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Scalar Top Production in e^+e^- Annihilation

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ABSTRACT

We discuss the pair production of light scalar top quarks in e^+e^- annihilation. The lighter mass eigenstate $\tilde{t}_1$ is a mixture of SU(2) doublet and singlet $\tilde{t}$ states and can have a vanishing coupling to the Z boson. We show that the present data on the total decay width of the Z^0 can put a bound on the $\tilde{t}_1$ mass for some values of the mixing angle, but even existing dedicated search for squarks leaves a large portion of the parameter space unconstrained. We include both initial-state QED and final-state QCD corrections in our results for the cross section; even quite far away from the threshold the total corrections can lie between -15% and $+70\%$.

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1. Introduction

A lot of effort has been devoted in recent years to the search for supersymmetric partners^[1] of the well-known quarks, leptons, and gauge bosons of the Standard Model. Unfortunately, no positive signal has so far been established. Very recently, the Mark II Collaboration at the SLC and the four LEP experiments have pushed^[2-6] the bounds on the masses of almost all charged sparticles, as well as for scalar neutrinos, to the vicinity of $\frac{1}{2}m_Z$. Furthermore, the CDF Collaboration has announced^[7] preliminary bounds on the mass of the gluino and of the squarks of about 150 GeV, assuming that the twelve squark states are degenerate in mass. However, to our knowledge *no* experimental bound on the superpartners of the top quark (stop) has ever been published. This criticism is not at all academic, since most models^[1] allow the lighter stop to be considerably lighter than the other squarks; its mass could even be quite close to the mass of the lightest supersymmetric particle (LSP), which is usually assumed to have no electromagnetic or color interactions and to be stable. In this case a light stop might be difficult to detect at hadron colliders.

Some limits on squarks from e^+e^- colliders allow the possibility that one flavor of squarks is lighter than the others. However, they still assume that the left and right squarks are degenerate. On the other hand, the two stop mass eigenstates are expected to have a substantial mass splitting. The lighter mass eigenstate is a mixture of the left and right stops and its coupling with the Z^0 can even *vanish* for a particular mixing angle.

In this note we discuss the production of a pair of light stops at e^+e^- colliders that operate at or above the Z resonance. The possible vanishing of the Z coupling forces us to include the photon-exchange contribution to the cross section even at $\sqrt{s} = m_Z$. A realistic estimate of the relative importance of γ - and Z -exchange contributions then necessitates the inclusion of initial-state radiation. We also include final state QCD corrections, which are quite a bit larger than for ordinary (fermionic) quarks.

The remainder of this paper is organized as follows. In Sec. II we briefly discuss the diagonalization of the stop mass matrix. In Sec. III we discuss stop production in Z decays; we find that some combinations of masses and mixing angles can already be excluded from the experimental upper bound on the total width of the Z boson. In Sec. IV we present results for the stop pair production cross section at and beyond the Z resonance. Finally, Sec. V contains a brief summary of our results and our conclusions.

2. Stop Mixing

The mass matrix for the two scalar tops in the basis $(\tilde{t}_L, \tilde{t}_R)$ can be written as^[8]

$$\mathcal{M}_t^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 & m_t a \\ m_t a & m_{\tilde{t}_R}^2 \end{pmatrix}, \quad (2.1)$$

where we have used the notation of Ref. 9, which we will follow throughout our paper. Here, a is expected^[1] to be of the order of the average squark mass. One can see from Eq. (2.1) that the mixing of the left and right stops can be substantial unlike the case for the superpartners of lighter quarks. The lighter physical eigenstate $\tilde{t}_1$ can be written as

$$\tilde{t}_1 = \tilde{t}_L \cos \theta_t + \tilde{t}_R \sin \theta_t, \quad (2.2)$$

where

$$\tan \theta_t = \frac{m_{\tilde{t}_1}^2 - m_{\tilde{t}_L}^2}{am_t} \quad (2.3)$$

and

$$m_{\tilde{t}_1}^2 = \frac{1}{2} \left[m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 - \sqrt{(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)^2 + 4a^2 m_t^2} \right]. \quad (2.4)$$

Since we are only interested in $\tilde{t}_1$ pair production, we can treat $m_{\tilde{t}_1}$ and θ_t as the free parameters of the problem, rather than the parameters of the mass

matrix (2.1). It is perhaps worth pointing out that supergravity models favor^[1] $m_{\tilde{t}_L}^2 > m_{\tilde{t}_R}^2$, i.e., $|\cos \theta_t| < 1/\sqrt{2}$; however, we will impose no such constraint.

The only nontrivial term in the Lagrangian that we need is the $Z\tilde{t}_1\tilde{t}_1^*$ coupling; it is given by^[9]

$$\mathcal{L} = -i \frac{g}{\cos \theta_W} Z^\mu \left(\frac{1}{2} \cos^2 \theta_t - \frac{2}{3} \sin^2 \theta_W \right) \tilde{t}_1^* \overleftrightarrow{\partial}_\mu \tilde{t}_1. \quad (2.5)$$

Notice that θ_t enters only as $\cos^2 \theta_t$; we can therefore restrict ourselves to the region $0 \leq \theta_t \leq \pi/2$.

3. Light Stop Production in Z Decays

The $Z \rightarrow \tilde{t}_1 \tilde{t}_1^*$ decay width is given by

$$\Gamma(Z \rightarrow \tilde{t}_1 \tilde{t}_1^*) = 6\Gamma(Z \rightarrow \nu_e \bar{\nu}_e) \cdot \left(\frac{1}{2} \cos^2 \theta_t - \frac{2}{3} \sin^2 \theta_W \right)^2 \beta^3 \left[1 + \frac{4\alpha_s}{3\pi} F(\beta) \right], \quad (3.1)$$

with $\beta = (1 - 4m_{\tilde{t}_1}^2/m_Z^2)^{1/2}$. The function $F(\beta)$, describing QCD radiative corrections, has been computed by Schwinger^[10] for the case of scalar QED. It is given by

$$\begin{aligned} F(\beta) = & \frac{1+\beta^2}{\beta} \left[4 \text{Li}_2\left(\frac{1-\beta}{1+\beta}\right) + 2 \text{Li}_2\left(-\frac{1-\beta}{1+\beta}\right) - 3 \ln \frac{2}{1+\beta} \ln \frac{1+\beta}{1-\beta} \right. \\ & \left. - 2 \ln \beta \ln \frac{1+\beta}{1-\beta} \right] - 3 \ln \frac{4}{1-\beta^2} - 4 \ln \beta \\ & + \frac{1}{\beta^3} \left[\frac{5}{4} (1+\beta^2)^2 - 2 \right] \ln \frac{1+\beta}{1-\beta} + \frac{3}{2} \frac{1+\beta^2}{\beta^2}, \end{aligned} \quad (3.2)$$

where $\text{Li}_2(x) = -\int_0^x dt \ln(1-t)/t$ is the Spence function. Note that we have written $F(\beta)$ in a somewhat more compact form than Schwinger, and have corrected a misprint in his Eq. (5-4.132).

It is illuminating to study the behavior of $F(\beta)$ in the limiting situations. In the high energy limit $\beta \rightarrow 1$, one has^[10]

$$F(\beta \rightarrow 1) = 3, \quad (3.3)$$

which is four times bigger than the corrections for fermionic quarks in the same limit. In the opposite limit of $\beta \rightarrow 0$, one has

$$F(\beta \rightarrow 0) \rightarrow \frac{\pi^2}{2\beta} - 2. \quad (3.4)$$

Here, the first term is universal; it is due to the color-Coulomb interaction of the $\tilde{t}_1$ squarks. For β very close to zero, the expression (3.1) becomes unreliable; in this region the effects of $(\tilde{t}_1 \tilde{t}_1^*)$ bound-state formation are felt. These effects have recently been discussed in Ref. 11. A simple interpolation formula for Eq. (3.2) is given in Ref. 10:

$$F(\beta) \simeq \frac{\pi^2}{2\beta} - \frac{1+\beta}{2} \left(\frac{\pi^2}{2} - 3 \right), \quad (3.5)$$

which reproduces the exact result within 1.5%.

The width (3.1) can be large enough to allow us to exclude some region in $(m_{\tilde{t}_1}, \theta_t)$ -space from the requirement that the total width of the Z is not too large. The present world average for this quantity is^[2]

$$\Gamma_Z = 2.496 \pm 0.016 \text{ GeV}, \quad (3.6)$$

while the Standard Model predicts^[12] $\Gamma_Z(\text{SM}) > 2.468 \text{ GeV}$. We can thus conclude that

$$\Gamma(Z \rightarrow \tilde{t}_1 \tilde{t}_1^*) < 55 \text{ MeV} \quad (95\% \text{C.L.}). \quad (3.7)$$

The corresponding constraint on $m_{\tilde{t}_1}$ and θ_t is depicted in Fig. 1. The dotted curve has been obtained without QCD corrections, while for the solid curves the

full expression (3.1), (3.2) has been used, with^[2] $\alpha_s(m_Z) = 0.115$. We see that inclusion of the corrections pushes the bound on $m_{\tilde{t}_1}$ beyond the reach of the TRISTAN Collider if θ_t is small. However, for $\theta_t \geq 0.5$ no significant bound can be given; recall that supergravity models favor $\theta_t > 0.7$. Note that this bound is independent of $\tilde{t}_1$ decay modes. Figure 1 also contains lines of constant $\Gamma(Z \rightarrow \tilde{t}_1 \tilde{t}_1^*) = 10 \text{ MeV}$, corresponding to a branching ratio of about 0.4%. This value is roughly equivalent to the bound obtained from dedicated squark searches at present^[2, 5, 6] (see the last section). We see that even this bound would leave quite a large window for light (in principle even massless) $\tilde{t}_1$. Finally, we also show lines for a partial width of 1 MeV. Such a bound would exclude unmixed squarks almost up to the kinematical limit; however, in 10% of total possible region of θ_t , and 20% of the region favored by supergravity models, one would still not be able to exclude a light $\tilde{t}_1$. Clearly, such a tight bound can only come from searches for explicit signals for $\tilde{t}_1$ production; we therefore now turn our attention to the relevant cross section.

4. The $e^+e^- \rightarrow \tilde{t}_1 \tilde{t}_1^*$ Production Cross Section

Inspection of Eq. (3.1) shows that even an infinitely accurate measurement of Γ_Z would not allow us to put an absolute bound on $m_{\tilde{t}_1}$; the reason is that $\Gamma(Z \rightarrow \tilde{t}_1 \tilde{t}_1^*)$ *vanishes* if

$$\theta_t \equiv \theta_t^0 = \arccos\left(\frac{2}{\sqrt{3} \sin \theta_W}\right) \simeq 0.98. \quad (4.1)$$

In order to obtain a realistic estimate of the $\tilde{t}_1$ production rate if $\theta_t \simeq \theta_t^0$, photon exchange contributions have to be included even for $\sqrt{s} = m_Z$. On the other hand, it is well known that the Z -exchange contribution at $\sqrt{s} \simeq m_Z$ is strongly affected by initial-state photon radiation; the γ -exchange contribution is much less affected, but we will see that even here the effect can be quite sizeable.

The effect of initial-state radiation is most easily included by introducing an energy-dependent e^+e^- luminosity function. For our purposes it is sufficient to use the $\mathcal{O}(\alpha_{\text{em}})$ expression, which reads after the summation of leading logarithms^[13]

$$L_{ee}(x) = \left[\beta_{\text{em}}(1-x)^{\beta_{\text{em}}-1} \left(1 + \frac{3}{4}\beta_{\text{em}} \right) - \frac{\beta_{\text{em}}}{2}(1+x) \right] \left[1 + \alpha_{\text{em}} \left(\frac{\pi}{3} - \frac{1}{2\pi} \right) \right], \quad (4.2)$$

where

$$\beta_{\text{em}} = \frac{2\alpha_{\text{em}}}{\pi} \left(\ln \frac{s}{m_e^2} - 1 \right). \quad (4.3)$$

In terms of this function, the total cross section for $e^+e^- \rightarrow \tilde{t}_1 \tilde{t}_1^*$ is given by

$$\sigma(e^+e^- \rightarrow \tilde{t}_1 \tilde{t}_1^*) = \int_0^1 dx L_{ee}(x) \hat{\sigma}(xs). \quad (4.4)$$

Here, $\hat{\sigma}(\hat{s})$ is the cross section without initial state radiation; it is given by

$$\begin{aligned} \hat{\sigma}(\hat{s}) = & \frac{\pi\alpha^2}{\hat{s}} \beta^3 \left[\frac{4}{9} - \frac{4}{3} c_t v_e \frac{\hat{s}(\hat{s} - m_Z^2)}{(\hat{s} - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \right. \\ & \left. + c_t^2 (v_e^2 + \frac{1}{16}) \frac{\hat{s}^2}{(\hat{s} - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \right] \cdot \left[1 + \frac{4\alpha_s(\hat{s})}{3\pi} F(\beta) \right], \end{aligned} \quad (4.5)$$

with $\beta = (1 - 4m_{\tilde{t}_1}^2/\hat{s})^{1/2}$, $v_e = -\frac{1}{4} + \sin^2\theta_W$, and

$$c_t = \frac{1}{\sin^2\theta_W \cos^2\theta_W} \left(\frac{1}{2} \cos^2\theta_t - \frac{2}{3} \sin^2\theta_W \right); \quad (4.6)$$

and $F(\beta)$ is given by Eq. (3.2). We have used $m_Z = 91.1$ GeV and $\Gamma_Z = 2.5$ GeV; for the strong coupling constant α_s we have used the one-loop expression for five massless flavors with $\Lambda = 100$ MeV, which gives $\alpha_s(m_Z^2) = 0.120$.

In Fig. 2, we show the total $\tilde{t}_1$ pair cross section as a function of the mixing angle, for $\sqrt{s} = m_Z$ and $m_{\tilde{t}_1} = 40$ GeV (a) and $\sqrt{s} = 150$ GeV, $m_{\tilde{t}_1} = 70$ GeV (b). Here, as well as in the subsequent figures, the dotted curves show the cross section without QED and QCD corrections; the long-dashed curves include initial-state radiation, but no QCD corrections, while the short-dashed curves do not include QED corrections but do include QCD corrections; finally, the full curves include both types of corrections.

As expected, initial-state radiation reduces the pure Z contribution at $\sqrt{s} = m_Z$ by about 30%; however, they are large ($\simeq -25\%$) even in Fig. 2(b). The reason is the strong β dependence of the cross section (4.5), which makes it very sensitive to small changes in $\hat{s}$. However, we see that in both cases the positive QCD corrections overcompensate the negative QED corrections. In Fig. 2(b), the net result is a correction of +22%, almost independent of θ_t . In Fig. 2(a), the net correction is only +5%, except for $\theta_t \approx \theta_t^0$; here the γ exchange contribution dominates, which is less affected by the negative initial-state correction, so that the total correction reaches +20% at $\theta_t = \theta_t^0$.

Fig. 2(a) shows that an integrated luminosity of several pb^{-1} will be necessary to derive lower bounds on $m_{\tilde{t}_1}$ which are valid for all values of θ_t . This is in sharp contrast to the case of either a pure left-handed or a pure right-handed squark; inclusion of nontrivial mixing is therefore crucial for any realistic analysis. Notice that even at 60 GeV above the Z resonance the cross section varies by a factor of 2.5 as θ_t is increased from 0 to θ_t^0 . Notice finally that the curves of Fig. 2(a) are almost symmetric around θ_t^0 , since γ - Z interference is very small here (zero if initial-state radiation is neglected). In contrast, Fig. 2(b) clearly shows the effect of positive (negative) γ - Z interference for $\theta_t < (>)\theta_t^0$.

In Fig. 3, we demonstrate the effect of the various radiative corrections on the energy dependence of the $\tilde{t}_1$ pair cross section. In both plots we have chosen $\theta_t = 1.25$; Fig. 3(a) is for $m_{\tilde{t}_1} = 30$ GeV and LEP I energies, and Fig. 3(b) for $m_{\tilde{t}_1} = 70$ GeV for LEP II energies and beyond.

Figure 3(a) shows an example of a *negative* total correction for $\sqrt{s} \lesssim m_Z$; the reduction of $m_{\tilde{t}_1}$ compared to the situation of Fig. 2(a) has decreased the relative importance of the positive QCD corrections, while the initial-radiation corrections depend only weakly on $m_{\tilde{t}_1}$ as long as $m_{\tilde{t}_1} < \frac{1}{2}m_Z$, unless $\theta_t \simeq \theta_t^0$. As a result, the total correction reaches -11% for $\sqrt{s} = 90$ GeV, which is however much smaller than the -30% correction that obtains for $\alpha_s = 0$; note also that we never found total corrections more negative than -15% for any realistic scenario. On the other hand, Fig. 3(a) shows that very large and positive total corrections are possible even quite far away from threshold, in regions where the QED corrections are also positive. As is well known,^[13] in the Z line shape this happens for $\sqrt{s} \gtrsim 92.5$ GeV; as a result, the total corrections reach $+70\%$ for $\sqrt{s} = 95$ GeV, $m_{\tilde{t}_1} = 30$ GeV.

A similar change of sign of the QED corrections also occurs for $m_{\tilde{t}_1} > \frac{1}{2}m_Z$, as depicted in Fig. 3(b). However, it occurs only at $\sqrt{s} \simeq 4m_{\tilde{t}_1}$; only for these large energies does the increase of the $1/\hat{s}$ factor in $\hat{\sigma}$, Eq. (4.5), overcompensate the reduction of β^3 when $\hat{s}$ is reduced by the emission of photons from the initial state. Notice also that the QCD corrections decrease with $\sqrt{s}$ for fixed $m_{\tilde{t}_1}$, while the QED corrections increase; as a result, the total correction in Fig. 3(b) is almost constant at 15% for a large range of $\sqrt{s}$.

Probably more important than the behavior at large $\sqrt{s}/m_{\tilde{t}}$ is the threshold behavior, which is quite strongly affected by both types of radiative corrections; the QED corrections are most negative, and the QCD corrections most positive, in the region of small β . Due to the $1/\beta$ singularity of Eq. (3.4) the QCD corrections win out, leading to a (formally) infinitely large correction factor in the limit $\beta \rightarrow 0$; recall, however, that for very small β bound-state effects become important. Nevertheless we can conclude that the net effect of the corrections is to significantly steepen the increase of σ with $\sqrt{s}$ in the region of small β .

Finally we would like to point out that for the case $m_{\tilde{t}_1} \simeq \frac{1}{2}m_Z$, which is of interest for the present round of LEP experiments, the threshold behavior strongly

depends on θ_t . This is demonstrated in Fig. 4, where we show the threshold behavior of the $\tilde{t}_1$ pair cross section for $m_{\tilde{t}_1} = 45$ GeV and three different values of θ_t ; all curves include full QED and QCD corrections.

If θ_t is far away from θ_t^0 , the cross section shows a clear peak in the vicinity of $\sqrt{s} = m_Z$; due to the β^3 factor in $\hat{\sigma}$, the peak is actually shifted towards somewhat larger values of $\sqrt{s}$. This is demonstrated by the uppermost curve, which is for $\theta_t = 1.2$.

If $|\theta_t - \theta_t^0| \simeq 0.1$, the cross section still has a small maximum at $\sqrt{s} \simeq 92$ GeV; however, the cross section quite soon reaches a minimum, after which the increase of the β^3 factor wins over the decrease of the Z exchange contribution, so that the cross section begins to rise again, eventually reaching a second maximum at $\sqrt{s} \simeq 140$ GeV. Finally, if $|\theta_t - \theta_t^0| \lesssim 0.05$, the first maximum disappears altogether; in this case the behavior of the cross section is similar to that depicted in Fig. 3(b). Notice that the initial-radiation effects always tend to wash out the first maximum, whereas the QCD corrections tend to make it more pronounced; since the peak appears at low β , the QCD corrections win, so that the net effect of the radiative corrections is to increase the height of the first maximum.

5. Summary and Conclusion

In this note we discussed in some detail the production of a pair of light scalar top quarks in e^+e^- collisions. After setting up our notation in Sec. II, we observed in Sec. III that the present bound on the total decay width of the Z boson already allows to put a bound of about 30 GeV on $m_{\tilde{t}_1}$, if $\tilde{t}_1$ is dominantly an SU(2) doublet. We also discussed the QCD corrections in this section, which are positive and turn out to be much larger than for ordinary quarks. Nevertheless the total Z decay width does not lead to a bound on $m_{\tilde{t}_1}$ in the theoretically favored case where $\tilde{t}_1$ has a large or even dominant SU(2) singlet component. In this case it is even possible that the $Z\tilde{t}_1\tilde{t}_1^*$ coupling vanishes altogether; this makes

it necessary to include the γ exchange contribution to the $\tilde{t}_1$ pair production cross section even for $\sqrt{s} \simeq m_Z$.

In Sec. IV we presented results for this cross section, emphasizing the need to include nontrivial stop mixing (*i.e.*, $\theta_t \neq 0$ or $\pi/2$) in any realistic analysis; varying θ_t can change the cross section by a factor of 300 for $\sqrt{s} = m_Z$, and by a factor of two or more even at LEP II energies. We also saw that it can drastically change the threshold behavior of the cross section if $m_{\tilde{t}_1} \simeq \frac{1}{2}m_Z$.

In these calculations we included both final-state QCD corrections, which are always positive, and initial-state QED corrections, which are negative close to threshold and for $\sqrt{s}$ at or just below m_Z , but become positive for larger $\sqrt{s}$. The net corrections are usually positive and can be very large (*e.g.*, +70% for $\sqrt{s} = 95$ GeV, $m_{\tilde{t}_1} = 30$ GeV and sizeable $Z\tilde{t}_1\tilde{t}_1^*$ coupling). The only exception to this rule occurs for $m_{\tilde{t}_1} \lesssim 35$ GeV and $\sqrt{s} \lesssim 92$ GeV, again for sizeable $Z\tilde{t}_1\tilde{t}_1^*$ coupling; however, the corrections are never more negative than -15%, leading to a still very large (≥ 50 pb) peak cross section. The corrections should thus make it easier to constrain (or discover!) the scalar top sector.

We close with some remarks concerning the actual signals for $\tilde{t}_1\tilde{t}_1^*$ production, closely following the discussion of Ref. 9. There it was pointed out that the one-loop induced decay $\tilde{t}_1 \rightarrow c\tilde{z}_1$, where $\tilde{z}_1$ denotes the lightest neutralino which we assume to be the LSP, always dominates four-body decays like $\tilde{t}_1 \rightarrow b\ell^+\nu\tilde{z}_1$. Furthermore, we now know^[2] that the three-body decays $\tilde{t}_1 \rightarrow b\tilde{\ell}^+\nu$, $b\ell^+\tilde{\nu}$ are kinematically forbidden if $\tilde{t}_1$ is light enough to be pair produced at LEP I. At present the relevant signature is thus a pair of c -quark jets plus missing energy-momentum.

Searches for such a specific final state have been made.^[5,6] In particular, the DELPHI Collaboration made analyses for the cases of relatively light and heavy $\tilde{z}_1$, both of which may be approximately translated to a bound

$$\Gamma(Z \rightarrow \tilde{q}\tilde{q}) \lesssim 10 \text{ MeV} . \quad (5.1)$$

As can be seen from Fig. 1, this result of a dedicated squark search is not sufficient to exclude a light stop for the mixing angle $0.8 \lesssim \theta_t \lesssim 1.2$.

The case where $m_{\tilde{z}_1}$ is close to $m_{\tilde{t}_1}$ deserves a comment; in this case we cannot always expect to see a peak in the cross section at $\sqrt{s} \simeq m_Z$, a trick that has been used^[6] by the DELPHI Collaboration in their search for the other sfermion in the case of small $m_{\tilde{f}} - m_{\tilde{z}_1}$. A more elaborate strategy will thus be needed.

The situation is somewhat more complicated for heavier $\tilde{t}_1$, which could be produced at LEP II. One will still be able to constrain the possible $\tilde{t}_1$ decay modes using the results of searches for charginos and charged sleptons; however, it will probably be quite difficult to improve the bound on sneutrino masses beyond the present limit of about 40 GeV, unless the sneutrino decays visibly. (One possibility is to look for $e^+e^- \rightarrow \gamma + \text{missing energy-momentum}$; however, one will need a large statistics to probe the region $m_{\tilde{\nu}} > m_Z/2$ in that way.) So even if strong bounds on chargino and slepton masses can be derived, one will have to search simultaneously for $\tilde{t}_1 \rightarrow c\tilde{z}_1$ and for $\tilde{t}_1 \rightarrow b\ell^+\tilde{\nu}$.

Some of our results are also relevant for other processes. Obviously, the same QCD corrections apply to the production of any colored scalars in e^+e^- annihilation. Similarly, our result that initial-state QED corrections can be large and negative even far away from the Z resonance applies to the pair production of any particles that are produced in a P -wave state; in particular, this means that the cross section for the pair production of scalar leptons, as well as of neutral fermions with pure axial couplings to the Z , can be reduced by up to 25% close to threshold.

Finally, in the threshold region, mixing between SU(2) doublet and singlet squarks can also be relevant for $\tilde{b}$ and $\tilde{c}$ squarks. The reason is that even if the off-diagonal entries in the mass matrix (2.1) are small, there can be sizeable mixing if the two diagonal entries are nearly degenerate; notice that all experimental bounds have been derived under the assumption of no mass splitting. Of course, the mass splitting between the two eigenstates will not be large in this case,

being of the order of the quark mass. The thresholds for the production of a light and a heavy eigenstate (via Z exchange only^[9]) as well as for two heavy eigenstates will therefore be crossed fairly soon after pair production of the light eigenstate becomes possible. Beyond the last of these thresholds, mixing will become irrelevant for the total squark cross section. However, since the mass splitting in the sbottom sector could easily be 10 GeV, the recent experimental bounds^[2, 5, 6] on scalar quarks can probably not strictly exclude the region $m_{\tilde{b}_1} \gtrsim 40$ GeV.

Notice that a good bound on $m_{\tilde{t}_1}$ would help to close a loophole through which a light top quark might have escaped detection:^[14] If $m_{\tilde{t}_1} < m_t - m_{\tilde{z}_1}$, the decay $t \rightarrow \tilde{t}_1 + \tilde{z}_1 \rightarrow c + \tilde{z}_1 + \tilde{z}_1$ becomes possible, which does not lead to isolated leptons and would therefore not have been detected in recent top searches.^[15] We therefore urge our experimental colleagues to search for $\tilde{t}_1$ production in spite of the complications that arise from $\tilde{t}_L$ - $\tilde{t}_R$ mixing.

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FIGURE CAPTIONS

- Fig. 1. Lines of constant $\Gamma(Z \rightarrow \tilde{t}_1 \tilde{t}_1^*)$ in the $(\theta_t, m_{\tilde{t}_1})$ plane. The solid (dotted) curves are with (without) QCD corrections. The lower curves in the region of small θ_t are for a width of 55 MeV, equal to the bound (3.7); the region below these curves is thus excluded by present data. The other curves are for widths of 10 MeV and 1 MeV, as indicated.
- Fig. 2. The θ_t dependence of the total $e^+e^- \rightarrow \tilde{t}_1 \tilde{t}_1^*$ cross section, for (a) $m_{\tilde{t}_1} = 40$ GeV, $\sqrt{s} = m_Z$ and (b) $m_{\tilde{t}_1} = 70$ GeV, $\sqrt{s} = 150$ GeV. The dotted curves show the tree level result. The long-dashed curves include initial-state QED corrections but no QCD corrections, while the short-dashed curves only include QCD corrections. The full curves include both types of corrections.
- Fig. 3. The energy dependence of the total $e^+e^- \rightarrow \tilde{t}_1 \tilde{t}_1^*$ cross section for $\theta_t = 1.25$ and (a) $m_{\tilde{t}_1} = 30$ GeV at LEP I energies, and (b) $m_{\tilde{t}_1} = 70$ GeV at LEP II energies and beyond. The meaning of the different curves is as in Fig. 2.
- Fig. 4. The dependence of the threshold behavior of the total $\tilde{t}_1 \tilde{t}_1^*$ cross section on the mixing angle θ_t , for $m_{\tilde{t}_1} = 45$ GeV. All curves include both QED and QCD corrections.

Fig. 1

Fig. 2(a)

Fig. 2(b)

Fig. 3(a)

Fig. 3(b)

Fig. 4