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**VACUUM STABILITY, WORMHOLES, COSMIC RAYS
AND THE COSMOLOGICAL BOUNDS ON m_t AND m_H**

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ABSTRACT

In the Standard Model, if the top quark mass m_t is larger than some critical value depending on the Higgs mass m_H , then we live in an unstable vacuum state corresponding to a local minimum of the effective potential. An experimental discovery of the top quark with m_t above this critical value would invalidate the standard version of the wormhole theory, according to which the vacuum energy should be zero at the absolute minimum of the effective potential. However, unless the top quark is much heavier than this, the lifetime of the unstable vacuum state is greater than the age of our part of the universe. In this paper we develop a stochastic approach to tunneling and apply it to examine the possibility that cosmic ray collisions may trigger vacuum decay and derive improved cosmological bounds on m_H and m_t .

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1. Introduction

The masses of the top quark and of the Higgs boson are still not determined. Unsuccessful direct searches for the top quark tell us that $m_t > 89\text{GeV}$ [1] and analyses of radiative corrections in the Standard Model suggest that $m_t = 130 \pm 30\text{GeV}$ [2]. Unsuccessful LEP searches for the Higgs boson tell us that $m_H > 41\text{GeV}$ [3], but electroweak radiative corrections do not yet improve this information about m_H . However, strong constraints on m_H and m_t can be obtained by renormalization group calculations and considerations of the effective Higgs potential. For example, an upper bound on m_H as a function of m_t is obtained by requiring the coupling constants of the Standard Model to remain finite up to a large renormalization scale (for a review and references see [4]). Moreover, radiative corrections to the effective Higgs potential in the theories with heavy fermions may lead to a formation of a new, deeper minimum of the effective potential, thus destabilizing our vacuum state. The requirement that our vacuum be metastable with a lifetime $> 10^{10}$ years leads to a lower bound on m_t as a function of m_H [5-8], as seen in the Figure.

As we argue in Section 3, this requirement is strengthened by one prevalent interpretation of wormhole theory, according to which the cosmological constant vanishes at the absolute minimum of the effective potential, which would imply that our present vacuum is absolutely stable. This requirement is also shown in the Figure. However, if one disregards this wormhole argument, one might suspect that this constraint is too strong, since if the probability of the spontaneous decay of the metastable vacuum is small enough (as is often the case [7, 9]), the transition to the true vacuum state can be avoided within our past light-cone.

However, the decay of the false vacuum could in principle be triggered by high energy accelerators or by cosmic ray collisions. This question was first addressed by Hut and Rees [10], who reassured the reader that high energy accelerators could not destabilize our vacuum since cosmic rays would have already done so. Some years later, Sher and Zaglauer [11] looked at the constraints on m_t and m_H that would be deduced from considering cosmic ray collisions. They required that enough energy exist in the collision to go over the barrier, and that the size of the interaction region be greater than the critical bubble. They concluded that cosmic rays would, in general, induce the transition if our vacuum was unstable. Arnold [12] refuted this conclusion by showing that the number of quanta required to be produced was large: $N > 100$. He noted that the perturbative cross-section for producing this large number of quanta would thus be suppressed by a factor of α^N , where α is some Yukawa coupling. This apparently eliminated the possibility that cosmic rays induce the transition.

Arnold's calculation was based on the interpretation of tunneling in terms of quantum statistics. This approach, called the stochastic approach to tunneling, was extensively used

earlier in the theory of tunneling in the expanding universe and even in the theory of the baby universe formation [13 - 16]. The stochastic approach gives less precise results than the Euclidean approach to tunneling in some cases, but it has the advantage of being very simple and intuitive, and it may work in several cases where the validity of the Euclidean approach is questionable [16].

Arnold [12] did not perform a complete quantitative investigation of the phase transitions stimulated by cosmic ray collisions. As we will see, this problem is very difficult and requires a detailed knowledge of the process of multiple Higgs boson production at energies many orders higher than those which can be probed at existing accelerators. The most rigorous way to study the influence of the cosmic rays on phase transitions would be to investigate the instanton solution, which describes tunneling, in the presence of colliding particles. This method was used in a series of recent papers on the nonperturbative electroweak processes with baryon number nonconservation [17]. The results obtained in ref. [17] are extremely interesting, even though their validity is still a matter of some controversy, both for the reason that the Euclidean methods used there need a more detailed justification and for the reason that the approximations used in [17] fail at very high energies. In this paper we will try to study the possibility that cosmic rays may trigger vacuum decay using much weaker assumptions about electroweak high energy interactions than those made in [17]. Our hope is that the more intuitive stochastic approach to tunneling which will be used below is enough to get some reliable results. In particular, it will be possible to show the domain of those values of m_t and m_H for which we can safely live in the false vacuum without being too much afraid of phase transitions triggered by cosmic rays. Our approach may provide also an alternative way to study other nonperturbative processes at high energies.

2. Effective Potential in the Standard Model

First of all, let us remember some facts about the effective potential of the Higgs field ϕ in the Standard Model. In the one-loop approximation [18] the effective potential is given (approximately) by

$$V = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 + B\phi^4\ln(\phi/M). \quad (1)$$

Here $B = (9g^4 + 6g^2g'^2 + 3g'^2 + 9\lambda^2 - 3g_1^4)/512\pi^2$, where g, g' and g_1 are the SU(2) gauge coupling, the U(1) gauge coupling and the top quark Yukawa coupling respectively. One can see immediately that if $B < 0$ then the vacuum is unstable. It has a maximum at some value of the field $\phi = \phi_1 > \phi_0$ and is unbounded from below at $\phi > \phi_1$. Here $\phi_0 = 246$ GeV is the value of the scalar field ϕ in the local minimum of $V(\phi)$ [5-7]. This leads to

the original bound on the top quark mass $m_t = \frac{1}{\sqrt{2}} g_V \phi_0$. To get more exact results, one should improve eq. (1) with the help of the renormalization group equation. An interesting feature of the renormalization group improved potential is that it may not be unbounded from below; a second minimum always forms at some point $\phi_2 > \phi_1$ [8]. The form of the improved potential for different values of m_t and m_H was obtained in [8] by means of computer calculations. One of the main results is that the minimum at $\phi = \phi_2$ is deeper than the minimum at $\phi = \phi_0$ for $m_t > 95 \text{ GeV} + 0.6 m_H$, as shown in the Figure.

3. Top Challenge to Wormholes

Of course, many people would not like even to consider the possibility that we live in an unstable vacuum state since this would be a scientific version of the doomsday prediction. However, after we learned that the universe as a whole is gravitationally unstable and that the idea of Einstein to make it static does not work, there is no reason to insist on absolute vacuum stability. Rather one should be happy if it is stable enough...

There exists another problem related to the assumption that our vacuum is unstable. Namely, it is very hard to understand why the vacuum energy now is vanishingly small. One may have an idea that it might be zero in the absolute minimum of the effective potential for reasons to be understood with a further development of elementary particle theory, but at the first glance it is absolutely incredible that there may exist any reason for the vacuum energy density to be smaller than $10^{-29} \text{ g.cm}^{-3}$ in a false vacuum state in which we live now if the top quark is heavy enough.

The most popular idea for the solution of the vacuum energy (cosmological constant) problem was proposed by Coleman [19] in the context of the baby-universe theory. This theory is based on the investigation of the role of Euclidean wormholes for fixing the coupling constants of the low-energy theory. It is claimed that according to the baby-universe theory the vacuum energy density should be exactly zero in a state corresponding to the absolute minimum of the effective potential $V(\phi)$, and that the wormhole effects do not modify the results of the standard perturbative calculations which fix the shape of $V(\phi)$ [19, 20].

But this means that according to [19, 20] $V(\phi_2) = 0$ for $m_t > 95 \text{ GeV} + 0.6 m_H$. However, the energy difference between $V(\phi_2)$ and $V(\phi_0)$ is much bigger than $\sim 10^{25} \text{ g.cm}^{-3}$. In such a case the energy density of our present unstable vacuum state $\phi = \phi_0$ would be more than 50 orders of magnitude bigger than the cosmological constraint $V(\phi_0) < 10^{-29} \text{ g.cm}^{-3}$. This means that either the top quark is light enough and we live in a stable vacuum, or the theory [19] is incorrect [21].

Of course, we are not going to say that there is no way to generalize the baby-universe theory or to suggest some other theory which may explain why the vacuum energy density

should be zero even if the vacuum is false (for a discussion of various possibilities see e.g. [21]), but it is hard to imagine that this can be done without use of the anthropic principle. In any case, the masses of the top quark and of the Higgs boson are still unknown, so now is the time to place one's bets! One of the purposes of our paper is to examine the question whether the chances that we live in the false vacuum are still high when cosmic ray collisions are taken into account.

4. Euclidean Methods in the Theory of Tunneling

One of the simplest and most elegant approaches to tunneling is the Euclidean one [22]. The main idea of this approach is that tunneling is a motion with imaginary energy, which is equivalent to motion in imaginary time, i.e. in Euclidean space. The probability of tunneling is proportional to $\exp(-S_E)$, where S_E is the Euclidean action corresponding to the tunneling trajectory. In other words, S_E is the instanton action, where the instanton is the solution of the Euclidean field equations describing tunneling.

A most instructive example is tunneling in a theory with the effective potential

$$V = \frac{1}{2} \mu^2 \phi^2 - \frac{1}{4} \lambda \phi^4. \quad (2)$$

This theory may serve as a very good model for the theory (1) at large ϕ , with λ in (2) equal to $-B \ln(\phi/M)$, where $\ln(\phi/M)$ corresponds to a typical value of the scalar field ϕ on the tunneling trajectory.

The tunneling trajectories (instantons) with the minimal action possess the $O(4)$ symmetry of Euclidean space [22]. The Euclidean equation for $O(4)$ symmetric tunneling is

$$d^2 \phi / dr^2 + (3/r) d\phi / dr = -dV/d\phi, \quad (3)$$

with the boundary conditions $\phi(r = \infty) = \phi_0$ and $d\phi/dr|_{r=0} = 0$. Here $r = \sqrt{x_i^2}$; the x_i are the Euclidean coordinates, $i = 1, 2, 3, 4$.

In the theory (2) with $m = 0$ equation (3) has a class of solutions which are called Fubini instantons [23]:

$$\phi(r) = 2\sqrt{\frac{2}{\lambda}} \left(\frac{\rho}{r^2 + \rho^2} \right), \quad (4)$$

where ρ is arbitrary. The corresponding Euclidean action does not depend on ρ ,

$$S_E = 2\pi^2 \int r^3 \left(\frac{1}{2} (d\phi/dr)^2 + V(\phi) \right) dr = \frac{8\pi^2}{3\lambda}. \quad (5)$$

The bubbles which appear after tunneling are also described by eq. (4) if one understands by r^2 its Minkowski counterpart $t^2 - r^2$. The probability of the bubble formation per unit four-volume can be estimated by the expression

$$P \sim \rho^{-4} \exp\left(-\frac{8\pi^2}{3\lambda}\right) \quad (6)$$

for $m = 0$.

A somewhat surprising fact is that for any nonvanishing m there are *no* instanton solutions in the theory (2). This does not mean, however, that there is no tunneling in this theory [24, 25]. The point is that for any nonvanishing m the minimum of tunneling action corresponds to the $\rho \rightarrow 0$ limit of the field configurations (4). But one can easily check that the action of these configurations differs from its limiting value (5) by $\Delta S_E \ll 1$ for $\rho \ll \frac{\sqrt{\lambda}}{m}$. This means that the Fubini instantons (4) with $\rho \ll \frac{\sqrt{\lambda}}{m}$ may play the role of instantons in the theory (2) with nonvanishing m as well.

To study tunneling in the Standard Model one should solve the Euclidean equations of motion for the renormalization group improved effective potential $V(\phi)$ [8, 12, 26]. However, an investigation of the simple model (2) often provides a very good approximation to the results of the complete analyses of tunneling in the theory with the improved potential. Therefore in the main part of the paper we will study tunneling in the model (2), but in the end we will give the results of a more complete investigation.

5. Stochastic Approach to Tunneling

Euclidean methods are elegant but rather formal and their interpretation and validity is not always clear. Therefore it is desirable to suggest an alternative approach which would clarify the physical meaning of the results obtained above and provide a way to investigate more complicated situations. In what follows we will study tunneling in the theory (2) by the stochastic methods developed in [12-16].

The main idea of the method consists of a specific interpretation of tunneling which we will illustrate on an example of tunneling in the theory (2) with $m = 0$.

A bubble of a classical field is formed only if it is sufficiently large and it contains a sufficiently big field ϕ . Indeed, if the size of the bubble is too small, the gradient term $\nabla^2 \phi$ in the equation of motion of the field ϕ is bigger than the term $|V'(\phi)|$, and the field ϕ inside the bubble does not grow. In the theory (2) this means that the inverse size of the bubble should satisfy the condition

$$r^{-2} \sim k^2 < \phi^{-1} |V'| = \lambda \phi^2 = \frac{1}{3} m^2(\phi) \equiv \frac{1}{3} V'''(\phi). \quad (7)$$

Let us estimate the probability of an event when vacuum fluctuations occasionally build up a configuration of the field satisfying this condition. In order to do it one should remember that the dispersion of quantum fluctuations of the field ϕ with $k < k_{max}$ is given by

$$\langle \phi^2 \rangle_{k < k_{max}} = \frac{1}{8\pi^2} \int_0^{k_{max}} \frac{k^2 dk}{\sqrt{k^2 + m^2}}. \quad (8)$$

For the massless field with $k_{max}^2 = \lambda \phi^2$ this gives

$$\langle \phi^2 \rangle_{k < k_{max}} = \frac{k_{max}^2}{16\pi^2} = \frac{\lambda \phi^2}{16\pi^2}. \quad (9)$$

This gives us an estimate of the dispersion of perturbations which may sum up to produce a field ϕ which satisfies the condition (7). Of course, this estimate is rather crude, since some long-wave perturbations can sum up to short wave ones and vice versa. But let us nevertheless use eq. (9) to evaluate the probability of satisfying (7). This can be done with the help of the Gaussian distribution:

$$P(\phi) \sim \exp\left(-\frac{\phi^2}{2 \langle \phi^2 \rangle_{k < k_{max}}}\right) = \exp\left(-\frac{8\pi^2}{\lambda}\right). \quad (10)$$

Note that the factor in the exponent in (10) to within a factor of 3 coincides with the Euclidean action S_E (5). Taking into account the very rough method we used to calculate the dispersion of the perturbations responsible for tunneling, the coincidence is rather impressive. (The disagreement by a factor of 3 shows that some fluctuations with momenta slightly bigger than $\sqrt{\lambda}\phi$ also give a contribution to the instanton (4). In order to check it one should consider the Fourier transform of $\phi(r)$ (4), see below. To take it into account in what follows we will just multiply the dispersion squared by the factor 3.) The agreement between the results of the simple method discussed above and a more complicated Euclidean approach becomes even more impressive if one remembers that most of the results obtained in the tunneling theory by Euclidean methods can easily be reproduced (with an accuracy of the coefficients of $O(1)$ in the exponent) by this simple method [16]. A similar method developed to a somewhat better extent in the context of the inflationary universe scenario [13-16] gives results for the tunneling probability which exactly coincide with the corresponding results obtained by the Euclidean approach.

6. Stochastic Approach to Tunneling and Multiple Particle Production

The results obtained above have a very useful interpretation [12]. Let us consider the Fourier transform of the instanton configuration (4) at the moment $x_4 = 0$ corresponding to the moment of tunneling $t = 0$:

$$\phi(k) = \int d^3x e^{ikx} \phi(x) = \frac{4\pi\rho}{k} \int \frac{\sin kr}{r^2 + \rho^2} = \frac{4\pi^2\rho}{k} \sqrt{\frac{2}{\lambda}} e^{-k\rho} \quad (11)$$

By generalizing the well-known expressions for the number N of particles [27] one can say that the classical scalar field configuration (4) corresponds to N overlapping particles of the original massless field ϕ , where

$$N = \frac{1}{2}(2\pi)^{-3} \int k \phi^2(k) d^3k = \frac{\pi^2}{\lambda} = \frac{3}{8} S_E. \quad (12)$$

This gives us a very interesting interpretation of the exponential suppression of tunneling. Tunneling with $S_E \gg 1$ is exponentially suppressed since it is improbable that $N \sim S_E \gg 1$ virtual particles with small momenta meet each other at the same point. (A slight difference between our result and the result of ref. [12] ($N = \frac{20}{3} S_E$) is due to a choice of a different surface of integration in [12], which does not correspond to the moment of the bubble formation $t = 0$).

This interpretation suggests also the way to study tunneling induced by collisions of elementary particles: In order to avoid the exponential suppression of the tunneling probability one should just create $N > \frac{3}{8} S_E$ scalar particles at a distance smaller than ρ from each other. For the theory of a massive scalar field ϕ (2) the corresponding distance should be smaller than $\sqrt{\lambda} m^{-1}$.

However, such a statement could be a gross overestimate of the probability to trigger vacuum decay by particle collisions, for the reasons to be discussed below.

First of all, the distribution of Higgs particles produced in the collision typically is not spherically symmetric. This happens even if the expression for the cross-section is spherically symmetric. Consider for example the process $e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}$: even though the process goes through one low partial wave, the final state particles event-by-event are distributed in narrow jets with angular width $O(\alpha_s)$. In order to estimate the possible importance of this effect let us see how our estimate for the probability of tunneling based on the stochastic approach will change if the particles produced in the collision have momenta $k < k_1$ in some direction and $k < k_2$ in the orthogonal directions, such that the asymmetry parameter is very big, $A = \frac{k_1}{k_2} \gg 1$.

In this case both k_1 and k_2 should satisfy the condition (7), which gives $\lambda\phi^2 > k_1^2$. On the other hand, instead of the integration in eq. (8) being symmetric in all directions, one should perform an integration from $-k_1$ to k_1 in one direction and from $-k_2$ to k_2 in two other directions. This gives a value of the dispersion squared which is approximately a factor $A^{-2} \ln A$ smaller than (9), for the same value of ϕ . Thus, our estimate for the probability of the asymmetric tunneling is

$$P \sim \exp(-S_E A^2 \ln^{-1} A) = \exp(-\frac{8\pi^2}{3\lambda} A^2 \ln^{-1} A), \quad (13)$$

and the number of the Higgs bosons which is necessary to trigger the asymmetric tunneling can be estimated as

$$N \sim \frac{3}{8} S_E A^2 \ln^{-1} A = \frac{\pi^2}{\lambda} A^2 \ln^{-1} A, \quad (14)$$

which could lead to a substantial suppression of the probability of the vacuum destabilization by particle collisions if e.g. $A \sim \frac{1}{\alpha}$.

Another possible source of problems is the energy distribution of the Higgs bosons produced in the collision. The improved effective potential typically has a maximum at some value of the field $\phi = \phi_1 \gg \phi_0$ and the absolute minimum at $\phi = \phi_2 \gg \phi_1$ [8]. If the momenta of the particles produced are in the interval $\sqrt{\lambda}\phi_1 < k < \sqrt{\lambda}\phi_2$, then one can use our estimates (13) and (14), properly modified to take into account the difference between the effective potential in the model (2) and the improved effective potential in the standard model. However, if all Higgs bosons produced in a collision have momenta bigger than $\sqrt{\lambda}\phi_2$, they will not trigger the phase transition at all! Thus, one should understand by N in eq. (14) the number of the Higgs particles produced with momenta in the interval $\sqrt{\lambda}\phi_1 < k < \sqrt{\lambda}\phi_2$.

Another interesting possibility appears if the largest momentum k_1 is still smaller than $\sqrt{\lambda}\phi_1$. Then instead of the estimate (10) one should have

$$P(\phi) \sim \exp(-S_E \frac{\lambda\phi_1^2}{k_1^2} A^2 \ln^{-1} A). \quad (15)$$

In this case the number of particles which is necessary to trigger the phase transition is even bigger than in eq. (14),

$$N \sim \frac{3}{8} S_E \frac{\lambda\phi_1^2}{k_1^2} A^2 \ln^{-1} A, \quad (16)$$

which entails a stronger suppression of the probability of vacuum destabilization.

One more problem is whether the process of multiple Higgs boson production effectively occurs at one point (or, to be more accurate, inside a domain of the same characteristic size as the instanton), or whether it occupies a big volume. If the effective size l of the domain where Higgs bosons are produced is big, this introduces an additional cut-off at $k \sim l^{-1}$ in the integral for the dispersion, which should lead to a stronger exponential suppression of the probability of the process. One could expect, however, that among the diagrams describing multiple particle production which may have a large total cross-section $\sigma \sim E_\phi^{-2}$: $E_\phi \sim m_W/\alpha$, there exist some diagrams describing point-like interactions with a small cross-section $\sigma \sim E^{-2}$ where E is the c.m.s. energy of colliding particles. The cross-section $\sim E_\phi^{-2}$ of multiperipheral interactions presumably will not lead to a sufficiently high

concentration of the Higgs bosons at one point, which is necessary to trigger the vacuum decay.

In order to get a more precise idea of a possible spatial distribution of the Higgs bosons produced in the cosmic ray collisions we will use an approach suggested recently by Goldberg [28]; see also a related paper by Cornwall [29]. Let us consider ultra high energy quark-antiquark annihilation with Higgs boson production. According to [28], this reaction in the $\lambda\phi^4$ theory looks like a chain reaction $q + \bar{q} \rightarrow \phi \rightarrow 3\phi \rightarrow 3^2\phi \rightarrow 3^3\phi \dots$. The first scalar particle produced has a very big energy E equal to the sum of energies of the two quarks, but zero momentum (in the c.m.s. of the two colliding quarks). Therefore the cross-section of the multiple Higgs boson production is spherically symmetric. This symmetry is spontaneously broken since each decaying Higgs boson produces three bosons moving away from the center with energy $O(E/3)$. Each of such bosons produces 3 new bosons with energy $O(E/9)$, etc. This process goes on until $N = O(E/m_H)$ nonrelativistic Higgs particles are produced. At first stages the process looks as a growth of a fractal, but the first few steps give negligible contribution to the size of the resulting configuration of particles $O(\frac{1}{3}m_H^{-1})$ [28]. At the last stage the decaying Higgs particles are nonrelativistic, so they move in all directions and collide with each other in a small domain of a size $O(\frac{1}{3}m_H^{-1})$, thus restoring the spherical symmetry of the particle distribution. Note, that this distribution corresponds to the process described by eqs. (15), (16) with $A = 1$ and $k_1 \sim m_H \sim \sqrt{\lambda}v$.

The method used in ref. [28] for the description of the multiple Higgs boson production also breaks down at some (extremely high) energy. However, the qualitative description of the process may remain correct even when the tree approximation used in [28] breaks down. In our opinion, the most vulnerable place of this picture is the size and the spherical symmetry of the Higgs boson distribution. Our results will be modified if the Higgs boson distribution will look not like a spherically symmetric ball but rather a fractal [30]. To be on the safe side, in what follows we will consider both the spherically-symmetric case, $A = 1$, $k_1 \sim m_H \sim \sqrt{\lambda}v$, and the case with $k_1 > \sqrt{\lambda}\phi_1$ and with a large asymmetry of the Higgs boson distribution, $A = O(1/\alpha)$ with $\alpha \sim 10^{-2}$, see eqs. (13), (14).

7. Vacuum Destabilization by Cosmic Rays

We now turn to the question whether our vacuum could be destabilized by cosmic ray collisions. Hut and Rees [10] assumed that the cross-section for cosmic rays to induce vacuum decay is geometrical, i.e., $\sim E^{-2}$, where E is the center of mass energy of colliding particles, and found that the number of events occurring in our past light cone is [10]

$$\left(\frac{10^{12} \text{ GeV}}{E} \right)^{6.2} \theta(7 \cdot 10^{10} \text{ GeV} - E) \quad (17)$$

(the only relevant collisions here are those of cosmic rays with each other; fixed target events will have too little energy to cause vacuum decay.) The power of 6.2 arises because the high-energy cosmic ray density n varies as $E^{-2.1}$, so $n^2\sigma$ varies as $E^{-6.2}$. The θ -function simply expresses the fact that no cosmic rays of higher energy have been seen, and it is expected that interactions with the microwave background should deplete the flux dramatically above this energy.

If N Higgs bosons are produced, each will have energy $\sim k_1 \sim E/N$. The factor of A in equations (13) and (14) is given by $A = k_1/k_2$. As mentioned in the previous section, we consider two possibilities:

(i) $A = 1$, $k_1 \sim m_H \sim \sqrt{\lambda}v < \sqrt{\lambda}\phi_1$ (where ϕ_1 is ϕ at the top of the barrier). In this case, eq. (16) gives

$$N \sim \frac{3}{8} S_E \frac{\phi_1^2}{v^2}. \quad (18)$$

(We use here equations derived for the theory (2) since the effective potential of the Standard Model can be well approximated by the theory (2) in the relevant region.)

(ii) $A = O(1/\alpha) \sim 10^2$, $k_1 > \sqrt{\lambda}\phi_1 \sim \frac{m_H}{\alpha}\phi_1$. Then eq. (14) gives

$$N \sim \frac{3}{8} S_E \alpha^{-2} \ln^{-1} \frac{1}{\alpha}. \quad (19)$$

It is now straightforward to find the number of events in our past light-cone. It should be emphasized that most numerical factors are irrelevant: if eq.(17) changes by 10^{10} , our results will be only marginally affected. For a given value of m_H and m_t , we find ϕ_1 from the effective potential and S from a tunneling calculations (see Arnold [11] for a simple description). This gives N and E , from which we calculate the number of events in our past light cone. As an example, suppose $S = 1000$, $m_H = 10^2 \text{ GeV}$; then in the case (i) $N \sim \frac{3}{8} S E \frac{\phi_1^2}{v^2}$, and $E = N m_H$. If $\phi_1 = 100 \text{ TeV}$, then eq. (19) gives $\sim 10^{14}$; if $\phi_1 = 1000 \text{ TeV}$, then eq. (19) gives zero (energy is not enough to produce N Higgs bosons). Clearly, our results are very insensitive to many numerical factors. We determine ϕ_1 and S from the full two-loop renormalization-group improved effective potential.

8. Bounds on m_t and m_H

In the light of the above discussion, we now examine the regions of m_t and m_H that we allowed under various hypotheses, as shown in the Figure. The line labelled LEP is the experimental lower limit $m_H > 41.6 \text{ GeV}$ recently established by the ALEPH collaboration [3], and the line labelled CDF is the experimental lower limit $m_t > 89 \text{ GeV}$ recently established by the CDF collaboration [1]. The dotted curve labelled NC surrounds the region of

(m_t, m_H) allowed at the $1 - \sigma$ level by a recent analysis [2] of electroweak data including the effects of radiative corrections. The dashed line labelled CW bounds the region of small (m_t, m_H) excluded by cosmological considerations of possible consequences of the existence of a local minimum of $V(\phi)$ at $\phi = 0$ without account taken of cosmic ray destabilization of the vacuum state [9,7,32,4]. The dashed line labelled NP bounds the region of large m_H in which the Standard Model couplings become nonperturbative above an energy scale $m_H \sim 10^{16}$ GeV. To the left of the dashed line S, our present electroweak vacuum is absolutely stable, whereas to the right of the dashed line US it would have decayed spontaneously somewhere within our past light-cone.

The discussion of the previous section has centered on the dash-dotted line labelled CR. It turns out that the assumptions (i) and (ii) on the Higgs boson distribution lead to values of m_t that only differ by 0(2) GeV for any given m_H , so we have only plotted one line. Below and to the right of this line cosmic ray collisions could possibly have triggered vacuum decay somewhere within our past light-cone. The region above and to the left of both of these lines is perhaps safe.

We see that within the region allowed by the LEP, CDF and NC constraints there are 3 distinct possibilities for the destiny of our universe. In the domain (1), our present electroweak vacuum is absolutely stable and neither we nor our descendants need worry about being exploded all of a sudden. In the domain (2), our present electroweak vacuum is unstable, but its partial lifetimes for both spontaneous decay and decay triggered by cosmic ray collisions exceed 10^{10} y. However, we do not exclude the possibility that some other mechanism might trigger more rapid vacuum decay. In the domain (3), our analysis indicates that cosmic ray collisions might trigger vacuum decay, but this problem deserves more thorough study. The boundary between the regions (2) and (3) is relatively insensitive to our assumptions on the distribution of the Higgs bosons produced in the cosmic rays collisions, but there are other uncertainties as we now discuss.

9. Relation of our Approach to other Methods of Investigation of Non-perturbative Electroweak Processes

There are many uncertainties associated with our approach. The main uncertainty is that we do not know much about multiple Higgs particle production at superhigh energies. But the main advantage is that we know much more about it than about *nonperturbative* processes accompanied by multiple Higgs particle production. We have essentially split the problem into two pieces: investigation of tunneling in the presence of many Higgs particles and the possibility of producing many Higgs particles with the desired properties. We already know a lot about the first piece, and we can hope gradually to understand more about the second one by the standard well-developed methods of the theory of multiple

particle production. *Prima facie*, it is encouraging that the two different assumptions (i) and (ii) of Section 7 gave very similar curves CR, but we cannot exclude the possibility that future investigations may yet modify this curve.

The situation here is very similar to the situation of nonperturbative electroweak processes with baryon number violation. In the first papers on this problem, such processes were considered in systems in thermodynamical equilibrium containing many Higgs bosons and vector particles [31]. This is analogous to the first (unambiguous) part of the investigation performed in our paper (Sections 2 - 6). On the other hand, the investigation of similar processes which may occur due to multiple particle production is much more complicated. The Euclidean approach used in most of the papers on nonperturbative electroweak processes with B-violation [17] is more conventional than the one used above, but all uncertainties there are built into a single Euclidean block, which hides them much better. For this reasons some attempts to subdivide the whole process into several pieces were made, see e.g. [32]. It would be interesting to study nonperturbative electroweak processes with B-violation [17] by the more simple and intuitive methods used in the present paper. One should study in particular the same issues which were very important for our investigation, namely whether multiple particle production is essentially point-like, and what will happen if one takes into account a possible non-sphericity of the distribution of particles produced after each particular collision.

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FIGURE CAPTION

Experimental, astrophysical and cosmological bounds on m_t and m_H . The line labelled LEP is the lower bound on m_H established in Ref. [3], and the line labelled CDF is the bound on m_t given in Ref. [1]. The dotted line labelled NC encloses the region allowed at 1 standard deviation by a recent analysis [2] of precision electroweak data including radiative corrections. The line CW is a cosmological lower bound on m_H for small m_t [9,7,32,4], and the line NP indicates where the Standard Model couplings become non-perturbative. Our present vacuum is stable to the left of the line S and has a spontaneous decay lifetime $< 10^{10}$ y to the right of the line US. The line CR bounds the region in which cosmic ray collisions might have triggered vacuum decay according to the assumptions discussed in the text.