ITP-SB-90-47
CERN-TH.5879/90

SUPERSTRING ANOMALIES IN THE SEMI-LIGHT CONE GAUGE

Fiorenzo Bastianelli, Peter van Nieuwenhuizen
Institute for Theoretical Physics
State University of New York at Stony Brook
Stony Brook, N.Y. 11794, USA

and

Antoine Van Proeyen *
Theory Division, CERN
CH-1211 Geneva 23, Switzerland

Abstract

We compute the Einstein, Lorentz and Weyl worldsheet anomalies and the super Poincaré space-time anomalies of the heterotic superstring, using path-integrals and the gauge $\Gamma^+ \Theta = 0$ to fix the local fermionic κ -symmetry. The regulators for the various Jacobians are all obtained from a recent general method of constructing consistent regulators. We do not use a special gauge for the metric. A general lemma for anomalies of algebraic and non-algebraic symmetries is obtained. Our results show that only for a certain class of metrics do all worldsheet and space-time anomalies cancel simultaneously after adding a suitable but complicated set of counterterms.

* Bitnet address: FGBDA19@BLEKUL11. On leave of absence from Instituut voor theoretische fysica, K.U. Leuven, B-3030 Belgium. Aangesteld navorser, N.F.W.O., Belgium.

Introduction

The Green-Schwarz (GS) string as a classical theory is manifestly space-time supersymmetric and Poincaré invariant. A still unresolved problem is its manifest super-Poincaré invariance at the quantum level. Recently progress has been made in the completely covariant quantization of the superparticle, but the extension of these results to the superstring does not seem obvious [1-8]. The only two successful quantization schemes used so far employ either the manifestly unitary light cone gauge [9], or the so-called semi-light cone gauge [10,11], both of which destroy the manifest rigid space-time super-Poincaré invariance. In the former there are no space-time anomalies. We shall use the latter approach, in which the local fermionic κ -symmetry [12] is fixed by the unitary gauge $\Gamma^+ \Theta = 0$, while the other local worldsheet symmetries are covariantly quantized. This raises the threat of worldsheet and/or space-time anomalies.

Recently, it was claimed by Kraemmer and Rebhan that anomalies in the semi-light cone approach do not cancel [13]. This would be fatal for the GS formulation of the superstring because it would mean that physical results are gauge dependent. Clearly this is an important problem and we have decided to check their computation. Our main result, eq. (28), although derived by a different method, agrees with theirs. On the other hand, M. Chu claimed that in a Hamiltonian formulation, the definition of the Lorentz generators can be changed in order $\hbar$ such that anomalies disappear [14]. In the conformal gauge, our work agrees with her results because then eq. (28) vanishes, as we explain in the conclusions. However, for our purposes, the conformal gauge is not in general sufficient because it cannot always be reached globally. In addition, it is not clear to us whether one may add any term of order $\hbar$ to the generators in Hamiltonian formalism, or only terms which correspond to adding local counterterms in Lagrangian formalism. Kraemmer and Rebhan used analytic regularization to obtain the leading part of the anomalies, and added further contributions obtained from heat kernel methods to obtain a worldsheet covariant answer. Kallosh and Morozov [11] studied worldsheet anomalies in the conformal gauge, and showed that they can be made to vanish by suitable field redefinitions, but they did not consider the space-time anomalies.

In computations of anomalies, regulators for the various fields are sometimes chosen rather arbitrarily, only being required to regulate certain divergent expressions without for example checking that they all produce the same kind of anomalies (for example consistent anomalies or covariant anomalies [15]). We shall obtain all our regulators for the various

fields from a recently constructed general method for the construction of consistent regulators [16-18] (for a recent application to the gravitational anomaly of a chiral boson see [19]).

In this letter, we shall compute *all* anomalies in a general gauge for the metric and investigate whether they can all be cancelled by varying the same set of local counterterms.

The model

The GS heterotic superstring is described by the following action * [20]

$$S = \int d^2x \left[-\frac{1}{2} g^{\alpha\beta} \Pi_\alpha^\mu \Pi_\beta^\mu + \epsilon^{\alpha\beta} \partial_\alpha X^\mu (\bar{\Theta} \Gamma_\mu \partial_\beta \Theta) + \frac{i\epsilon}{\sqrt{2}} \psi^\Lambda \epsilon_-^\alpha \partial_\alpha \psi^\Lambda \right] \quad (1)$$

where ψ^Λ are 32 heterotic fermions, Θ is a $d = 10$ Majorana Weyl space-time spinor with 16 real components (which are $d = 2$ anticommuting scalars) and $\Pi_\alpha^\mu = \partial_\alpha X^\mu - \bar{\Theta} \Gamma^\mu \partial_\alpha \Theta$. The action has the following symmetries: $d = 2$ local Einstein (ξ^α), Lorentz (λ), Weyl (σ) symmetry

$$\begin{aligned} \delta\Theta &= \xi^\alpha \partial_\alpha \Theta, \quad \delta X^\mu = \xi^\alpha \partial_\alpha X^\mu, \quad \delta\psi^\Lambda = \xi^\alpha \partial_\alpha \psi^\Lambda - \frac{1}{2}(\sigma - \lambda)\psi^\Lambda \\ \delta e_\alpha^a &= \xi^\beta \partial_\beta e_\alpha^a + (\partial_\alpha \xi^\beta) e_\beta^a + \lambda e_\alpha^a + \sigma e_\alpha^a \end{aligned} \quad (2)$$

local fermionic κ -symmetry ("Siegel symmetry")

$$\delta\Theta = \Gamma_\mu \Pi_\alpha^\mu e_-^\alpha \kappa_+, \quad \delta X^\mu = \bar{\Theta} \Gamma^\mu \delta\Theta, \quad \delta e_\alpha^+ = 4e_\alpha^{+\bar{\kappa}} + \nabla_+ \Theta \quad (3)$$

and rigid target super-Poincaré symmetry

$$\delta\Theta = \epsilon + \frac{1}{4} \Lambda_{\mu\nu} \Gamma^\mu \Gamma^\nu \Theta, \quad \delta X^\mu = a^\mu - \bar{\Theta} \Gamma^\mu \epsilon + \Lambda^\mu{}_\nu X^\nu \quad (4)$$

We will assume that $\Gamma^+ \kappa_+ = 0$. In a covariant quantization of the κ -symmetry this is obviously not allowed, but we will fix this symmetry in a unitary way and covariantly quantize the other gauge symmetries. Restricting κ_+ to satisfy $\Gamma^+ \kappa_+ = 0$ removes the linear

* Our $d = 10$ Dirac matrices Γ^μ are all real and satisfy $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$ with $\eta^{\mu\nu} = (-1, +1, \dots, +1)$. They are given by $\Gamma^0 = -i\tau_2 \otimes I_{16}$, $\Gamma^9 = \tau_1 \otimes \tau_3 \otimes I_8$ and $\Gamma^i = \tau_1 \otimes \begin{pmatrix} \gamma^i & \\ & \gamma^i \end{pmatrix}$ for $i = 1, 8$, where the 8×8 matrices γ^i for $i = 1, 7$ are antisymmetric and real and satisfy $\{\gamma^i, \gamma^j\} = -2\delta^{ij}$, while $\gamma^8 = I_8$. Furthermore $\bar{\Theta} = \Theta^T \Gamma^0$ and $\Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^9 \pm \Gamma^0)$. The chirality matrix $\Gamma^{11} = \Gamma^0 \Gamma^1 \dots \Gamma^9$ equals $-\tau_3 \otimes I_{16}$, and our Θ 's with $\Gamma^+ \Theta = 0$ are of the form $(0, 0, \Theta_a, 0)$ where $a = 1, 8$. Finally $\epsilon_{01} = \epsilon_{4-} = \epsilon^{+-} = 1$

dependence of the several components of the gauge symmetry. Since the κ -symmetry is now irreducible, we do not need an infinite tower of ghosts for ghosts, and we can fix it by requiring $\Gamma^+ \Theta = 0$. In the gauge $\Gamma^+ \Theta = 0$, the κ -ghosts do not propagate. Instead of adding a gauge fixing and a ghost term for this gauge we can equally well fix the κ -symmetry classically since classical gauge fixing and quantization commute for algebraic symmetries [21]. Of course, this will require compensating κ -transformations to be added to the other symmetries in order to maintain this gauge and this in turn will in general modify the ghost action. The local bosonic symmetries are fixed by choosing a given arbitrary background for the vielbein, which we again denote by e_a^α . Accidental cancellations of parts of the anomalies in the conformal gauge [13] are hereby avoided. After integrating out the Lorentz and Weyl ghosts the coordinate ghosts form a b, c system of conformal weight $j = 2, -1$ in a general background, and the quantum action becomes [10,11]

$$S = \int d^2x \, e \left\{ -\frac{1}{2} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu + 2\sqrt{2}i(\nabla_- X^+) \Theta_a \nabla_+ \Theta_a + \frac{i}{\sqrt{2}} \psi^A \nabla_- \psi^A + b_{++} \nabla_- c^+ + b_{--} \nabla_+ c^- \right\} \quad (5)$$

where the Weyl and Lorentz covariant derivatives are introduced in the following way. If $A_\pm$ transforms under Weyl and Lorentz symmetry as $\delta A_\pm = -J_\pm(\sigma \pm \lambda) A_\pm$ then $\nabla_\pm = e_\pm^\alpha(\partial_\alpha \mp J_\pm \omega_\alpha)$ with $\omega_\alpha = e^{-1} e_{\alpha\alpha} e^{\beta\gamma} \partial_\beta e_{\gamma}{}^a$. For our fields $J_\pm = 0$ for X^μ and Θ_a , $J_- = \frac{1}{2}$ for ψ^A , and $J_+ = -1$ for $c^\mp$ and e_β^A . In order that Θ have a non-vanishing kinetic term we shall require that X^+ have a non-vanishing background u^+ satisfying $\nabla_- u^+ = e_-^\alpha \partial_\alpha u^+ \neq 0$. For the one-loop anomalies, we retain only the part of (5) quadratic in quantum fields and replace $\nabla_- X^+$ in (5) by $\nabla_- u^+$.

The anomalies can be computed in two ways: either by varying the external fields, and evaluating the Jacobians of the compensating variations of the quantum fields, or by making a BRST change of the quantum integration variables in the path-integral. These two approaches should give the same results. (For an explicit demonstration of their equality in the case of the spinning string in superspace see [22]. In general this equality holds under certain conditions [23] which are satisfied in our case.) We shall choose the former and vary the background fields. The Jacobians J are then uniquely determined from the obvious covariant transformation rules of the quantum fields which keep the action invariant.

The recipe yielding consistent regulators for a field theory given by $\mathcal{L} = \frac{1}{2} \phi^T T \phi$

(where $T\mathcal{O}$ is thus the kinetic matrix) is as follows [16,17]. First one adds massive Pauli-Villars (PV) fields χ with mass term $\frac{1}{2} M \chi^T T \chi$. If these mass terms are chosen such that they respect certain symmetries, these symmetries are without anomalies. The remaining symmetries yield potential anomalies, which are proportional to the variation of the mass term, namely

$$\Delta n = \text{Tr} \left[J \exp(\mathcal{O}^n / M^2) \right] \quad (6)$$

where $n = 1$ for X , and $n = 2$ for all other fields. (This achieves that all regulators have a Dalemberertian along the diagonal.) J is here the transformation matrix of the fields, as we will only consider field-independent T matrices. (For field-dependent T matrices there are corrections [17,18].) The crucial point of this result is that it is not the kinetic matrix which is to be used for the regulator but rather T^{-1} times the kinetic operator, where T is the (background field-dependent or independent) mass matrix. In our case we shall choose local Lorentz and Weyl invariant mass terms for Θ_a , ψ^A and b, c so that these fields yield Einstein anomalies. For X^μ we choose an Einstein invariant mass term, but move the resulting Weyl anomaly to the Einstein sector by adding the variation of a suitable counterterm [15,24]. Then we shall analyze the sum of all these Einstein anomalies.

The fields ψ^A , $b_{\pm\pm}$, $c^\pm$, Θ_a' can all be treated as anticommuting b, c systems [25], with conformal spins $J_- = \frac{1}{2}$, $J_+ = 2$, $J_+ = -1$, $J_+ = \frac{1}{2}$, respectively. (We redefine Θ by absorbing a factor $(\nabla_- X^+)^{\frac{1}{2}}$ and use $\Theta' = (\nabla_- X^+)^{\frac{1}{2}} \Theta$ as an independent variable. This leads to the assignment of $J_+ = \frac{1}{2}$ to Θ' .)

Anomalies in b, c systems

A general b, c system of spin $(j, 1-j)$ and Lagrangian

$$\mathcal{L} = -ie \, b \nabla_+ c \quad (7)$$

has the following background symmetries

$$\begin{aligned} \delta b &= \xi^\alpha \partial_\alpha b - j(\sigma + \lambda)b, & \delta c &= \xi^\alpha \partial_\alpha c - (1-j)(\sigma + \lambda)c, \\ \delta e_\alpha{}^a &= \xi^\beta \partial_\beta e_\alpha{}^a + (\partial_\alpha \xi^\beta) e_\beta{}^a + \lambda e_\alpha{}^b e_b{}^c + \sigma e_\alpha{}^a. \end{aligned} \quad (8)$$

For systems with the opposite chirality replace ∇_+ by ∇_- in (7) and $(\sigma + \lambda)$ by $(\sigma - \lambda)$ in (8). We regulate the loops by the following action for PV fields B, C and $\bar{B}, \bar{C}$

$$\mathcal{L} = -ie \, B \nabla_+ C + i \, \bar{B} \partial_- \bar{C} - im \left((e_-)^j \bar{B} \bar{B} + (e_-)^{1-j} C \bar{C} \right). \quad (9)$$

where $(g = |\det g_{\mu\nu}|)$

$$\begin{aligned} \mathcal{O} &= \frac{1}{\sqrt{g}} \partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu + V^\mu \partial_\mu + S \\ I &= m^2 + S - \frac{1}{2} D_\mu V^\mu - \frac{1}{4} V_\mu V^\mu - \frac{1}{6} R \\ I_\mu &= \frac{1}{2} (\partial_\mu - V_\mu) I + \frac{1}{12} D^\nu (\partial_\mu V_\nu - \partial_\nu V_\mu). \end{aligned} \quad (15)$$

Here R is the Ricci scalar ($R = R^\lambda_{\nu\rho\lambda} g^{\nu\rho}$; $R^\lambda_{\nu\rho\sigma} = \partial_\rho \Gamma^\lambda_{\nu\sigma} + \dots$). To derive this lemma we pull the factor e^{ikx} to the left, extract a factor $e^{(-\frac{k^2}{m^2})}$ with $k^2 = g^{\mu\nu} k_\mu k_\nu$ from the left-hand side of the exponential and expand the rest using the Baker-Campbell-Hausdorff formula, keeping only terms that survive in the limit $m^2 \rightarrow \infty$. Since the result must be covariant (the trace is built from covariant objects), we list all possible structures which can appear by dimensional analysis, compute the non-covariant leading terms and covariantize successively. The above result includes as special cases many previous lemmas [27,18].

Applying this lemma to (13) where $g^{++} = 0$, $g^{+-} = \frac{1}{2}$, $g^{--} = -h_{++}$, we get

$$An(\xi^\alpha) = \pm \left[(12j^2 - 12j + 2) \frac{1}{24\pi} \int d^2x (\xi^- + h_{++} \xi^+) \partial_-^3 h_{++} - \frac{m^2}{2\pi} \int d^2x \xi^+ \partial_- h_{++} \right] \quad (16)$$

where for bosons the $+$ sign holds, while for fermions (the only cases we need) the $-$ sign is needed. The m^2 -dependent term can be eliminated by introducing several PV regulators [16], and will not be considered any further. The m^2 -independent term is a spin-dependent coefficient $\pm(12j^2 - 12j + 2)$ times the variation of the Polyakov gravitational WZNW action

$$S_P(h_{++}) = \frac{-1}{48\pi} \int d^2x (\partial_-^2 h_{++}) [\partial_- (\partial_+ - h_{++} \partial_-)]^{-1} \partial_-^2 h_{++}. \quad (17)$$

(The form of [28] is obtained by substituting $h_{++} = \frac{\partial_+ f}{\partial_-}$ into (17).) The result for the other chirality is obtained by simply replacing the indices $+$ and $-$. This concludes our discussion of general b , c systems. We now apply these results to the various fields of the superstring.

Computation of anomalies

The computation of the anomalies for the X^μ fields is standard. For example using the Fujikawa variable $\tilde{X}^\mu = e^{\frac{1}{2}} X^\mu$ and the mass term $m^2 \tilde{X}^\mu \tilde{X}_\mu$ we get the usual Weyl anomaly which corresponds to the variation of the effective action

$$\Gamma_X = d \frac{-1}{96\pi} \int d^2x c R \frac{1}{e \nabla^2} e R = d \{ S_P(h_{++}) + S_P(h_{--}) + local terms \} \quad (18)$$

The $m = 0$ part of the action is invariant under (8) (with b, c replaced by B, C and $\delta\tilde{B} = \delta\tilde{C} = 0$). The mass term is Weyl and Lorentz invariant but breaks Einstein invariance. We introduce the variables [26] $e_+^+ = e_+^+$, $e_-^- = e_-^-$, $h_{++} = \frac{e_+^+}{e_+^+}$, $h_{--} = \frac{e_-^-}{e_-^-}$ for the background vielbein (where e_α^a has the lower index curved), such that $h_{\pm\pm}$ is Weyl and Lorentz invariant, while under these symmetries $\delta e_{\pm\pm} = (\sigma \mp \lambda) e_{\pm\pm}$. Further we introduce the Fujikawa variables $\tilde{B} = (e_-^-)^{1-j} B$, $\tilde{C} = (e_-^-)^{1-j} C$ for the quantum fields and rewrite the PV action as

$$\mathcal{L} = -i\tilde{B}(\partial_+ - h_{++}\partial_- - (1-j)(\partial_- h_{++}))\tilde{C} + i\tilde{B}\partial_+\tilde{C} - im(\tilde{B}\tilde{B} + \tilde{C}\tilde{C}) \quad (10)$$

Defining $D_+^j = (\partial_+ - h_{++}\partial_- - j(\partial_- h_{++}))$ we get (on the basis $\tilde{B}$, $\tilde{C}$, $\tilde{B}$, $\tilde{C}$)

$$\mathcal{O} = \begin{pmatrix} \partial_- & \partial_- \\ D_+^{1-j} & \partial_- D_+^{1-j} \end{pmatrix}, \quad \mathcal{O}^2 = \begin{pmatrix} \partial_- D_+^j & \partial_- D_+^{1-j} \\ D_+^{1-j} \partial_- & D_+^{1-j} \partial_- \end{pmatrix} \quad (11)$$

The consistent regulator for the field b of spin j is then $\mathcal{O}^2 = \partial_- D_+^j$, while for c of spin $1-j$ we must use $\partial_- D_+^{1-j}$. In terms of twiddled variables, the background symmetry reads

$$\begin{aligned} \delta\tilde{B} &= \xi^\alpha \partial_\alpha \tilde{B} + j(\partial_- \xi^- + h_{++} \partial_- \xi^+) \tilde{B}, \quad \delta\tilde{C} = \xi^\alpha \partial_\alpha \tilde{C} + (1-j)(\partial_- \xi^- + h_{++} \partial_- \xi^+) \tilde{C}, \\ \delta h_{++} &= \xi^\alpha \partial_\alpha h_{++} + \partial_+ \xi^- + h_{++}(\partial_+ \xi^+ - \partial_- \xi^-) - h_{++}^2 \partial_- \xi^+, \quad \delta\tilde{B} = \delta\tilde{C} = 0 \end{aligned} \quad (12)$$

The Weyl and Lorentz symmetries are manifestly preserved since the mass term is invariant under them. The Einstein anomaly, however, is non-vanishing:

$$An(\xi^\alpha) = STr \left[\left(\xi^\alpha \partial_\alpha + j(\partial_- \xi^- + h_{++} \partial_- \xi^+) \right) \exp \left(\frac{\partial_- D_+^j}{m^2} \right) \right] + (j \rightarrow 1-j) \quad (13)$$

where the derivatives in the j -dependent term of the Jacobian only act on ξ . Traces of this type can be obtained from the following lemma

$$\begin{aligned} \text{Lemma:} \quad & \lim_{m^2 \rightarrow \infty} Tr \left[\left(a + b^\mu \partial_\mu \right) \exp \left(\frac{\mathcal{O}}{m^2} \right) \right] \\ & \equiv \lim_{m^2 \rightarrow \infty} \int d^2x \int \frac{d^2k}{(2\pi)^2} e^{-ikx} (a + b^\mu \partial_\mu) \exp \left(\frac{\mathcal{O}}{m^2} \right) e^{ikx} \\ & = \int d^2x \frac{\sqrt{g}}{4\pi} (aI + b^\mu I_\mu) \end{aligned} \quad (14)$$

with $e\nabla^2 = \partial_\mu e g^{\mu\nu} \partial_\nu$. The local counterterms can be explicitly exhibited by first decomposing $R = R^+ + R^-$ where $R^+ = -2(d_- \omega_+ + \omega_+ \omega_-)$ and $R^- = 2(d_+ \omega_- - \omega_+ \omega_-)$ with $d_\pm = e_\pm^\alpha \partial_\alpha = e^{-1} e_\mp^\alpha (\partial_\pm - h_{\pm\pm} \partial_\mp)$ and $\omega_\pm = e_\pm^\alpha \omega_\alpha = \mp e^{-1} [(\partial_\pm e_\mp^\alpha) - \partial_\mp (h_{\pm\pm} e_\mp^\alpha)]$. Using $e g^{\alpha\beta} = \frac{1}{1-h_{--}h_{++}} \begin{pmatrix} -2h_{--} & 1+h_{++}h_{--} \\ 1+h_{++}h_{--} & -2h_{++} \end{pmatrix}$ we obtain

$$\begin{aligned} c\nabla^2 &= 2e\nabla_+ \nabla_- = 2e(d_+ - \omega_+) d_- = 2e\nabla_- \nabla_+ = 2e(d_- + \omega_-) d_+ \\ &= 2(\partial_- - \partial_+ h_{--})(1 - h_{++} h_{--})^{-1} (\partial_+ - h_{++} \partial_-) \\ &= 2(\partial_+ - \partial_- h_{++})(1 - h_{++} h_{--})^{-1} (\partial_- - h_{--} \partial_+) \\ eR^+ &= -2(\partial_- - \partial_+ h_{--})(1 - h_{++} h_{--})^{-1} (\partial_- h_{++}) + c\nabla^2 \ln(e_-) \\ eR^- &= -2(\partial_+ - \partial_- h_{++})(1 - h_{++} h_{--})^{-1} (\partial_+ h_{--}) + c\nabla^2 \ln(e_+). \end{aligned} \quad (19)$$

In this equation all derivatives act to the far right. The effective action becomes

$$\begin{aligned} \int d^2x eR \frac{1}{e\nabla^2} eR^+ &= 2 \int d^2x \left((\partial_-^2 h_{++}) (\partial_- (\partial_+ - h_{++} \partial_-))^{-1} \partial_-^2 h_{++} + \text{loc. terms} \right) \\ \text{loc. terms} &= (1 - h_{++} h_{--})^{-1} \left[h_{--} (\partial_- h_{++})^2 - (\partial_+ h_{--}) (\partial_- h_{++}) \right. \\ &\quad + (\partial_- h_{++}) (\partial_- - h_{--} \partial_+) \ln(e_- e_+) \\ &\quad + [(\partial_- h_{++}) (\partial_- - h_{--} \partial_+) + (\partial_+ h_{--}) (\partial_+ - h_{++} \partial_-)] \ln(e_-) \left. \right] \\ &\quad + \frac{1}{2} \ln(e_- e_+) c\nabla^2 \ln(e_-) \end{aligned} \quad (20)$$

where the derivatives in $(\partial_\mp h_{\pm\pm})$ do not act beyond the brackets. To obtain this result we have rewritten $(\partial_- - \partial_+ h_{--})$ in R^+ as $-(\partial_+ - \partial_- h_{++}) h_{--} + \partial_- (1 - h_{++} h_{--})$, and then partially integrated with the operator $(\partial_+ - \partial_- h_{++})$. Adding to this result the terms with R^- , obtained by replacing all $+$ with $-$, we obtain (18) in a convenient basis [29, 24]. Subtracting the local terms in (20) amounts to shifting the Weyl anomaly to the Einstein anomaly (since $h_{\pm\pm}$ are manifestly Weyl and Lorentz invariant).

We get the following result for the sum of all integrated anomalies

$$\Gamma^{\text{tot}} = \left(d - 26 + \frac{32}{2} \right) S_P(h_{--}) + \left(d - 26 + \frac{8}{2} \right) S_P(h_{++}) = -12 S_P(h_{++}) \quad (21)$$

(for $d = 10$). This contains the contribution of d fields X^μ in both sectors, the $b_{\pm\pm}$, $c^\pm$ sector contributing to the $h_{\mp\mp}$ term, the 32 fields ψ^A contributing to the h_{--} term, and the 8 fields Θ_a contributing to the h_{++} term. (In the latter two cases there is a factor $\frac{1}{2}$ because $b = c$ in (7) so that we do not have to add the $(j \rightarrow 1 - j)$ in (13).)

Clearly an Einstein anomaly is left. It can be transformed in a Weyl- Lorentz anomaly by adding back the local counterterms in (20), which replaces $S_P(h_{++})$ by $eR\nabla^{-2}R^+$ as in (20). This Weyl- Lorentz anomaly can be cancelled by adding yet another counterterm depending on $\nabla_- u^+$ [13]. The effective action reads then

$$\Gamma^{\text{eff}} = \frac{12}{96\pi} \int d^2x eR \left(\frac{1}{e\nabla^2} eR^+ + \ln \nabla_- u^+ \right) \quad (22)$$

Now the Einstein background symmetry is manifest. Under a local Lorentz and Weyl transformation the expression between brackets in (22) is invariant (use that $e\nabla^2$ is invariant and obtain δR^+ from (19)). The remaining variation, of eR in (22), equals $e\nabla^2 2\sigma$, and partially integrating with $e\nabla^2$ the variation vanishes upon using the equation of motion of the background field u^+ , namely $\nabla_+ \nabla_- u^+ = (d_+ - \omega_+) d_- u^+ = 0$,

$$\nabla_-^2 \ln \nabla_- u^+ = 2\nabla_- \nabla_+ \ln \nabla_- u^+ = 2(d_- + \omega_-) \frac{1}{d_- u^+} d_+ d_- u^+ = -R^+ \quad (23)$$

Hence, the effective action in (22) is free from any worldsheet anomalies.

At this stage all that is left to do is to check for possible anomalies in the super Poincaré algebra (4). We shall compute these directly using (6), (11) and (14). Note that the effective action (22) was a convenient way of summarizing the world-sheet anomalies. But there is no guarantee that its variation would yield the correct super Poincaré anomalies. In the gauged fixed theory we need to add compensating κ transformations to preserve the gauge fixing condition. We have $\kappa = -\frac{1}{2\nabla_- X^+} (\Gamma^+ \epsilon + \Gamma^i \theta \Lambda_{-i})$. We exhibit the susy and translation transformation rules

$$\begin{aligned} \delta \Theta_a &= \epsilon_a + \frac{1}{\sqrt{2}} \left(\frac{\nabla_- X^i}{\nabla_- X^+} \right)^i \gamma^{ia} \epsilon_a \\ \delta X^- &= a^- - i\sqrt{2} \Theta_a \epsilon_a + i \left(\frac{\nabla_- X^i}{\nabla_- X^+} \right) \Theta_a \gamma^i \epsilon_a \\ \delta X^+ &= a^+, \quad \delta X^i = a^i - 2i \Theta_a \gamma^i \epsilon_a \end{aligned} \quad (24)$$

and Lorentz transformation rules

$$\begin{aligned} \delta \Theta_a &= -\frac{1}{2} \left(\frac{\nabla_- X^i}{\nabla_- X^+} \right) \Lambda_{-j} \gamma^i \epsilon_a \gamma^j \Theta_b + \frac{1}{4} \Lambda_{ij} \gamma^i \epsilon_a \gamma^j \Theta_b + \frac{1}{2} \Lambda_{+-} \Theta_a \\ \delta X^+ &= \Lambda_{-j} X^j - \Lambda_{+-} X^+ \\ \delta X^- &= \frac{-i}{\sqrt{2}} \left(\frac{\nabla_- X^i}{\nabla_- X^+} \right) \Lambda_{-j} \Theta_a \gamma^i \epsilon_a \gamma^j \Theta_b + \Lambda_{+-} X^j + \Lambda_{+-} X^- \\ \delta X^i &= \Lambda^i_{-} X^- + \frac{i}{\sqrt{2}} \Lambda_{-j} \Theta_a \gamma^i \epsilon_a \gamma^j \Theta_b + \Lambda^i_{-j} X^j + \Lambda^i_{+-} X^+ \end{aligned} \quad (25)$$

Many Jacobians, which we can read off from (24, 25) are off-diagonal with respect to our diagonal regulators and therefore do not contribute to the anomalies. The cases we have to analyse explicitly are the Λ_{-+} and Λ_{-j} transformations. We begin by noting that Θ' (but not Θ !) is invariant under Λ_{-+} transformations and transforms into a term proportional to a Lorentz generator under Λ_{-j} transformations

$$\delta\Theta'_a = \frac{1}{2} \left(\frac{\nabla_- X^i}{\nabla_- X^+} \right) \Lambda_{-i} \Theta'_a - \frac{1}{2} \left(\frac{\nabla_- X^i}{\nabla_- X^+} \right) \Lambda_{-j} \gamma^i_{ab} \gamma^j_{ab} \Theta'_b, \quad (26)$$

where the first term comes from the transformation in X^+ , and the second from the compensating κ transformation. They combine to a term proportional to $(\gamma^{[i}\gamma^{j]})_{ab}$ which is antisymmetric in 10 dimensions. Since the Θ' regulator is diagonal in spinor space, there is no space-time Lorentz anomaly for the Θ' fields at all. Since also the X regulators are diagonal in space-time, the X fields are also free from space-time Lorentz anomalies. Note that these results hold whether or not the background X^i is constant at all.

Thus there are no space-time super-Poincaré anomalies before introducing the local counterterms. The local terms in (20) do not change this because the field in this expression are invariant, but then we introduced the second term in (22). Splitting $\delta\phi = \delta\phi_{\text{back}} + \delta\phi_{\text{quantum}}$ we have

$$\delta u^+ = a^+ + \Lambda_{-j} u^j + \Lambda_{-+} u^+, \quad \delta \nabla_- u^+ = \Lambda_{-j} \nabla_- u^j + \Lambda_{-+} \nabla_- u^+ \quad (27)$$

and thus

$$\delta \Gamma^{\text{eff}} = \frac{12}{96\pi} \int d^2x \, eR \left(\Lambda_{-+} + \Lambda_{-j} \frac{\nabla_- u^j}{\nabla_- u^+} \right) \quad (28)$$

Conclusions

We have computed all worldsheet and space-time anomalies, which is clearly necessary for a conclusive answer (for example, the counterterm $eR \ln \nabla_- u^+$ in eq. (22) shifted the anomalies from the worldsheet to the space-time sector). In addition, we have used general metrics, because absence of anomalies for the subclass of conformal metrics does not prove that there are no anomalies, even if one does know in advance that Einstein anomalies cancel, because one cannot always reach the conformal gauge globally. Our results for the worldsheet anomalies agree with Kraemer and Rebhan. Kraemer and Rebhan argue that since the counterterms manifestly break space-time Lorentz symmetry, there should be a space-time Lorentz anomaly. We actually computed the contributions to the space-time Lorentz anomalies. The regulated Jacobians gave no contribution due to our choice of Θ' as

variable. However, varying the background scalar field u^μ in the counterterm, we found the candidate anomaly in (28). The problem is now: when is (28) vanishing?

Suppose we restrict our discussion to worldsheets with Minkowski signature and trivial topology (σ running from $-\infty$ to $+\infty$). Writing R according to (19) as

$$\nabla^2 \left[-\frac{1}{\partial_+ - h_{++} \partial_-} \partial_- h_{++} - \frac{1}{\partial_- - h_{--} \partial_+} \partial_+ h_{--} + \ln e_- - e_+ \right] \quad (29)$$

where the expression between square brackets is a scalar, we would obtain vanishing space-time Lorentz anomalies if we could partially integrate. The reason is that if one could partially integrate ∇_+ from $\nabla^2 = 2\nabla_+ \nabla_-$ onto the scalar function $\frac{d_- u^j}{\partial_- u^+}$, this last term would cancel because the background u^μ satisfies its classical equation of motion

$$\begin{aligned} d_+ \frac{d_- u^j}{\partial_- u^+} v &= (d_- u^+ d_+ d_- u^j - d_- u^j d_+ d_- u^+) (d_- u^+)^{-2} \\ &= (d_- u^+ \nabla_+ d_- u^j - d_- u^j \nabla_+ d_- u^+) (d_- u^+)^{-2} = 0 \end{aligned} \quad (30)$$

The problem, however, is that one cannot always forget the boundary terms. Suppose one could cast the metric in conformal form. Then $R = \nabla^2 \ln \rho$ and if ρ were to tend to one at infinity, we could partially integrate. However not all metrics in Minkowski space can be obtained from such conformal metrics by a coordinate transformation which reduces to the unit transformation at infinity. In string theory, where one must integrate over all metrics, this might imply that anomalies remain. Of course, in string theory one works in Euclidean space, and Θ' has a zero mode Θ'_0 . If there were no further operators, integration over Θ'_0 would yield zero. If vertex operators make the integration over Θ'_0 non-trivial, one would have to study (28) for Euclidean metrics. In that case, even the R term gives a contribution, and furthermore we must use patches to go to the conformal gauge. It is not clear to us how in that case the anomalies could cancel.

Acknowledgements

We thank A. Rebhan for a useful discussion. One of us (A.V.P.) would like to thank the Institute for Theoretical Physics at Stony Brook for its hospitality.

References

- [1] U. Lindström, M. Roček, W. Siegel, A. van de Ven and P. van Nieuwenhuizen, *Phys. Lett.* **B224** (1989) 285.
- [2] M. Green and C. Hull, *Phys. Lett.* **B225** (1989) 57.
- [3] R. Kallosh, *Phys. Lett.* **B224** (1989) 275.
- [4] M.B. Green and C.M. Hull, *Mod. Phys. Lett.* **A5** (1990) 1399 and contribution to the conference "Strings '90".
- [5] A. Miković, M. Roček, W. Siegel, P. van Nieuwenhuizen, J. Yamron and A. van de Ven, *Phys. Lett.* **B235** (1990) 106;
- [6] E.A. Bergshoeff, R. Kallosh and A. Van Proeyen, preprint CERN-TH.5788/90, to be published in *Phys. Lett. B*; R.E. Kallosh, preprint CERN-TT.5789/90, to be published in *Phys. Lett. B*.
- [7] J. Fish and M. Henneaux, Bruxelles preprint ULB-TH2-80-04.
- [8] F. Bastianelli, G. Delius and E. Laenen, *Phys. Lett.* **B229** (1989) 223.
- [9] M. Green and J. Schwarz, *Nucl. Phys.* **B243** (1984) 285.
- [10] S. Carlip, *Nucl. Phys.* **B284** (1987) 365.
- [11] R. Kallosh and A. Morozov, *Int. J. Mod. Phys.* **A3** (1988) 1943.
- [12] W. Siegel, *Phys. Lett.* **128B** (1983) 397.
- [13] U. Kraemmer and A. Rebhan, *Phys. Lett.* **B236** (1990) 255.
- [14] M. Chu, University of Florida preprint UFIFT-HEP-90-9.
- [15] W. Bardeen and B. Zumino, *Nucl. Phys.* **B244** (1984) 421.
- [16] A. Diaz, W. Troost, P. van Nieuwenhuizen and A. Van Proeyen, *Int. J. Mod. Phys.* **A4** (1989) 3959.
- [17] W. Troost, P. van Nieuwenhuizen and A. Van Proeyen, *Nucl. Phys.* **B333** (1990) 727.
- [18] M. Hatsuda, W. Troost, P. van Nieuwenhuizen and A. Van Proeyen, *Nucl. Phys.* **B335** (1990) 166.
- [19] F. Bastianelli, Stony Brook preprint ITP-SB-90-63.
- [20] For details one may consult, for example, section 2 of U. Lindström, M. Roček, W. Siegel, P. van Nieuwenhuizen and A. van de Ven, *Nucl. Phys.* **B330** (1990) 19.
- [21] P. van Nieuwenhuizen, *Phys. Rev.* **D24** (1981) 3315.
- [22] F. Bastianelli and P. van Nieuwenhuizen, *Phys. Rev.* **D41** (1990) 2525.
- [23] J. Kowalski-Glikman and P. van Nieuwenhuizen, Stony Brook preprint ITP-SB-90-57.
- [24] M. Sanielevici, G. Semenoff and Y.S. Wu, *Nucl. Phys.* **B312**(1989) 197.
- [25] D. Friedan, E. Martinec and S. Shenker, *Nucl. Phys.* **B271** (1986) 93.
- [26] V. Knizhnik, A. Polyakov and A.B. Zamolodchikov, *Mod. Phys. Lett.* **A3** (1988) 819.
- [27] P. Bouvknegt and P. van Nieuwenhuizen, *Class. Qu. Gr.* **3** (1986) 207; A. Ceresole, P. Pizzochero and P. van Nieuwenhuizen, *Phys. Rev.* **D39** (1989) 1567.
- [28] A. Polyakov, *Mod. Phys. Lett.* **A2** (1987) 893.
- [29] H. Leutwyler, *Phys. Lett.* **153B** (1985) 65.