Peculiar features of proton electromagnetic form factors

E. Tomasi-Gustafsson\textsuperscript{1}, A. Bianconi\textsuperscript{2} and Y. Wang\textsuperscript{3}

\textsuperscript{1} CEA, IRFU, SPhN, Saclay, 91191 Gif-sur-Yvette Cedex, France
\textsuperscript{2} Università degli Studi di Brescia and INFN, 25133 Brescia, Italy
\textsuperscript{3} CNRS/IN2P3, Institut de Physique Nucléaire, UMR 8608, 91405 Orsay, France

Abstract
Electromagnetic hadron form factors are fundamental quantities which describe the internal structure of the hadron. Experimental programs are ongoing or foreseen at the world facilities to increase the precision and/or to extend the kinematical range of the measurements. The collected data call for a unified interpretation of form factors in the scattering (space-like) and annihilation regions (time-like). In the time-like region form factors are complex functions and their meaning is not so obvious. The imaginary part, driven by unitarity, possibly contains information on the early process of the hadron formation. We focus here on a peculiar oscillatory behavior recently observed in the data on electron-positron annihilation from the BABAR collaboration, in the near threshold region, carrying information on the early stages of the hadron formation.

1. Introduction
Electromagnetic hadron form factors (FFs) are fundamental quantities which describe the internal structure of the hadron (for a recent review see Ref. \cite{1}). Since decades the knowledge of electromagnetic form factors (EMFFs) of hadrons, their measurement and their description has been considered essential to access the internal structure of hadrons and to understand their dynamical properties as the charge and magnetic distributions. Surprising features have been observed during the last years, renewing the interest in the field. New observations in polarization experiments, have been possible due to experimental developments: high intensity polarized electron beams, high luminosity colliders, large solid angle, high resolution spectrometers and detectors, and in particular, proton and neutron polarimeters in the GeV region. FFs are generally measured through electron-proton elastic scattering, assuming that the interaction occurs through the exchange of one photon that carries a space-like (SL) four-momentum $q^2$ and are functions of one variable, only. The crossing symmetry related annihilation reactions, $e^+ + e^- \leftrightarrow \bar{p} + p$, access the time-like (TL) region of momentum transfer. Being FFs analytical functions, the scattering and annihilation regions are strongly related and nucleon models are required to incorporate the necessary analytical properties to describe FFs in all the kinematical region.

Due to unitarity, FFs are real functions in the scattering and complex functions in the annihilation region. The Phragmén-Lindelöf theorem for analytical functions requires them to coincide at the limit for $|q^2| \to \infty$ and then the imaginary part has to vanish.

Theoretically, FFs enter explicitly in the coupling of a virtual photon with the hadron electromagnetic current, and can be directly compared to hadron models which describe dynamical properties of hadrons. They are experimentally accessible through the knowledge of the differential cross section and the polarization observables. Efforts are presently directed, on one side, to increase the precision and, on the other side, to extend the kinematical range of the measurements.

The electromagnetic vertex $\gamma' \to hh$ (h is any hadron) is defined by two structure functions, which, in turn, are expressed in terms of $(2S + 1)$ FFs, $S$ being the hadron spin. Assuming parity and time-invariance, Protons (and neutrons) have two FFs, electric $G_E$, and magnetic $G_M$, which are
normalized at $q^2 = 0$ to the static values of the charge $G_E(0) = 1$ and of the magnetic moment, $G_M(0) = \mu$.

In the TL region where a hadron pair is formed by or annihilated into a virtual photon, the unpolarized cross section contains the squared moduli of the two FFs. The angular distribution allows, in principle, for the individual determination of FFs, but until now the luminosity has not been sufficient and the experimental results assume $|G_E| = |G_M|$ or $G_E = 0$. The recent data by the BABAR collaboration [2, 3] cover a region from the $\bar{p}p$ threshold to $q^2 \approx 36 \text{ GeV}^2$. Thirty data points have been extracted in the region $q^2 < 10 \text{ GeV}^2$, with a relative error lower than 10%. Regular structures have been found in these data [4]. These results are illustrated below and are interpreted in terms of an interplay between two steps in the annihilation process: the hadron formation from three bare quarks, taking place on a time scale $1/\sqrt{q^2}$, and a relatively small perturbation associated to rescattering processes taking place on a larger time scale.

Different theoretical models have been developed to describe the electromagnetic structure of the hadrons and applied to the calculation of SL FFs. Some of them can be applied in all kinematical region, as vector meson dominance [5] or dispersion relations [6], where the complex nature of TL FFs arises naturally.

Recently a model was suggested to interpret nucleon electromagnetic FFs both in SL and TL regions [7] that gives a qualitative explanation for the observed oscillations. It assumes that in $e p$ elastic scattering or in the $e^+ e^- \rightarrow \bar{p} + p$ annihilation a large quantity of energy (mass) and momentum is concentrated in a small volume creating a strong gluonic field, i.e., a gluonic condensate of clusters with a randomly oriented chromo-magnetic field. Applied to the scalar part of the field, it explains the observed additional suppression of the electric FF, and leaves unchanged the predictions from quark counting rules for the magnetic FF. Similarly, in TL region, above the physical threshold, $q^2 \geq 4M^2$ ($M$ is the proton mass), the vacuum state created at the collision, transfers all the energy to a S-wave state with total spin 1, consisting in at least six massless valence quarks, a set of gluons and a sea of current $q\bar{q}$ quarks. The quarks as partons have no structure ($|G_E| = |G_M| = 1$), which may explain the observed point-like behavior of FFs at threshold. Then, the current quarks (antiquarks) absorb gluons and transform into constituent quarks (antiquarks). This model has one free parameter, in principle calculable, and it is expected to apply starting from moderate values of the momentum transfer. It gives a qualitative understanding of the experimental data, suggesting a generalization of the FF definition, where the meaning of FF in $e^+ e^-$ annihilation is the time evolution of the charge distribution of the newly formed hadron system.

2. Proton form factors: experimental status

An overview of selected data and models on proton form factors (FFs) is given in Fig. 1. For a complete review of data and references, see [1]. The physics content is highlighted below, following the $q^2$ axis.

2.1 The space-like region: $q^2 \leq 0$

Traditionally $e p$ elastic scattering is considered as the preferred way to investigate the internal structure of the proton. Assuming one-photon exchange and due to the $J^{PC} = 1^{--}$ nature of the virtual photon, the unpolarized cross section for electron hadron elastic interaction has a characteristic dependence on $\cot^2 \theta$ ($\theta$ is the electron scattering angle in the laboratory (Lab) system).

The measurement of the differential cross section at fixed $Q^2$, for different angles allows to extract the electric and magnetic FFs as the slope and the intercept, respectively, of this linear distribution. This is called the Rosenbluth method [8]. Backward $eN$-scattering determined by the magnetic FF only, that is weighted by a factor $\tau = Q^2/(4M^2)$ ($M$ is the proton mass) which makes the determination of
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Fig. 1: World data on proton FFs as a function of $q^2$. SL region: $G_M$ data (blue circles), $G_E$ data (red triangles) from unpolarized measurements [10] and from polarization measurements (green stars) [14]. Prediction of Ref. [7] for $G_E(G_M)$ solid, green (black) line). TL region: $|G_E| = |G_M|$ world data for $q^2 > 4M^2$ and prediction for $G_M$ from Ref. [7] (black, solid line). The prediction from Ref. [5] is the orange, dash-dotted line.

$G_E$ more imprecise when $Q^2$ increases.

Since the pioneering experiments of Hofstadter [9] several measurements of the unpolarized cross section for $ep$ elastic scattering have been performed. Radiative corrections, which depend on $\varepsilon$ and $Q^2$ too, become larger with $Q^2$, reaching up to 50%. They have been generally applied at first order in the electromagnetic fine constant $\alpha$, beyond the Born approximation ($\alpha^3$).

From unpolarized cross section measurements the determination of $G_E$ and $G_M$ has been done up to $Q^2 \simeq 8.8$ GeV$^2$ [10] and $G_M$ has been extracted up to $Q^2 \simeq 31$ GeV$^2$ [11] under the assumption that the electric FF vanishes ($G_E = 0$) or that it equals the magnetic FF, $G_M$, scaled by the proton magnetic moment $G_E = G_M/\mu$ (full circles in In Fig. 1).

Polarization phenomena were studied and developed by the Kharkov school since the mid of last century. In Refs. [12, 13] it was first pointed out that the polarized cross section contains the interference of the amplitudes, giving access to the sign of FFs (while the unpolarized cross section contains FFs squared) and being more sensitive to a small $G_E$ contribution. It was applied by the JLab GEp collaboration in a series of experiments, measuring precisely the ratio of the electric to magnetic FFs up to $Q^2 = 8.9$ GeV$^2$ [14] (green triangles-down in In Fig. 1) which is directly related to the ratio of the transverse over longitudinal polarization in the scattering plane of the recoil proton.

The Akhiezer-Rekalo method for the measurement of the FFs ratio gave very precise results, as it was expected, because, at first order, the beam helicity as well as the analyzing power of the proton polarimeter cancel, reducing the systematic errors. But the experiment showed also a surprising behavior: a monotone decreasing of the FFs ratio when $Q^2$ increases. An extrapolation of this tendency at large $Q^2$ may lead to the ratio passing through zero and even becoming negative. As $G_M$ is supposed to be well known from the unpolarized cross section, the present understanding is that $G_M$ follows a dipole ($Q^{-4}$) behavior and that $G_E$ follows a steeper decreasing. Recent unpolarized experiments confirm that unpolarized experiments give a FFs ratio consistent with unity (with a larger error as $Q^2$ increases) whereas polarized experiments deviate from unity as $Q^2$ increases. The reason has likely to be attributed to the contribution of higher order radiative corrections (for a recent discussion see Ref. [15]). Note that unpolarized data, selected in experiments where radiative corrections did not exceed
Fig. 2: Fits of Babar data, according to the four parametrizations (a) $F_R$, (b) $F_S$, (c) $F_{SC}$, (d) $F_{RP}$ (see text). For each insert: (top) the data of Babar are plotted, together with the parametrization (blue, dashed line) and the global fit (solid line); (bottom) the difference of the data and the parametrization are shown, together with the corresponding damped oscillation fit (solid red line). In Figs. b), c), and d), $F_R$ is shown for comparison (green dashed line).

20%, also show a deviation of the ratio from unity (see Fig. 9 of Ref. [1]).

It is expected that data on individual FFs in the TL region will help to clarify this issue.

2.2 The time-like region: $q^2 \geq 0$

The near threshold region is particularly intriguing. Several experiments have been performed. The Coulomb factor, plays a specific role at threshold, compensating the phase-space relative velocity. It turns out that the extrapolation of the cross section to threshold is consistent with $|G_E^p(4M^2)| = |G_E^p(4M^2)| = 1$, as in the case of a pointlike fermion [16].

The recent data on the generalized FF obtained by the BABAR collaboration [2, 3] from the initial state radiation reaction $e^+ + e^- \rightarrow \bar{p} + p + \gamma$ are very precise and extend with continuity from the threshold to $q^2 \simeq 37 \text{ GeV}^2$.

These data are well reproduced by the function $F_R$ proposed in [17] that we will use as reference, see Table 1. Other parametrizations have been proposed in the literature: - following scaling pQCD rules, $F_S$, [18]. - introducing a correction for $\alpha_s$ [19, 20] $F_{SC}$, and - a two-pole function $F_{RP}$ in frame of ADS/CFT [21]. Their forms and parameters are reported in Table 1.

In the following, instead of $q^2$, the relevant variable is taken as the 3-momentum $p$ of one of the two hadrons in the frame where the other one is at rest: $p = \sqrt{E^2 - M^2}$ with $E = q^2 / (2M) - M$. In this case, the structures shown by the data, become regularly spaced, indicating that a simple rescattering mechanism takes place. Therefore, we suggest that the annihilation process occurs in two steps, formation and rescattering, corresponding to different time scales. A consequence of this assumption is that the measured FFs can be fitted by a function: $F(p) = F_0(p) + F_{\text{osc}}(p)$ that is the sum of a regular "background" $F_0$ and an oscillating function $F_{\text{osc}}(p)$.

Since the rescattering mechanism is not known, we cannot identify the sources of rescattered waves, but we may gain some clue on their space distribution. Let $r$ be the space variable that is conjugated to $p$ via three-dimensional Fourier transform. We may identify $r$ as the distance between the centers of the two forming or formed hadrons, in the frame where one is at rest. Let $M_0(r)$ and $M(r)$ be the Fourier transforms of the regular background fit and of the complete fit, Fig. 3. The most relevant feature is that $M_0(r)$ decreases by 7 orders of magnitude for $r$ ranging from 0 to 2 fm. The decrease is regular and almost constant on a semilog scale. $M_0(r)$ is steep near the origin, too. It can
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Table 1: Fit functions and parameters

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Model</th>
<th>$A \pm \Delta A$ [GeV]$^{-1}$</th>
<th>$B \pm \Delta B$ [GeV]$^{-1}$</th>
<th>$C \pm \Delta C$</th>
<th>$D \pm \Delta D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[17]</td>
<td>$</td>
<td>F_R</td>
<td>= \frac{\mathcal{A}}{(1 + q^2/m_1^2) [1 - q^2/0.71]^2}$</td>
<td>$0.05 \pm 0.01$</td>
<td>$0.7 \pm 0.2$</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{A} = 7.7$ GeV$^{-4}$, $m_1^2 = 14.8$ GeV$^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[18]</td>
<td>$</td>
<td>F_S</td>
<td>= \frac{\mathcal{A}}{(q^2)^2 \log^2(q^2/\Lambda^2)}$</td>
<td>$0.05 \pm 0.01$</td>
<td>$0.7 \pm 0.2$</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{A} = 40$ GeV$^{-4}$, $\Lambda = 0.45$ GeV$^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[19, 20]</td>
<td>$</td>
<td>F_{SC}</td>
<td>= \frac{\mathcal{A}}{(q^2)^2 \left[ \log^2(q^2/\Lambda^2) + \pi^2 \right]}$</td>
<td>$0.05 \pm 0.01$</td>
<td>$0.6 \pm 0.2$</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{A} = 72$ GeV$^{-4}$, $\Lambda = 0.52$ GeV$^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[21]</td>
<td>$</td>
<td>F_{TP}</td>
<td>= \frac{\mathcal{A}}{(1 - q^2/m_1^2)(2 - q^2/m_2^2)}$</td>
<td>$0.1 \pm 0.01$</td>
<td>$1.0 \pm 0.2$</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{A} = 1.56$, $m_{1,2}^2 = 1.5, 0.77$ GeV$^2$</td>
<td></td>
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be interpreted by the fact that both $F_0(p)$ and its transform $M_0(r)$ are expression of that short distance quark-level dynamics [22, 23] that permits exclusive $\bar{p} p$ production at the condition that the final quarks and antiquarks are formed within a small region. Near threshold, the size of this region is $\leq 0.1$ fm, much smaller than the standard hadron size.

In the right panel of Fig. 3 $M(r)$ is superimposed to $M_0(r)$. These two functions coincide for $r < 0.7$ fm, indicating that the physical reason of the data oscillation is related to processes occurring at a scale: $r \simeq 0.7$-1.5 fm, corresponding to the largest annihilation probability in the phenomenological $\bar{p} p$ interactions in the near-threshold region [24]. At a distance of 1 fm, the relevant part of rescattering must involve physical or almost physical hadrons that annihilate into groups of 2-10 mesons.

3. Conclusions

After giving a fast review on the world data on proton FFs, in SL and TL regions, we have illustrated a systematic modulation pattern in the TLFF measured by the BABAR collaboration in the range $q^2 < 10$ GeV$^2$. This modulation presents periodic features with respect to the momentum $p$ associated with the relative motion of the final hadrons. It suggests an interference effect involving rescattering processes at moderate kinetic energies of the outgoing hadrons. Such processes take place when the centers of mass of the produced hadrons are separated by $\simeq 1$ fm. For this reason at least a relevant part of rescattering must consist of interactions between phenomenological or almost phenomenological protons and antiprotons.

Precise measurements in the near threshold region are ongoing at BESIII (BEPCII), on the proton as well as on the neutron, bringing a new piece of information. The measurement of TL FFs in a large $q^2$ range will be possible at PANDA (FAIR). In the SL region, programs at MAMI (Mainz) and at Jefferson Lab are planned to increase the precision and/or the kinematical range of the data. These experimental efforts will motivate the development of those models that apply to the whole kinematical region, for a unified vision of TL and SL form factors bringing to a better understanding of the internal dynamics of the hadron electromagnetic structure.
Fig. 3: (Left) Fourier transform of the background $M_{0}(r)$ (Right): total $M(r)$ (solid line) and $M_{0}(r)$ (dashed line) for comparison (linear vertical scale).

References