Tests of QCD from jets on the Z^0 peak.

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Abstract

We present a QCD analysis of some recently published data on jet production in Z^0 decays. We find that at the Z^0 energy one can analyse also jet shape variables which were disregarded at lower energy e^+e^- experiments because of their large hadronization sensitivity. We also discuss the possibility of extracting from the jet data certain detailed features of the colour structure of the radiative corrections to three jet production.

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1. Introduction

The beginning of the operations of the e^+e^- colliders on the Z_0 peak, SLC and LEP, together with the first results they gave on the structure of hadronic events, have opened the possibility of performing new QCD tests in e^+e^- collisions. This is important not only for testing the validity of QCD as the fundamental theory of strong interactions, but also to test to which extent we can trust our use of QCD to model hadronic processes.

Because of the large theoretical uncertainties in e^+e^- jet physics (due to the large size of the radiative corrections and hadronization effects) an increase in the center of mass energy, with the corresponding reduction in the value of the strong coupling constant, improves the situation drastically. Furthermore, hadronization corrections decrease as an inverse power of the center of mass energy. Therefore, while at lower energy e^+e^- experiments, one was forced to consider jet quantities which have small hadronization sensitivity, at LEP one can consider a much larger class of quantities.

e^+e^- annihilation provides the simplest testing ground for perturbative QCD. Unlike hadronic collisions and deep inelastic scattering, where physical predictions must be given in terms of hadron structure functions, in e^+e^- physics there is a large class of quantities which depend only upon the strong coupling constant α_s . QCD tests in e^+e^- experiments can therefore be viewed as measurements of α_s . Consistency among the different measurements (as well as consistency with the values of α_s measured in different contexts) would provide a very convincing test of the underlying theory.

The simplest quantity which is calculable, and which can be directly related to α_s , is the hadronic width of the Z^0 . The perturbative prediction for this quantity is believed to be very sound^[1]. Non-perturbative hadronization corrections are believed to be highly suppressed for this quantity. One expects therefore that the uncertainties in the theoretical prediction are solely due to higher order perturbative effects. The LEP data show good agreement of the predicted and measured value of the hadronic width^[2]. The experimental errors, however, are very large, and it is difficult to turn this measurement into a determination of α_s .

The next simplest quantities for which QCD makes a definite prediction are

jet related measurements. The capability of making definite predictions for the distribution of infrared safe quantities relies upon the assumption of the softness of the hadronization process. Jet predictions should therefore be considered less sound than the prediction of the hadronic width. It is however important to test their validity, since the same assumption enters also in many calculations of hadronic processes.

The radiative corrections to jet quantities are quite large, typically of the order of 30%, for some quantities up to 50% at LEP energy. Therefore we expect theoretical uncertainties of the order of 10-20%, coming only from higher order corrections.

Perturbation theory is incapable of describing the whole production process. The formation of hadrons must be modeled in some way, and one must estimate the possible effects of hadronization on jet related quantities. One way to do this is by switching on and off the hadronization in a shower Monte Carlo. It turns out that, in most hadronization models, hadronization corrections to infrared safe quantities are suppressed by a single power of the annihilation energy. Therefore at LEP one expects moderate hadronization effects. We point out that hadronization effects should not be estimated using matrix element Monte Carlo models^[3]. In fact, in these models, the hadronization stage must also contain a large amount of higher order effects^[4]. This is because they do not include perturbative production of more than four partons.

2. QCD analysis

All four LEP experiments and MARK II have published studies on the structure of the hadronic final states (ref. [5] through [9]). Many of these studies report comparisons between data and Monte Carlo predictions. In ref. [8], detailed QCD studies have been performed, which show evidence in favour of the running of the strong coupling constant. A complete QCD study of all shape variable distributions has not been performed yet. However, in ref. [9], a very complete experimental study of a large number of shape variables has been given, together with tables of data unfolded from detector effects. Four of the variables examined in ref. [9] are infrared insensitive, and are therefore directly calculable in terms of the strong coupling constant alone. We will now perform a QCD analysis of these data. We will also

include the data on three cluster multiplicity, taken from the first of ref. [8]. We will show that a fairly consistent picture emerges. The need for the radiative corrections is clearly shown by the data. The introduction of hadronization corrections improves the consistency of the overall picture. However, unlike the case of lower energy e^+e^- collisions, the importance of hadronization effects is much smaller than the radiative corrections themselves, as it should be in the asymptotic regime. It is then interesting to ask whether the pattern of the radiative corrections one observes is specific of QCD. In order to answer this question we have attempted to see whether one can use the data to constrain the colour factors that appear in the formulae for the radiative corrections to shape variables. We found that it is very difficult to answer this question in full generality using only the variables we have considered. In spite of this, we will show that some information on the specific colour structure of the radiative corrections is present in the data.

We consider the following four jet shape variables: thrust, oblateness, F-major and C-parameter. The definition of these variables is as given in ref. [4]. The C-parameter is related to H_2/H_0 of reference [9] by $C = 1 - H_2/H_0$. In addition we consider the cross section for three jet production with the E_0 cluster definition[10]. These variables are all 3-jet dominated. Therefore in perturbative QCD they receive the lowest order contribution from the three parton production process ($\mathcal{O}(\alpha_s)$), and the $\mathcal{O}(\alpha_s^2)$ corrections are known[11].

We illustrate in detail the case of thrust. The analysis of the other variables is performed in an analogous way.

Thrust is defined as

$$t = \frac{\sum_a |\vec{p}_a \cdot \vec{n}_t|}{\sum_a |\vec{p}_a|} \quad (2.1)$$

where a runs over all the final state particles, and $\vec{n}_t$ (the thrust axis) is defined as the axis which maximizes the right-hand side of eq. (2.1). The kinematical range of t depends on the number of final particles. For two particles in the final state $t = 1$, for three particles $2/3 \leq t \leq 1$, for four particles $1/\sqrt{3} \leq t \leq 1$ and for an arbitrary number of particles in the final state $1/2 \leq t \leq 1$. The differential cross section at order α_s^2 is given by

$$\frac{1}{\sigma_0} \frac{d\sigma}{dt} = \frac{\alpha_s(\mu)}{2\pi} A_t(t) + \left(\frac{\alpha_s(\mu)}{2\pi} \right)^2 \left(A_t(t) 2\pi b_0 \log \frac{\mu^2}{S} + B_t(t) \right) \quad (2.2)$$

where σ_0 is the leading order cross section for the reaction $e^+e^- \rightarrow$ hadrons and

$$b_0 = \frac{33 - 2n_f}{12\pi}. \quad (2.3)$$

n_f is the number of quark flavours and $A_t(t)$, $B_t(t)$ are two universal functions depending only upon the definition of t and the perturbative structure of the theory.

In order to compare the theoretical distribution with the experimental one, formula 2.2 should be multiplied by a factor $\frac{1}{1+\alpha_s/\pi}$ (this corresponds to substituting σ_0 with the one loop corrected total hadronic cross section). At lowest order in α_s this is equivalent to the replacement of the function B with $B - 2A$.

We have chosen to extract α_s from the quantity

$$M_t^{(\text{exp})} = \sum_{i \in I} < 1 - t >_i \frac{N_i}{N_{\text{tot}}} \quad (2.4)$$

where the sum is taken over a suitable subset I of the experimental bins, $< 1 - t >_i$ is the central value of $(1 - t)$ over the i -th bin and N_i/N_{tot} is the corresponding fraction of hadronic events. Notice that the quantity $M_t^{(\text{exp})}$ is very close to the mean value of $(1 - t)$. This is the case provided the bins in I are sufficiently small and correspond to a significant range of t . We choose this quantity because the correct mean value of $(1 - t)$ was not available in the published OPAL data. The theoretical expression $M_t^{(\text{QCD})}$ has the form

$$M_t^{(\text{QCD})} = \frac{\alpha_s(\mu)}{2\pi} A_t + \left(\frac{\alpha_s(\mu)}{2\pi} \right)^2 \left(A_t 2\pi b_0 \ln \mu^2/s + B_t - 2A_t \right) \quad (2.5)$$

where

$$A_t = \sum_{i \in I} < 1 - t >_i \int_i A(t) dt \quad (2.6)$$

and analogously for B_t . The expression (2.5) is valid as long as the bins in I do not contain the two jet region ($t = 1$). We restricted the bin range in order to be always in the kinematic region of three jets. By considering the data in ref. [9] we choose $0.72 < t < 0.97$.

We have evaluated $\alpha_s(M_Z)$ from the data using three different approximations for the theoretical value of M_t :

1. using the cross-section at leading order in α_S ;
2. using the expression at next-to-leading order of eq. (2.5);
3. correcting the right-hand side in eq. (2.5) with a correction factor which takes into account the hadronization of partons.

For each of these three different approximations we evaluate $\alpha_S(M_Z)$ by solving eq. (2.5) for $\alpha_S(\mu)$. After that we have run $\alpha_S(\mu)$ to M_Z according to the two-loop renormalization group equation. Therefore the resulting determination of $\alpha_S(M_Z)$ can be represented as a function $a(\mu, M_t^{(\text{exp})})$. In order to assess the theoretical uncertainty due to the scale dependence we choose the values $\mu = M_Z/4, M_Z/2, M_Z$.

The determinations of $\alpha_S(M_Z)$ are shown in fig. 1. In this figure the central values correspond to the case $\mu = M_Z/2$, $\alpha_S(M_Z) = a(M_Z/2, M_t)$. The error bars have been determined by considering the range of $a(\mu, M_t^{(\text{exp})})$ when μ varies between $M_Z/4$ and M_Z and M_t between $M_t^{(\text{exp})} + \Delta M_t^{(\text{exp})}$ and $M_t^{(\text{exp})} - \Delta M_t^{(\text{exp})}$, where $\Delta M_t^{(\text{exp})}$ is the experimental error. Thus the error bars in fig. 1 give an estimate of both the experimental error and the theoretical uncertainties due to unknown higher order QCD effects. The correction due to the effect of hadronization, calculated using JETSET-72^[12] and HERWIG-43^[13], was given to us by J. W. Gary^[14]. In fig. 1 we show the determination of $\alpha_S(M_Z)$ coming from the mean value (in the sense of eq. (2.4)) of thrust, oblateness, F-major and C-parameter, together with the determination of $\alpha_S(M_Z)$ coming from the three cluster multiplicity in the E_0 -scheme [10]. The corresponding values of $\alpha_S(M_Z)$ are reported in Table 1. In the case of the leading order evaluation (fig. 1a) the theoretical contribution to the error bar is simply due to the running of α_S from μ to M_Z . It must be noticed that at the leading order the value of $\alpha_S(M_Z)$ obtained from oblateness is strongly inconsistent with the others. The experimental data clearly manifest the need for radiative corrections. The values of $\alpha_S(M_Z)$ displayed in fig. 1b are much more consistent and with smaller errors than those in fig. 1a. The smaller errors are due to the reduction of the scale dependence when going to next-to-leading order (the full, all order result, would be μ independent). Oblateness is an exception to the above pattern, because of its large and negative α_S^3 corrections. It also happens that in part of the range of μ that we have considered there is no solution for α_S in the case of oblateness. When this happens we simply set μ to the minimal scale

$\mu_{\min}$ in the range $M_Z/4 < \mu < M_Z$ for which there is a solution. In particular, for the next-to-leading determination of $\alpha_S(M_Z)$ from oblateness without hadronization corrections, and for the upper end of the experimental error, we have $\mu_{\min} = 0.44 M_Z$. When we include the hadronization correction, even for the central value of the experimental determination we have $\mu_{\min} = 0.56 M_Z$.

The sign of $\mu^2 da/d\mu^2$ is the same as the α_S^3 coefficient in eq. (2.5). The order α_S^3 corrections to oblateness are negative, while for all the others they are positive. This means that for oblateness the upper end of the error bar in fig. 1b corresponds to the small scale choice, and the lower end to the high scale. For the other quantities the pattern is inverted. From fig. 1b one can see that the low scale choice allows for a better description of the data than the choice $\mu = M_Z$. This might be an indication that the typical energy scale involved in the strong process of three jet production is somewhat smaller than the beam energy. Consequently the choice of a small μ would make the perturbation expansion converge faster. On the other hand, from fig. 1c-d, we can see that if we include non-perturbative hadronization corrections the choice $M_Z/2 < \mu < M_Z$ is perfectly adequate.

We have considered in our analysis only the average values of the shape variables. Of course more information is contained in the differential distributions. However we find that the value of α_S determined bin by bin are all very close to the value obtained from the average of the shape variable, but have larger statistical errors. In fig. 2 for example we show the bin by bin determination of α_S from the thrust parameter. This is the reason why we have chosen to present only the results obtained from the average values.

We now turn our attention to the effects of hadronization. In fig. 3 we show effect of the hadronization corrections to $<1-t>$, evaluated using HERWIG-43 and JETSET-72, as a function of the C.M.-energy. The hadronization corrections are estimated by switching on and off the hadronization stage after the full shower development. The figures show the ratio of the values of $<1-t>$ estimated with and without the hadronization stage. At 91 GeV these effects are $\sim 12\%$, while at 40 GeV they are $\sim 30\%$. They have a behaviour which is roughly $\sim 1/E_{\text{C.M.}}$. The expected hadronization corrections at LEP-2 energy are only of order 5–6%. Since $<1-t>$ is proportional to α_S in leading order, also the measured value of α_S will show a similar sensitivity to the hadronization corrections.

In Table II we report the hadronization corrections for all the variables^[14].

Correcting the perturbative result for hadronization effects brings about a better agreement with the data. From a theoretical point of view, however, there is no sound justification in doing so. In fact, there is simply no sound theory of power suppressed effects in jet production processes. One may also worry about the fact that by tuning the many parameters of Monte Carlo models in order to fit the data, one is biasing the possible results of the tests of the short distance theory. We observe, for example, that the value we are using now for the hadronization correction to thrust is twice as large as the one reported in ref. [4]. This is due to the fact that the results we are using now were obtained with the HERWIG parameters retuned to LEP data^[9]. One can therefore suspect that the tuning itself may have eaten away some physics effects which showed deviations from the QCD formulas, and were not necessarily given by hadronization. On the other hand, the fact that two different Monte Carlo models give similar results is reassuring. Furthermore, we observe that if we try to extract a value of α_S from fig. 1, at next to leading order, which is compatible with all the variables we examined, we would obtain similar values, whether or not we correct for hadronization effects. The results given in Table I roughly suggest the value $\alpha_S(M_Z) = .12 \pm .01$.

The most important indication that emerges from fig. 1 is that at LEP energies genuine perturbative QCD processes at order α_S^2 become more important than hadronization effects. This was not the case at PETRA energies.

3. The colour structure of radiative corrections

It would be interesting to see whether more detailed features of QCD, with its colour structure, do emerge from the data. In fact in all jet production processes of order α_S^2 , three independent combinations of colour factors arise. This is the case for 4-jet dominated variables and for α_S^2 corrections to 3-jet dominated ones. The colour factor that arises at order α_S is C_F , and at order α_S^2 we have C_F^2 , $C_F C_A$ and $C_F T_R$ (where in $SU(3)$ we have $C_F = 4/3$, $C_A = 3$, $T_R = n_f/2$).

Attempts to separate the various colour components from the 4-jet structure have already been performed^[16]. We would like to point out that in principle the same kind of information can also be extracted from 3-jet dominated shape vari-

ables. Therefore we have calculated separately the three colour components of the B coefficients (as defined in eq. 2.2)

$$B = B_{C_F} + B_{C_A} + B_{T_R} \quad (3.1)$$

where B_{C_F} is the term proportional to C_F^2 , B_{C_A} is proportional to $C_F C_A$ and B_{T_R} is proportional to $C_F T_R$.

We have evaluated the three components of eq. 3.1 by numerical integration using a modified version of the program developed by one of the authors in ref. [4]. We obtained histograms of the various components of $(1-t)B(t)$. The histograms were then approximated with expressions of the form

$$(1-t)B(t) = \sum_i C_i f_i(1-t) \quad (3.2)$$

where f_i are functions consisting of powers of $(1-t)$ and $\ln(1-t)$ and C_i are coefficients. In order to avoid the singularity due to the kinematic boundary between the three and four parton region, we restricted ourselves to the three parton region $2/3 < t < 1$. The coefficients C_i have been determined by performing a best χ^2 fit to the integral of the histogram. This is the correct procedure because the histogram value in each bin is the mean value of $(1-t)B(t)$ in the bin. We always found good fits with small χ^2 's. The errors of the numerical integration were of the order of 3 - 5% in each bin. We estimated that the error for the fitted function is of the order of a percent if we are not too close to the kinematic boundaries. This is more than adequate for phenomenological applications. However one should not use our formulae to extract the limiting behaviour of B near the kinematic boundaries. We performed analogous fits to a number of other shape variables. In Table III we report the results of our fits. The fits have the form

$$B(x) = \sum_i C_i F_i(x) \quad (3.3)$$

where $F_i(x) = f_i(x)/x$. We have $x = 1-t$ for thrust, $x = o$ for oblateness, and so on, and B is defined in a way analogous to eq. (2.2). In the table we give the functions F_i as well as the coefficients C_i for thrust, oblateness, C-parameter, F-major and N_3 (the number of three clusters in the E_0 scheme versus y_{cut}). Notice that for N_3 , which is an integrated quantity, the fitting functions F_i are different. The meaning

of B is in this case

$$\frac{\sigma(y_{\text{cut}})}{\sigma_0} = \frac{\alpha_S(\mu)}{2\pi} A_{N_3}(y_{\text{cut}}) + \left(\frac{\alpha_S(\mu)}{2\pi} \right)^2 \left(A_{N_3}(y_{\text{cut}}) 2\pi b_0 \log \frac{\mu^2}{S} + B_{N_3}(y_{\text{cut}}) \right). \quad (3.4)$$

For F-major, C-parameter and N_3 we report also the fit to the leading order coefficient A . For the oblateness the A term is equal to the one for F-major, and for thrust one can use the analytic formula given in ref. [15].

In principle, it would be nice to perform a fit to the three colour coefficients, and see if they are consistent with the QCD values. In practice, we found this not possible, at least by using only the five variables we have considered. There is, however, some information that can be extracted. In fig. 4 we show what happens to the determinations of α_S when the value of C_A is changed, keeping fixed C_F and T_R (notice that also b_0 must be changed accordingly). We do not apply any hadronization correction, since it is not at all clear how would they change as C_A changes. We can see that the values $C_A = 2$ and $C_A = 4, 5$ are less consistent than $C_A = 3$. It is difficult to quantify precisely the degree of inconsistency, since it depends upon the method we have chosen to determine the theoretical errors. We can, however, explain qualitatively what happens in fig. 4. For $C_A = 2$, we see that the cluster multiplicity becomes inconsistent with the thrust and the C-parameter. This is not because of the shift in the central values, but rather the effect of a reduction in the theoretical error. In fact, with smaller C_A the radiative corrections become smaller, the scale sensitivity becomes also smaller, and one expects smaller higher order effects. For $C_A = 4$, we see that the oblateness becomes inconsistent with the other values. It happens that oblateness has a large negative correction coming from the C_F^2 term which is partly compensated by a positive $C_A C_F$ term. At larger values of C_A the radiative correction to oblateness becomes then smaller, and the corresponding theoretical error is reduced. Also, the data requires a negative correction to oblateness, and therefore larger C_A values are disfavoured.

We attempted to see whether the specific colour structure of QCD emerges also from the distributions of the shape variables. In fig. 5, for example, we show the determination of $\alpha_S(M_Z)$ from the 3-cluster multiplicity performed at various values of y_{cut} , for both standard QCD and the case $C_A = 0$, $C_F = 4/3$, $T_R = n_f/2$. We can see that when going towards moderately small values of y_{cut} the $C_A = 0$ model becomes strongly disfavoured. Not only does one obtain large values of α_S ,

but also can one no longer use the “natural” range $M_Z > \mu > M_Z/2$, since very often there is no solution for α_S in this range (we have chosen in these cases the maximum value of μ for which a solution exists). This can be understood in the following way. As y_{cut} goes to zero the C_F^2 term goes to negative infinity (this is a manifestation of the Sudakov double logarithm). This is moderated by a large positive $C_A C_F$ term. Therefore in standard QCD this Sudakov type behaviour starts dominating at smaller values of y_{cut} . Fig. 5 then shows that the data clearly favour the moderated Sudakov behaviour required by standard QCD. We have also attempted to see what happens if one tries to fit the three cluster multiplicity for a choice of the colour factors corresponding to an “abelian” model of QCD, which corresponds to $C_A = 0$, $C_F = 4/3$, $T_R = C_F n_f$. We find that the fit in this case is even worse than the one shown in fig. 5.

In ref. [16] certain four jet distributions^[17] have been used to exclude a fictitious abelian model of QCD. Of course there are many other more compelling facts that exclude this possibility. We therefore prefer to state the problem in terms of excluding certain ranges of values for the various colour coefficients that arise at order α_S^3 . It is interesting then to point out that also from the 3-jet dominated shape variables one can extract information similar to the one obtained in ref. [16] using 4-jet variables.

4. Conclusions

In conclusion, we have performed a QCD analysis of some LEP data on 3-jet dominated shape variables that have recently become available. We have found that, unlike what happens at PETRA energy, the pattern of radiative corrections shows that we are reaching the asymptotic regime, with the leading terms reasonably larger than the next-to-leading terms, which in turn are larger than hadronization effects. This therefore indicates that a full analysis of a large class of shape variables is possible at LEP, and there is no need to restrict the analysis to particularly well behaved variables. We also attempted to see whether certain features of the colour structure of the underlying theory do emerge from the data. Although it seems impossible to extract each independent colour factor combination from the limited number of variables that we examined, we have seen that some restrictions on the

colour factors can be obtained. In particular, we have shown that certain choices of the colour factors, corresponding to an "Abelian" theory, are strongly disfavoured by the data.

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Figure Captions

- Fig. 1: The various determinations of $\alpha_S(M_Z)$. Each determination in each group corresponds to the value of $\alpha_S(M_Z)$ extracted (in the order) from thrust, oblateness, F-major, C-parameter, and N_3 for $y_{\text{cut}} = .08$.
- Fig. 2: Leading (crosses) and next-to-leading (diamonds) determinations of $\alpha_S(M_Z)$ from each bin of the thrust distribution measured by the OPAL collaboration.
- Fig. 3: Hadronization correction factor (hadronic over partonic) for the average value of $(1 - t)$ versus the CM energy.
- Fig. 4: The various next-to-leading determinations of $\alpha_S(M_Z)$ from thrust, oblateness, F-major, the C-parameter, and N_3 , for different values of the colour factor C_A .
- Fig. 5: The determinations of $\alpha_S(M_Z)$ from N_3 versus y_{cut} , for $C_A = 3$ (standard QCD) and $C_A = 0$.

	$\alpha_s(M_Z)$			
	Leading	L+NL	L+NL+HJ	L+NL+HH
t	0.205 +0.066 -0.041	0.142 +0.019 -0.015	0.132 +0.017 -0.013	0.132 +0.017 -0.013
o	0.007 +0.013 -0.011	0.098 +0.041 -0.020	0.163 +0.028 -0.058	0.160 +0.026 -0.057
f_m	0.159 +0.035 -0.025	0.122 +0.011 -0.009	0.117 +0.010 -0.008	0.118 +0.010 -0.008
c	0.188 +0.052 -0.034	0.137 +0.015 -0.012	0.125 +0.013 -0.010	0.126 +0.013 -0.011
N_3	0.142 +0.027 -0.020	0.113 +0.009 -0.007	0.120 +0.010 -0.008	0.117 +0.009 -0.007

Table I: Determinations of $\alpha_s(M_Z)$ from shape variables.

$\sqrt{s}$ (GeV)	10	20	30	40	50	60	70	91	150	200
$\langle 1-t \rangle$	3.15	2.02	1.55	1.35	1.25	1.20	1.17	1.12	1.094	1.034
	2.092	1.656	1.396	1.279	1.220	1.179	1.156	1.118	1.068	1.042
$\langle o \rangle$	1.026	0.754	0.707	0.707	0.719	0.749	0.760	0.793	0.870	0.883
	0.825	0.757	0.720	0.722	0.741	0.755	0.774	0.802	0.865	0.889
$\langle f_m \rangle$	2.31	1.60	1.32	1.20	1.14	1.12	1.10	1.07	1.04	1.01
	1.601	1.358	1.205	1.135	1.099	1.075	1.067	1.049	1.019	1.005
$\langle c \rangle$	2.93	1.98	1.58	1.39	1.28	1.24	1.20	1.15	1.10	1.01
	1.949	1.629	1.403	1.295	1.231	1.193	1.168	1.129	1.072	1.050
N_3	1.82	1.20	0.954	0.895	0.872	0.904	0.898	0.917	1.012	0.956
$y_{\text{cut}} \approx 0.08$	1.62	1.155	0.944	0.925	0.930	0.926	0.941	0.953	0.977	0.985

Table II : Hadronization corrections factors to the mean value of shape variables from HERWIG-43 (upper rows) and JETSET-72 (lower rows) at different energies.

$B = \sum_i C_i F_i$							
$F_1 = \frac{1}{x}, F_2 = 1, F_3 = x, F_4 = x^2, F_5 = \frac{\log x}{x}, F_6 = \frac{\log^2 x}{x}, F_7 = \frac{\log^3 x}{x}$							
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Thrust							
$x = 1 - t$	B_{C_F}	-865.34	1698.3	-1862.1	868.20	-521.17	-57.672
	B_{C_A}	1936.9	-4520.0	6882.7	-7095.9	1103.8	257.28
$\frac{2}{3} < t < 1$	B_{T_R}	1936.9	-4520.0	6882.7	-7095.9	1103.8	257.28
F-major	A	-4.1188	32.222	-73.192	74.591	9.4910	0
$x = f_m$	B_{C_F}	15868.6	-28517.2	22482.8	-10857.2	9998.75	2074.28
$0 < f_m < \frac{1}{\sqrt{3}}$	B_{C_A}	-147.595	1843.58	-3309.27	1825.29	399.623	80.6826
	B_{T_R}	2039.55	-3959.89	3792.08	-2330.92	1196.80	304.041
Oblateness	B_{C_F}	5776.88	-10419.9	7500.28	-2632.08	3367.24	624.618
$x = o$	B_{C_A}	4115.82	-5092.80	1162.38	248.891	3136.19	701.876
$0 < o < \frac{1}{\sqrt{3}}$	B_{T_R}	2959.06	-5662.94	5307.52	-3158.52	1744.32	419.629
C-parameter	A	-5.709	4.811	-6.430	3.665	5.271	0
$x = C$	B_{C_F}	-22.95	301.2	-874.1	591.0	207.9	103.3
$0 < C < \frac{3}{4}$	B_{C_A}	772.7	-1255	580.3	-165.3	741.6	101.0
	B_{T_R}	-150.8	186.0	53.52	-73.67	-187.6	-3.071
$F_1 = 1, F_2 = y, F_3 = y^2, F_4 = y^3, F_5 = \log^2 y, F_6 = \log^3 y, F_7 = \log^4 y$							
N_3	A	1.4062	-11.348	20.152	2.1322	-88420	-93761
$0 < y < \frac{1}{3}$	B_{C_F}	-52.749	380.97	-972.63	983.96	-35.709	-40.139
	B_{C_A}	5.1794	-69.503	55.315	302.29	-21.097	-19.963
	B_{T_R}	1.6887	-15.987	125.00	-274.02	7.5706	7.1083

Table III: Coefficients for the fits to the shape variable B terms defined in eqs. (2.2), (3.1) and (3.4). For F-major, the C-parameter and N_3 we also give the fit to the A terms.

- Fig. 1 -

- Fig. 3 -

- Fig. 2 -

- Fig. 5 -

- Fig. 4 -