The Alternating Gradient Proton Synchrotron.

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1. - Introduction and History of the CERN Proton Synchrotron Group.

The purpose of this paper is to describe in physical terms the new principle that has extended the maximum energy to which proton synchrotrons can be built nowadays. By using the principles of the constant gradient machines, of which the Cosmotron at Brookhaven and the Bevatron at Berkeley are examples, I will try to outline the differences between the old and new type machines and the facts that have been discovered in developing the new idea.

But first I would like to outline the history of the Group which has carried out most of this development work in Europe. In June of 1952 at a conference in Copenhagen the newly formed CERN Council decided to start an engineering design study for the construction of a 10 GeV proton synchrotron that was to be a scaled up version of the Cosmotron. The principles of this machine were well known at this time and so the design study was thought to be a simple extrapolation of engineering design. By July 1952 a design group of three people was set up with Dr. O. DAHL in charge and they were asked to have the study ready by November 1953.

In August of 1952 DAHL, WIDERÖE and GOWARD visited Brookhaven to see the Cosmotron at first hand and to talk to the staff that had built it. They found not only the Cosmotron but a new idea for making cheaper and higher energy machines. This idea was so attractive that in October 1952 the CERN Council decided to drop the engineering study for a 10 GeV proton synchrotron and ask the group to undertake a theoretical study on the new idea. The energy of the new machine was fixed at about 30 GeV since it seemed at that time that a 30 GeV new style machine would cost about the same as a 10 GeV old style machine.

In December 1952 the Proton Synchrotron (PS) group was formed to augment the original staff of three and, for convenience, the group was split up into four subgroups charged with investigating the main features of the

new idea. The theoretical study group, working mainly on questions of orbit stability, had its headquarters in Harwell, England, and some of its staff in Copenhagen. The magnet studies were concentrated in Paris in Professor GRIVET's laboratory. The radio frequency problems were studied at Heidelberg in Professor BOTHE's institute, and the headquarters of the whole group was at Dr. DAHL's Institute at Bergen. For the next ten months the group worked on the various problems and met together from time to time to discuss results.

By October 1953 enough was known about the machine to make it possible to present a tentative design to the Council of CERN. In America meanwhile the Brookhaven team and another at M.I.T. Boston had been working on the same problems. So at the end of October a conference was held at the Institute of Physics of the University of Geneva to which were invited representatives of Brookhaven, Berkeley, M.I.T. and the Mid-Western Universities. The PS group of CERN gave a series of lectures on its researches over the previous year and presented a consistent and, I think, practical design for a 30 GeV Alternating Gradient (A. G.) Proton Synchrotron. The Brookhaven team had also covered the same ground and had arrived at an almost identical design.

At the time of this conference the CERN Council agreed that the PS group should centre itself in Geneva as this was going to be the final home of CERN and the State of Geneva and the University kindly offered temporary accommodation for the group. By December 1953 most of the group had moved to Geneva and the real work of designing the machine started.

We plan that the construction of the machine will take about six years. A start has already been made on the final site at Meyrin which is near the French frontier, west of Geneva. We hope to have our final laboratories and main buildings for the machine completed at the beginning of 1956 when we will move to the permanent site. The manufacture of the magnet, which is the most important part of the machine and the one which determines the time scale for construction, will start in the middle of 1955 and parts will arrive on the site in the middle of 1956. The whole machine should be installed by the end of 1959 and the first trials started at that time.

We hope to build up the staff from the present 50 to about 70 by the end of 1954, 120 by the end of 1955 and reach our peak staff of 130 during 1956. Thereafter there will be a fall off in staff as the machine reaches completion and finally a large part of the design staff will be replaced by nuclear physics staff using the machine for high energy nuclear physics experiments.

2. - Principles of the Constant Gradient Proton Synchrotron.

Although the principles of the constant gradient (C. G) machine are well known it is useful to have these in mind when considering the differences between the two types of machines. The Cosmotron is taken as an example of a C. G.

machine. Protons are accelerated from an energy of 3.5 MeV, at which energy they are introduced into the machine from an electrostatic generator, to about 2.5 GeV. The acceleration is at a constant radius of 10 metres in a ring shaped magnetic field. The magnetic field is increased from the injection field of 270 gauss to a peak field of 14000 gauss in 1 s so keeping the particles at constant radius as their energy is increasing.

We first consider the stability of the free transverse oscillations, or betatron oscillations, at constant energy. If the lines of magnetic field are bowed outwards from the centre of the machine so that there is a fall off in the magnetic field with radius then for a particle displaced above or below the horizontal plane of symmetry there is a restoring force towards this median plane. Therefore for all negative values of the field gradient dB/dr there will be some focusing force in the vertical direction. If we define,

$$n = -\frac{r}{B} \frac{dB}{dr},$$

then the condition for vertical stability is that $n > 0$. In the horizontal plane there is an equilibrium radius for a particle defined by its energy as,

$$B_0 r_0 = \frac{\sqrt{E^2 - E_0^2}}{ec},$$

that is, a particle at energy E in a magnetic field B_0 will describe a circle of radius r_0 round the machine. A particle placed at some greater radius r_1 in a slightly decreased field B_1 will oscillate about the equilibrium orbit circle providing the product $B_1 r_1$ is greater on the outside of the circle than $B_0 r_0$ and smaller on the inside. The limit of stability is reached when the product $B_1 r_1$ is the same as $B_0 r_0$ or when,

$$\frac{d(Br)}{dr} = 0.$$

In terms of the field gradient n this is,

$$n = 1.$$

So we find that for stability of the free oscillations in both vertical and horizontal directions we must have,

$$0 < n < 1.$$

We now consider the oscillations of particles at different energies in the machine. As already noted there is a certain circle of radius r_0 where the

magnetic field is B_0 and round which particles of a certain energy E_s will circulate without oscillation. E_s is called the equilibrium energy. Now the magnetic field is increasing with time and to keep particles at a constant radius their energy must be increased by a certain amount eV per revolution. The method of increasing their energy is usually by acceleration in an electric field fed by a radio frequency power generator making available more than the required energy gain per revolution. We assume that the frequency of this generator is sympathetically matched to the magnetic field so that particles going round the equilibrium orbit at a velocity corresponding to their ever increasing energy always take a revolution time equal to the periodic time of the accelerating voltage. Then at some phase of the accelerating voltage waveform particles can find a voltage that will give them the correct energy gain per revolution providing the peak voltage available is greater than that required by the particles. There are of course two phases, one on either side of the sinusoidal accelerating voltage waveform, where the correct energy gain per revolution can be found. The stable one, called the equilibrium or stable phase, is discovered as follows. A particle at an energy greater than the equilibrium energy will describe a circle of larger radius than the equilibrium radius and so take longer to go round one revolution. But on the other hand it is going at a greater velocity than the equilibrium particle and so takes less time to go round. It happens in a synchrotron that the increase in revolution time due to the larger radius is greater than the decrease in revolution time due to the increased velocity, so the radius effect wins. Now if this particle at greater energy takes longer to go round then it finds itself after one revolution at a different phase of the accelerating waveform. The stable phase angle is then on that side of the sinewave that makes this new phase angle correspond to less energy gain per revolution for this particle. Then as the particle goes round it gains less energy than the equilibrium particle and finally has the same energy. A similar argument applies to particles with less initial energy than the equilibrium particle and consequently at smaller radius than the equilibrium radius. In this way particles oscillate in energy, phase and radius about the equilibrium values. The frequency of this oscillation is very slow compared with the free oscillations so that the analyses for the free oscillation can be carried out assuming constant energy.

It is of interest to note that if the frequency of the accelerating voltage is not correct for the equilibrium particle at its equilibrium radius then there must exist some new equilibrium radius at which the particle can pick up the correct amount of energy without oscillating in phase. Providing this new equilibrium radius is inside the vacuum chamber then some particles will be accelerated, but if at any time during the 1 second accelerating period the equilibrium orbit goes outside the vacuum chamber then all particles are lost. This condition determines a frequency tolerance on the accelerating voltage.

frequency throughout the cycle and means that elaborate arrangements have to be made to keep the frequency and the magnetic field in step.

3. - Principles of the Alternating Gradient Proton Synchrotron.

In recent years the cost of proton synchrotrons has mounted rapidly until nowadays a machine whose energy satisfies a nuclear physicist costs as much as a modern jet bomber. It became increasingly clear that if further progress was to be made in high energy nuclear physics by experiments using artificially accelerated particles some new principle must be found that would cheapen the cost per GeV. It was lucky for CERN that just at the time a European machine was being considered this new principle was discovered. The problem was simple enough. A cheaper machine could be built if the amplitudes of the free and forced oscillations of the accelerating particles could be decreased in some way so that the vacuum chamber size and the cross-section of the magnet ring could be reduced. The simplest way to reduce the amplitude of the free oscillations is to increase the frequency by increasing the restoring force, and although this is easy to achieve in the vertical direction by increasing the magnetic field gradient, the condition for horizontal stability is violated if n exceeds unity.

The new principle discovered by CHRISTOPHILOS and COURANT, LIVINGSTON and SNYDER increases the frequency of the betatron oscillations by alternating the sign of the gradient of the magnetic field. The structure of the magnet is no longer uniform round the ring with a constant gradient but is broken up into sectors whose gradient is alternatively positive and negative.

Equations of motion: $\ddot{\varrho} = \omega_0^2 n \varrho$, $\ddot{z} = \omega_0^2 n z$

Solutions for radial motion:

focusing field, $n = -$

$$\varrho = A \sin \omega_0 n^{\frac{1}{2}} t + B \cos \omega_0 n^{\frac{1}{2}} t$$

defocusing field, $n = +$

$$\varrho = C \sinh \omega_0 n^{\frac{1}{2}} t + D \cosh \omega_0 n^{\frac{1}{2}} t.$$

Particle motion (sector length = $\pi/2$).

Fig. 1. - Motion of particles in an A.G. machine.

In Fig. 1 are given the magnetic field equations and the differential equations for particles circulating in these fields. The solutions to these equations in the case of the horizontal motion for instance show that when n is negative the sector is focusing for this motion, but that when n is positive the solutions give an exponential divergence of the particle away from the equilibrium orbit. Nevertheless under certain conditions it is possible to achieve overall focusing properties over a number of sectors as is shown in the lower

illustration. Since the old restriction of $n=1$ no longer applies n can now be a very large number so that the wavelength of the oscillation can be reduced to the length of a few sectors and the number of cycles of betatron oscillation per revolution can be increased from 0.7 to the order of 10. There is now no difference in the focusing conditions for horizontal and vertical motions since what is a focusing sector for horizontal motion is a defocusing sector for vertical motion and vice versa so that the magnet structure looks the same for both motions with a phase shift of one magnet sector.

We can divide up the magnet structure into identical periods consisting in the simplest case of a focusing followed by a defocusing sector which repeat an integral number of times round the magnet ring. A general transfer matrix can be written for this magnet period relating the end values for the displacement and derivative of a particle to the starting values,

$$\begin{pmatrix} y_1 \\ \dot{y}_1 \end{pmatrix} = T \begin{pmatrix} y_0 \\ \dot{y}_0 \end{pmatrix}.$$

There also exists a special solution, called the Floquet solution, which can be written,

$$\begin{pmatrix} y_{f1} \\ \dot{y}_{f1} \end{pmatrix} = \lambda \begin{pmatrix} y_{f0} \\ \dot{y}_{f0} \end{pmatrix}$$

where λ is the Floquet multiplier. It can be shown that the only possible oscillatory solutions are when,

$$\lambda = e^{\pm i\mu},$$

where μ is the phase shift of the oscillation per magnet period. Applying these solutions to the general matrix gives,

$$\text{Tr}(T) = (T_{11} + T_{22}) = 2 \cos \mu,$$

from which the stability limits are,

$$\cos \mu = \pm 1 \quad \text{or} \quad \frac{\text{Tr}(T)}{2} = \pm 1.$$

In Fig. 2 these limits have been drawn on a plane with axis n_1 and n_2 which are the n values in the focusing and defocusing sectors. Outside the two lines $\cos \mu_z = +1$ and -1 there is vertical instability and outside the other two lines $\cos \mu_r = -1$ and $+1$ there is horizontal instability. The final stable area for both motions is shaped like, and in fact called, a « necktie ». The waveforms of free oscillation drawn in Fig. 1 are for a machine operating in the middle of this stable area where $\mu_z = \mu_r = \pi/2$, i.e. where there is a

phase shift of $\pi/2$ radians every magnet period and so four magnet periods per betatron wavelength.

We now come to the stability of the energy oscillations. A particle with the equilibrium energy or momentum goes round the machine in a circle as in the constant gradient machine. But due to the rapid fall-off of the magnet field with radius in a defocusing sector, and the equally rapid increase in magnet field with radius in a focusing sector, the equilibrium radius for a particle with say greater momentum than the equilibrium particle is no longer a smooth curve along the sectors but a discontinuous series of straight lines as is shown in Fig. 3. The radial position of these straight lines corresponds to where the product $B \cdot r$ is appropriate to the energy of the particle and they lie either side of the continuous equilibrium orbit for the particle of correct energy. The particle cannot of course follow these discontinuous lines and particles on the new equilibrium orbit are found to describe a nearly sinusoidal motion of wavelength equal to the magnet period displaced outwards at a greater

Fig. 3. - Momentum compaction.

Fig. 2.

radius than the old equilibrium orbit in the case of particles with greater energy. The important fact now emerges that the displacement of the equilibrium orbit in the alternating gradient machine for a given energy difference is far smaller than in a constant gradient machine and the term used to express this effect is « momentum compaction ». For a constant gradient machine we have

$$\frac{\Delta r/r_0}{\Delta p/p_0} = \frac{1}{1-n} \cong 2.5,$$

but for the alternating gradient machine we have

$$K = \frac{\Delta r/r_0}{\Delta p/p_0} = \frac{1}{n} \frac{2}{\varphi} \left(\frac{1}{\coth \varphi - \cot \varphi} \right),$$

where $\varphi = (\pi/2M) \cdot n^{\frac{1}{2}}$ and M is the number of magnet periods per revolution. For a typical A.G. machine the « momentum compaction factor » $K = 0.04$ gives an improvement of about 60 to 1 over the C.G. machine and this is just as important as the reduction in the betatron oscillation amplitude already noted in favour of the new principle.

However this improvement brings with it certain difficulties or at least differences in the conditions of stability of the energy oscillations. It has already been described how particles in a C.G. machine oscillate stably about a certain phase of the waveform of the accelerating voltage. It was pointed out that the increased radius of a particle with greater energy than the equilibrium particle caused the particle to take longer to go round one revolution, and that this effect more than offsets the decrease in revolution time due to the greater velocity of this particle. In the A.G. machine, due to the momentum compaction effect, the increase in radius is so small for a particle with greater energy than the equilibrium particle that the velocity effect wins in the non-relativistic region and the stable phase is on the other side of the accelerating voltage waveform, as it is in a Linear Accelerator where there is only a velocity effect. However, when the particles reach relativistic velocities the velocity effect becomes smaller and smaller and finally the radius effect wins again. Thus at some time during the acceleration cycle the stable phase angle changes from one side of the accelerating voltage waveform to the other, from the Linac stable phase angle to the synchrotron stable phase angle. The particle energy at which this change over takes place is called the « Transition Energy » and in order to preserve the full beam intensity it is essential to shift the phase of the accelerating voltage waveform by π radians fairly quickly at this critical energy.

This summary completes the main physical features of the new idea and the differences in principle between A.G. machines and C.G. machines. We must now look at the difficulties that rise when the new idea is examined in greater detail and an attempt is made to realise an alternating gradient machine in practice. What has been described above was covered in the original paper of COURANT, LIVINGSTON and SNYDER. What follows was uncovered by the development work of the American groups at Brookhaven and at M.I.T. and independently by the CERN PS group in Europe.

4. - Modifications to the Stability Criteria due to Errors of Construction.

In an A.G. proton synchrotron the magnet is divided up into a number of supposedly identical sectors. These sectors are physically separate units and have to be arranged round a circle. If the units are perfect in themselves there will still be some error in the alignment of the units round a perfect circle. Also the n values in the various sectors will not be identical, as manufacturing

and material tolerances will cause random variations about the correct value.

In Fig. 4 these sorts of errors are analysed into equivalent perturbations that act on a particle as it goes round the machine. It can be seen that the mechanical errors of alignment give rise to perturbations that are independent of the displacement and derivative of the particle as it goes by the incorrectly placed sector whereas an error in the n value of a sector or in its physical length gives rise to a perturbation that is linearly proportional to the displacement of the particle or its derivative, or both. There is one other type of error not shown in Fig. 4 which is of importance in the operation of the machine. If one sector is twisted out of the plane of the orbit, then coupling terms arise between the horizontal and vertical betatron oscillation which in some circumstances can produce serious effects.

TYPE OF ERROR	FORM OF ERROR	EQUIVALENT PERTURBATION
DISPLACEMENT OF EQUILIBRIUM ORBIT IN ONE SECTOR		 $e \sim s$
TILT OF EQUILIBRIUM ORBIT IN ONE SECTOR		 $e \sim s$
INCORRECT 'n' VALUE IN ONE SECTOR		 $e \sim s, \frac{\delta n}{n}$ $e \sim s, \frac{\delta n}{n}$
INCORRECT LENGTH OF ONE SECTOR		 $e \sim s, \frac{\delta \theta}{\theta}$ $e \sim s, \frac{\delta \theta}{\theta}$

Fig. 4.

RESONANCE

Fig. 5. - Resonance in a linear system.
Perturbation $\dot{e} = \text{constant}$.

The effect of the first « constant » type of perturbation, due to misalignments is shown in Fig. 5 for the simple case of one perturbation per revolution. If there are an integral number of betatron cycles per revolution (called Q) then the perturbation excites a resonance since it occurs at the same phase of the betatron oscillation each turn. If we adjust the parameters of the machine so that it operates between these resonant points, then there exists an orbit for a particle that repeats itself each revolution allowing for the perturbation. This is illustrated for this simple case in the second diagram of Fig. 5. This special orbit is called the « Closed Orbit » and it can be shown that all other orbits are free oscillations about the closed orbit. At a resonant value the displacement of the closed orbit from the

ideal orbit becomes infinite as is shown in the last picture.

The second type of perturbation, illustrated in Fig. 6, amplifies the free oscillations about the closed orbit. At the resonant values, that is integral

Q values, the perturbations excite oscillations whose amplitude becomes infinite, and the same thing happens when there are an integer plus a half cycles per

Fig. 6.

with the vacuum chamber walls due to «blow up» of the betatron oscillations. As in the C.G. machine the closed orbit must, of course, lie within the vacuum chamber throughout the whole acceleration cycle.

We can now illustrate all these effects on the original stability diagram of Fig. 2. In Fig. 7 only the main resonances have been drawn in for a typical machine. In Fig. 8 the small black diamond shown in Fig. 7 is magnified and the sub-resonances drawn in. Also illustrated in this figure are the resonances due to the effect of twisted sectors mentioned above that occur when,

$$Q_R + Q_V = \text{integer},$$

where Q_R and Q_V are the number of betatron cycles per revolution in the horizontal and vertical directions respectively. Finally in Fig. 9 the largest of the available safe areas called the «Working Diamond» is further magnified

at the so-called «Sub-resonances». It can be shown that these resonances and sub-resonances do not occur at single values of Q but that there is instability over a narrow band of Q values about the resonant ones. At the edge of these «Stop Bands» the amplitude of oscillation rises linearly to infinite values and inside the stop bands the increase in amplitude is exponential. Between the stop bands the amplitude of oscillation beats up and down quite stably at a maximum amplitude shown in the last illustration of Fig. 6.

It is clear from these physical features that an A.G. machine must work between the stop bands with a Q value of integer plus a quarter or three quarters. If for any reason during the acceleration cycle the Q value approaches a stop band then most of the particles will be lost by collision

Fig. 7. - Stability diagram (resonance lines drawn in).

and all the stop bands drawn in. The working point is then in the centre of this working diamond and it must stay near the centre during the whole of the acceleration cycle.

Now the question of stability of the betatron oscillations looks very different from the original picture given in Fig. 2. The large stable area has been subdivided by unstable bands until only a very small area is really safe and the tolerances imposed on the errors of construction of the machine are very tight indeed.

For a typical machine the length of the working diamond expressed as a ratio of the mean n value at the working point is

$$\frac{\Delta n_1}{n} = 5\%$$

and the width of the diamond in the same units is

$$\frac{\Delta n_2}{n} = 1\%.$$

The width of the stop bands is determined by the random errors in the sector n values or in sector lengths and to keep the stop bands reasonably narrow, so that the safe area of the working diamond is not further reduced, it is necessary to limit the random n errors to less than 1% of the mean n value.

Fig. 9.

Fig. 8. - Part of stability diagram (main, sub and twist resonances).

The A.G. machine is very sensitive to mechanical misalignment errors. If in a C.G. machine we displace one sector of the magnet of length l by an amount a then the peak amplitude of oscillation due to this displaced sector is approximately

$$\hat{y}_{CG} = \frac{l \cdot a}{r_m} \cdot \sqrt{1 - n},$$

where r_m is the mean radius of the magnet ring. The same length sector displaced by the same amount in an A.G. machine gives a peak amplitude of oscillation of

$$\hat{y}_{AG} = \frac{l \cdot a}{r_m} \cdot \sqrt{n} \cdot W,$$

where W is a waveform correction factor due to the non-sinusoidal waveforms in A.G. machines. For a typical machine where $\mu = \pi/3$, W is about 4. Allowing for the much higher n of the A.G. machine compared with the C.G. machine and the different waveforms the ratio

$$\frac{y_{AG}}{\lambda_{CG}} \cong 100.$$

In practice to keep the peak deviation of the closed orbit within 0.5 cm of the centre of the vacuum chamber the magnet units of an A.G. machine have to be aligned to within ± 0.05 cm of a perfect circle. Since the mean radius of a 25 GeV machine is about 100 metres this means measuring the position of the magnet units to within 1 part in 10^5 . As has already been emphasized if the closed orbit at any time wanders outside the vacuum chamber then all particles are lost, not just a fraction of them.

5. - Design Data and Layout of a 25 GeV Proton Synchrotron.

In Table I are listed the significant parameters of the most recent CERN design of a 25-GeV machine. The energy is only limited by financial consi-

TABLE I. - *Parameters of a 25 GeV proton synchrotron.*

Maximum energy	$E = 25$ GeV
Maximum magnetic field on the orbit	$B_0 = 12\,000$ gauss
Radius of curvature of magnet units	$r_0 = 7\,178$ cm
Mean radius of orbit	$r_m = 9\,490$ cm
Field index	$n = 278$
Phase shift of betatron oscillations per magnet period	$\mu = 0.8$
Number of betatron cycles per revolution	$Q = 6.25$
Transition energy (total)	$E_{TR} = 6.0$ GeV
Injection energy (kinetic)	$E_1 = 50$ MeV
Injection magnetic field	$B_1 = 140$ gauss
Time of rise of magnetic field	$t = 1$ s
Pulse repetition	$T = 5$ s
Harmonic order of radiofrequency	$m = 32$
Radiofrequency range	5.1 to 16.3 MHz

derations and the peak magnetic field by reasons of constancy of n during the acceleration cycle. The values of n and μ have been chosen as the result of a comprehensive study of all the limitations imposed by the physical features of such a machine. The transition energy is determined by the need to get passed the rapid change in the velocity of the particles while they are ap-

proaching relativistic velocities and the need to allow the energy oscillations to be damped down so that discreet bunches of particles are formed before the phase of the accelerating voltage is shifted. The injection energy is chosen as high as is practical using known Linac design information. The Cosmotron of 10 m radius can use an electrostatic generator giving 3.5 MeV protons and inject at a magnetic field of 270 gauss. With our machine of 100 m radius the injection energy is 50 MeV at a magnet field of 140 gauss which is only half the Cosmotron value.

Bearing in mind that the remanent fields in the magnet are about 40 gauss and that eddy current effects are of the order of tens of gauss this injection

Fig. 10. - Magnet structure.

Fig. 11. - Building layout.

field is rather low. But without constructing a very high energy Linac similar to the new Harwell Linac it is not possible to go any higher. The rise time of the magnetic field is determined by the peak kVA of the generator supplying the power which for a 1 s rise time is about 28000 kVA. The rise time could be as long as 2 s without any detrimental effect. The repetition rate of the magnet cycle is a question of the mean generator power and more importantly of the rate of the output pulses from a nuclear physics point of view. If we ask for at least one pulse every 5 s, as is delivered by the Cosmotron, then the mean generator power is about 1.7 MW. Finally the harmonic order of the accelerating voltage frequency is chosen

as high as possible so that the amplitude of the energy oscillations is as small as possible consistent with the practical performance of the material used in the accelerating cavities whose properties deteriorate at frequencies much higher than 20 MHz.

In Fig. 10 the structure of the magnet is shown diagrammatically with breaks between the magnet units for placing the accelerating cavities and other correcting lenses. The cross-section of the magnet is also illustrated.

Fig. 11 shows the shape of the buildings to house the proton synchrotron.

6. - Conclusions.

This completes an introduction to the new A.G. proton synchrotron showing how the idea started, the original optimism and the subsequent disenchantment as the development work proceeded. We are left with a new type of machine that offers a method of getting very high energy particles cheaper than by any other artificial means but at the expense of imposing very tight tolerances in the construction of the machine. Nevertheless the fact that both Brookhaven and CERN are going ahead with this type of machine shows that there are reasonable hopes that the tolerances can be met in practice.

APPENDIX

1. - Comparison between the Beam Intensities of an Alternating Gradient Proton Synchrotron and a Constant Gradient Proton Synchrotron of the same Vacuum Chamber Aperture.

It is of general interest to calculate the beam intensity accepted into a constant gradient proton synchrotron of the same aperture and therefore roughly of the same cost as the present CERN design of alternating gradient proton synchrotron. At least such a study will answer the question why do we proceed with the A.G. design despite all the difficulties with construction tolerances rather than make a more established and better understood machine.

There are certain common features with either machine. The injection energy cannot be any different from 50 MeV since it is determined by the injection field which should not go lower than the present 140 gauss. Similarly there is no reason to suppose that the vacuum pressure will be any different in the two machines if the same size chamber is used.

2. - Injected Beam Diameter Inside the Vacuum Chamber.

We will take the same value for the « Acceptance » of the injection Linac that has been used for the A.G. machine, namely

$$A_L = 0.5 \cdot 10^{-3} \text{ cm radians}$$

(e.g. spot radius of 0.5 cm and incoherent half angle of divergence of 10^{-3} radians).

The cross-section of the beam, if it is correctly introduced inside the vacuum chamber of the C.G. machine, is given by

$$\text{vertical height of beam} = \hat{y}_v^2 = \frac{r_m}{n^{\frac{1}{2}}} \cdot A_L,$$

$$\text{horizontal width of beam} = \hat{y}_r^2 = \frac{r_m}{(1-n)^{\frac{1}{2}}} \cdot A_L,$$

where r_m is the mean radius of the vacuum chamber.

Substituting appropriate values for the C.G. machine, we find

$$\hat{y}_v = 2.4 \text{ cm} \quad \text{and} \quad \hat{y}_r = 2.95 \text{ cm}.$$

These are amplitudes so the peak to peak beam dimensions are

$$\text{height} = 4.8 \text{ cm} \quad \text{and} \quad \text{width} = 5.9 \text{ cm}$$

and the cross-section is elliptical.

Remembering that the vacuum chamber has internal dimensions of height $\times$ width = 8 cm $\times$ 12 cm, we see that the beam will nicely fit inside with some room to spare all round.

In the case of the A.G. machine the equations giving the beam cross-section are more complicated and for a machine with $n = 278$, $\mu = 0.8$ the beam dimensions work out to be

$$\text{height} \times \text{width} = 2 \text{ cm} \times 2 \text{ cm}.$$

3. - The Possibility of Multiturn Injection.

The beam current in the Cosmotron is greatly increased by using multi-turn injection so that particles are accepted into the machine during several revolutions. It is obvious that with a beam 5.9 cm wide in a vacuum chamber of 6 cm half width there is no question of more than one turn injection. However, it is worth while to calculate the rate at which the equilibrium orbit shrinks in each revolution if the particles are not being accelerated. The

shrinkage per revolution is given by

$$\frac{\Delta r}{r_0} = \frac{dB}{dt} \cdot \frac{2\pi \cdot r_m}{c\sqrt{2(E_K/E_0)}} \cdot \frac{1}{B_0} \cdot \frac{1}{(1-n)},$$

where E_K is the kinetic energy of the injected particles,

B_0 is the magnetic field at injection,

E_0 is the rest energy of the proton.

For $dB/dt = 14\,000$ gauss/s, $r_m = 9\,490$ cm, $B_0 = 140$ gauss and $E_K = 50 \cdot 10^3$ eV, we find

$$\Delta r = 9.3 \text{ cm}.$$

4. - Comparison of the Current Accepted by the Two Machines.

We can write the proton intensity accepted by a machine in terms of the injected intensity as

$$I = \eta_s \cdot \eta_{\text{gas}} \cdot \eta_T \cdot \eta_R \cdot I_0,$$

where the η are dimensionless coefficients due to the following reasons,

η_s is the loss coefficient due to the combined synchrotron and betatron oscillation amplitudes losing particles to the vacuum chamber walls;

η_{gas} represents the loss of particles due to gas scattering;

η_T is the loss coefficient due to passing through the transition energy in the case of the A.G. machine;

η_R is the loss coefficient due to deviations in the «closed orbit» at injection due to starting magnetic field errors.

We now proceed to calculate these coefficients for each machine.

5. - η_s , the Synchrotron Oscillation Coefficient.

The amplitude of radial oscillation during the synchrotron phase oscillation in a C.G. machine is given by

$$\frac{\Delta r_s}{r_0} = 1.18 \left[\frac{r_0 \cdot r_m \cdot \dot{B}}{E_K \cdot m \cdot n(1-n) \cdot 10^8} \right]^{\frac{1}{2}},$$

where r_0 and r_m are the radius of curvature and mean radius of the particles; $\dot{B} = dB/dt$; m = harmonic order of the accelerating voltage frequency. The units are gauss, cm, eV.

If we use the fundamental frequency of the accelerating voltage, i.e. $m = 1$,

then $\Delta r = 236$ cm. If we use $m = 32$, the 32nd harmonic as in the A.G. machine, then $r = 40$ cm. In either case the oscillation amplitude is much larger than the vacuum chamber width.

To calculate η_s we imagine that particles are injected continuously at the correct energy and radius during one revolution. Working with a stable phase angle of 30° , half of these particles fall outside the stable phase swing in both machines. At one quarter cycle later in the synchrotron period all the particles that originally lay at one radius, but distributed round π radians in angle, have rotated so that they are bunched into a narrow angle around the machine, but spread out in radius. The peak value of the radial spread is just the synchrotron swing Δr_s , calculated above. All those particles that now lie outside the vacuum chamber are of course lost and some allowance has to be made for the extra radial swing of the betatron oscillations set up at injection and already calculated above as $\hat{y}_r = 2.95$ cm.

From this model we find

$$\eta_s = \frac{a - \hat{y}_r/2}{\Delta r_s}$$

(where a is the horizontal half width of the vacuum chamber), and for the values already calculated $\eta_s = 0.1$. Even using the high harmonic order of 32, then 90% of the particles are lost during the first synchrotron oscillation cycle.

In the case of the A.G. machine, due to the momentum compaction, the synchrotron oscillation amplitude is only 0.5 cm and all the particles are held inside the vacuum chamber.

6. - η_{gas} , the Gas Scattering Coefficient.

Injecting at 50 MeV instead of 3.5 MeV as in the Cosmotron, the gas scattering losses are very much smaller even though the vacuum chamber is small and the pressure, $1 \cdot 10^{-7}$ mm Hg, rather worse. The method of calculating the gas scattering amplitudes and the probability of survival of a particle is given in *Rev. Sci. Instr.*, 24, no. 9, p. 836. Using this method we find that

$$\eta_{\text{gas}} = 0.5$$

if we assume that the median plane of the particles at injection can lie up to 1 cm away from the geometric plane and that this effect gives an effective vertical half aperture of 3 cm for the vacuum chamber. Thus due to gas scattering at injection we lose about 50% of the injected particles.

In the A.G. machine there is virtually no loss due to gas scattering.

7. - η_r , the Transition Energy Loss Coefficient.

This coefficient only applies to the A.G. machine and must be a guess at this stage. We guess that perhaps half of the particles may be lost going through the transition energy, but we have reason to believe that very few will be lost

in practice. Nevertheless we take

$$\eta_T = 0.5.$$

8. - η_R , the Injection Magnetic Field Error Coefficient.

The injection field is 140 gauss and we assume that we can measure this to ± 0.5 gauss, i.e. about $\pm 0.3\%$ accuracy. The equivalent radial shift of the equilibrium orbit due to this field is given by

$$r_R = \frac{r}{n} \frac{dB}{B_0},$$

and for the C.G. machine $\Delta r = 37$ cm (for the Cosmotron it is only 5 cm). If we construct the magnet ring out of individual sectors of length l , as was done for the Cosmotron magnet, then the equilibrium radius in these sectors will be distributed at random about the correct radius within the limits ± 37 cm.

We can now work out the peak amplitude of betatron oscillation due to a shift of the equilibrium orbit in one of these sectors. For the C.G. machine,

$$\hat{y} = \frac{2\pi}{\lambda} \cdot l \cdot \Delta r_r = \sqrt{1-n} \frac{l \cdot r_r}{r_m}$$

and for $l = 25$ cm, $\Delta r = 37$ cm, we find $\hat{y} = 0.054$ cm. For N such sectors in the complete magnet ring in which the injection fields are distributed at random, we can simply write that the peak amplitude of oscillation after one revolution is

$$|\hat{y}|, = \hat{y} \sqrt{N} = \hat{y} \sqrt{\frac{2\pi r_0}{l}}$$

and for the C.G. machine $|\hat{y}|, = 2.27$ cm.

After one revolution all the errors repeat again and there is no randomness between successive turns. If the machine is working with one cycle of betatron oscillation (i.e. $n = 1$) per revolution, then the amplitude builds up linearly to infinity. If on the other hand $n = 0.75$, then there is one half betatron cycle per revolution and the amplitude built up on the first revolution is reduced again to zero on the next revolution and the closed orbit deviation is half the amplitude after one revolution. Since in C.G. machines $n = 0.6$, we assume that the closed orbit deviation is roughly equal to the amplitude after one revolution.

The net result of the errors due to injection field is to make the effective size of the vacuum chamber smaller than the actual size in the horizontal plane.

We can therefore calculate a new value for the synchrotron loss coefficient,

η_s , and call it η'_s and define the loss coefficient due to the errors in the injection field as

$$\eta_x = \frac{\eta'_s}{\eta_s}.$$

For the C.G. machine values already assumed we find that

$$\eta_E = 0.5.$$

In the case of the A.G. machine similar errors in the injection field give a peak closed orbit deviation of 0.5 cm and this does not cause any loss of the beam.

9. - Mechanical Errors in Aligning the Sectors of the Magnet.

It has been shown that the A.G. machine is about 100 times as sensitive to errors in the alignment of the magnet sectors to a perfect circle as is the C.G. machine. Alignment errors present no great problems with the C.G. machine, but if the specified tolerance is not kept on these misalignments in the A.G. machine, then the closed orbit quickly approaches the vacuum chamber walls and all the particles are lost. It is difficult to express this idea as a loss coefficient since its value is either 1 if the tolerances are maintained or 0 if they are not. All one can express is an alignment coefficient η_A for the C.G. machine which is $0.01\eta_A$ for the A.G. machine. The A.G. machine has just got to be aligned correctly by some means.

10. - Conclusion.

In Table II the various loss coefficients calculated for the two machines have been tabulated and the final accepted current worked out. We see with the rather optimistic conditions and simple methods used in this evaluation that a C.G. machine with the same size vacuum chamber as the latest CERN A.G. machine will only accept about 5% of that current accepted by the A.G. machine. This surely is sufficient justification for going ahead with the A.G. machine.

As a matter of interest the loss coefficients have been calculated for a C.G. machine with twice the horizontal aperture of the present A.G. machine. The accepted current is still only 36% of that accepted by the A.G. machine. The C.G. machine magnet cost for this double aperture would be over twice that for the A.G. machine magnet and the vacuum chamber and vacuum pump costs are also doubled. Furthermore the magnet generators cost more to supply the extra field. In fact such a machine would be appreciably more expensive than the A.G. machine, but not by a factor of 2.

TABLE II.

Kind of Machine	Alternating gradient	Constant gradient	Constant gradient
Vacuum chamber dimensions . . .	8 cm $\times$ 12 cm	8 cm $\times$ 12 cm	8 cm $\times$ 24 cm
Injected beam dimensions (for $A_L = 0.5 \cdot 10^{-3}$ cm radians)	2 cm $\times$ 2 cm	4.8 cm $\times$ 5.9 cm	4.8 cm $\times$ 24 cm
Injection time (revolutions)	1	1	3
Injected current	I_0	I_0	$3I_0$
Synchrotron oscillation amplitude (32 nd harmonic)	0.7 cm	40 cm	40 cm
Synchrotron oscillation coefficient η_s	1	0.1	0.15
Gas scattering coefficient: η_{gas} (pressure = 10^{-5} mm Hg)	1	0.5	0.5
Transition energy coefficient η_{Tr}	0.5	1	1
Injection field coefficient η_R (error = 0.5 gauss in 140)	1	0.5	0.8
Alignment coefficient η_A	$0.05\eta_A$	η_A	η_A
Accepted current (normalised to that for A.G. machine)	1	0.05	0.36

Lastly, quite apart from comparisons of intensity between the two types of machines, there is an important practical difficulty with C.G. machines of large radius. For $n = 0.7$ and $r_0 = 100$ m, $dB/dr = 0.01$ gauss cm^{-1} at an injection field $B_0 = 140$ gauss. Over a vacuum chamber 10 cm wide therefore the field variation at injection time is only 0.1 gauss and if we ask that n should be constant to 10% over the aperture of the vacuum chamber, then we have to be able to measure and guarantee magnetic fields to better than 0.01 gauss.

This is far from possible and represents probably the greatest practical reason for abandoning C.G. machines at these high energies.

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