Scanned with permission from the publisher:

A realistic theory of mesons / Jouvét, B

CERN-56-04

Published in Nuovo Cimento 2 (1955) no 4, pp 941-968

From Carmen Vasini <vasini@sif.it>

To Jens Vigen <Jens.Vigen@cern.ch>

Subject Re Request for permission to reproduce published material

Dear Jens Vigen,

we are glad to grant you permission to reproduce the material requested provided full credit is given to the source

Kindest regards

Carmen Vasini

B JOUVET

1955

N 4 del *Supplemento* al Vol 2, Serie X,
del *Nuovo Cimento* pag 941 968**A Realistic Theory of Mesons.**

B JOUVET

Institut Henri Poincaré Paris (+)

(ricevuto il 7 Gennaio 1955)

CONTENTS — § 1 Introduction — § 2 The theories of nuclear interactions *a* The theories F (theory of FERMI, or « pair theories ») *b* The theories Y (theory of Yukawa, or « mesons theories ») *c* The theory Y+F *d* The realistic theory of elementary particles (theory R) *e* The general Yukawa theories — § 3 General Equations and Methods *a* Particles and coupling *b* Regularization, normalization and renormalization *c* The primitive interaction kernels — § 4 Physical meaning of the interactions: the mesons *a* Form of the interaction kernels *b* Existence and properties of mesons *c* The mesons equations *d* The free mesons *e* The mesons in beta decay — § 5 The realistic meson theory and the Yukawa theories *a* The Yukawa charged nuclear forces *b* The mesons theory of beta decays — § 6 Generalisation of the realistic theory *a* The renormalizations *b* The fermion mesons *c* The problem of four fields coupling — § 7 Conclusions

1 — Introduction

The explanation of « elementary » particles structure, and therefore the prediction of their characteristics (mass, spin, coupling constants with the various fields) has been the object of much research proceeding mainly from two concepts:

— The concept which we call « realistic », which is an attempt to explain the structure of elementary particles, considered as formed in a definite manner

(+) At present with the Theoretical Study Division of CERN (European Organization for Nuclear Research) at the Institute for Theoretical Physics University of Copenhagen

by certain spin $\frac{1}{2}$ particles, supposed more elementary. The main difficulty lies in the way to state, and then to resolve, the problem of the binding of fermions without being given the medium of the boson fields *a priori* ⁽¹⁾

— At the beginning of research in this field, this first concept did not appear to be satisfactory, and more formal methods proceeding from a phenomenological concept of elementary particles were able, if not to explain the relations between the particles, at least to give the form of the equations of the particles supposed free and without structure ⁽²⁾. Attempts have been made for several years to propose new frames and new principles to represent or deduce either the masses and spins or the coupling of the elementary particles (non local theories, spin or charge space, mass operators) ⁽³⁾

It is the purpose of this paper to present a new form of realistic concept of mesons unlike the previous proposed models ⁽¹⁾. We limit our examples to the pseudo scalar interactions. We shall show that in order to unify Yukawa theory of nuclear forces and Fermi theory of beta decay (Sect 2) the choice of a Fermi coupling between the various pairs of fermions (nucleons and leptons) allows us, when using the regularization and a certain summing up of the terms in the *S* matrix (Sect 3), to demonstrate the existence of mesons (bosons) whose masses and coupling constants with the various fermion pairs are expressed with the properties of the fermions only (Sect 4). The construction of a theory of Yukawa type equivalent to the realistic theory allows us to give the rules for the treatment of every term of the *S* matrix. For certain values of the regularization constants, Yukawa theory is renormalizable (Sect 5). Various problems concerning the elementary particles are suggested (Sect 6). This work is the generalization of a more restricted attempt to explain the photon by postulating a Fermi vector coupling between the electron and neutrino pairs (attempt to which we refer here as « V^0 ») ⁽⁴⁾

⁽¹⁾ L. DE BROGLIE: *Theorie des particules à spin (methode de fusion)* (Paris, 1943); C. N. YANG and E. FERMI: *Phys Rev*, **76**, 1739 (1949); G. WENTZEL: *Phys Rev*, **79**, 710 (1950); W. HEISENBERG: *Zeits f Naturf*, **5a**, 251, 367 (1950); *Physica*, **19**, 897 (1953); H. YUKAWA: p. 287 in *L. de Broglie, physicien et penseur* (Paris, 1952).

⁽²⁾ A. PROCA: *Journ de Phys*, **7**, 347 (1936); M. FIERZ: *Helv Phys Acta*, **12**, 4 (1939); W. PAULI and M. FIERZ: *Proc Roy Soc*, **173**, 211 (1939).

⁽³⁾ M. BORN: *Rev Mod Phys*, **21**, 463 (1949); B. T. DARLING: *Phys Rev*, **80**, 460 (1950); H. YUKAWA: *Phys Rev*, **91**, 415, 416 (1953); S. WATANABE: *Phys Rev*, **91**, 771 (1953); PAIS: *Physica*, **19**, 869 (1953); A. PROCA: *Journ de Phys*, **15**, (1954); O. HARA and T. MARUMORI: *Nuovo Cimento*, **12**, 309 (1954).

⁽⁴⁾ B. JOUVET: *Journ de Math*, **33**, 201 (1954) or *These de doctorat, Paris* (1954); A summary of V^0 and of the present paper is published in *Compt Rend Ac Sci*, **237**, 1642 (1953); **238**, 55, 327, 454 (1954); **239**, 1121, 1267 (1954).

2 – The Theories of Nuclear Interactions

Every theory of the nuclear interactions tries to explain, with the smallest number of concepts, the five interactions (at least):

- 1) Interactions nucleon nucleon: (nuclear force)
- 2) Interactions nucleon leptons (e , ν , μ): (β decays)
- 3) Interactions nucleons π mesons: (diffusion, absorption or emission)
- 4) Interactions leptons π mesons: (π mesons decay)
- 5) Interactions leptons leptons: (μ mesons decay)

In quantum theory this problem reduces to the choice of what particles and what couplings between these particles can be considered as existing fundamentally

We review briefly various solutions

a) The theories F (theory of Fermi, or «pair theories») ⁽⁵⁾ – The existence of fermions and couplings between the pairs of fermions is enough to provide an explanation of the interactions 1, 2, 5, but this theory does not consider the interactions 3 and 4

b) The theories Y (theories of Yukawa, or «mesons theories») – The first research on elementary particles led YUKAWA to invent the meson and meson coupling with nucleons, foreseeing interaction 3 and explaining 1. For some years, the lack of experimental precision and the confusion of π mesons with μ mesons permitted the hope of explaining the interactions 2 and 5 by coupling meson fermions, giving also rise to the interactions 4 and 5. This hope no longer exists

c) The theory Y + F – Since the theories *a)* and *b)* are valid for some interactions and not for others, we are led to postulate that there exist fundamental mesons and fermions coupled with YUKAWA coupling. This postulate explains interactions 3, 4 and 1. Then, to explain the Fermi interactions 2 and 5, we also postulate the existence of a fundamental coupling between the fermion pairs. Thus, the interactions are the result of two types of coupling F or Y, with F larger than Y for the interactions 2 and 5, and Y larger than F for interaction 1. However, to be consistent, it would be necessary to show that the Fermi interactions result effectively from the existence of Fermi coupling. But one does not know how to do this because Fermi coupling is not renormalizable in the sense of DYSON. Then, for lack of a theoretical proof

⁽⁵⁾ See for bibliography: M. TANNENWALD: *Phys. Rev.*, **86**, 332 (1952)

that the corrections to Fermi coupling due to the closed loops of fermions containing unknown constants are negligible, one uses the experimental fact that the Fermi interactions appears to be almost independent of the energy. However this empirical argument seems to us not sufficiently convincing to justify this bicephal theory.

Then a convenient treatment of the Fermi coupling has led us to the possibility of constructing a new type of theory.

d) The realistic theory of elementary particles (theory R) – We postulate that there exist only fermions (nucleons and leptons), and Fermi coupling between the various pairs of fermions. According to the realistic concept of elementary particles, one must be able to deduce from this hypothesis alone not only the form of the interactions 2 and 5 but also the existence of boson particles and the form of their coupling with fermions, and then the interactions 3 and 4, and therefore 1. To show that this program is realisable is the purpose of this paper. It will appear that the theory R is equivalent to a more general Yukawa theory than the usual one.

e) The generalized Yukawa theories – One can avoid the difficulties concerning the form and the value of the Fermi interaction deduced from the usual Y theory by postulating the existence of heavy boson mesons coupled more strongly with leptons and more weakly with nucleons than the mesons now known. Such an hypothesis, and the many other which are necessary to explain the high energy nuclear features (hyperons...), often appear arbitrary, and thus it is preferable to have a theory of particles. However the price we must pay in the present R theory is the reject of Dyson renormalization method and the introduction in the theory of a finite number of arbitrary constants, in term of which the masses and coupling constants of the boson particles are expressed.

3 – General Equations and Methods

a) Particles and coupling – The only particles that we suppose to exist fundamentally are the fermions forming two families, nucleon and leptons. We suppose they have spin $\frac{1}{2}$. We suppose that the only neutral lepton is the neutrino, and denote the leptons by $L_0, L_1, L_2, \dots, L_l$ ($L_0 = \text{neutrino}$).

Let n and p denote the neutron and proton, and N_i the particles of the nucleon family (n, p , hyperons).

In this paper we are interested only in the charged pairs of fermions. All the coupling constants in the charged Fermi coupling are supposed to have the same value, g , of unknown numerical value.

Two pairs which are identical or charge conjugate are supposed uncoupled.

The coupled pairs are chosen so that the charge is conserved in elementary processes, and the Lagrangian is Hermitian

In precise examples, we will utilise the interaction Lagrangian:

$$(3\ a1) \quad \sum_{i=1}^l g\{(\bar{\psi}_n \gamma_5 \psi_n)(\bar{\psi}_{L_i} \gamma_5 \psi_{L_0}) + (\bar{\psi}_n \gamma_5 \psi_n)(\bar{\psi}_{L_0} \gamma_5 \psi_{L_i})\} + \\ + \sum_{i < j}^l g\{(\bar{\psi}_{L_i} \gamma_5 \psi_{L_0})(\bar{\psi}_{L_0} \gamma_5 \psi_{L_j}) + (\bar{\psi}_{L_0} \gamma_5 \psi_{L_i})(\bar{\psi}_{L_j} \gamma_5 \psi_{L_0})\}$$

(Charged pseudoscalar couplings, omission of hyperons)

b) Regularization, normalization and renormalization – As is well known, the quantum theory of fields, is not usable if one does not add certain principles to allow a non ambiguous treatment of the infinite expressions. These principles, which often appear as rules for calculus because their meaning is unknown, are the principles of renormalization and otherwise of regularization which is associated with a «normalization» for theories which are not renormalizable. This last occurs in the present case

We call «regularization», the method which allows us, first to separate in a covariant way the infinities in an ultraviolet divergent expression and, second, to substitute for these infinities some arbitrary constants. This is a mathematical operation, always possible, which associates to every Feynman graph an explicit expression containing a finite number of arbitrary constants. One can imagine that this method supplies a deeper understanding of the infinities problem. The ways to perform the regularization though generally equivalent, are not alike in the concepts that they represent ⁽⁶⁾

After regularization we must interpret the physical meaning of the resulting expressions, by studying the effect of the arbitrary constants in various concrete physical situations. Two cases can occur:

— The first possibility is that the theory is «Complete» which means that all the arbitrary constants appearing in the theory can be interpreted as corrections to the fundamental constants of the theory, generally contained in the Lagrangian and in the waves normalizations. The existence of invariances of the theory under certain transformations generally permits us to eliminate or to give definite values to some regularization constants (gauge invariances of Maxwell Dirac theory, or Fermi V^0 coupling ⁽⁴⁾)

— The other possibility is that the theory is «Incomplete» so that the regularization constants represent not only corrections to the constants of the

⁽⁶⁾ W. PAULI and VILARS: *Rev. Mod. Phys.*, **13**, 203 (1941); W. PAULI: *Feld quantisierung*, F. T. H. lectures (Zürich, 1950-1951); L. DE BROGLIE: *Compt. Rend. Ac. Sci.*, **229**, 401 (1949); W. GÖTTINGER: *Phys. Rev.*, **89**, 1012 (1953)

theory but also introduce new types of physical concepts not foreseen in the foundations of the theory (Lagrangian); these concepts are new fields or particles and new forms of coupling between the fields (example, the arbitrary meson-meson coupling appearing in the PS theory of mesons)

Once this interpretation has been made an infinite sum of terms depending on an infinite number of arbitrary constants appears in the calculus of a given effect up to an infinite power of the coupling constants. Thus the fundamental constants of the theory are corrected by an infinite number of arbitrary corrections. For this reason, these corrections are not usable. In order to avoid this difficulty the « normalization » principle is defined ⁽⁷⁾

— Principle of normalization for complete theories. The arbitrary constants are chosen so that the corrections they give to the fundamental constants of the theory (contained in the Lagrangian), for any order in the development of the S matrix, are null. For example, in meson theory the meson mass and the coupling constant of mesons with nucleons are normalized by postulating that the nucleon closed loops carried by the meson ($B^i(p^2)$) satisfy the condition $(B^i(p^2)/(p^2 + \pi^2))_{(p^2 + \pi^2) \rightarrow 0} \rightarrow 0$; therefore:

$$B^i(-\pi^2) = 0, \quad \left(\frac{\partial B^i(p^2)}{\partial p^2} \right)_{-\pi^2} = 0$$

This method is used in electrodynamics when one postulates the gauge invariance of the Lagrangian, which is equivalent to postulating the nullity of photon mass in order to eliminate in the S matrix the corrections to the photon mass.

When no more than three wave functions are coupled in the same coupling as in Maxwell-Dirac electromagnetism, the normalization procedure can be expressed mathematically as the conservation of the poles and residues appearing in every degree of approximation of the S matrix.

— Principle of normalization for incomplete theories. One method for the treatment of the physical quantities not included in the Lagrangian consists in completing this Lagrangian by introducing convenient terms containing new constants to be determined by experiments (example, the introduction

⁽⁷⁾ We do not at all think that the actual arbitrary regularization and normalization process constitute a satisfactory solution of the infinities problem. But for want of a renormalization method able to treat the fundamental problems of particle structure (mass and coupling), the proposed method is equivalent to Dyson's, when it applies and otherwise, must give satisfactory results if the true, but now unknown, series of corrections to the fundamental constants of the theory are strongly convergent. See, on these problems, the LORENZ-KAMERLINGH-ONNES memorial papers in *Physica*, **19**, (1953).

of λq^4 coupling in meson theory) Another way is to retain the first constant of a new type appearing in the S matrix and to normalize the other constants of that type so that the corrections they add to the first constant are null

However, the true difficulty of the normalization of theories is the physical definition of the constants It consists in the choice of the most convenient physical conditions under which a certain interaction has a given value This difficulty appears more important when there exists a coupling between more than three wave functions, as in the meson meson coupling of the Yukawa theory and in Fermi couplings; these cases will be discussed in Section 6

c) *The primitive interaction kernels* – The « pair theories » ⁽⁵⁾ consider only the lowest order interaction terms between two nucleons, via lepton fields The usual treatment of the Fermi theory takes into account only the first order term in g for the nucleon lepton interaction On the contrary, let us define the « primitive interaction kernels », (UV) , represented in Feynman graphs as the sum of all the « chains » constituted with any number of closed loops having only two vertices, and connecting a fermion line at a vertex where the pair is U to another (or the same) fermion line at another vertex where the pair is V ⁽⁸⁾

As examples, with the coupling postulated in (3 a), the charged pairs can be:

$$U, V = (np), \quad (L_0, L_1), \quad , \quad (L_0, L_f)$$

Then we have

$$(3\ c1) \quad \left\{ \begin{array}{l} (UU) = \sum_{\substack{w \neq u \\ x \neq u \\ x \neq w}} \text{diagram 1} + \sum_{\substack{w \neq u \\ x \neq u \\ x \neq w}} \text{diagram 2} + \sum_{\substack{w \neq u \\ x \neq u \\ y \neq x \\ y \neq w}} \text{diagram 3} + \\ (UV) = \text{diagram 4} + \sum_{\substack{w \neq u \\ w \neq v}} \text{diagram 5} + \sum_{\substack{w \neq v \\ x \neq v \\ w \neq x}} \text{diagram 6} + \end{array} \right.$$

⁽⁸⁾ S. KAMEFUCHI and H. UMEZAWA: *Prog Theor Phys*, **8**, 579 and **9**, 529 (1953), have shown that such an iteration of loops with two vertices allows us to treat the « second kind » interactions, including the Fermi interactions However our paper differs from theirs in the method of renormalization, in the demonstration of the possibility of treating completely this interactions, and mainly in the physical interpretation, since these authors have not seen that the Fermi theory is, in the whole, equivalent to the Yukawa theory

This iteration method, is a generalization of the damping theory of Heitler (*Proc Camb Phys Soc*, **37**, 291 (1941)) and Wilson (*Proc Camb Phys Soc*, **37**, 301 (1941))

The S matrix elements due to these interactions between free particles are

$$(3\ c2) \quad S(UV) = \int d^4p J_u^i(p) [\gamma + \gamma^2 \sum_{\substack{w \neq u \\ w \neq v}} K_w^i(p) + \gamma^3 \sum_{\substack{w \neq u \\ x \neq v \\ w \neq x}} K_w^i(p) K_x^i(p) + \dots] J_v^i(-p)$$

$$\text{with } J_u^i(p) J_v^i(-p) = (\bar{a}_\alpha(p_\alpha) O^i a_\beta(p_\beta)) (\bar{a}_\gamma(p_\gamma) O^i a_\delta(p_\delta)) \delta^4(p_\alpha - p_\beta + p) \delta^4(p_\gamma - p_\delta - p),$$

and an analogous formula for $S(UU)$

Here, the p_λ are the momenta of the four fermions denoted by the indices $\alpha, \beta, \gamma, \delta$, paired as $U = (\alpha, \beta)$ and $V = (\gamma, \delta)$. The operator O^i is one of the five Dirac matrix tensors (S, V, T, A, P). The constant $\gamma = ig/2\pi$. The functions $K_w(p)$, associated with the closed loop formed with the pair $W = (\lambda, \mu)$, are

$$(3\ c3) \quad \begin{cases} K_w^i(p) \equiv K_{\lambda\mu}^i(p) = \int d^4x \exp[-ipx] K_{\lambda\mu}^i(x), \\ K_{\lambda\mu}^i(x-x') = P(J_{\lambda\mu}^i(x), J_{\mu\lambda}^i(x'))_0; \quad J_{\mu\lambda}^i(x) = \bar{\psi}_\lambda(x) O^i \psi_\mu(x) \end{cases}$$

Then, one primitive interaction kernel in impulse space is

$$(3\ c4) \quad (UV(p)) = \gamma + \gamma^2 \sum_{\substack{w \neq u \\ u \neq v}} K_w(p) + \gamma^3 \sum_{\substack{w \neq u \\ x \neq v \\ w \neq x}} K_w(p) K_x(p) + \dots$$

In this paper it is not necessary to specify the tensor index i

The algebraically simple form of the kernels results from the fact that the whole impulse is carried in one point of the vertices. More complicated integral equations appear when we take into account the radiative connections between different loops, that is to say, when we are concerned with closed loops having more than two vertices. This case will be considered in Section 5.

It is simpler to utilise the equations of which the kernels are solutions rather than the series. One shows easily, from the properties of the Feynman kernels, or by direct substitution, that the primitive interaction kernels satisfy the equations (in impulse space)

$$(3\ c5) \quad \begin{cases} (UU) = \gamma \sum_{w \neq u} (UW) K_w, \\ (UV) = \gamma + \gamma \sum_{w \neq v} (UW) K_w = (VU) = \gamma + \gamma \sum_{w \neq u} (VW) K_w \end{cases}$$

The solutions are

$$(3.66) \quad \left\{ \begin{array}{l} (UU) = \frac{1}{D} \left\{ \gamma^2 \sum_{u \neq v} K_u + 2\gamma^3 \sum_{\substack{x < y < u \\ x, y \neq v}} K_x K_u + 3\gamma^4 \sum_{\substack{x < y < u \\ x, y, u \neq v}} K_x K_y K_u + \dots \right\} \\ (UV) = \frac{1}{D} \left\{ \gamma + \gamma^2 \sum_{v \neq u \neq w} K_u + \gamma^3 \sum_{\substack{y < w \\ y, w \neq u \neq v}} K_y K_u + \dots \right\} \\ D = \left\{ 1 - \gamma^2 \sum_{y < u} K_y K_u - 2\gamma^3 \sum_{x < y < u} K_x K_y K_u - \dots \right\} \end{array} \right.$$

The series appearing in these expressions have a finite number of terms if the number of fermion pairs that we consider is finite.

In the solutions we have omitted arbitrary distributions which appear in the factor $1/D$, (the expression should be $1/D + c\delta(D)$ where c is an arbitrary constant). However as we have shown in V^0 , this constant is determined by the physical condition that the interactions must be causal, which is true because, the pair exchange is causal. This effect is to shift the poles of the kernel by an arbitrarily small imaginary quantity, as in the definition of the A_π functions.

4 - Physical Meaning of the Interactions: the Mesons

a) *Form of the interaction kernels* - After regularization, every closed loop having two vertices is a function of the total momentum energy of the pair and of two arbitrary constants. Without any loss of generality one can suppose that these two constants express some properties of the fermion pair forming the loop. Then for the loop formed with the pair $W = (\lambda, \mu)$, one has

$$(4.1) \quad K_w = K_{\lambda\mu} = K_{\lambda\mu}(p^2, C_{\lambda\mu}^1, C_{\lambda\mu}^2) \\ (p = \text{impulse, } C^1 \text{ and } C^2 \text{ the reg. const.})$$

For some interval of values $p_a^2 < p^2 < p_b^2$, the function $K_w(p^2)$ can be approximated with a better approximation when the C^i are greater and the interval smaller.

$$(4.2) \quad K_w(p^2) \approx iF_w(p^2 + M_w)$$

The unidimensional constant F_w and the constant M_w (dimension M^2) are functions of the C^i constants and generally depend weakly on the interval. In this approximation (4.2), the kernel (UU) is the ratio of two polynomials,

the degree of the denominator D being n , and of the numerator $n-1$, where n is the number of different pairs ($u = 1, 2, \dots, n$). Then one can write

$$(4\ a3) \quad (UU) \approx i \sum_{l=1}^n \frac{A_{uu}^l}{p^2 + \pi_l^2}$$

The numerators of the kernels (UV) with $u \neq v$ have degree $n-2$, and the denominators are the same. Thus

$$(4\ a4) \quad (UV) \approx i \sum_{l=1}^n \frac{A_{uv}^l}{p^2 + \pi_l^2} \quad \text{with} \quad \sum_{l=1}^n A_{uv}^l \approx 0$$

The last equality is not important, because it is due to the approximation (4 a2). In a more general way, let us define independently of the approximation (4 a2)

$$(4\ a5) \quad (UV(p^2)) = \frac{N_{uv}(p^2)}{D(p^2)} \quad \text{for any } u \text{ and } v = 1, 2, \dots, n$$

$$(4\ a6) \quad D(-\pi_l^2) = 0; \quad A_{ui}^l = \frac{1}{i} [(UV(p^2)) (p^2 + \pi_l^2)]_{(p^2 + \pi_l^2) \rightarrow 0}$$

The constants π_l^2 and A_{uv}^l defined in this way are calculable functions of fermion masses and G^i constants of all the pairs, and thus depend only on the properties of the fermions.

b) Existence and properties of mesons — We limit now the discussion of the preceding equations in postulating that the constants A_{uv}^l and π_l^2 are positive real quantities.

It is then clear that the poles of the kernels (zeros of D) can be interpreted, according to the identity (in the approximation (4 a2)) or to the similarity (in the general case (4 a6)) of the kernels (UV) with the kernels that would give a Yukawa theory postulating the exchange of mesons of mass π_l between the fermions.

But, and this point is essential, to be able to interpret the kernels (UV) as resulting from exchange of mesons, one must be able to define the coupling constant between the π_l meson and the fermion pair u , G_u^l , through the relations

$$(4\ b1) \quad A_{uv}^l = G_u^l G_v^l$$

Now this is possible only if there exist the identities

$$(4\ b2) \quad (A_{ui}^l)^2 = A_{uu}^l A_{ii}^l,$$

between the constants A_{ui}^l

From (4 a5) and (4 a6), one sees that this identity is satisfied if

$$(4\ b3) \quad [(N_{uv}(p^2))^2 - (N_{uu}(p^2))(N_{vv}(p^2))] = 0 \quad \text{when} \quad (p^2 + \pi_i^2) = 0$$

We show in the Appendix that, resulting from the hypothesis of the couplings (3 a) (namely that the coupling between identical pairs is null, and that the coupling constant between different pairs is the same constant g), this identity is satisfied. The reason is that the quantity in brackets in (4 b3) is proportional to the function $D(p^2)$ which, by definition (4 a6) is null at every pole, independently of the complex value of the constants A_{uv}^i and π_i^2 . This result is also independent of the tensor form of the coupling. It is possible that there exists a more general choice of the relations between the Fermi coupling constants, which allows the possibility of interpreting the kernel with meson concepts, but the essential point we want to underline here is that the equality of the Fermi constants for different pairs of fermions (np ev, np $\mu\nu$, ev $\mu\nu$), which is verified experimentally, implies the existence of bosons through which the Fermi interactions occur.

c) *The mesons equations* – The field operator of meson must be some combination of the field operators of fermion, since the functional space is determined by the eigen vectors of the free Hamiltonian which is function of only the fermion fields. Let us write the S matrix for the scattering of two fermions:

$$(4\ c1) \quad S \sim \int \sum_{uv} J_u J_v' (UV(x - x')) dx dx'$$

In the approximation (4 a2) one has

$$(4\ c2) \quad (UV(x - x')) \approx \sum_i G_u^i G_v^i Y_i(x - x'),$$

where the Y_i functions are the Yukawa potentials associated with mesons π_i . Then

$$(4\ c3) \quad S \approx \int \sum_u J_u \sum_i G_u^i \Pi_i(x) dx,$$

with the definition of the meson field operator $\Pi_i(x)$

$$(4\ c4) \quad \Pi_i(x) = \int Y_i(x - x') (\sum_v G_v^i J_v(x')) dx'$$

We have

$$(4\ c5) \quad (\square - \pi_i^2) \Pi_i(x) = \sum_v G_v^i J_v(x)$$

The exact form of the meson field operator, independently of the approximation (4 a2) is given in Sect 5

d) *The free mesons* - In the S matrix formalism we cannot represent the free mesons (present in $\pm$ infinite time), and the mesons appear always as intermediate states. Physically such a situation seems satisfactory since we do not know stable mesons. However, as Prof H UMEZAWA has pointed out to us, if the poles are real and positive quantities ($\pi^2 > 0$), the addition of an arbitrarily small imaginary quantity required to define the Δ_F functions implies the existence of outgoing and ingoing free waves which are not null at plus or minus infinity, which is inconsistent with the S matrix formalism. The way to switch off the meson waves at infinity is to introduce a finite imaginary part in the mass definition giving rise to an exponential decrease of the wave amplitude, associated with an increasing of the free pairs of fermions. Such a situation appears in this theory if, and only if, the masses π_i^2 are larger than the energy necessary to allow a real creation of some fermion pair; indeed the $K_{\alpha\beta}(p^2)$ functions present a finite imaginary part when $-p^2 > (m_\alpha + m_\beta)^2$, and then the D function is a complex one ⁽⁹⁾. If further information on this problem makes this interpretation correct, then to avoid in V^0 theory ⁽⁴⁾ the presence of photons at infinity, it would be necessary to have a finite imaginary part of the mass. According to this theory, this would mean that the neutrino mass is not absolutely null. However, this has no effect on the gauge invariance, which results only from the vectorial form of the coupling and not from the nullity of the neutrino or photon mass.

Another possible way to attack the problem of particle structure is to utilise some non-adiabatic method (TAMM-DANCOFF or BETHE SALPETER). However if it is possible to obtain the masses of the mesons in this way it is not possible to deduce also the meson fermion coupling constants. Furthermore, we have difficulties in this method, in interpreting the part of the mass operator having complex value.

This difficulty seems connected with the fact that one cannot consider the mesons as formed by fermions « bound » together in the usual sense of this word, which implies the notion of persistence of every constituent. In our concept, the meson is successively in every possible state of virtual pairs of fermions, and it appears as some « fusion » state due to the fluctuation of the pairs. The same difference exists in the prediction of the mass of « bound » or « fused » particles. The mass of the particle resulting from the binding of two particles is, for ordinary couplings, not very far from the sum of the masses of the constituents. In fusion, on the contrary, one has no idea of

⁽⁹⁾ The presence of complex poles in fermion self energy contributions, and mathematical difficulties thereby arising have been indicated by G. FEIDMAN: *Proc. Roy. Soc.*, A **223**, 112 (1954).

the resulting mass, and the intuitive idea that the mass is some mean value of the masses of pairs is wrong, as can be shown by the example of the photon for which the mass is independent of the masses of electron, proton and other charged pairs treated on the same footing as the neutrino pair

c) The mesons in beta decay – As a result of the preceding discussions (Sect 2, and 2 b), the beta decay necessarily occurs through the meson fields. Precise experiments would be able to detect the mesonic influences. They can be a modification to the lifetime due to interaction of the virtual mesons with the meson, nucleons, coulomb field of nucleus, or a possible bremsstrahlung of the mesons before the decay. We note that the actually known mesons are not sufficient to explain the experimental result (2 b)

5 – The Realistic Meson Theory and the Yukawa Theories

We treat now precise examples, using the Lagrangian (3 a1)

a) The Yukawa charged nuclear forces – Let us suppose, as a first approximation, that the regularization constants of the lepton loops are infinite

$$(5\ a1) \quad K_{li} = \text{infinite}$$

and that the only other pair is the charged nucleon pair (np). Then the primitive interaction kernels are

$$(5\ a2) \quad (L_i L_j) = 0, \quad (NL_i) = 0, \quad (NN) = -\frac{l\gamma}{1 + l\gamma K_N},$$

where we denote $L_i = (L_i, L_0)$, $i = 1, 2, \dots, l$; and $N = (np)$. In this approximation (5 a1) there exists interaction between the nucleons and not between the leptons nor between leptons and nucleons. We see that D generally has one pole. Thus the theory may be equivalent to the usual Yukawa theory of charged nuclear force.

α) One method of treating the interactions – In a general way, we write the K_N function in the form

$$(5\ a3) \quad K_N(p^2) = a + b(p^2 + \pi^2) + (p^2 + \pi^2) R(p^2), \quad R(-\pi^2) = 0$$

the constants a and b being functions of the C_N^1 and C_N^2 regularization constants of the nucleon loop (the exact form of these functions depend upon the method of regularization, and it is not necessary to make this dependence precise here). In this formula we supposed that the meson mass is known. This is without justification since it is a function of the C_N^i . Let us, however, choose the C_N^i

constants in such a way that we have

$$(5\ a4) \quad a = -1/l\gamma, \quad b = i/(G^2)$$

Then

$$(5\ a5) \quad (NN) = iG^2 \frac{1}{(p^2 + \pi^2)(1 - iG^2 R(p^2))}$$

We recognize here the interaction kernel produced by the exchange of a meson, the propagator of which is modified in the first order in G^2 by the vacuum polarization in a Yukawa theory in which the meson is coupled with the nucleon pair by the constant $2\pi G$. It is the Dyson function Δ'_F in which we have omitted the terms proportional to degrees in G higher than the second. We denote this function as Δ'^1_F .

Then, in the same way as for the photon (Chap. 9 A in V⁰) one can show that, from the equality $(NN) = iG^2 \Delta'^1_F$, (see equation (5 a5)), the value of a S matrix element in R theory, represented by the sum of every graph formed with certain open lines of nucleons and certain closed loops of nucleons having more than two vertices but connected from one vertex to another by « chains » having any number of closed loops with two vertices, of lepton and nucleon type, is equal to the value of a corresponding S matrix element, in the Yukawa theory, formed by the same open lines of nucleons and the same nucleon closed loops having more than two vertices, but connected from vertex to vertex by first order connected meson lines having the value Δ'^1_F . It results from this theorem that, in the (5 a1) approximation the R theory is identical to the Yukawa theory ignoring the lepton fields.

β) A second way of treating the interactions – Although the first method is able to lead to the usual Yukawa theory, we think that this treatment is not physically correct because it implies the *a priori* knowledge of the meson mass and it chooses the constants in order to obtain exactly the usual Yukawa theory. Rather, let us write

$$(5\ a6) \quad (NN) = iA/U(p^2, g, C_N^1, C_N^2)$$

and define the experimental constants (A and π^2) by

$$(5\ a7) \quad \left\{ \begin{array}{l} U(-\pi^2, g, C_N^1, C_N^2) = 0 \rightarrow \pi^2 = \pi^2(g, C_N^1, C_N^2), \\ \left[\frac{dU}{dp^2} \right]_{(p^2 + \pi^2 = 0)} = 1 \rightarrow A = A(g, C_N^1, C_N^2) \end{array} \right.$$

Conversely, the regularization constants C_N^i are

$$(5\ a8) \quad C_N^i = C_N^i(\pi, G, g)$$

To be able to generalise the equivalence theorem between the R theory and some Yukawa theory Y_R , as we have done in the preceding method, let us define the free meson propagator $(1/X(p^2))$ by the condition that the interaction kernel between two nucleons, due to the exchange of this meson, the propagation of which is corrected to the first order in G^2 by the vacuum polarization terms conveniently normalized, be identical with the primitive interaction kernel (NN) of the realistic theory

$$(5\ a9) \quad \left\{ iG^2 X(p^2) \left[1 - iG^2 \frac{1}{R(p^2)} (p^2 + \pi^2) / X(p^2) \right] \right\} = \\ = \left\{ (NN) = iG^2 \frac{1}{U(p^2, g, C_N^1, C_N^2)} \right\}$$

From this there results

$$(5\ a10) \quad \Lambda(p^2) = U(p^2) + iG^2 R(p^2) (p^2 + \pi^2)$$

The function $X(p^2)$ is generally not equal to $(p^2 + \pi^2)$, because

$$(5\ a11) \quad \left[\frac{d^2(X(p^2))}{(dp^2)^2} \right]_{-\pi^2} = \left[\frac{d^2 U}{(dp^2)^2} + 2iG^2 \frac{dR}{dp^2} \right]_{-\pi^2},$$

is different from zero, except for particular values of the C_N^i constants (that of the preceding case) (*). Thus

$$(5\ a12) \quad X(p^2) = (p^2 + \pi^2)(1 + Z(p^2, \pi^2)), \quad Z(-\pi^2, \pi^2) = 0$$

and the Hamiltonian of the free meson is generally not a linear function of π^2 . Then the meson mass renormalization method is not suitable and we are obliged to use the normalization. The charge renormalization can still be made with Dyson formalism. Also there results from the condition (5 a9) that the Yukawa theory Y_R , in which the free meson equation is defined by

$$(5\ a13) \quad X(p^2)II(p) = 0,$$

where $X(p^2)$ is given by (5 a10), is identical to the realistic theory. From this identity, one obtains immediately a simple way to treat every effect deducible

(*) See correction in the Note added at the end of the paper

from the S matrix. The R theory is then renormalizable, or normalizable, according to the values that we give to the first regularization constants C_A^i . If it is renormalizable, it is equivalent (in this approximation (5 a2)) to the usual Y theory, and if not one sees that the difference from the usual Y theory lies in the non linearity of the meson equation, mostly affecting the results in the virtual meson processes.

b) The mesons theory of beta decays - There exist nucleon leptons and leptons leptons interactions when the function K_{L_i} are finite. A new approximation of the R theory is

$$(5\ b1) \quad |\gamma K_{L_i}| \gg 1 \text{ and finite,} \quad \gamma K_{L_i} \approx \text{independent of } p^2 \text{ for } |p^2| < M_v^2$$

We shall show that this approximation gives rise to the π meson theory of PS beta decay.

In this approximation the D function has only one pole with low mass. We interpret this pole as the π meson. Taking for K_y the same function as in (5 a), one sees that

$$(5\ b2) \quad \left(\frac{G_{L_i}}{G_{L_j}} \right)^2 = \frac{N_{L_i} \cdot i_i}{N_{L_j} \cdot L_j} \approx \left(\frac{K_{L_j}}{K_{L_i}} \right)^2,$$

G_{L_i} = coupling constant between π meson and $L_i L_0$ pair

One obtains the experimental G constants for coupling of π meson with $(\mu\nu)$ and $(e\nu)$ pairs in choosing

$$(5\ b3) \quad \gamma K_{L_1} \sim 5 \cdot 10^9, \quad \gamma K_{L_2} \sim 1.5 \cdot 10^8$$

Evidently, in such an approximation, the Fermi interaction resulting from the exchange of the π meson alone is too weak, mainly the Fermi interaction between the lepton pairs. This results from neglecting the p^2 dependance of K_{L_i} functions, which gives rise to new and heavy bosons more strongly coupled with leptons and less with nucleons.

However, although this is a very crude approximation, one sees appearing a law for the cut off which seems to have a profound significance: For the pairs of a same family, the magnitude of the function K_u is larger when the mass of the pair is smaller. One can derive the behaviour of this law from a theorem of Heisenberg ⁽¹⁰⁾ which implies the impossibility of having a local, causal and covariant interaction between null mass particles. Then if we postulate a Fermi coupling between two pairs of fermions, one having a null mass, the interaction between two null mass fermions through the virtual pairs

⁽¹⁰⁾ W. HEISENBERG: in *L. de Broglie, physicien et penseur* (Paris, 1953), p. 283

(which is causal, local and covariant) must be null. This is the case only when the null mass closed loops have an infinite value. Then, the regularization constants must be chosen so that they tend to infinity when the pair mass tends to zero. This theorem applies quite generally and permits us

- (i) to explain the photon structure,
- (ii) to forbid the two neutrino decay of π^0 meson,
- (iii) to show the impossibility of a local interaction between photons

6 – Generalization of the Realistic Theory

a) The renormalization – From the equivalence theorem, between the R theory, and the Y_R -theory we are led to normalize or renormalize according to the general scheme of sect. 3. Physically we preserve the properties of mass and coupling constants of the new particles which result in the summing of a certain class of graphs. Mathematically, this preservation is equivalent to the hypothesis that the poles and residues occurring in this sum of graphs must remain the same when we take into account the other graphs of the S matrix. However, it is not clear in this method why the known beta interaction have almost the same value which appears rather independent of the exchanged energy. This point can be cleared up only by a better knowledge of all the mesons through which this interaction occurs. Another way to renormalize, given by KAMEIUCHI-UMEZAWA (*), starts, on the contrary, preserving the Fermi constant so that at zero energy the Fermi interaction is equal to the Fermi constant. However one does not understand the physical meaning of the terms introduced in the Lagrangian for renormalization. Furthermore, the authors are not concerned with the poles of the kernels, and we do not know if the corresponding masses are also renormalized *de facto*. In R theory we obtain an equality of the Fermi interactions for low energy, by postulating that every $\gamma K_\alpha(p^2)$ function has the same value when p^2 is null.

In our method we dispose of $2L$ arbitrary constants if the g constant is given for every coupled pair (L = number of unlike pairs); then there are (at least) L mesons each coupled with all the L pairs so that we have $L(L+1)$ experimental constants to determine the value of the $2L$ arbitrary constants. Therefore, there must exist relations between the various mesons and one can hope that a better knowledge of the light mesons will help us in research on the heavy mesons. As the R theory gives a relation between the number of fermion pairs and the number of bosons, so the Y theory, when it postulates many bosons of the same tensorial nature, must also postulate a corresponding number of fermion pairs to be renormalizable (*).

(*) These points are more precisely discussed in the Note at the end of the paper

b) The fermion mesons - In the same way as in the preceding sections, one can also build a realistic theory for the complex structure of fermion mesons formed by a definite fusion of certain bosons and certain fermions supposed more elementary and given *a priori*. An example of such a meson appears in the problem of π nucleon scattering, and is due to the pole produced by the iteration of nucleon self energy graphs. However as in sect. 4 *d*, this pole can be interpreted as a new meson (here an hyperon, Y) only in the S matrix formalism which allows to define the coupling form $(\psi_Y O^5 \psi_N) \varphi_\pi G_{Y\Lambda}^\pi$.

The general argument that we have given about the bosons applies also in this case so that the mass of these new fermions must be complex. This can happen either if the mass of the fermion meson is larger than the mass of its constituents or if the constituents are also meson (complex) particles. The problem of renormalization in this case has already been resolved by the Dyson method. Apparently, the only difficult problem lies in the choice of the most important fundamental particles and in a satisfactory treatment of the strong coupling pion nucleon. Thus one can hope that if this rather long calculations can be carried out one will be able to construct by degrees all the mesons by postulating some fundamental Fermi coupling between a very restricted number of «most elementary» particles (electron, neutrino, proton and perhaps neutron) (*).

c) The problem of four fields coupling ⁽¹¹⁾

α) The Fermi couplings - In the main, we have shown that the Fermi coupling is equivalent to a sum of Yukawa Lorentz type coupling between the given fermions and new boson particles (mesons) always unstable. There appears also an additional coupling term between the meson wave functions (except in V^0 case) as the $\lambda \varphi^4$ usual term of Y theory, introduced by MATTHEWS.

Let us now start from the same Lagrangian, and make a Fierz transformation ⁽¹²⁾; then the coupled lepton and nucleon pairs will be reorganized in mixed lepton nucleon pairs coupled together with different coupling constants and different tensor forms, giving a new form for the Lagrangian. The new meson theory that one can deduce from this new Lagrangian must be identical to the preceding and then there must exist relations between the new and old types of mesons. For the coherence of the S matrix formalism, these new mesons must have masses larger than the proton mass in order to prevent the presence of such meson at infinity by the imaginary part of the mass.

(*) See Note added at the end of the paper.

⁽¹¹⁾ We are indebted to H. UMEZAWA for informing us that S. KAMEFUCHI has published a paper on this subject (*Prog. Theor. Phys.*, **11**, 273 (1954)). This review is not arrived in Paris, and it seems from *Phys. Abstract*, 10789 (1954), that this author has not reached the physical conclusion given here.

⁽¹²⁾ FIERZ: *Zeits. f. Phys.*, **104**, 553 (1937).

The hypothesis of such mesons has been introduced in the papers of TANIKAWA and TANAKA ⁽¹³⁾ These mesons decay in nucleon plus lepton, $(n, p, Y, \dots) \rightarrow (e, \nu, \mu, \dots)$, according to the value of their mass. These authors showed also that only scalar and pseudoscalar mesons of this type are sufficient to deduce the Yang-Tiomno interactions. Then, according to our theory, one can show that the S and PS Fermi couplings between the mixed pairs (nucleon and lepton) quite suffice to explain the known charged interactions. The R theory allows us to investigate the existence and the properties of the neutral Fermi couplings. Among the neutral Fermi couplings that experiment indicates, certainly, the Vectorial and PS exist which give rise to the photon and π^0 meson⁽⁴⁾. The existence of θ mesons which may be scalar can be explained in two different ways. The first way is to derive their existence from S Fermi couplings (charged and neutral) between the fermion pairs. In this model the resulting meson will be called « primitive » like the π mesons.

A second way to construct S mesons exists in the PS mesons theory:

$\beta)$ *The Matthews term in PS meson theory* – Indeed, for the $\lambda\varphi^4$ term, one can try to apply the same treatment that we have done for the Fermi coupling. However, a new difficulty arises from the fact that one can iterate the meson closed loops in every vertex in many different ways, because one can group in pairs the four mesons lines of a vertex in three different ways. The same difficulty occurs also in Fermi interaction when instead of the Lagrangian that we have considered one uses the sum of the three different Lagrangians obtained by Fierz transformations. However, due to the apparent success of the iterative method in Fermi interaction, we shall suppose that the combination problem of iteration of graphs also has a solution in the meson-meson interaction. Then, as in the Fermi coupling treatment, there appear two arbitrary constants, the λ constant of coupling, and the cut off constant in meson closed loops with two vertices c_π . Pole and its residue produced by the iteration method are interpreted as due to a new meson having a certain mass and a coupling constant with PS meson pair. This meson is scalar and its structure is different from the S meson constructed with the S Fermi coupling. We call it a « secondary » meson, and to recall its PS Fermi coupling origin, we note it S_{ps} .

As an application, if the λ constant is large, there is no reason to neglect it in the calculation of the nuclear potential. Its action is important as soon as there are more than two mesons in flight; but then the most important quantity is the pion S_{ps} coupling constant. In the same way this constant must be introduced in the pion nucleons scattering, before the usual iteration of nonadiabatic methods.

⁽¹³⁾ TANIKAWA: *Prog. Theor. Phys.* **3** 338 (1948); TANAKA: *Prog. Theor. Phys.* **9**, 169 (1953).

As a conclusion of this rather imprecise study we want to point out the fact that it seems generally possible to substitute for a more than three field coupling an equivalent description, using additional field quantities and new coupling containing no more than three fields

7 - Conclusion

To summarize this paper, we have shown that, with the use of regularization and a certain way of ordering and summing the matrix elements, the Fermi coupling can be completely treated. The main result is the interpretation of the method in terms of new particle concepts. Then we derive from the existence of Fermi coupling between fermion pairs the properties of boson mesons through which the Fermi interactions must occur. The extension of this method and of the physical interpretation can be made, and one reaches the conclusion that every meson, boson or fermion, can be built from the smallest number of fundamental particles (electron, neutrino, proton, and perhaps neutron), requiring no essentially new principle, or new methods. However, the use of regularization, introducing in the theory arbitrary constants to be determined by experiment need to be improved. It is in research on the law of the regularization constants (whose qualitative form is given) and in its further interpretation that one can make progress. Most of the problems underlined in Sect. 6 require further works. We shall treat in our next publications more precise examples of the applications of this theory:

- The PS theory (structure of mesons, bosons and fermions, charge symmetry and independence of nuclear interactions)
- The electromagnetic properties of mesons (charge and photo decays)
- The V, PV, T interactions (allowed meson coupling forms, gauge invariances)

* * *

The author is indebted to Prof. H. UMEZAWA for pointing out his prior results ⁽⁸⁾ on Fermi coupling treatment, and for his valuable discussions on this paper. Thanks are also due Prof. L. DE BROGLIE and A. PROCA for a critical reading of the French manuscript. We should like to thank ELIZABETH SCOTT for helping in translation.

APPENDIX

Demonstration of formula (4 b3)

The kernel equations are (3 c5)

$$a) \quad (UU) = \gamma \sum_{v \neq u} (UV) K_v,$$

$$b) \quad (UV) = \gamma + \gamma \sum_{w \neq v} (UW) K_w = (VU) = \gamma + \gamma \sum_{w \neq u} (VW) K_w,$$

then

$$\begin{aligned} (UU) &= \gamma \left[(UV) K_v + \sum_{u \neq v} (UW) K_u - (UU) K_u \right] \\ &= \gamma (UV) K_v + ((UV) - \gamma) - \gamma (UU) K_u, \end{aligned} \quad \text{from } b)$$

or

$$(UV) (1 + \gamma K_u) = \gamma + (UU) (1 + \gamma K_u)$$

and by permutation of u and v

$$(VU) (1 + \gamma K_u) = \gamma + (VV) (1 + \gamma K_v)$$

The product of these two equations is

$$\begin{aligned} (UV)^2 (1 + \gamma K_u) (1 + \gamma K_v) &= (UU) (VV) (1 + \gamma K_u) (1 + \gamma K_v) + \\ &+ \gamma^2 + \gamma [(UU) (1 + \gamma K_u) + (VV) (1 + \gamma K_v)] \end{aligned}$$

and using $a)$

$$[(UV)^2 - (UU) (VV)] (1 + \gamma K_u) (1 + \gamma K_v) = \gamma^2 + \gamma^2 \left[\sum_w ((UW) + (VW)) K_w \right]$$

writing

$$\begin{aligned} (UV) &= \frac{N_{uv}}{D} \quad \text{for every } u \text{ and } v; \\ (N_{uu}^2 - N_{uu} N_{uu}) &= \gamma^2 \left\{ \frac{D + \sum_w (N_{uw} + N_{vw}) K_w}{(1 + \gamma K_u)(1 + \gamma K_v)} \right\} D \end{aligned} \quad \text{q e d}$$

NOTE ADDED IN PROOF

I meant to have a paper (R) published in *Nuovo Cimento* under the title of « A Realistic Theory of Mesons » Owing to an unexpected delay, two other papers presented at the Pisa conference, vid ,

(E): Les Couplages de Fermi et la Théorie des Bosons

(F): L'exclusion des Fantômes

will be issued before the first paper (R). I therefore endeavoured to write both of them so that they might be thoroughly understood without precise reference to (R).

I am going to deal with the alterations and generalizations that have occurred

1 - On the mesons equations

Corrections and additions to the Sect. 5a and 6a of (R). The paper (E) is a general mathematical demonstration of the Equivalence of the R theory to a certain Yukawa theory, when no real meson is present. This theorem has been very shortly indicated at the end of sect. 5ax of (R). The application of (E) permits to show that the mesons equations are linear functions of the Dalemberertian, and to clarify their meaning and the methods of renormalization.

Aa) One pair; first order normalization

Let us first correct the wrong result of non linearity of mesons equations (R. 5a β) which comes from the fact that we did not utilise the explicite form of the (NN) kernel. The equation (R. 5a9), (which is the condition $(UV_F) = (UV_F^1)$ of (E)), has the form

$$(N1.1) \quad \left\{ (NN) = \frac{\gamma'}{1 - \gamma K_N} \right\} = \left\{ X(p^2) \left(1 - \frac{iG^2}{i^2 \bar{G} \bar{K}_N^R / \bar{X}(p^2)} \right) = iG^2 A_F^1 \right\}$$

We use here the notation γ' for $(-l\gamma)$, and K_N^R for the normalized (or renormalized) expression for the closed loop B_N^2 such that $K_N^R(-\pi^2) = 0$ and $((\partial/\partial p^2) K_N^R(p^2))_{-\pi^2} = 0$. Please remark that the (NN) kernel is identical to the kernel which would result from a direct coupling between nucleon pairs with the coupling constant $g' = -l\gamma$. Expressing K_N^R in term of the regularized expression for closed loop B_N^2 , the function K_N , it follows

$$(N1.2) \quad K_N^R(p^2) = K_N(p^2) - K_N(-\pi^2) - K_N'(-\pi^2) (p^2 + \pi^2) \\ K_N'(-\pi^2) = \left(\frac{\partial}{\partial p^2} K_N(p^2) \right)_{-\pi^2}$$

Then, from (N1.1) and (N1.2) one obtains

$$(N1.3) \quad \{ iG^2 A_F = iG^2 / X(p^2) \} = \frac{\gamma'}{1 - \gamma' (K_N - K_N^R)},$$

$$(N1.4) \quad \frac{X(p^2)}{iG^2} = \frac{1}{\gamma'} + (K_N^R(p^2) - K_N(p^2)) = \\ = \frac{1}{\gamma'} - K_N(-\pi^2) - K_N'(-\pi^2) (p^2 + \pi^2),$$

properties:

- They have 2 vertices from which emerge external pairs; the pairs carried by enclosing line are of u type at one vertex and of v type at the other
- They have q other vertices between which internal pairs are exchanged
- The irreducibility is the following topological property: Let x_u and x_v be the positions of the two « external » vertices in a two dimensional mapping; then, every open line crossing the loop $B_{uv}^{2,q}$ between these two points, and cutting lines of the loop only at the vertex points other than x_u and x_v , does not ever cross only one vertex

Let us then define the functions $\tilde{K}_{uv}$ represented by the sum of all these closed loops, $\tilde{B}_{uv}^2 = \sum_q B_{uv}^{2,q}$; if $u=v$ the smallest q is null and if $u \neq v$ the smallest q is two, or three when Furry's theorem applies

The « General interaction kernels in R theory », are then functions of the $\tilde{K}_{uv}$; in a same way as in (R 3c), one obtains

$$(N17) \quad (\tilde{U}V_R) = \gamma_{uv} + (\tilde{U}W_R)\tilde{K}_{wx}{}'_{xv},$$

we denote here $\gamma_{uv} = ig_{uv}/2\pi$, g_{uv} being the coupling constant of the two pairs u and v (for an example of this calculation in the case of 2 pairs, see: V° Chap VIII, F) Let us now define the « General normalized interaction kernel in the Yukawa theory », $(\tilde{U}V_R)$ by the formula

$$(N18) \quad (UV_R) = (UV_R^0) + (\tilde{U}W_R)\tilde{K}_{wx}^R(\Lambda V_R^0),$$

in which, (UV_R^0) denotes the « bare mesons interaction kernel »

$$(N19) \quad (UV_R^0) = \sum_l iG_u^l G_v^l / \Lambda_l(p^2), \quad \Lambda_l(p^2) = p^2 + \pi_l^2,$$

and $\tilde{K}_{wx}^R$ the general normalized closed loop functions, obtained from $\tilde{K}_{wx}$. Let us explain this calculation. If the general closed loop functions, $\tilde{K}_{uv}$, have the same types of divergences as the first order closed loops K_u used in previous sections, they contain $2L(L+1)/2$ arbitrary constants after a regularization of these expressions in the whole. One can thus determine, in normalization, all the constants, and this is equivalent to impose two conditions to every K_{uv}^R functions. As before, one has

$$(N110) \quad \tilde{K}_{uv} - \tilde{K}_{uv}^R = F_{uv}(p^2 + b_{uv}) \equiv \text{binomial } P_{uv}$$

P_{uv} is the infinite part of $\tilde{K}_{uv}$ before regularization; after it, F_{uv} and b_{uv} are arbitrary constants; P_{uv} is a pure imaginary quantity. The condition

$$(N111) \quad (\tilde{U}V_R) = (\tilde{U}V_R),$$

determines the mesons of the equivalent Yukawa theory. After some manipulations of the equations (N1, 7, 8, 11) one gets

$$(N1\ 12) \quad (U V_r^0) = \gamma_{uv} + (U W_r^0) P_{ws} \gamma_{sv}$$

It results that the field equations of mesons are linear functions of p^2 . The bare meson field operator, Π^l is defined by

$$(N1\ 13) \quad \sum_l G_u^l \Pi^l = \sum_v (U V_r^0) J_v,$$

which gives the well known result when using the explicit form (N1 9)

As a conclusion, let us remark that, if the series of approximations of the unrenormalized closed loop expressions is strongly convergent, then P_{uu} ($u \neq v$) are smaller than P_{uu} (the first order P_{uv} ($u \neq v$) is of the same order of magnitude than second or third order P_{uu}). If one neglects them, one gets the relations between the coupling constants and the masses of the various mesons, underlined in Ba. This will perhaps provide an experimental test of the convergence of series expansions

2 - On the Heisenberg theory of elementary particles ⁽¹⁾

It is useful to discuss some common features and differences between the Heisenberg theory and mine, since both are related to a realistic concept of elementary particles. The Heisenberg way starts from the only neutrino field, non linearly coupled with itself through a Fermi type coupling, and calculates the various excited and compound states of this field (deductive way)

We use, on the contrary, an inductive approach:

— In a first step, mostly developped in (V⁰) and (R), we determine the Fermi couplings between the various fermions, supposed elementary, in such a way that the resulting bosons are the experimentally known particles (photon, pions, ...)

— The second step will consist, when we suppose given the bosons and the «most elementary» fermions, in constructing the fermions mesons, in such a way that they are coupled by the Fermi coupling determined in step 1

⁽¹⁾ I am now referring to a series of papers which were unfortunately unknown to me when I sent paper (R) to be published: W. HEISENBERG: I *Nachr. d. Göttinger Akad. Wiss.*, S. III (1953); II *Zeits. f. Naturf.*, **9a**, 292 (1954); III To be published (in collaboration with F. KORTER and H. MITTER). I am very indebted to Prof. HEISENBERG for giving me a preprint of his last paper (III), of which I discuss mostly here

The solution of this problem, must indicate what are (or is) the most elementary fermion fields, and their coupling, (ν , e , p , n , or ν alone?)

Other differences appear for the treatment of the equations

The Heisenberg equation and the associated method leads to finite results. Every closed loop is finite, because it must always be calculated with the use of a propagator $S(p)$ very strongly regularized as a consequence of the non linear equation. The masses are calculated by using non adiabatic methods, but, as pointed out in R 4d, Heisenberg is led to follow a way similar to that we used in (V^o) and (R), to calculate the coupling constant (fine structure constant). A phenomenon analogous to our « Deutino », (see V^o, Chap. XII, XIII), appears also in his solution of the null mass boson problem (first term of equations 67 and 68 of III).

Now for a synthesis of our two theories ⁽²⁾. Let us postulate that, in the end, my way of approach will show that one can actually construct every particle with the only neutrino field, conveniently coupled with some Fermi type couplings (according to the Heisenberg idea). In this model, one must be able to show (step 2), that, starting from neutrino field and various boson fields given « a priori », one obtains all the fermions which are coupled according to the Fermi coupling calculated in step 1. But, this is equivalent to start from the general propagator of neutrino field, (more precisely from the fermion interaction kernel between two events where two pairs interact), which contains the various excited states of the neutrino.

The Fermi couplings that we have calculated in step 1, and that the step 2 must explain, have the following properties:

In all the cases when it was possible to compare two Fermi constants, they were found to have the *same absolute value*. This property has been indicated in (F) as resulting from the charged Fermi interactions measurements; but this is not the most convincing test, since the experiments give the value of the interactions and not the value of the coupling constants. The previous property results mostly

a) from the equality of magnitude of electric charge of p and e ;

b) from the equality of magnitude of mesonic charge of p and n ⁽³⁾.

In two cases these constants have *opposite signs*.

⁽²⁾ We give here some details about the arguments given in the paper (F).

⁽³⁾ Let us remark here that the charge independence of nuclear forces, $(G_{nn}^0)^2 = (G_{pp}^0)^2 = -(G_{nn}^0)(G_{pp}^0) = \frac{1}{2}(G_{np}^{\pm})^2$, that we have deduced in *Compt. Rend. Ac. Sc.* t. 239 p. 1267 (1954) from the coupling

$$\sum_{i=1}^{\text{ept}} g \left\{ (\bar{\psi}_n \gamma^5 \psi_p)(\bar{\psi}_p \gamma^5 \psi_n) + ((\psi_n \gamma^5 \psi_n) - (\bar{\psi}_p \gamma^5 \psi_p)) (\bar{\psi}_i \gamma^5 \psi_i \pm \bar{\psi}_i \gamma^5 \psi_p) \right\} + \text{c.c.}$$

in using the approximation of (R 5a1), results only from the fact that the Fermi constants of the pairs (nn) and (pp) have the same absolute value, and opposite signs. The com

It results from the last property that step 2 leads to the existence of imaginary Fermi couplings (Ghost couplings), which appear necessarily associated with the convenient couplings. Then, either one must exclude the Heisenberg idea, or there must exist in nature some kind of « Ghost coupling exclusion principle ». I admitted in (F) the second eventuality. The demonstration of this principle has to be formulated in accordance with the general formalism of quantum mechanics. Some consequences of this principle are investigated in (F).

3 - A new frame for mesons theory

We shall briefly review the general restrictions imposed to the relations between the various mesons, as it results from the papers (R) and (F). We use a phenomenological classification of « absolute » (A) and « non absolute » (N A) selection rules (S R).

- a) The Lorentz group S R - (A)
- b) The Isotopic spin S R - (N A)

A deeper reason for occurrence of isotopic spin quantum number, has been explained by the Fermi coupling ⁽³⁾: further calculations can now show the limits of its validity.

- c) The allowed meson couplings (A)

If one admits the postulate that there exist only the five types of Fermi couplings (S, V, T, A, PS) between certain fermions, then the resulting mesons can have only a few types of couplings with the fermion pairs; for example the PV coupling of PS mesons is excluded.

- d) The Causality S R - (N A)

The cut-off law, though quantitatively arbitrary, is however restricted by the causality condition, (R 5b). This restriction is more stringent when the particles resulting from decay have null masses; however, this S R does not apply when the interactions are non local. An interesting example is given by the μ -meson decay ⁽⁴⁾; one cannot have the process $\mu \rightarrow e + B^0 \rightarrow e + (\nu^* + \nu)$ (B^0 means some neutral boson) and, in the same order of the coupling constants, the process $\mu \rightarrow \nu + B^{\pm} \rightarrow \nu + (e + \nu^*)$ is allowed ($B^{\pm}$ are some charged mesons). In high order processes, one can however obtain interactions between the neutrino pair and the $(e\mu)$ pair, by exchanging two charged bosons, which give non-

parison of the Fermi constants of neutral and charged couplings can be done by the calculation of the masses of charged and neutral pions, when the number of charged and neutral lepton pairs are better known.

⁽⁴⁾ We are indebted to Dr L. MICHEL for an interesting discussion on this point.

local interactions. When the masses of the resulting particles are not null, the causality S R is only qualitative. We think that one must impute the preference decay $\pi \rightarrow \mu + \nu$ rather than $\pi \rightarrow e + \nu$ to a law of this type. Everything goes as if there existed some kind of «stepping law», which favours decays giving rise to the smallest lost of mass. Perhaps the Heisenberg theory can explain this fact quantitatively.

e) The Ghost S R - (N A)

We indicated in (F) a tentative application of this law in a simplified model, which shows also an illustration of a complicated interaction which weakens the S R. It is also a problem to determine if these S R can be expressed as a consequence of some phenomenological new quantum number (Gell Mann's Strangeness, for example).

f) The Charge conservation - (A)

We introduce the charge conservation as a separate principle. But a realistic theory should be able to demonstrate this principle, in time.

B JOUVET

1955

N 4 del *Supplemento* al Vol 2, Serie X,
del *Nuovo Cimento* pag 941-968