Archives

PHOTOS-a universal Monte Carlo
for QED radiative corrections in decays.

ELISABETTA BARBERIO^{*}, BOB VAN EIJK^{**} and ZBIGNIEW WAS^{***}
CERN, Geneva.

ABSTRACT

In this paper we present an algorithm for the Monte Carlo simulation of QED single photon radiative corrections in decays. The algorithm is implemented in an independent package written in FORTRAN77. The program is universal, i.e., it allows for easy interface with "any" program generating decays of "any" particle. The program can be used to estimate the size of the QED bremsstrahlung in the leading-logarithmic (collinear) approximation. The proper soft photon behavior is also reproduced. The program is designed such that the inclusion (if necessary) of the exact, process dependent, $\mathcal{O}(\alpha)$ matrix element is *possible* and straightforward. The program provides final states with their full topology, including kinematical effects due to massive particles. For Z decays into a pair of light fermions an extensive comparison of our algorithm with programs based on exact $\mathcal{O}(\alpha)$ matrix element was performed.

* Permanent address, Fachbereich Physik, Universität Siegen, 5900 Siegen, Fed. Rep. Germany.

** Also supported by the 'Netherlands Organization for scientific Research (NWO), The Netherlands.

*** Permanent address, Institute of Nuclear Physics, 30-059 Kraków, ul. Kawory 26a, Poland.

PROGRAM SUMMARY

Title of the program: PHOTOS, version: 1.3

Computer: IBM 3090, VAX; *Installation:* CERN

Operating system: VM/CMS,VMS,ULTRIX

Programming language used: FORTRAN 77, program is available also in the PATCHY format

High speed storage required: 5000 words

No. of bits in a word: 32

Peripherals used: Line printer

No. of cards in combined program and test deck: 1589

Keywords: radiative corrections, heavy leptons, resonances, particle decays, Monte Carlo simulation, quantum electrodynamics, leading-log approximations

Nature of physical problem: Most of the decay modes of particles or resonances are accompanied by the QED real photon corrections. The presence of a hard photon may affect shape of the measured distributions, detection efficiencies and/or experimental selection criteria.

Method of solution: The Monte Carlo simulation is suited best for studying the combined process of production and decay of unstable particles including QED corrections and detector efficiencies.

Restrictions on the complexity of the problem: The QED $O(\alpha)$ corrections are implemented in the leading-logarithm approximation with proper soft photon behavior. Emission of non-collinear and hard photons is simulated, but the validity of distribution may be restricted.

Typical running time: 140 events/sec (VAX 8350) for the $Z \rightarrow \mu^+ \mu^-$ decay mode.

PROGRAM DESCRIPTION

1. Introduction.

QED (bremsstrahlung) corrections are well known (see eg. [1,2]) to affect measurements of decay properties. Hard photons may also affect selection criteria and, indirectly, measured branching ratios. The complete answer to the size of the bremsstrahlung correction can be obtained from a complete Monte Carlo study, which takes into account all classes of QED bremsstrahlung, that is, (i) bremsstrahlung in production (ii) bremsstrahlung in decays, (iii) detector properties.

Monte Carlo programs for the determination of radiative corrections in both production and decay have not previously been available. In some cases such corrections may be negligible. Attempts to introduce bremsstrahlung corrections in decays will require construction of separate programs for each decay channel of any particle or resonance. In the era of high statistics measurements, which starts with LEP, τ -, c- and b-factories, requirements for precision increase significantly. QED corrections in decays of many particles will have to be first estimated and later carefully calculated. The detailed knowledge of the kinematics of such bremsstrahlung in decay configurations is also essential in studies for the development of dedicated detectors, eg. for LHC or SSC.

The aim of this paper is to present a utility program which implements an algorithm for QED bremsstrahlung. It can be combined with any M.C. program generating decays. The only requirement is that the particle codes and common format as defined by the Particle Data Group [3,4] have to be used in the host program.

The outline of the paper is as follows. In chapter 2 we recall the basic elements of the structure of QED allowing for the construction of a universal algorithm for radiative corrections in decays. In chapter 3 we describe the phase space

parametrization of the decay. In chapter 4 we give a functional description of the algorithm used in our program. Chapter 5 is devoted to the discussion of the approximations in the algorithm and effects of the exact $\mathcal{O}(\alpha)$ matrix elements, which may be added later using (channel dependent) rejection techniques. The decay $Z \rightarrow \mu^+ \mu^-$ is given as an example. Chapter 6, concludes the paper.

2. Structure of QED corrections.

Let us recapitulate those properties of QED corrections which are independent of a particular process and are essential for the construction of our algorithm. The first relevant property is that of the factorization of bremsstrahlung kernels^{*} in the leading logarithm approximation [7]. We can write this in the form of the following generic formula

$$d\sigma(P(p) \rightarrow ch(q_1)Y_2(q_2)Y_3(q_3)\dots\gamma(k)) \approx d\sigma(P(p) \rightarrow ch(q_1+k)Y_2(q_2)Y_3(q_3)\dots) \frac{\alpha}{\pi} \frac{1+(1-x)^2}{2x} dx \frac{1}{\left(1 - \cos\theta \sqrt{1 - \frac{m_{ch}^2}{m_p^2}}\right)} d\cos\theta \frac{d\phi}{2\pi}. \quad (2.1)$$

P denotes the decaying particle, ch is the leading charged decay product, whose spin we assumed to be $\frac{1}{2}$. The $Y_i(q_i)$ denote other decay products which are either neutral or "slow" charged particles. The angles θ and ϕ define the direction of the bremsstrahlung photon with respect to ch , and x is the fraction of the energy carried by the photon with respect to the $(\gamma + ch)$ system. The parent P and charged particle ch masses are denoted by m_p and m_{ch} .

^{*} This factorization or rather semi-factorization extends also in some cases to exact $\mathcal{O}(\alpha)$ calculations. It was exploited in the algorithm of MUSTRAAL [5] for the muon pair production and later in Monte Carlo for Z and W decays into a pair of quarks and a gluon [6].

The intuitive meaning of the formula (2.1) is that the charged P decay product ch , fragments into itself and a photon according to the corresponding Lipatow-Altarelli-Parisi [8,9,10] splitting function. We do not integrate over the angular distribution of the photon but keep only those terms that properly account for the leading log result.

The $\mathcal{O}(\alpha)$ bremsstrahlung corrections alone lead to infrared singularities and indeed the integrated contribution from the formula (2.1) is infinite. However, the singularities are cancelled by virtual corrections which are infinite as well, but carry the opposite sign. From the Lee-Nauenberg-Kinoshita theorem [11] we know that by summation of virtual and bremsstrahlung corrections large mass logarithms[†] are cancelled. At the level of our approximation this means that the virtual corrections are of the same size as the real bremsstrahlung ones but with the opposite sign. We can extend the formula (2.1) to include the Born level cross section and virtual corrections:

$$d\sigma(P(p) \rightarrow ch(q_1)Y_2(q_2)Y_3(q_3)\dots(\gamma(k))) \approx \frac{d\sigma(P(p) \rightarrow ch(q_1)Y_2(q_2)Y_3(q_3)\dots)\delta(\vec{k})(1+N(\epsilon)) + d\sigma(P(p) \rightarrow ch(q_1+k)Y_2(q_2)Y_3(q_3)\dots)\theta(\vec{k}-\epsilon)}{\pi \frac{1+(1-x)^2}{2x}} dx \frac{1}{\left(1 - \cos\theta \sqrt{1 - \frac{m_{ch}^2}{m_p^2}}\right)} d\cos\theta \frac{d\phi}{2\pi}. \quad (2.2)$$

Here we introduced a cut-off on the photon energy ϵ . All bremsstrahlung below energy ϵ is integrated and summed over together with the virtual corrections and is absorbed into the factor $N(\epsilon)$.

The formula (2.2) would be sufficient if we were only interested in obtaining the leading logarithmic approximation for bremsstrahlung. However, as we want

[†] We exclude vacuum polarisation effects from our consideration. The bulk of these effects can be usually incorporated into the overall normalisation of the branching ratio.

to control systematic errors due to neglected terms, we will impose the following requirements on the matrix element and phase space to be generated:

- the full phase space has to be covered,
- 4-momentum conservation has to be rigorously obeyed, including all mass effects,
- the approximate matrix element used in the algorithm has to be known *exactly* and given in an explicit analytical form.

3. Parametrization of the phase space.

Before we restructure formula (2.2) let us define variables used to parametrize the phase space:

- k is the photon energy in the P rest frame.
- θ_1, ϕ_1 are angles defining the Y direction in the P rest frame. The Y is equal to Y_2 for the two body decay modes and denotes the sum of all Y_i otherwise.
- θ_2, ϕ_2 are angles defining the direction of the photon momentum with respect to ch in the rest frame of the $(ch, \sum_i Y_i)$ system (thus excluding the photon).
- For a two-body decay this completes the parametrization of the phase space. In case of a multi-body final state we can complete the parametrization of the phase space with the introduction of M_Y , the invariant mass of the $\sum_i Y_i$ system and any set of variables defining sub-phase space ω_Y of Y_i in the $\sum_i Y_i$ rest frame.

Let us start with the parametrization of the phase space for the Born cross section. We will use the Bjorken convention [12].

$$d\sigma^{Born}(P \rightarrow chY) = |M_B|^2 \frac{1}{m_P 16(2\pi)^2} \frac{\lambda^{1/2}(m_P^2, m_{ch}^2, M_Y^2)}{m_P^2} d\cos\theta_1 d\phi_1. \quad (3.1)$$

Here $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. For two particle final states, M_Y is the mass of $Y = Y_2$. For a multi-body final state, the phase space given by the

formula (3.1) is completed by the factor $dM_Y^2 d\omega_Y$, where $d\omega_Y$ denotes the “decay phase space” of the system Y and dM_Y^2 the integral over its mass. For each final state fermion, a factor $2m_f$ should be added to the phase space expression and, if there are any identical particles a statistical factor should be also included.

The differential cross-section for the decay $P \rightarrow chY\gamma$ takes the following form:

$$d\sigma(P \rightarrow chY\gamma) = |M|^2 \frac{1}{m_P 32(2\pi)^5} \frac{\lambda^{1/2}(m_P^2(1 - \frac{2k}{m_P}), m_{ch}^2, M_Y^2)}{m_P^2(1 - \frac{2k}{m_P})} k dk d\cos\theta_1 d\phi_1 d\cos\theta_2 d\phi_2, \quad (3.2)$$

with the photon energy constraint: $0 < k < k_{max} = \frac{m_P}{2} - \frac{(m_{ch} + M_Y)^2}{2m_P}$. Note also that in case of multi-body final states the same extension of the phase space as in the Born case has to be performed.

We can rewrite expression (3.2), and factorize the matrix element squared $|M|^2$ into the Born matrix element $|M_B|^2$ and a bremsstrahlung factor $f(k, c_2, \phi_2)$. This factorization is valid in the leading log approximation. We obtain formula for the *raw* cross section:

$$d\sigma^R(P \rightarrow chY(\gamma)) = |M_B(\cos\theta_1, \phi_1)|^2 \frac{1}{m_P 32(2\pi)^5} f(k, \cos\theta_2, \phi_2) \frac{\lambda^{1/2}(m_P^2, m_{ch}^2, M_Y^2)}{m_P^2} k dk d\cos\theta_1 d\phi_1 d\cos\theta_2 d\phi_2, \quad (3.3)$$

which can be rewritten as

$$d\sigma^R(P \rightarrow chY(\gamma)) = d\sigma^B(P \rightarrow chY) f(k, \cos\theta_2, \phi_2) k dk d\cos\theta_2 d\phi_2. \quad (3.4)$$

The factor $f(k, \cos\theta_2, \phi_2)$ is a function given to $\mathcal{O}(\alpha)$ in leading-logs and for ch of

spin $\frac{1}{2}$ we derive

$$f(k, \cos \theta_2, \phi_2) = \frac{(2\pi)^2}{m_P^2} \left\{ \delta\left(\frac{k}{m_P}\right) \frac{m_P}{k} \left(1 - \frac{\alpha}{\pi} N\right) + \right. \quad (3.5)$$

$$\left. \Theta(k - \epsilon) \frac{\alpha}{\pi} \left(1 + \left(1 - \frac{k}{2k_{\max}}\right)^2 \frac{m_P^2}{2k^2} \frac{2}{1 - \cos \theta_2} \sqrt{1 - \frac{m_{2A}^2 m_P^2}{(m_P^2 + m_{2A}^2)^2}}\right) \right\},$$

with

$$N = \frac{1}{\sqrt{1 - \frac{m_{2A}^2 m_P^2}{(m_P^2 + m_{2A}^2)^2}}} \ln \left(\left(1 + \sqrt{1 - \frac{m_{cA}^2 m_P^2}{(m_P^2 + m_{cA}^2)^2}} \right)^2 \frac{(m_P^2 + m_{cA}^2)^2}{m_{cA}^2 m_P^2} \right) \quad (3.6)$$

$$\left(\ln \frac{k_{\max}}{\epsilon} - \frac{3}{4} + \frac{\epsilon}{k_{\max}} - \frac{\epsilon^2}{4k_{\max}^2} \right).$$

If we assume that the exact matrix element for a certain decay channel is known, we can correct for possible deviations due to the leading logarithm approximation used in eq. (3.3). The ratio between the exact expression (3.2) and the approximation (3.3) is given in a form of a weight:

$$W = \frac{|M|^2}{|M_B|^2} f(k, c_3, \phi_2), \quad (3.7)$$

where the Jacobian factor (J) reads:

$$J = \frac{\lambda^{1/2} \left(1, \frac{m_{2A}^2}{m_P^2} \left(1 - \frac{3\epsilon}{m_P} \right), \frac{M_Y^2}{m_P^2} \left(1 - \frac{3\epsilon}{m_P} \right) \right)}{\lambda^{1/2} \left(1, \frac{m_{2A}^2}{m_P^2}, \frac{M_Y^2}{m_P^2} \right)}. \quad (3.8)$$

In the case of $k = 0$ the $|M|^2$ should be understood as the sum of Born, virtual and, integrated up to energy ϵ , real photon contributions. The function $f(k, \cos \theta_2, \phi_2)$ was chosen such that $W \rightarrow 1$ as the ch mass goes to zero and for $0 \ll 1 - \cos \theta_2 \ll 1$ (leading log).

Let us now return to the discussion of the formula (3.4). The expression forms the basis of the algorithm presented in this paper. For the generation of events one can simply use the Born level algorithm and then supply it with the photon variables (k, θ_2, ϕ_2) generated according to the distributions (3.4-3.6). The leading-log content of these formulae is equivalent to that of (2.2). Namely, that the bremsstrahlung constitutes a process of fragmentation of the charged P decay product into itself and a photon, according to the Altarelli-Parisi-Lipatow splitting function. However, in this case we exactly control the approximate distributions we use. Note that the generation of the photon variables depends only on the electromagnetic charge and spin of ch and does not depend on the properties of the particular process under consideration.

One has to be careful when applying this algorithm. It is assumed that the Born matrix element M_B is a slowly varying function of the phase space. Special care has to be taken in the case of the cascade decays. For example one should apply the algorithm presented here twice, for the $\tau^+ \rightarrow \rho^+ \nu$ and $\rho^+ \rightarrow \pi^+ \pi^0 \nu$, rather than in one step to the $\tau^+ \rightarrow \pi^+ \pi^0 \nu$, because of the strong variation of the matrix element in the presence of the ρ resonance in the $\tau^+ \rightarrow \pi^+ \pi^0 \nu$ 3 body phase space.

4. Description of the Monte Carlo algorithm.

In this section we will present the PHOTOS Monte Carlo algorithm. The algorithm for the treatment of photonic corrections to the kinematics of a general multi-body decay of any particle is based on the manipulation of four-momenta of the decay particles as in the absence of any radiation. Clearly, the decay products of P may be obtained applying any kind of package that is able to generate complete decays.

The program is organized as follows:

- (1) We calculate two angles $\cos \theta_1, \phi_1$ (defining ch direction) and the masses of ch

and spectator system $\sum_i Y_i$ from the four-momenta of the P decay products defined in the P rest frame.

- (2) According to the distribution (3.5) we generate a bremsstrahlung photon with energy k and direction $\cos \theta_2, \phi_2$.
- (3) If the generated photon energy is zero, we leave the original event unmodified and go to point (9) of the algorithm. Otherwise, through the unique relation between the parametrization of the Born level phase space with the photon variables $k, \cos \theta_2, \phi_2$, and phase space including photon, we determine the complete kinematical configuration of the bremsstrahlung event. This is defined indirectly by the parametrization of the phase space given in chapter 3.
- (4) The modified momentum q of the charged particle ch in the rest frame of the $(ch, \sum_i Y_i)$ system is obtained through the relation

$$q = \frac{\lambda^{1/2} \left(m_P^2 \left(1 - \frac{2k}{m_P} \right), m_{ch}^2, M_Y^2 \right)}{2m_P \sqrt{1 - \frac{2k}{m_P}}} \quad (4.1)$$

- (5) We rotate all four-momenta of P decay products to have the momentum of ch anti-parallel to the third axis of the reference frame.
- (6) We boost the charged particle (and its subsequent decay products) along the third reference axis, reducing its momentum to $-q$. A second boost is performed, in which resultant momentum of the Y system is reduced to q .
- (7) We construct the photon 4-momentum from variables $k, \cos \theta_2, \phi_2$. The complete event is constructed in terms of four-momenta in the rest frame of the decay particles, excluding the photon.
- (8) Next, a series of Lorentz transformations is performed which are the same for all P decay products. First, they are rotated in order to get the photon along third axis of the reference frame. Next, all particles are boosted to

the P rest frame. The boost is followed by a rotation to get the resultant momentum of Y along the third axis*. A final rotation restores the original direction (as defined at the Born event) of Y.

- (9) An iterative procedure is applied in the case where the charged multiplicity is greater than one, that is, more than one particle is able to radiate a photon. Since our algorithm treats only $\mathcal{O}(\alpha)$ corrections, *simultaneous* emission of photons from the several charged decay products is *excluded*. Thus the iteration over charged particles is ended as soon as the first photon emission occurs or when no charged particles are left. In the latter case, PHOTOS leaves the event untouched. To compensate for the reduced probability of the program to reach further iterations of the algorithm we increase accordingly the probability of the hard bremsstrahlung. We divide the raw hard photon probability (see formula 3.6) by the probability of not generating a photon in all previous iterations.

- (10) If the exact matrix element for a certain decay channel is known, we may correct for the possible deviations of our algorithm from the exact results with the help of the weight (3.7). If an event is rejected, PHOTOS leaves the event untouched. Optionally, if the calculation of the weight, as in the case with the simple universal weight presented later in this Section does not require all information about the event, the rejection can be done without the construction of the event, and thus some computer time can be saved.

— Let us note that for the subsequent decay products of Y_i (or ch) we perform a simplified transformation to save computer time. It is defined using original and final directions of Y_i (ch) only. In this way we introduce a spurious Wigner rotation and lose control over the overall orientation of the Y_i (ch) decay products in Y_i (ch) rest frames. In the case of cascade decays with spin

* In a multi-body (> 2) decay, in addition to the transformation sequence described above we make two rotations to assure that the azimuth of the Y constituents, with respect to the Y-axis, is preserved after introduction of the bremsstrahlung corrections.

degrees of freedom included, the more complicated transformation, identical to transformation on mother particles, would have to be used.

The single universal weight is defined in our program. It is always smaller than one and we perform rejection according to it after point (3) of the above algorithm. With the help of this rejection we correct the distribution in the soft photon region and for the differences between Altarelli-Parisi kernels for possible different spins, s , of ch .

Our principal weight reads:

$$W = W_1 W_2 W_3, \quad (4.2)$$

where

$$W_1 = \frac{\beta_1 (1 - \cos \theta_2) \beta_0}{\beta_0 (1 - \cos \theta_2) \beta_1}, \quad (4.3)$$

$$W_2 = \left(1 - \frac{1 - \beta_1^2}{G_s (1 - \cos^2 \theta_2 \beta_1^2)}\right) \left(\frac{1}{2} + \frac{\cos \theta_2}{2} \beta_1\right) \quad (4.4)$$

and for decay products of different spin s we use

$$W_3 = \begin{cases} \frac{1 - \frac{h}{k_{max}}}{1 - \frac{h}{k_{max}} + \frac{1}{2} \left(\frac{h}{k_{max}}\right)^2} & \text{for } s = 0; \\ 1 & \text{for } s = \frac{1}{2}; \\ \frac{1 - \frac{h}{k_{max}} + \frac{1}{2} \left(\frac{h}{k_{max}}\right)^2 - \frac{1}{2} \left(\frac{h}{k_{max}}\right)^3}{1 - \frac{h}{k_{max}} + \frac{1}{2} \left(\frac{h}{k_{max}}\right)^2} & \text{for } s > \frac{1}{2} \end{cases} \quad (4.5)$$

and

$$G_s = \begin{cases} 1 & \text{for } s = 0; \\ \frac{1}{2} \left(1 - \frac{k}{k_{max}} + \frac{1}{1 - \frac{k}{k_{max}}}\right) & \text{for } s = \frac{1}{2}; \\ 1 & \text{for } s > \frac{1}{2} \end{cases} \quad (4.6)$$

where, we define

$$\beta_1 = \sqrt{1 - \frac{4m_{ch}^2 \left(1 - \frac{2k}{m_P}\right)}{m_P^2 \left(1 - \frac{2k}{m_P} + \frac{m_{ch}^2 - M_P^2}{m_P^2}\right)^2}}, \quad (4.7)$$

$$\beta_0 = \sqrt{1 - \frac{m_{ch}^2 m_P^2}{(m_P^2 + m_{ch}^2)^2}}. \quad (4.8)$$

Let us now explain briefly the components of the weight W . The first two components W_1 and W_2 correct our original distribution in the soft photon region.^{*} Effectively, in this region matrix element actually used in the generation is

$$M = M_B \left(\frac{q_1 \cdot \varepsilon}{q_1 \cdot k} - \frac{P \cdot \varepsilon}{P \cdot k} \right). \quad (4.10)$$

The photon polarization is denoted ε , and appropriate summation over ε is assumed in the calculation of the cross sections. Weight W_3 accounts for the difference between the Altarelli-Parisi kernel for spin $\frac{1}{2}$ particle, and kernels for the other spins.

Any further improvements on distributions can be performed only for the separate modes and they require inclusion of the corresponding $\mathcal{O}(\alpha)$ matrix elements. In the M.C. it would be realized easily with help of additional rejection.

^{*} Note that in the case when the algorithm is used for multi-charged final states the weights $W_1 W_2$ has to be modified to reproduce the infrared limit of the distribution. An additional weight (calculated from the four-momenta defined in the P rest-frame) has to be introduced:

$$W_{multi} = \frac{\sum_i \left| Q_i \frac{q_{1,i} \cdot \varepsilon}{q_{1,i} \cdot k} + Q_i \frac{q_{2,i} \cdot \varepsilon}{q_{2,i} \cdot k} + \dots \right|^2}{\sum_i Q_i^2 \left| \frac{q_{1,i} \cdot \varepsilon}{q_{1,i} \cdot k} \right|^2 + Q_i^2 \left| \frac{q_{2,i} \cdot \varepsilon}{q_{2,i} \cdot k} \right|^2 + \dots}, \quad (4.9)$$

where $Q_1, Q_2, \dots$ denote charges of the decay products of P . In the calculation of this weight, use of REAL*8 variables in kinematics, may be important if a ratio of the mother particle mass to the mass of charged decay product is large. We decided not to include this weight into our program. Note also, that this weight will not necessarily be smaller than 1.

5. Systematic error of the algorithm due to neglected terms.

To discuss the effects of neglected terms, we will present a detailed comparison of our algorithm with that of MUSTRAAL [5], which includes effects of complete $\mathcal{O}(\alpha)$ QED corrections in the reaction $e^+e^- \rightarrow Z, \gamma \rightarrow \mu^+\mu^-$. By suppressing the initial state bremsstrahlung and quantum interference between bremsstrahlung from initial and final states, we obtain an algorithm similar to the one adopted by PHOTOS. This is no surprise, as the algorithm described here was constructed on the basis of a modernized MUSTRAAL-type algorithm, the single photon option of the KORALZ program [13]. (The difference between the single photon option of the KORALZ and MUSTRAAL programs consists of a clearer and more precise treatment of masses and τ spin effects.)

Our algorithm differs from that of MUSTRAAL or the single photon option in KORALZ in two ways. The first difference is purely technical: MUSTRAAL is by construction, a stand-alone program and it does not perform transformations of previously generated Born events. The second is of a more fundamental nature and reflects the fact that MUSTRAAL includes the complete matrix element. From a technical point of view, the algorithm of PHOTOS applied to the $e^+e^- \rightarrow \mu^+\mu^-$ events is identical to MUSTRAAL.[†] However, the weights and some distributions used in the generation are different. To generate exactly the same distributions with PHOTOS as with MUSTRAAL we have to replace in formula (3.5)

$$\frac{2}{1 - \cos \theta_2 \sqrt{1 - \frac{m_e^2 m_p^2}{(m_p^2 + m_e^2)^2}}} \quad (5.1)$$

by

$$\frac{1}{1 - \cos \theta_2 \sqrt{1 - \frac{m_e^2 m_p^2}{(m_p^2 + m_e^2)^2}}} + \frac{1}{1 + \cos \theta_2 \sqrt{1 - \frac{m_e^2 m_p^2}{(m_p^2 + m_e^2)^2}}} \quad (5.2)$$

[†] Note that there is an error in the description of the MUSTRAAL program [5]. Contrary to the statement made there, the angles defined in the Born distributions have a physical meaning of the direction of respectively μ^+ or μ^- in the LAB system.

In addition, the three components of the weight (4.2) have to be replaced by the following expressions:

$$W_1 = \frac{1 - \cos^2 \theta_2 \beta_0^2}{1 - \cos^2 \theta_2 \beta_1^2}, \quad (5.3)$$

$$W_2 = 1 - \frac{1 - \beta_1^2}{G_s} \left(\frac{1 - \cos \theta_2 \beta_1}{1 + \cos \theta_2 \beta_1} + \frac{1 + \cos \theta_2 \beta_1}{1 - \cos \theta_2 \beta_1} \right), \quad (5.4)$$

and

$$W_3 = \frac{\left(1 - \frac{1}{2} \frac{k}{k_{\max}} (1 - \cos \theta_2 \beta_1)\right)^2}{1 + \left(1 - \frac{k}{k_{\max}}\right)^2}. \quad (5.5)$$

The G_s factor does not require any modification. We have checked, with a sample of 10^6 events, that once the modifications listed above are made, there are no differences between results from PHOTOS and MUSTRAAL.

In the example presented above it was rather simple to introduce the complete $\mathcal{O}(\alpha)$ matrix element because of its factorization property. This is not always true. The factorization which allows us to represent leading-log or infrared corrections in the form of a convolution of a Born cross section and a structure function distribution, breaks $\mathcal{O}(\alpha)$ when we incorporate mass terms ($m^2 \simeq s$) [14]. Even in the ultrarelativistic limit, this kind of treatment of exact matrix elements has its limitations. If helicities of the decay particles would be measurable, (as in the case of, e.g., $Z \rightarrow \tau^+ \tau^-$) then there exists a nonvanishing correction in the $m_\tau/m_Z \rightarrow 0$ limit, which breaks $\mathcal{O}(\alpha)$ factorization. This correction is fortunately very small. We can safely neglect it until more than 10^6 Z decays into τ pairs have to be considered.

In [6], which includes a description of another MUSTRAAL-inspired algorithm, it was shown that the two choices of the kinematics, in our language corresponding to the emission of the photon from the μ^- or μ^+ , are in fact closely related to the polarization of the emitted photon. In the soft photon limit, according to the formulae (5.2-5.5), the probability for the emission of a photon collinear to the μ^+ to be “emitted” by the μ^- is thus 50% ! In this case “emitted” means simply the

choice of the corresponding branch of kinematics. Fortunately, when the photon energy diminishes, the difference between the two kinematical choices tends to disappear anyway.

We now present the numerical results of the tests of our program. In all figures, the ratios of distributions generated with the PHOTOS and MUSTRAAL Monte Carlo programs are presented. The solid lines correspond to the normal mode of the PHOTOS operation. In the case of the dotted line we have included effects due to the weight w_{multi} given by formula (4.9). We present the following plots:

1. Photon energy spectrum.
2. Photon angular distribution with respect to μ^+ , for photons' energy below 1 GeV.
3. Photon angular distribution with respect to μ^+ , for photons' energy above 1 GeV.
4. Transverse distribution of all produced particles, as defined in Ref. [6].

As we can see the agreement is excellent within the expected accuracy. For the weighted events, only in the transverse distribution there is a statistically significant difference between PHOTOS and MUSTRAAL (10^6 events). The statistical fluctuations on the histograms are sizeable because most of the cross section is placed in the first, and eventually last, bins.

Let us stress that the algorithm described here, is definitely limited to the leading-log approximation (with soft photon improvements). One should not be impressed too much by the excellent agreement between PHOTOS and MUSTRAAL. To some extent it reflects the similarity between the two programs. In fact, the weight (4.2,4.9) of the PHOTOS program was chosen to minimize the differences. The discrepancies between PHOTOS and $\mathcal{O}(\alpha)$ results are expected for hard, non-collinear photons. Our approximation can indeed be quite imprecise. For example, in the $\tau \rightarrow \pi \bar{\nu}$ decay the big logarithm is $\ln \frac{m_\tau}{m_\pi} \simeq 3$. We may expect a systematic error in our approach to be on the order of 30%.

Finally, let us remark that in the present version of the program the Jacobian factor of eq. (3.8) is not included in the weight. It may be useful to introduce it for the regions near the edges of the phase space where the mass terms become important. Thus, one should not apply our algorithm to calculate the distributions relevant for the measurement of the neutrino mass in τ, μ decays without further discussion and improvements.

6. Conclusion.

The present program represents an example of a Monte Carlo generator for QED single photon radiative corrections in decays. It has the following properties:

- (1) It can be combined with any Monte Carlo for simulation of decays, provided that this host program is based on the concept of Standard Common definitions of the Particle Data Group.
- (2) It conserves both energy and momentum.
- (3) Leading-log (collinear) and infrared limits are properly reproduced in the QED bremsstrahlung matrix element.
- (4) Systematic errors due to neglected terms are well understood.

The program is supplemented with a demonstration program and output of a test run. Detailed instructions how to run the program are given in Ref. [15]. To run the example program the JETSET [16] version 7.2 [17] has to be available.

Acknowledgements Useful discussions with S. Jadach, A. M. Lutz and B. Kniehl are acknowledged. Thanks go to U. Stiegler who helped to debug some of the formulae. One of the authors (Z.W.), acknowledge warm hospitality of the following laboratories: CERN-LAA Geneva, CPPM Marseille and MPI Munich, where parts of this work were done, and support from the Polish Government Grants CPBP.01.09, CPBP.01.03 and RRI.14.1.

REFERENCES

1. S. Jadach, et al. "The τ polarization measurement", in "Z physics at LEP 1", ed. G. Altarelli, R. Kleiss C. Verzegnassi, CERN report, CERN 89-08 (1989) vol 1, p. 235.
2. A. Ali, Z. Z. Aydin, Nuovo Cim. **43A** (1978) 270, S. Behrends et al. Phys. Rev. **D32** (1985) 2468.
3. M. Aguilar-Benitez et al. (Particle Data Group), 'Review of Particle Properties', Phys. Lett. **B239** (1990) 1.
4. T. Sjöstrand, et al. "QCD Generators", in "Z physics at LEP 1", ed. G. Altarelli, R. Kleiss C. Verzegnassi, CERN report, CERN 89-08 (1989) vol 3, p. 143.
5. F. A. Berends, S. Jadach, R. Kleiss, Comput. Phys. Comm. **29** (1983) 185.
6. R. Kleiss, Phys. Lett. **B180** (1986) 400.
7. V. N. Baier, V. S. Fadin, V. A. Khoze, E. A. Kuraev, Phys. Rep. **78** (1981) 293.
8. G. Bonneau, F. Martin, Nucl. Phys. **B27** (1971) 381.
9. L. N. Lipatov, Sov. J. Nucl. Phys. **20** (1975) 94.
10. G. Altarelli, G. Parisi, Nucl. Phys. **B126** (1977) 298.
11. T.D Lee, M. Nauenberg, Phys. Rev. **133** (1964) B1549, T. Kinoshita, J. Math. Phys. **3** (1962) 650.
12. J. D. Bjorken, S. D. Drell, Relativistic Quantum Mechanics, McGraw-Hill Book Company, 1964
13. Z. Wąs, Acta Phys. Polon. **B18** (1987) 1099.
14. S. Jadach, Z. Wąs, Comp. Phys. Comm. **36** (1985) 191.
15. E. Barberio, B van Eijk, Z. Wąs, PHOTOS, version 1.3 User Guide and Reference Manual, Long Write-up W5049 and CERN/LAA-MSL/90-04.
16. T. Sjöstrand, Comp. Phys. Commun. **27** (1982) 243, Comp. Phys. Commun. **28** (1983) 229, and M. Bengtsson, T. Sjöstrand, Comp. Phys. Commun. **43** (1987) 367.
17. M. Bengtsson, T. Sjöstrand, "Jet fragmentation and e^+e^- annihilation" CERN program library ref no: W5035.

Fig 1 :
Ratio PHOTOS/MUSTRAAL for energy distribution

Fig 3 :
Ratio PHOTOS/MUSTRAAL for angular distribution
 $k > 1 \text{ GeV}$

Fig 4 :
Inclusive transverse momentum distribution
of all particles for $Z^0 \rightarrow \mu^+\mu^-$

Fig 2 :
Ratio PHOTOS/MUSTRAAL for angular distribution
 $0.046 \text{ GeV} < k < 1 \text{ GeV}$

The Lund Monte Carlo - JETSET version 7.2
** Last date of change: 25 Nov 1989 **

PHOTOS, Version: 1.3
Released at: 6/ 5/90
PHOTOS QED Corrections in Particle Decays
Monte Carlo Program - by E. Barberio, B. van Eijk and Z. Was

Event No.	Particle Parameters	Parent(s)	Daughter(s)	Px	Py	Pz	E	Inv. M
0	1	0	3	37.24	21.82	14.34	45.50	1.35
0	2	0	3	-37.24	-21.82	-14.34	45.50	1.35
0	3	94	1	0	0	0	91.00	91.00
0	4	4	3	0	0	0	91.00	91.00
0	5	4	3	0	0	0	91.00	91.00
0	6	21	4	16.16	10.85	14.25	45.71	6.85
0	7	21	4	16.16	10.85	14.25	45.71	6.85
0	8	21	4	16.16	10.85	14.25	45.71	6.85
0	9	21	5	16.16	10.85	14.25	45.71	6.85
0	10	21	5	16.16	10.85	14.25	45.71	6.85
0	11	21	5	16.16	10.85	14.25	45.71	6.85
0	12	21	5	16.16	10.85	14.25	45.71	6.85
0	13	21	5	16.16	10.85	14.25	45.71	6.85
0	14	21	5	16.16	10.85	14.25	45.71	6.85
0	15	21	5	16.16	10.85	14.25	45.71	6.85
0	16	21	5	16.16	10.85	14.25	45.71	6.85
0	17	21	5	16.16	10.85	14.25	45.71	6.85
0	18	21	5	16.16	10.85	14.25	45.71	6.85
0	19	21	5	16.16	10.85	14.25	45.71	6.85
0	20	21	5	16.16	10.85	14.25	45.71	6.85
0	21	21	5	16.16	10.85	14.25	45.71	6.85
0	22	21	5	16.16	10.85	14.25	45.71	6.85
0	23	21	5	16.16	10.85	14.25	45.71	6.85
0	24	21	5	16.16	10.85	14.25	45.71	6.85
0	25	21	5	16.16	10.85	14.25	45.71	6.85
0	26	21	5	16.16	10.85	14.25	45.71	6.85
0	27	21	5	16.16	10.85	14.25	45.71	6.85
0	28	21	5	16.16	10.85	14.25	45.71	6.85
0	29	21	5	16.16	10.85	14.25	45.71	6.85
0	30	21	5	16.16	10.85	14.25	45.71	6.85
0	31	21	5	16.16	10.85	14.25	45.71	6.85
0	32	21	5	16.16	10.85	14.25	45.71	6.85
0	33	21	5	16.16	10.85	14.25	45.71	6.85
0	34	21	5	16.16	10.85	14.25	45.71	6.85
0	35	21	5	16.16	10.85	14.25	45.71	6.85
0	36	21	5	16.16	10.85	14.25	45.71	6.85
0	37	21	5	16.16	10.85	14.25	45.71	6.85
0	38	21	5	16.16	10.85	14.25	45.71	6.85
0	39	21	5	16.16	10.85	14.25	45.71	6.85
0	40	21	5	16.16	10.85	14.25	45.71	6.85
0	41	21	5	16.16	10.85	14.25	45.71	6.85
0	42	21	5	16.16	10.85	14.25	45.71	6.85
0	43	21	5	16.16	10.85	14.25	45.71	6.85
0	44	21	5	16.16	10.85	14.25	45.71	6.85
0	45	21	5	16.16	10.85	14.25	45.71	6.85
0	46	21	5	16.16	10.85	14.25	45.71	6.85
0	47	21	5	16.16	10.85	14.25	45.71	6.85
0	48	21	5	16.16	10.85	14.25	45.71	6.85
0	49	21	5	16.16	10.85	14.25	45.71	6.85
0	50	21	5	16.16	10.85	14.25	45.71	6.85
0	51	21	5	16.16	10.85	14.25	45.71	6.85
0	52	21	5	16.16	10.85	14.25	45.71	6.85
0	53	21	5	16.16	10.85	14.25	45.71	6.85
0	54	21	5	16.16	10.85	14.25	45.71	6.85
0	55	21	5	16.16	10.85	14.25	45.71	6.85
0	56	21	5	16.16	10.85	14.25	45.71	6.85
0	57	21	5	16.16	10.85	14.25	45.71	6.85
0	58	21	5	16.16	10.85	14.25	45.71	6.85
0	59	21	5	16.16	10.85	14.25	45.71	6.85
0	60	21	5	16.16	10.85	14.25	45.71	6.85
0	61	21	5	16.16	10.85	14.25	45.71	6.85
0	62	21	5	16.16	10.85	14.25	45.71	6.85
0	63	21	5	16.16	10.85	14.25	45.71	6.85
0	64	21	5	16.16	10.85	14.25	45.71	6.85
0	65	21	5	16.16	10.85	14.25	45.71	6.85
0	66	21	5	16.16	10.85	14.25	45.71	6.85
0	67	21	5	16.16	10.85	14.25	45.71	6.85
0	68	21	5	16.16	10.85	14.25	45.71	6.85
0	69	21	5	16.16	10.85	14.25	45.71	6.85
0	70	21	5	16.16	10.85	14.25	45.71	6.85
0	71	21	5	16.16	10.85	14.25	45.71	6.85
0	72	21	5	16.16	10.85	14.25	45.71	6.85
0	73	21	5	16.16	10.85	14.25	45.71	6.85
0	74	21	5	16.16	10.85	14.25	45.71	6.85
0	75	21	5	16.16	10.85	14.25	45.71	6.85
0	76	21	5	16.16	10.85	14.25	45.71	6.85
0	77	21	5	16.16	10.85	14.25	45.71	6.85
0	78	21	5	16.16	10.85	14.25	45.71	6.85
0	79	21	5	16.16	10.85	14.25	45.71	6.85
0	80	21	5	16.16	10.85	14.25	45.71	6.85
0	81	21	5	16.16	10.85	14.25	45.71	6.85
0	82	21	5	16.16	10.85	14.25	45.71	6.85
0	83	21	5	16.16	10.85	14.25	45.71	6.85
0	84	21	5	16.16	10.85	14.25	45.71	6.85
0	85	21	5	16.16	10.85	14.25	45.71	6.85
0	86	21	5	16.16	10.85	14.25	45.71	6.85
0	87	21	5	16.16	10.85	14.25	45.71	6.85
0	88	21	5	16.16	10.85	14.25	45.71	6.85
0	89	21	5	16.16	10.85	14.25	45.71	6.85
0	90	21	5	16.16	10.85	14.25	45.71	6.85
0	91	21	5	16.16	10.85	14.25	45.71	6.85
0	92	21	5	16.16	10.85	14.25	45.71	6.85
0	93	21	5	16.16	10.85	14.25	45.71	6.85
0	94	21	5	16.16	10.85	14.25	45.71	6.85
0	95	21	5	16.16	10.85	14.25	45.71	6.85
0	96	21	5	16.16	10.85	14.25	45.71	6.85
0	97	21	5	16.16	10.85	14.25	45.71	6.85
0	98	21	5	16.16	10.85	14.25	45.71	6.85
0	99	21	5	16.16	10.85	14.25	45.71	6.85
0	100	21	5	16.16	10.85	14.25	45.71	6.85
0	101	21	5	16.16	10.85	14.25	45.71	6.85
0	102	21	5	16.16	10.85	14.25	45.71	6.85
0	103	21	5	16.16	10.85	14.25	45.71	6.85
0	104	21	5	16.16	10.85	14.25	45.71	6.85
0	105	21	5	16.16	10.85	14.25	45.71	6.85
0	106	21	5	16.16	10.85	14.25	45.71	6.85
0	107	21	5	16.16	10.85	14.25	45.71	6.85
0	108	21	5	16.16	10.85	14.25	45.71	6.85
0	109	21	5	16.16	10.85	14.25	45.71	6.85
0	110	21	5	16.16	10.85	14.25	45.71	6.85
0	111	21	5	16.16	10.85	14.25	45.71	6.85
0	112	21	5	16.16	10.85	14.25	45.71	6.85
0	113	21	5	16.16	10.85	14.25	45.71	6.85
0	114	21	5	16.16	10.85	14.25	45.71	6.85
0	115	21	5	16.16	10.85	14.25	45.71	6.85
0	116	21	5	16.16	10.85	14.25	45.71	6.85
0	117	21	5	16.16	10.85	14.25	45.71	6.85
0	118	21	5	16.16	10.85	14.25	45.71	6.85
0	119	21	5	16.16	10.85	14.25	45.71	6.85
0	120	21	5	16.16	10.85	14.25	45.71	6.85
0	121	21	5	16.16	10.85	14.25	45.71	6.85
0	122	21	5	16.16	10.85	14.25	45.71	6.85
0	123	21	5	16.16	10.85	14.25	45.71	6.85
0	124	21	5	16.16	10.85	14.25	45.71	6.85
0	125	21	5	16.16	10.85	14.25	45.71	6.85
0	126	21	5	16.16	10.85	14.25	45.71	6.85
0	127	21	5	16.16	10.85	14.25	45.71	6.85
0	128	21	5	16.16	10.85	14.25	45.71	6.85
0	129	21	5	16.16	10.85	14.25	45.71	6.85
0	130	21	5	16.16	10.85	14.25	45.71	6.85
0	131	21	5	16.16	10.85	14.25	45.71	6.85
0	132	21	5	16.16	10.85	14.25	45.71	6.85
0	133	21	5	16.16	10.85	14.25	45.71	6.85
0	134	21	5	16.16	10.85	14.25	45.71	6.85
0	135	21	5	16.16	10.85	14.25	45.71	6.85
0	136	21	5	16.16	10.85	14.25	45.71	6.85
0	137	21	5	16.16	10.85	14.25	45.71	6.85
0	138	21	5	16.16	10.85	14.25	45.71	6.85
0	139	21	5	16.16	10.85	14.25	45.71	6.85
0	140	21	5	16.16	10.85	14.25	45.71	6.85
0	141	21	5	16.16	10.85	14.25	45.71	6.85
0	142	21	5	16.16	10.85	14.25	45.71	6.85

Vector Sum: 0 0 0 91.00 0 0 0 91.00

==== after PHOTOS: ===									
Event No. 1									
Particle Parameters									
0	1	2	3	4	5	6	7	8	9
Type	Parent(s)	Daughter(s)	Px	Pz	E	Inw.	M.		
1	0	37,24	-21.82	-14.34	45.50	1.35			
2	0	37,24	-21.82	-14.34	45.50	1.35			
3	0	37,24	-21.82	-14.34	45.50	1.35			
4	1	2	0.00	0.00	91.00	91.00			
5	94	4	0.00	0.00	91.00	91.00			
6	1	3	37.00	14.25	45.29	2.65			
7	4	3	37.00	14.25	45.29	2.65			
8	4	3	37.00	14.25	45.29	2.65			
9	4	3	37.00	14.25	45.29	2.65			
10	16	9	36.90	14.45	45.41	1.35			
11	16	9	36.90	14.45	45.41	1.35			
12	17	11	-36.51	-21.93	-14.42	45.14	3.95		
13	10	18	-0.46	0.24	0.16	0.57	0.00		
14	10	18	-0.46	0.24	0.16	0.57	0.00		
15	21	15	-15.36	-9.46	-4.99	18.77	1.35		
16	12	13	-21.15	-16.24	-3.85	26.12	1.10		
17	14	15	-16.24	-3.85	26.12	1.10			
18	19	13	-5.36	-3.75	5.76	6.97	0.01		
19	25	20	-10.94	-21.05	-13.41	43.91	1.35		
20	25	20	-10.94	-21.05	-13.41	43.91	1.35		
21	25	25	-0.30	0.63	0.16	0.37	0.00		
22	25	25	-0.30	0.63	0.16	0.37	0.00		
23	25	25	-0.30	0.63	0.16	0.37	0.00		
24	25	25	-0.30	0.63	0.16	0.37	0.00		
25	25	25	-0.30	0.63	0.16	0.37	0.00		
26	25	25	-0.30	0.63	0.16	0.37	0.00		
27	25	25	-0.30	0.63	0.16	0.37	0.00		
28	25	25	-0.30	0.63	0.16	0.37	0.00		
29	25	25	-0.30	0.63	0.16	0.37	0.00		
30	25	25	-0.30	0.63	0.16	0.37	0.00		
31	25	25	-0.30	0.63	0.16	0.37	0.00		
32	25	25	-0.30	0.63	0.16	0.37	0.00		
33	25	25	-0.30	0.63	0.16	0.37	0.00		
34	25	25	-0.30	0.63	0.16	0.37	0.00		
35	25	25	-0.30	0.63	0.16	0.37	0.00		
36	25	25	-0.30	0.63	0.16	0.37	0.00		
37	25	25	-0.30	0.63	0.16	0.37	0.00		
38	25	25	-0.30	0.63	0.16	0.37	0.00		
39	25	25	-0.30	0.63	0.16	0.37	0.00		
40	25	25	-0.30	0.63	0.16	0.37	0.00		
41	25	25	-0.30	0.63	0.16	0.37	0.00		
42	25	25	-0.30	0.63	0.16	0.37	0.00		
43	25	25	-0.30	0.63	0.16	0.37	0.00		
44	25	25	-0.30	0.63	0.16	0.37	0.00		
45	25	25	-0.30	0.63	0.16	0.37	0.00		
46	25	25	-0.30	0.63	0.16	0.37	0.00		
47	25	25	-0.30	0.63	0.16	0.37	0.00		
48	25	25	-0.30	0.63	0.16	0.37	0.00		
49	25	25	-0.30	0.63	0.16	0.37	0.00		
50	25	25</							