nu \gamma_5 L O_\mu R]; \quad \gamma_\nu \sigma_{\mu\alpha} \gamma^\nu = 0.$$

The diagrams of Fig. 10c also give no contribution. The off-diagonal self-energy is of the form

$$\Sigma^{\beta\gamma}(p) = A(p^2) \not{p} L + B(p^2) m_\delta m_\beta \not{p} R + C(p^2) (m_\delta L + m_\beta R) \quad (2.76)$$

where to lowest order in the weak coupling, A, B and C are independent of the external masses $m_{\gamma,\beta}$. Inspection of the couplings in Fig. 10c shows that a term proportional to $\sigma_{\mu\alpha} \gamma_5$ can arise only from the exchange of one W and one H (unphysical Higgs), but this contribution is symmetric in $m_{\beta,\gamma}$:

$$E_{\mu}^{\alpha}(10c) \sim \int \frac{d^4 r}{(2\pi)^4} f(r^2, (r-p)^2, (r-p-k)^2) m_{\alpha} [\gamma_{\mu}, \gamma_{\mu}] \gamma_5 \\ \times [r^2 A(r^2) + m_{\beta}^2 m_{\delta}^2 B(r^2) + (m_{\beta}^2 + m_{\delta}^2) C(r^2)] . \quad (2.77)$$

The imaginary part needed for CP violation is antisymmetric under the exchange $\beta \leftrightarrow \gamma$, and hence vanishes in the sum over intermediate states (β, γ) .

Finally we are left with the diagram of Fig. 10b, which we study by first examining the effective $\bar{q}_{\gamma} q_{\beta} \gamma$ vertex of Fig. 11. If $\Gamma_{\mu}^{\gamma\beta}$ is the one particle irreducible part in Fig. 11a, we may use the Ward-Takahashi identity to write

$$k^{\mu} \Gamma_{\mu}^{\gamma\beta}(p, k) = e Q_{\gamma} [\Sigma^{\gamma\beta}(p+k) - \Sigma^{\gamma\beta}(p)] \\ \Gamma_{\mu}^{\gamma\beta}(p, k) = e Q_{\gamma} \frac{\partial}{\partial p^{\mu}} \Sigma^{\gamma\beta}(p) + O(k) \quad (2.78)$$

where Q_{γ} is the charge of q_{γ} . So the effective $\bar{q}_{\gamma} q_{\beta} \gamma$ vertex may be separated into a part determined by the self-energy, Eq. (2.76), and the k -dependent parts of the diagrams of Fig. 12. Figure 12g can be ignored, and the other diagrams are all convergent. Furthermore, GIM⁽¹¹⁾ tells us that they vanish in the limit of degeneracy of exchanged quarks q_{δ} . As discussed in Ref. 12, the GIM cancellation is equivalent to a subtraction of the q_{δ} propagator. Then the relevant $O(k)$ contributions are still convergent after removal of one W propagator:

$$\Delta_{\mu\nu}^W \rightarrow i g_{\mu\nu} / m_W^2$$

The diagrams of Figs. 12c to 12f contain an additional boson propagator in the loop integration, and Fig. 12b has an implicit factor m_W^{-2} for the Higgs coupling. Therefore the diagrams of Figs. 12b to 12f must be $O(m_W^{-4})$. Dropping γ -matrices we have

$$E_{\mu}^{\gamma\beta}(12b \text{ to } 12f) \sim \frac{e g^2}{16\pi^2} m_{\delta}^2 \frac{k_{\mu} p_{\nu}}{(m_W^2 - p^2)} \times \left[\frac{1}{m_W^2} \text{ or } \frac{1}{m_W^2 - p^2} \right] \quad (2.79)$$

up to a possible logarithm. For the diagram of Fig. 12a, removal of the W propagator leaves a fermion loop integral with no additional factor of m_W^{-2} . However, the term linear in k then vanishes by symmetric integration over the loop momentum. To obtain a non-vanishing contribution we must integrate over the W propagator and

we again obtain an additional suppression:

$$E_{\mu}^{\gamma\beta}(12a) \sim \frac{eg^2}{16\pi^2} \frac{m_s^2}{(m_W^2 - p^2)} k p \ln \left(\frac{m_s^2}{m_W^2 - p^2} \right) \quad (2.80)$$

The result is completely analogous to that found in Ref. 12 for the (quark mass-shell) $\bar{s}u\gamma$ vertex: while the charge radius is $O(g^2 \ln m_q^2)$, the induced tensor and pseudotensor terms were found to be suppressed by an extra factor of m_q^2/m_W^2 .

Inserting Eqs. (2.80) and (2.79) into the diagram of Fig. 10b, we obtain for the leading contribution to the dipole moment

$$E_{\mu}^{\alpha}(12) \sim \frac{eg^4}{16\pi^2} m_{\alpha} \frac{m_{\rho,\gamma}^2}{m_W^2} \int \frac{d^4 r}{(2\pi)^4} \left[\frac{1}{m_W^2} \text{ or } \frac{1}{m_W^2 - p^2} \right] \\ \times \frac{[K, \gamma_{\mu}] \ln \left[(p^2 - m_W^2)/m_s^2 \right] p^2}{(p^2 - m_W^2)^2 (p^2 - m_{\rho}^2) (p^2 - m_s^2)}$$

The convergence is such that the propagator in brackets may again be replaced by m_W^{-2} . The remaining integral gives another factor of m_W^{-2} with a possible extra factor of $\ln^2 (m_W^2/m_q^2)$. Thus when we put in the appropriate angle factors (assuming $|c_i^2| \approx 1$) we get a contribution to the dipole moment which is at most

$$\left| \frac{D}{e} \right|_n = \left| \frac{D}{e} \right|_{u,d} \sim m_{u,d} \frac{G_F \propto (m_t^2 - m_c^2)(m_b^2 - m_s^2)}{\pi^3 m_W^4} \\ \times \ln^2 \left(\frac{m_W^2}{m_t^2, m_b^2} \right) s_1^2 s_2 s_3 \sin \delta \\ \sim (10^{-26} \text{ to } 10^{-27}) \frac{m_u}{m_n} \frac{m_t^2 m_b^2}{m_W^4} \ln^2 \left(\frac{m_W^2}{m_t^2, m_b^2} \right) \text{ cm.} \quad (2.81)$$

The analysis of the part of $E_{\mu}^{\gamma\beta}$ which depends on the self-energy insertions $\Sigma^{\gamma\beta}$ shown in Fig. 13 does not alter the estimate (2.81).

In formula (2.81) $m_{u,d}$ is a measure of the chiral symmetry breaking fraction of the neutron mass, which is probably $O(m_n/10)$. In the above discussion we have implicitly assumed that all quark masses are small compared with W-masses. If the top and bottom were actually of order m_W -- and why not? -- the mass-dependent factor in Eq. (2.81) would be replaced by a complicated function which we suspect to be $O(1)$. In that case formula (2.81) suggests

$$\left| \frac{D}{e} \right|_n \sim (10^{-27} - 10^{-28}) \text{ cm} \quad \text{for } m_{t,b} \sim m_W, \quad (2.82a)$$

On the other hand, if we take $m_t \sim m_b$ near the present experiment lower limit, we get

$$\left| \frac{D}{e} \right|_n \sim (10^{-30} - 10^{-31}) \text{ cm} \quad \text{for } m_{t,b} \sim 5 \text{ GeV}. \quad (2.82b)$$

Since the superweak¹⁰⁾ estimate is¹⁴⁾ $0(10^{-29})$ cm, the values (2.82) mean the KM model prediction is not very distinctive. Within the context of the model, knowledge of m_t and m_b would enable a better prediction of the neutron electric dipole moment -- and vice versa.

3. CONCLUSIONS AND DISCUSSION

We like the KM model because it introduces CP violation in a natural way, as a result of weak mixing between the quarks analogous to the Cabibbo angle in the GIM model. Thus two puzzles are reduced to one. Looked at another way, it gives a *raison d'être* to the fifth and sixth quarks, needed in the simplest Weinberg-Salam theory to cancel anomalies and restore renormalizability if there is a heavy lepton. Unfortunately it is a very well concealed theory, at least as long as the top and bottom degrees of freedom are hidden, and naked top and bottom states are not produced in e^+e^- annihilation or elsewhere. We found:

- a) That superweak results for common K decays are reproduced to high accuracy [cf. Eqs. (2.38) and (2.39)] whatever the masses of the top and bottom quarks.
- b) That in rare K decays, direct CP violating effects in the amplitudes may be comparable with superweak CP violating effects. Unfortunately the branching ratios for these decays are so small [cf. Eqs. (2.49) and (2.52)] that these effects seem essentially unobservable.
- c) There are in principle observable deviations from superweak predictions for charmed particle decays [cf. Eq. (2.68)]. Since no discovery of a charmed particle has yet been confirmed, checking results like Eq. (2.68) will clearly not be done in the near future.
- d) The neutron electric dipole moment is very sensitive to the masses of the top and bottom quarks.

$$\left| \frac{D}{e} \right|_n \approx \left\{ \begin{array}{l} (10^{-27} - 10^{-28}) \text{ cm} \\ (10^{-30} - 10^{-31}) \text{ cm} \end{array} \right\} \quad \text{for } m_{t,b} \sim \left\{ \begin{array}{l} m_W \\ 5 \text{ GeV} \end{array} \right\}.$$

Hence a measurement of this quantity would help us tell experimentalists where to look for top and bottom particles. Conversely, if such states were discovered, the above expression for the dipole moment would serve as a test of the validity of the KM model.

In principle the model also predicts small deviations from the GIM model and Cabibbo universality for couplings of the u, d, s and c quarks. This is because there is some "leakage" of weak charge to the t and b quarks, characterized by the angle factors s_2 and s_3 in Eq. (2.5). A search for such deviations would be valuable: unfortunately deviations from Cabibbo predictions for the u quark are experimentally no greater than the uncertainties due to radiative corrections and nuclear uncertainties. Checks on the dominance of strange particles in the decays of charmed particles would be important. Unfortunately the decay amplitudes are subject to large SU(3) or SU(4) symmetry violations. A good place to look might be the vector form factors in $D \rightarrow K + \ell + \nu$, $D \rightarrow \pi + \ell + \nu$ decays. At $m(\ell\nu) \approx 0$ these deviate from symmetry predictions only in second order in symmetry breaking, and a check of the GIM prediction

$$\frac{\langle \pi | J^{wk} | D \rangle}{\langle K | J^{wk} | D \rangle} \approx \tan \theta_c \quad \text{at} \quad q^2 = 0$$

would be least unreliable theoretically.

Probably this model cannot be confirmed or refuted by looking at topless and bottomless states. Unfortunately we have found no constraints on the masses of the top and bottom quarks. Our ignorance does not exclude their being anywhere between lower limits of (3-4) GeV and $\sim m_W$ or m_Z , or beyond. Hence it may prove difficult to expose naked top and bottom particles. Needless to say, there is no reason known why 8 or more quarks should not exist.

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APPENDIX

In this appendix we discuss Higgs structures which could give CP violation as in the KM model. One method would clearly be to follow Lee²⁶⁾ and introduce two or more Higgs multiplets, whose vacuum expectation values had "clashing" phases, introducing the requisite complex phases into the weak interaction matrix U of Eq. (2.1). However, we are motivated to restrict to a minimum the spectrum of unobserved (and almost unobservable) Higgs bosons, and introduce CP violation with just the original⁶⁾ doublet Higgs fields.

The neutral Higgs couplings to quarks can be written in the general form

$$\begin{aligned} & \left(\frac{H+v}{v} \right) \left[\bar{P}_c^R G_P P_c^L + \bar{P}_c^L G_P^\dagger P_c^R \right] \\ & + \left(\frac{H+v}{v} \right) \left[\bar{N}_c^R G_N N_c^L + \bar{N}_c^L G_N^\dagger N_c^R \right] \end{aligned} \quad (A.1)$$

where $P^{L(R)}$ and $N^{L(R)}$ are the multiplets of left-handed (right-handed) quarks of charges $2/3$ and $-1/3$ respectively, v is the vacuum expectation value of the Higgs field, G_P and G_N are coupling matrices which are not necessarily real, and the subscripts c indicate that the quark fields are Cabibbo rotated to the basis where charged weak currents are diagonal. The Higgs couplings (A.1) can be diagonalized by introducing transformations to quark states with well-defined masses:

$$\left. \begin{aligned} P_c^L &= U_P P^L, & N_c^L &= U_N N^L, \\ P_c^R &= V_P P^R, & N_c^R &= V_N N^R. \end{aligned} \right\} \quad (A.2)$$

The interactions (A.1) then become

$$\begin{aligned} & \left(\frac{H+v}{v} \right) \left[\bar{P}^R V_P^\dagger G_P U_P P^L + \bar{P}^L U_P^\dagger G_P^\dagger V_P P^R \right] \\ & + \left(\frac{H+v}{v} \right) \left[\bar{N}^R V_N^\dagger G_N U_N N^L + \bar{N}^L U_N^\dagger G_N^\dagger V_N N^R \right] \end{aligned} \quad (A.3)$$

which is supposed to be diagonal: we can choose the phases of $U_{P,N}$ and $V_{P,N}$ such that

$$V_P^\dagger G_P U_P = M_P \quad , \quad V_N^\dagger G_N U_N = M_N \quad (A.4)$$

where $M_{P,N}$ are the diagonal mass matrices for P,N type quarks. Going to the W^+ coupling which starts off life proportional to

$$W_\mu^+ \bar{P}_c^L \gamma^\mu N_c^L$$

this transforms under Eq. (A.2) to become

$$W_\mu^+ \bar{P}_c^L \gamma^\mu U_P^\dagger U_N N_c^L \quad (A.5)$$

so that the matrix U of Eq. (2.1) is given by

$$U = U_P^\dagger U_N \quad (A.6)$$

Consider finally the (unphysical) charged Higgs couplings. From Eq. (A.3) we see the H^+ has couplings proportional to

$$H^+ (\bar{P}_c^R G_P N_c^L) - H^+ (\bar{P}_c^L G_N^\dagger N_c^R) \quad (A.7)$$

which become

$$H^+ [\bar{P}^R (V_P^\dagger G_P U_N) N^L - \bar{P}^L (U_P^\dagger G_N^\dagger V_N) N^R] \quad (A.8)$$

under the rotations (A.2). Using (A.4) and (A.6) the expression (A.8) can be rewritten as

$$H^+ [\bar{P}^R M_P U N^L - \bar{P}^L U M_N N^R] \quad (A.9)$$

It is easy to see from the above analysis that we can introduce CP violation as in the KM model with just the usual Higgs doublet, and without any extra CP violations due to Higgs couplings. For example, if we set

$$U_P = V_P = V_N = I, \quad G_P = M_P, \quad G_N = M_N U, \quad (\text{A.10})$$

with U an arbitrary unitary matrix, then the conditions (A.4), (A.6) relating the Higgs coupling matrices $G_{P,N}$ and the weak interaction matrix U are clearly satisfied, and the couplings (A.9) of the (unphysical) Higgs bosons have the expected relation to the W couplings.

We should re-emphasize that *a fortiori* the CP violation scheme of Lee²⁶⁾, using more than one Higgs multiplet, could be employed for the KM model. In that case there would be extra CP violating effects in the Higgs sector of the model. These we have ignored, for example in the discussion of the neutron electric dipole moment in Section 2.7.

Figure captions

- Fig. 1 : Contributions to the $K^0-\bar{K}^0$ mass mixing matrix.
- Fig. 2 : Feynman diagrams leading to CP violation in amplitudes for K^0 , $\bar{K}^0$ decays.
- Fig. 3 : The 2π intermediate state in $K \rightarrow 2\gamma(+)$.
- Fig. 4 : Quark loop contribution to $K \rightarrow 2\gamma(\pm)$.
- Fig. 5 : Phenomenological model for $K \rightarrow 2\gamma(-)$.
- Fig. 6 : Intermediate states in $K \rightarrow 2\mu(+)$.
- Fig. 7 : Intermediate states in $K \rightarrow 2\mu(-)$.
- Fig. 8 : Contributions to quark electromagnetic form factors.
- Fig. 9 : The effective $\bar{q}_\alpha q_\alpha \gamma$ vertex separated into (a) irreducible and (b) one particle reducible parts.
- Fig. 10 : Diagrams contributing to the electric dipole moment of q_α : $E_\mu^{\gamma\beta}$ is the effective $E_\mu^{\gamma\beta}$ vertex.
- Fig. 11 : Decomposition of $E_\mu^{\gamma\beta}$ into (a) vertex and (b) self-energy parts.
- Fig. 12 : Contributions to the vertex part of $E_\mu^{\gamma\beta}$.
- Fig. 13 : Contributions to the self-energy part of $E_\mu^{\gamma\beta}$.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG.5

FIG.6

FIG.7

FIG.8

FIG.9

FIG.10

FIG.11.

FIG. 12

FIG. 13