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INTERPRETATION OF RESULTS FROM HIGH ENERGY e^+e^- COLLISIONS

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1. Measurements of the Z^0 mass: implications for the top quark mass

The first precision measurements at the high-energy colliders SLC and LEP were of the Z^0 mass. Table 1 shows the LEP measurements reported [1] at the Neutrino Conference held at CERN in June 1990, compared with the published SLC value. Also shown is the average of the LEP measurements, which has a statistical error of 9 MeV and a common LEP beam energy scale error of 30 MeV. The latter is measured by storing protons and positrons simultaneously at a nominal beam energy around 20 GeV, and measuring the difference in their velocities via their different radiofrequencies. From the results in Table 1 we extract [1]

$$m_Z = 91.172 \pm 0.031 \text{ GeV} \quad (1)$$

whose implications for the top quark mass we now discuss.

The top quark is an essential feature of the Standard Model [2] A third doublet of quarks $(t, b)_L$ is needed alongside the known $(u, d)_L$ and $(c, s)_L$ to cancel the triangle anomalies generated by the three lepton doublets $(\nu_e, e)_L$, $(\nu_\mu, \mu)_L$ and $(\nu_\tau, \tau)_L$. There is independent evidence from measurements of the forward-backward asymmetry in $e^+e^- \rightarrow b\bar{b}$ that the left-handed b quark has electroweak isospin $-\frac{1}{2}$ [3]

$$I_3(b_L) = -0.6 \pm 0.25 \quad (2)$$

Moreover, the absence of flavour-changing neutral currents in b decay, $b \not\rightarrow (s \text{ or } d) + (\ell^+ \ell^-)$, etc., tells that a third quark doublet must exist. Otherwise, the tree-level quark neutral current would be

$$\bar{d}_c \gamma^\mu L d_c + \bar{s}_c \gamma^\mu L s_c \ni \bar{d} \gamma^\mu L b + \bar{s} \gamma^\mu L b + (\text{h.c.}) \quad (3)$$

where $L \equiv \frac{1}{2}(1 - \gamma_5)$, which conflicts with data on $B \rightarrow (\ell^+ \ell^-) + X$ decay.

However, there is no model-independent prediction for m_t . LEP provides a model-independent lower bound $m_t > 46 \text{ GeV}$ from total cross-section measurements, the absence of "spherical" multijet events due to $t\bar{t}$ production and decay, and the absence of hadronic

events with isolated leptons [4]. There are also stronger direct limits from $p\bar{p}$ colliders, but these rely on the model-dependent decay branching ratio for $t \rightarrow b + \ell + \nu$. Within the Standard Model, the absence of $e + \text{jet}(s) + \not{p}_T$ events tells the UA2 collaboration [4] that $m_t > 67 \text{ GeV}$, whilst an unsuccessful combined search for $e + \text{jet}(s) + \not{p}_T, \mu + \text{jet}(s) + \not{p}_T, e^+e^- \text{ or } \mu^+\mu^-$ or $e^+\mu^\mp + \not{p}_T$ events tells the CDF collaboration [5] that $m_t > 89 \text{ GeV}$. It is worth noting that, as reported at this meeting [5], there is one $e\mu + \not{p}_T$ event which could perhaps be W^+W^- pair-production, or ...?

The LEP measurement of m_Z can be combined with lower-energy electroweak measurements to estimate m_t indirectly via radiative corrections [6,7]. These include vacuum polarization, vertex and box diagrams. One takes $\alpha_{em} = 1/137.0359895(61)$ and $G_\mu = 1.166389(22) \times 10^{-5} \text{ GeV}^{-2}$ from μ -decay experiments, and uses other measurements to determine $\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2$. For example the vector boson masses are given by

$$m_W^2 \sin^2 \theta_W = m_Z^2 \cos^2 \theta_W \sin^2 \theta_W = \frac{A}{1 - \Delta r} \quad (4)$$

where

$$A = \frac{\pi \alpha}{\sqrt{2} G_\mu} = 37.2802(3) \text{ GeV}^2 \quad (5)$$

and the radiative correction Δr is due to vacuum polarization diagrams [8]. It can be decomposed as

$$\Delta r = \Delta \alpha_{em} - \cot^2 \theta_W \Delta \rho + \hat{\Delta r} \quad (6)$$

where

$$\Delta \alpha_{em} = 0.0602 + \frac{40}{9} \frac{\alpha}{\pi} \ln m_Z/92 \text{ GeV} \pm 0.0009 \quad (7)$$

is the variation in the effective value of α_{em} between $Q^2 \simeq m_e^2$ and $Q^2 = m_Z^2$,

$$\Delta \rho \ni \frac{3G_\mu m_t^2}{8\pi^2 \sqrt{2}} \quad (8)$$

reflects electroweak isospin violation due to $m_t \neq m_b$, and the residual piece $\hat{\Delta r}$ also depends

on m_t :

$$\Delta r \ni \frac{\sqrt{2}G_\mu m_W^2}{8\pi^2}(\cot^2 \theta_W - \frac{1}{3}) \ln \frac{m_t^2}{m_W^2} \quad (9)$$

This latter can be compared with the sensitivity of Δr to m_H :

$$\Delta r \ni \frac{\sqrt{2}G_\mu m_W^2}{16\pi^2} \left\{ \frac{11}{3} \ln \frac{m_H^2}{m_W^2} - \frac{5}{6} \right\} \quad (m_H \gg m_W) \quad (10)$$

On the basis of these formulae and low-energy experiments, it was predicted [6] that a precision measurement of m_Z would enable one to predict

$$m_t \simeq 95 \text{ GeV} + 66(91.6 \text{ GeV} - m_Z) \pm 35 \text{ GeV} \quad (11)$$

and we now discuss the present status of this prediction using the electroweak data announced at the Neutrino 90 Conference [1].

For the comparison with the vector boson mass (VBM) measurements, we divide [9] the low-energy (LE) data into three sectors. The $\nu - q$ sector is dominated by deep inelastic νN and $\bar{\nu} N$ scattering, which are analyzed using the parton model and QCD. The principal uncertainty in analyzing these data is the appropriate value of the charm quark mass m_c to use: we take [10] $m_c = 1.45 \text{ GeV}$, which is close to the world's best value ($1.35 \pm 0.05 \text{ GeV}$) [11], consistent with the total cross-section data themselves ($1.5 \pm 0.3 \text{ GeV}$) [10] and consistent with data on dimuon production ($1.34 \pm 0.21^{+0.25}_{-0.04} \text{ GeV}$) [12]. From the $\nu - q$ data we extract the $u_{L,R}$ and $d_{L,R}$ effective couplings to the Z^0 , and thence

$$\sin^2 \theta_W = 0.229 \pm 0.004 \quad (12)$$

for $m_t = 100 \text{ GeV}$, with an additional error ± 0.004 if m_c is left free. As seen in Fig. 1, the central value of $\sin^2 \theta_W$ from the $\nu - q$ sector depends somewhat on m_t .

The $\nu - e$ sector is dominated by recent data on $\sigma(\bar{\nu}_\mu e)/\sigma(\nu_\mu e)$ [13], which yield

$$\sin^2 \theta_W = 0.240 \pm 0.012 \quad (13)$$

before the inclusion of radiative corrections. After their inclusion, the preferred value of $\sin^2 \theta_W$ decreases for $m_t > 60 \text{ GeV}$ [6,7,9].

The $e - q$ sector includes several sets of data: those on parity violation in atoms, $eN \rightarrow eX$ scattering, $\mu N \rightarrow \mu X$ scattering, and $e^+e^- \rightarrow \bar{q}q$ at lower-energy e^+e^- colliders [9]. The dominant role is now played by the Boulder experiment [13] on parity violation in Cesium, which gives

$$\delta \sin^2 \theta_W = \pm 0.007 \pm 0.003 \quad (14)$$

The first error in (14) is experimental, and the latter is theoretical. The central value of $\sin^2 \theta_W$ is also sensitive to m_t .

Figure 1 shows the $1 - \sigma$ ranges of $\sin^2 \theta_W$ as functions of m_t allowed by the different LE sectors. The LE data alone indicate [14]

$$m_t = 106^{+61}_{-\dots} \text{ GeV} \quad (15)$$

providing a non-trivial upper limit on m_t , but not a lower limit.

The latter is provided by the vector boson mass (VBM) measurements. The LE value of m_Z (1) gives the tight anticorrelation between $\sin^2 \theta_W$ and m_t shown in Fig. 2 [14]. Also shown is the range of $\sin^2 \theta_W$ allowed by the two most precise measurements of m_W/m_Z [15]:

$$\begin{aligned} m_W/m_Z &= 0.8775 \pm 0.0051 & (\text{CDF}) \\ &0.8831 \pm 0.0055 & (\text{UA2}) \end{aligned} \quad (16)$$

which together give

$$\sin^2 \theta_W = 0.225 \pm 0.007 \quad (17)$$

The VBM data by themselves give

$$m_t = 142^{+44}_{-55} \text{ GeV} \quad (18)$$

Combined with LE data, they give two independent lower bounds on m_t : one from the m_W/m_Z measurements (16) and the other from the incompatibility between the Z^0 measurement (1)

[14]

$$m_t = 122^{+41}_{-32} \text{ GeV} \quad (21)$$

A novelty is the fact that the precision electroweak data give for the first time [14] an independent indication on m_H , as seen in Fig. 5:

$$1.8 \text{ GeV} < m_H < 6 \text{ TeV} \quad (68\%c.l.) \quad (22)$$

which is mainly due to requiring consistency between the m_Z and ZD measurements, although a role is also played by the measurements of m_W/m_Z . We can expect these determinations of m_t (21) and m_H (22) to be improved by future precision measurements at LEP, as discussed in the next section.

The LEP measurements of Γ_{inv} are important not only for determining the number of light neutrinos, but also for constraining various massive dark matter candidates [17-19]. In the Standard Model,

$$\Gamma_Z = N_\nu \Gamma_\nu + 3\Gamma_L + \Gamma_h \quad (23)$$

and the peak hadronic cross-section is

$$\sigma_0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_e}{\Gamma_Z} \cdot \frac{\Gamma_h}{\Gamma_Z} (1 - \delta_{rc}) \quad (24)$$

where δ_{rc} is an electromagnetic radiative correction. Combining (23) and (24), we see that the peak cross-section changes by 13% for $\Delta N_\nu = 1$. Thus with good statistics ($\gtrsim 10^4 Z^0$) and good control of the total beam luminosity ($\lesssim 1\%$) one can hope to measure Γ_{inv} with a precision $\Delta N_\nu \lesssim 0.1$. Indeed, the LEP number reported at Neutrino 90 is [1]

$$N_\nu = 2.95 \pm 0.11 \quad (25)$$

averaged over all four experiments.

and the LE (particularly $\nu - q$) data at low m_t . They also give two more upper bounds on m_t : from compatibility between the LE data and m_Z on the one hand, and m_W/m_Z on the other hand. This is not yet the complete story, as recent LEP precision measurements of Z^0 decay widths add more information, as we now discuss.

2. Z^0 decay widths: implications for the top and Higgs masses, and for dark matter

LEP measurements [1] of the total Z^0 decay width Γ_Z , the hadronic decay width Γ_h , the leptonic decay width (assuming lepton universality) Γ_ℓ , the ratio Γ_h/Γ_ℓ , the invisible decay width Γ_{inv} , and the resulting equivalent number of neutrino species are shown in Table 2. Combined, they already yield $\Delta \sin^2 \theta_W \sim 0.003$ for any fixed value of m_t , and are more precise than all the $\nu - e$ and $e - q$ experiments, as seen in Fig. 2. As also seen there, measurements of Z^0 decays (ZD) are still less precise than measurements of m_Z , but will improve in precision as the Z^0 decay statistics improve.

Figure 3 shows some examples of Z^0 decay measurements compared with the theoretical predictions [16] as functions of m_t , including the uncertainties due to m_Z (1), m_H and α_s . Combining the Z^0 decay data with the measurement of m_Z , we find [14]:

$$m_t = 162^{+77}_{-111} \text{ GeV} \quad (19)$$

on the basis of LEP data alone.

The best estimate [14] of m_t comes from combining the LE, VBM and ZD data, as seen in

$$m_t = 127^{+24}_{-30} \text{ GeV}, \quad \sin^2 \theta_W = 0.2273^{+0.0033}_{-0.0031} \quad (20)$$

Fig. 4:

for $m_H = m_Z$. If m_H is allowed to vary between its direct experimental lower bound $m_H \simeq 40$ GeV and the upper theoretical expectation $m_H \simeq 1 \text{ TeV}$, this range is expanded somewhat to

This precision means [17] that any new neutral particle with neutrino-like couplings to the Z^0 must be very heavy, so that phase space suppresses its contribution to Γ_{inv} : $\Delta N_\nu < 0.17$ at the $2 - \sigma$ level. In particular, for Dirac neutrinos ($\nu \neq \bar{\nu}$) one finds

$$m_{\nu_D} > 44 \text{ GeV} \quad (26)$$

and for Majorana neutrinos ($\nu = \bar{\nu}$) one finds

$$m_{\nu_D} > 38 \text{ GeV} \quad (27)$$

As for sneutrinos, the conjectured supersymmetric spin-0 partners of sneutrinos, one finds

$$m_{\tilde{\nu}} > 42 \text{ GeV} \quad (28)$$

Conventional calculations of the relic abundances of massive particles then tell us [17] that such massive neutrinos cannot constitute the bulk of the dark matter in the Universe:

$$\Omega_{\nu_D} < 2 \times 10^{-5}, \quad \Omega_{\nu_M} < 2 \times 10^{-3} \quad (29)$$

unless $m_{\nu_D, \nu_M} \gg m_Z/2$. As for sneutrinos, the lower limit (28) is inconsistent with the upper limit $m_{\tilde{\nu}} < 10 \text{ GeV}$ imposed by unsuccessful underground searches using ^{76}Ge detectors [20]. Thus LEP has already killed three of the prime candidates for dark matter.

This type of LEP constraint on dark matter particles can be extended to other candidates, such as massive neutrino-like particles whose couplings to the Z^0 are suppressed by some mixing angle factor [18]. As we see in Fig. 6, Majorana particles of this type must have [18]

$$m_M > 14 \text{ GeV} \quad (30)$$

if they are to close the Universe for a present-day Hubble expansion rate $h_0 = 50 \text{ km}^{-1} \text{ s}^{-1} \text{ Mpc}^{-1}$, and Dirac particles must have [18]

$$m_D > 19 \text{ GeV} \quad (31)$$

if they are to close the Universe and also be consistent with the underground searches using ^{76}Ge detectors [20]. As for the lightest neutral spin- $\frac{1}{2}$ particles χ of supersymmetry, a coalition

of LEP and other data enforce [19]

$$m_\chi \gtrsim 10 \text{ to } 13 \text{ GeV} \quad (32)$$

Dark matter enthusiasts should caveat LEP!

Some information is also available from LEP and the SLC on Z^0 decays into heavy quarks [21]: $Z^0 \rightarrow \bar{b}b, \bar{c}c$, using either the semileptonic decay signature (b or c) $\rightarrow (e \text{ or } \mu) + \nu + X$, or the $D^* \rightarrow D\pi$ signature for charm. The available results are shown in Table 3. More accuracy will be required to probe the Standard Model beyond the tree level, and for this better knowledge will be needed of the branching ratios for the chosen signatures. There is also one measurement [22] of the forward-backward asymmetry for $Z^0 \rightarrow \bar{b}b$ decay: $A_{FB}^{b\bar{b}} = 0.145 \pm 0.52$ if one neglects $\bar{B}_0 B_0$ mixing. Here again, more accuracy will be needed to probe the Standard Model.

3. Future precision measurements: closing in on m_t and m_H

There are several other ways in which LEP will be able to probe the Standard Model beyond the tree level, which we now review [8].

3.1. FORWARD-BACKWARD LEPTON ASYMMETRY

The forward-backward asymmetry for leptons on the Z^0 peak is given at the tree level by

$$A_{FB} = \frac{3g_V^2 g_A^2}{g_V^2 + g_A^2} = \frac{3(1 - 4 \sin^2 \theta_W)^2}{1 + (1 - 4 \sin^2 \theta_W)^2} \quad (33)$$

It is quite a good approximation to the radiation corrections to use (33) with $\sin^2 \theta_W$ replaced by an effective $\sin^2 \theta_W$ on the Z^0 peak. We can deduce from (33) a sensitivity

$$\delta A_{FB} \simeq 24(1 - 4 \sin^2 \theta_W) \delta \sin^2 \theta_W \simeq 2 \cdot \delta \sin^2 \theta_W \quad (34)$$

for $\sin^2 \theta_W \simeq 0.23$, so that measuring A_{FB} with a precision of 0.003, which would require $10^6 Z^0$'s, would give a precision ± 0.0015 in $\sin^2 \theta_W$. The situation so far [1] is shown in Table

4, and corresponds to

$$\overline{\sin^2 \theta_W} = 0.2314 \pm 0.0035 \quad (35)$$

Given the expected difference between $\sin^2 \theta_W$ as defined by $1 - m_W^2/m_Z^2$ and $\overline{\sin^2 \theta_W}$, this value is completely compatible with the data discussed in previous sections, but the asymmetry data by themselves are not yet competitive. However, with increased statistics, the measurement of A_{FB} will refine significantly our present “fix” on m_t , as seen in Fig. 7.

3.2. τ POLARIZATION

This is measurable in many modes of τ decay: $e\bar{\nu}\nu, \mu\bar{\nu}\nu, \rho\nu, \alpha_1\nu$ and especially $\pi\nu$. The tree-level polarization is

$$P_\tau = \frac{2g_V g_A}{g_V^2 + g_A^2} = \frac{4(1 - 4\sin^2 \theta_W)}{1 + (1 - 4\sin^2 \theta_W)^2} \quad (36)$$

and has a sensitivity

$$\delta P_\tau \simeq 8 \cdot \delta \sin^2 \theta_W \quad (37)$$

Hence a measurement of P_τ with a precision of 0.02, which seems feasible with $10^6 Z^0$'s, would give $\delta \sin^2 \theta_W = \pm 0.003$. However, no measurements of P_τ have yet been reported. One way of expressing the interest in P_τ is shown in Fig. 8. The Standard Model gives a very tight correlation between the allowed values of A_{FB} and P_τ that is independent of m_t . Any deviation from this line would be evidence for physics beyond the Standard Model.

3.3. POLARIZATION ASYMMETRY

With polarized $e^\pm$ beams, as planned at the SLC and hoped for at LEP, one can measure more asymmetries, including for example the polarized forward-backward asymmetry [23]

$$A_{FB}^{ff} = \frac{3}{4} A_f \frac{(1 - P^+ P^-) A_e + (P^+ - P^-)}{(1 - P^+ P^-) + (P^+ - P^-) A_e} \quad (38)$$

where

$$A_f \equiv \frac{2g_V g_{A_f}}{g_V^2 + g_{A_f}^2} \quad (39)$$

Most interesting [24], however, is likely to be the overall left-right polarization asymmetry

$$A_{LR} = \frac{\sigma(L) - \sigma(R)}{\sigma(L) + \sigma(R)} = A_e = \frac{2(1 - 4\sin^2 \theta_W)}{1 + (1 - 4\sin^2 \theta_W)^2} \quad (40)$$

which has a sensitivity

$$\delta A_e \simeq -8 \cdot \delta \sin^2 \theta_W \quad (41)$$

to $\sin^2 \theta_W$. Assuming $\delta A_e = \pm 0.003$ which should be feasible with 10^6 polarized Z^0 's, one obtains $\delta \sin^2 \theta_W = 0.0004$, which is much more precise than from the forward-backward asymmetry or τ polarization.

3.4. MEASURING m_W

It is apparent from Fig. 2 that a precise determination of m_W would make a big contribution to fixing m_t and m_H via radiative corrections. The most precise determination m_W will come from the $e^+e^- \rightarrow W^+W^-$ threshold measured at LEP. There are in principle four different ways to measure m_W there that can be combined: (a) the threshold cross-section, (b) the end-point of the $W \rightarrow e\nu$ spectrum, (c) the jet-jet invariant mass from $W \rightarrow \bar{q}q$, (d) the $e - \nu$ invariant mass from $W \rightarrow e\nu$. It is believed that a combination of these methods could determine m_W with a precision $\delta m_W \leq 100$ MeV.

The potential impact [23] of this measurement is shown in Fig. 9. With the expected experimental errors in A_{LR} and m_W that are shown, it should be possible to fix independently m_t and m_H if direct searches do not get there first!

4. Searches for a Higgs boson

The dominant process for Standard Model Higgs production in Z^0 decay is $Z^0 \rightarrow H + (Z^{0*} \rightarrow \bar{q}q, \bar{\nu}\nu, \ell^+\ell^-)$ [25], with $Z^0 \rightarrow H + \gamma$ [26] also important for $m_H \gtrsim 50$ GeV. The total theoretical cross-section for $Z^0 \rightarrow H + Z^{0*}$ at the resonance peak is shown in Fig. 10. The dominant Z^{0*} decay is to $\bar{q}q$, which is difficult to detect. The cleanest signature would be from $Z^{0*} \rightarrow \ell^+\ell^-$, but this has a small branching ratio. So far, the greatest sensitivity has been obtained with $Z^{0*} \rightarrow \bar{\nu}\nu$. If $0 \leq m_H \lesssim 100$ MeV, the Higgs is long-lived and may decay outside the detector. If 100 MeV $\lesssim m_H \leq 2m_\mu$, the dominant H decays are to e^+e^- and $\gamma\gamma$. If $2m_\mu < m_H < 2m_c$, the dominant ($\gtrsim 75\%$) H decays contain at least two charged tracks ($\mu^+\mu^-, \pi^+\pi^-, K^+K^-, \dots$). If $2m_c < m_H$, multihadron $H \rightarrow$ heavy $\bar{q}q$ pair decays dominate ($\gtrsim 90\%$).

The ranges of m_H excluded so far by the different experiments are [27]:

$$\begin{array}{ll} \text{ALEPH [28]:} & 0 \leq m_H < 41.6 \text{ GeV} \\ \text{DELPHI [29]:} & 2m_\mu < m_H \leq 26 \text{ GeV} \\ \text{L3 [30]:} & 3 \text{ GeV} < m_H < 32 \text{ GeV} \\ \text{OPAL [31]:} & 0 \leq m_H < 32 \text{ GeV} \end{array} \quad (42)$$

The negative result of one of these searches for $m_H > 10$ GeV is shown in Fig. 11*. Future searches on the Z^0 peak should be able to get above $m_H = 50$ GeV, and searches at LEP II should be able to go beyond $m_H = 80$ GeV.

How do these bounds (42) compare with theoretical constraints on m_t and m_H ? There are three types of such constraints [32], illustrated in Fig. 12.

Finiteness of couplings The top quark Yukawa couplings λ_t and the Higgs self-coupling are not asymptotically free, which means that they increase at higher energies. One could then either demand that the couplings remain perturbative up to some cut-off Λ [33], or simply that they remain finite at scales up to Λ [34]. In practice, there is not much difference between these

* There have also been negative searches [27] for the associated production of neutral supersymmetric Higgs bosons in Z^0 decay, with sensitivity up to ~ 40 GeV.

two variants, because the couplings have an approximate infra-red fixed point. However, the bound does depend sensitively [35] on Λ . For example, one finds for

$$\begin{array}{llll} \Lambda = m_P, & 10^{10} \text{ GeV}, & 1 \text{ TeV}, & m_H: \\ m_H \lesssim 200 \text{ GeV}, & 250 \text{ GeV}, & 500 \text{ GeV}, & 650 \text{ GeV} \end{array} \quad (43)$$

The last of these is equivalent to the lattice bound [36] that $m_H \lesssim 1/\Delta$, where $\Delta = 1/\Lambda_{\text{lattice}}$ is the lattice spacing. We have chosen $\Lambda = 10^{16}$ GeV in Fig. 12.

Cosmological entropy generation

There is a lower bound on m_H in the region of small m_t , the so-called Coleman-Weinberg [37] region. If one supposes in the effective tree-level Higgs potential

$$V(H) = -\frac{1}{2}\mu^2 |H|^2 + \lambda_H |H|^4 \quad (44)$$

that $\mu^2 = 0$, then radiative corrections determine the Higgs v.e.v. v and fix [37]

$$m_H^2 = m_{CW}^2 = \frac{3\alpha_{em}v^2}{4\sin^2\theta_W} \left(1 + \frac{1}{2}\sec^4\theta_W - 0\left(\frac{m_f}{m_W}\right)^4\right) \quad (45)$$

which gives $m_H \sim 9.3$ GeV when $\sin^2\theta_W = 0.23$, increasing to $m_H = 10.4 \pm 0.3$ GeV when two-loop corrections are included [38], if $m_f \ll m_W$. In fact, we know that m_t is not negligible, and this value becomes

$$m_H = m_{CW} \times \sqrt{1 - \left(\frac{m_t}{83 \text{ GeV}}\right)^4} \quad (46)$$

for large $m_t \leq 83$ GeV. (We will see shortly what happens for larger m_t .) The value (45) or (46) is not necessarily a lower bound on m_H , since our electroweak vacuum would be stable for [39]

$$m_H \geq m_{LW} = m_{CW}/\sqrt{2} \quad (47)$$

However, it turns out that the electroweak phase transition in the early Universe would generate too much entropy if m_H were between m_{LW} and a value close to m_{CW} [40]. Therefore, in Fig. 12, we have taken (46) as the lower bound on m_H for $m_t \leq 83$ GeV, marked by CW.

(Meta)stable Higgs potential When m_t is sufficiently large for fixed m_H , the effective Higgs potential acquires a second minimum at some large Higgs v.e.v. which is energetically preferred over the conventional electroweak vacuum. The requirement [41] that this lower minimum does not appear at $| < 0 | H | 0 > | < \Lambda$ is also shown in Fig. 12. The requirement that our vacuum be the global minimum of the effective potential may be imposed by wormhole dynamics [42], in which case one should lie above and to the left of the line S in Fig. 12. However, even an unstable vacuum would be phenomenologically acceptable if the Universe would have cooled into it during its expansion and then stayed there for at least 10^{10} y. This metastability requirement on spontaneous decay of the electroweak vacuum is also shown in Fig. 12. There is one other danger to be addressed, though, namely the possibility that a high-energy cosmic ray collision or other event could have triggered vacuum decay [43]. Perturbative electroweak dynamics could presumably [44] not have stimulated vacuum decay, but this appears possible [42] via strong non-perturbative electroweak interactions below and the right of the line marked CR in Fig. 12.

We see that the direct lower bounds on m_t and m_H , and the $1 - \sigma$ radiative correction constraints mentioned in Section 2 are compatible with either of three possibilities. Our Universe could be absolutely stable, or it could be vulnerable to cosmic ray collisions in the future, or it should have collapsed already if some ideas about strong electroweak interactions at high energies were correct. It will be interesting to see which option Nature has chosen!

5. Tests of QCD

These have been made at LEP by using the rates of 2-, 3-, 4- and even 5-jet events, by measuring energy-energy correlations, and by looking at angular correlations in 4-jet events [45]. The first two classes of test prove the running of the QCD coupling α_s as predicted on the basis of asymptotic freedom, as seen in Fig. 13, while the latter confirm the spin and colour structure of non-Abelian QCD. The resulting determinations of α_s include many systematic uncertainties, some of which were previously neglected, related to fragmentation and to the choices of renormalization scale and jet recombination scheme. Some of the determinations of $\alpha_s^{\overline{MS}}(m_Z)$ reported at the Neutrino 90 Conference [45] are shown in Table 5. On the basis of

these, the rapporteur on QCD at that conference concluded that [45]

$$\alpha_s^{\overline{MS}}(m_Z) = 0.115 \pm 0.016 \quad (48)$$

which we will combine with electroweak data in the analysis of GUT predictions in the last section.

6. Beyond the Standard Model

A remarkable feature of the LEP data is their high level of consistency with supersymmetric GUT and string ideas. In a supersymmetric GUT, one expects the $SU(3)$, $SU(2)$ and $U(1)$ couplings to become equal at some large mass-scale m_{GUT} , from which they are renormalized to different values at present energies. From the value of $\alpha_s^{\overline{MS}}(m_Z)$ (48) measured at LEP, one can use a minimal supersymmetric GUT to calculate [46]

$$\sin^2 \theta_W^{\overline{MS}}(m_Z) = 0.235 \pm 0.004 \quad (49)$$

as seen in Fig. 14, to be compared with the value

$$\sin^2 \theta_W^{\overline{MS}}(m_Z) = 0.233 \pm 0.001 \quad (50)$$

extracted from low-energy data. (This $\overline{MS}$ definition of $\sin^2 \theta_W$ is expected [8] to be about 3% larger than the mass-shell definition of $\sin^2 \theta_W$ used in Sections 1 and 2.) To judge the significance of the minimal supersymmetric GUT prediction (49), we note that the minimal non-supersymmetric GUT prediction is [47]

$$\sin^2 \theta_W^{\overline{MS}}(m_Z) = 0.216 \pm 0.003 \quad (51)$$

which is certainly incompatible with the LEP data.

Moreover, in a minimal supersymmetric GUT with $m_b = m_\tau$ at m_{GUT} , as in many simple models [48], one can calculate the renormalization of the ratio m_b/m_τ using the LEP values of α_s and m_t to give [46]

$$m_b = 5.2 \pm 0.3 \text{ GeV} \quad (52)$$

on mass-shell, as seen in Fig. 15, to be compared with the potential model estimate

$$m_b = 5.0 \pm 0.2 \text{ GeV} \quad (53)$$

This prediction relies on the fact that there are only three generations, and was even used long before the LEP determination of N_ν to predict [48] that $N_\nu = 3$.

Even more strikingly, the value of m_{GUT} calculated in a minimal supersymmetric GUT (see Fig. 14) is about 2×10^{16} GeV, which is very close to the unification scale $m_{SU} \sim 10^{17}$ GeV predicted by string models [49]. One can even obtain $m_{GUT} \simeq m_{SU}$ either by adjusting the parameters (moduli) of string compactification to an effective four-dimensional theory, or by adding a few particles to the minimal supersymmetric GUT [46]. In general, such a modification of the spectrum would give a value of $\sin^2 \theta_W$ that agreed less well with experiment, as seen in Fig. 16. However, there do exist particular choices of extra representations that do retain the successful prediction (49). But in these cases m_b turns out about $\frac{1}{2}$ GeV higher than the successful prediction (52). Therefore it seems that the data would prefer us to adjust the moduli, if we want to obtain $m_{GUT} = m_{SU}$, although this is not necessary in some models [50].

One other comment on the successful prediction (49) of $\sin^2 \theta_W$ is in order here. The experimental value (50) was extracted using only the radiative corrections in the Standard Model, and one might wonder whether the preferred value of $\sin^2 \theta_W$ (or m_t) could be substantially altered by including supersymmetric loop diagrams. In fact, we find [51] that for $m_{\tilde{g}} \gtrsim 150$ GeV as suggested by the CDF Collaboration [5], and $\tan \beta \gtrsim 1.6$ as required by the unsuccessful LEP search for a light Higgs boson [27], the preferred value of $\sin^2 \theta_W$ is essentially unchanged, while the preferred central value (upper limit) on m_t might be reduced by at most 10(20) GeV.

Supersymmetric GUTs and string even give two independent reasons why m_t should be large, as suggested by the analysis of electroweak radiative corrections. One reason is that

the large Yukawa coupling λ_t of a heavy top quark could even be responsible for electroweak symmetry breaking. This is because a large λ_t could generate the negative mass-squared term $-\mu^2/2$ required in the effective Higgs potential (44) [52]. In most supersymmetric models, the many spin-0 masses-squared are initially positive at some large renormalization scale $\sim m_{GUT}$ or m_{SU} , but the physical values are then renormalized by gauge and Yukawa couplings. The effect of gauge couplings is to increase spin-0 masses-squared as one descends to lower energies, but Yukawa couplings tend to decrease them. In particular, a large Yukawa coupling of the Higgs to the top quark could drive its mass-squared negative as required (44). This is possible for

$$90 \text{ GeV} < m_t < 170 \text{ GeV} \quad (54)$$

depending on the initial ratio of the scalar m_0 and gaugino masses $m_{\frac{1}{2}}$ at m_{GUT} or m_{SU} .

Quite independently of this argument, string models also give a reason why m_t should be large. There are many different string models with different characteristics, but it is a generic feature that non-zero Yukawa couplings are comparable to gauge couplings

$$\lambda = c \cdot g \quad (55)$$

at the Planck scale, where c is a model-dependent constant. Specifically, in the flipped $SU(5)$ GUT one finds [50] that

$$\lambda_t = \sqrt{2} \cos \theta_t \cdot g \quad (56)$$

where θ_t is some as yet undetermined mixing angle. The leading-order renormalization group equation for λ_t is

$$\frac{d\lambda_t}{dt} = \frac{\lambda_t}{8\pi^2} \left[3\lambda_t^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{39}{90}g_Y^2 \right] \quad (57)$$

which has an approximate infra-red fixed point

$$\lambda_{t,IR}^2 = \frac{1}{3} \left(\frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{39}{90}g_Y^2 \right) \quad (58)$$

Since the initial value (56) of λ_t is less than $\lambda_{t,IR}$ (58), so also is the physical value of λ_t . This

means that

$$m_t < \frac{\sqrt{2}m_w}{g_2} \lambda_{t,R} = m_W \sqrt{\frac{2}{3}} \left[\frac{8}{3} \left(\frac{\alpha_3}{\alpha_2} \right) + \frac{3}{2} + \frac{13}{18} \tan^2 \theta_W \right]^{\frac{1}{2}} \quad (59)$$

which is about 220 GeV. Actual computations using the full renormalization group equations give [50] a slightly stronger upper bound

$$m_t < 190 \text{ GeV} \quad (60)$$

and one expects m_t to be more than half this value unless the mixing angle factor $\cos \theta_t$ in (56) is very small.

A value of m_t near the middle of the range preferred by LEP data corresponds to $\lambda_t \simeq 1/\sqrt{2}$ in the Standard Model, whereas the electron with mass 0.51 MeV has $\lambda_e = 1.5 \times 10^{-6}$. Such a small Yukawa coupling is technically natural, in that it is stable with respect to radiative corrections, but its origin is more of a puzzle than a large value of m_t . From the point of view of supersymmetric GUTs and string models, the key problem of the fermion mass spectrum is not

WHY IS THE TOP QUARK SO HEAVY?

but rather

WHY ARE THE OTHER QUARKS SO LIGHT?

ALEPH	DELPHI	L3	OPAL	MARK II
91.194(16)	91.182(21)	91.148(17)	91.163(17)	91.14(12)
91.172(9)(30)(scale error)				

Table 1
Measurements of m_Z (GeV)

	ALEPH	DELPHI	L3	OPAL	MARK II
Γ_Z (GeV)	2.498(31)	2.495(40)	2.502(33)	2.497(34)	2.42($^{45}_{35}$)
Γ_h (GeV)	1.753(27)	1.731(52)	1.733(44)	1.804(44)	1.56($^{28}_{24}$)
Γ_t (MeV)	84.4(1.3)	83.2(3.0)	84.3(1.3)	83.1(1.9)	
Γ_h/Γ_t	20.77(39)	20.8(0.6)	20.56(60)	21.72($^{71}_{65}$)	18.9($^{3.8}_{3.0}$)
Γ_{inv} (MeV)	492(25)	514(46)	516(42)	474(43)	460(100)
N_ν	2.80(12)	2.93(21)	3.04(14)	3.09(19)	2.8(0.6)

Table 2
Precision measurements at the Z^0 peak

	ALEPH	DELPHI	L3	MARK II	Standard Model
Γ_{bb}^* (MeV)	377(52)		378($^{17}_{16}$)(15)(33)	430($^{210}_{170}$)	378(2)
Γ_{cc}^* (MeV)	259(104)	282(56)(55)			297(3)

Table 3
Measurements of Z^0 decays into heavy quarks, including both statistical and systematic errors.

	ALEPH	L3	OPAL	MARK II
g_V^2	0.0013(9)	0.0040(26)	0.0011(21)	0.05(30) ⁽²⁴⁾
g_A^2	0.251(3)	0.250(5)	0.250(6)	0.31(31) ⁽⁴³⁾ ₍₃₀₎

Table 4
Measurements of Z^0 couplings to leptons

	DELPHI	L3	OPAL	MARK II
Jet rates	0.114(3)(4)(12)	0.114(5) ⁽¹³⁾ ₍₆₎	0.116(16)	0.123(9)(5)
Energy-energy correlations	0.106(6)(7)			

Table 5
Measurements of α_s at the Z^0 peak,
including both statistical and systematic errors.

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FIGURE CAPTIONS

- 1) One-standard-deviation error bands [14] on $\sin^2\theta_W$ as functions of m_t for the low-energy (LE) data. We show the $\nu - q$ sector, the $\nu - e$ and $e - q$ sectors combined, and the region allowed by the complete set of LE data. We have assumed $m_c = 1.45$ GeV and $m_H = m_Z$.
- 2) One-standard-deviation error bands [14] on $\sin^2\theta_W$ as functions of m_t assuming $m_H = m_Z$ for the high-energy (HE) data. We show the bands corresponding to the LEP measurements of the Z mass (m_Z) and Z decays (ZD). The error on $\sin^2\theta_W$ from measurements of m_W/m_Z is shown as a vertical error bar. Shown is also the region allowed by the complete set of HE data.
- 3) Comparison [16] of theoretical and experimental values of (a) Γ_Z , (b) Γ_t as functions of m_t .
- 4) χ^2 distributions [14] as functions of m_t assuming $m_H = m_Z$ for the $\nu - q$, VBM, HE, LE and all data.
- 5) $\Delta\chi^2 = 1$ contour in the (m_H, m_t) plane [14]: note the 68.3 % C.L. lower and upper limits on m_H of 1.8 GeV and 6 TeV, respectively. All the data are included.
- 6) Values [18] of mass m and Z^0 coupling strength $\sin^2\phi_Z$ relative to a conventional neutrino consistent with LEP [1] and giving $\Omega = 1$ for $h_0 = 0.5$ or 0.7 : (a) for Majorana fermions and (b) for Dirac fermions. In the latter case the constraint from underground ^{76}Ge experiments [20] is also shown.
- 7) Sensitivity [8] of a possible future lepton forward-backward asymmetry measurement A_{FB} to m_t and m_H .
- 8) Correlation [8] between values of A_{FB} and P_τ expected in the Standard Model, independently of m_t .
- 9) Implications [23] of possible A_{LR} and m_W measurements for m_t and m_H .
- 10) Peak cross-section [8] for $e^+e^- \rightarrow Z^0 \rightarrow H + Z^0$.

11) Upper limit on the number of $Z^0 \rightarrow H + Z^0$ events in one experiment [28] compared with the theoretical predictions as a function of m_H .

12) Bounds [42] on m_t and m_H due to cosmological entropy generation (CW) [40], stability [41] of the Higgs potential (S), metastability with a lifetime $> 10^{10} y$ (MS) and decay triggered by cosmic rays (CR) [43, 44].

13) Evidence from LEP for the running of the QCD coupling strength α_s [45].

14) Unification scales estimated [46] by integrating the two-loop renormalization group equations from $\bar{m} = m_Z$ upwards. The dotted lines are those of m_{32} , where $\alpha_3 = \alpha_2$. The solid line is that of m_{GUT} , where $\alpha_3 = \alpha_2 = \alpha_1$. The LEP ranges of $\sin^2 \theta_W^{MS}(m_Z)$ (2) and $\alpha_3(m_Z)$ (3) are shown on the vertical and horizontal axes respectively. The error bars emanating from the $m_{GUT} = 2 \times 10^{16}$ GeV point indicate the effects of varying the supersymmetry breaking parameters m_0 and $m_{1/2}$.

15) Values of m_b estimated [46] by integrating the two-loop renormalization equations including Yukawa couplings, assuming equal couplings for the b quark and the τ lepton at m_{GUT} . The solid (dashed) lines are for $\tan\beta = 2(5)$. Also shown is the domain of $(\alpha_3(m_Z), m_t)$ allowed by the LEP data for $\sin^2 \theta_W^{MS}(m_Z)$ and m_t , and assuming the GUT relation [46] between $\sin^2 \theta_W^{MS}(m_Z)$ and $\alpha_3(m_Z)$.

16) Values of $\sin^2 \theta_W^{MS}(m_Z)$ and $\alpha_3(m_Z)$ obtained [46] assuming $m_{GUT} = m_{SU} = 4 \times 10^{17}$ GeV and incorporating $[n, m]$ extra (3,1) and (3,2) representations of $SU(3) \times SU(2)$ at some intermediate scale m_I . The tickmarks on the [4,2] line indicate the corresponding values of m_b . The LEP ranges of $\sin^2 \theta_W^{MS}(m_Z)$ and $\alpha_3(m_Z)$ are indicated on the axes.

- Figure 1 -

- Figure 2 -

- Figure 3a -

- Figure 3b -

- Figure 4 -

- Figure 5 -

- Figure 8 -

- Figure 9 -

- Figure 6 -

- Figure 7 -

- Figure 10 -

- Figure 11 -

- Figure 12 -

- Figure 13 -

- Figure 16 -

- Figure 14 -

- Figure 15 -