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Notes on the Design of Distributed Amplifiers.

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(ricevuto il 9 Dicembre 1955)

Summary. — A study of distributed amplifiers is made, taking into account the self capacity of the coils and the effect of using anode and grid lines with different propagation speeds. The latter effect can be useful in avoiding the large increase in gain of the m -derived filter type amplifier at the upper end of the band. A possible procedure for a practical design is suggested, as well as an experimental method for the determination of the characteristics of the two lumped lines.

1. — Introduction.

The theory of distributed amplifiers, which is based on an idea of PERCIVAL ⁽¹⁾, was developed primarily by GINZTON *et al.* ⁽²⁾. Later, other workers published designs ⁽³⁾, but a more detailed description has never been presented.

This paper is limited to pointing out some details of design which may be useful in constructing amplifiers fast enough for general use in nuclear physics. For a more general description, and for points which are not treated here, the original paper by GINZTON *et al.* should be consulted.

A method for the measurement of the characteristics of the lumped lines which form the grid and anode circuits will be described briefly. The measu-

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⁽¹⁾ W. S. PERCIVAL: *British Patent Specification* No. 460, 562.

⁽²⁾ E. L. GINZTON, W. R. HEWLETT, J. H. JASBERG and J. D. NOE: *P.I.R.E.*, **36**, 956 (1948).

⁽³⁾ For a description of the theory and a list of references see I. A. D. LEWIS and F. H. WELLS: *Millimicrosecond Pulse Techniques* (London, 1954).

rement of the properties of entire lines provides more information than can be obtained from measurements of single components.

2. - Analysis of an Amplifier Section.

M-derived low pass filters are commonly used in distributed amplifiers because of their well known advantages over constant K filters ⁽⁴⁾.

Fig. 1.

For a more complete analysis the distributed capacity of the coil should be taken into account, because the filter characteristics can be greatly changed in the high frequency region. The element to be studied in this case is shown in Fig. 1a.

More accurately, the self capacity of the half coil should be taken into account, but this may be neglected unless the frequencies considered approach the half coil resonance.

The filter is equivalent to the element shown in Fig. 1b, where the stray inductance L_t is shown in the shunt arm. In the following, we suppose the parasitic inductance included in the term $((1 - m^2)/4m)L_k$, where m is a design parameter.

C is the self capacity of the coil, C_t the appropriate tube capacity, including parasitic capacities.

The characteristic impedance of the element (1a) is:

$$(1) \quad Z = \sqrt{\frac{L_k}{C_k}} \cdot \sqrt{\frac{1 - (\omega/\omega_c)^2}{1 - (\omega/\omega_r)^2}} = R_0 \sqrt{\frac{1 - (\omega/\omega_c)^2}{1 - (\omega/\omega_r)^2}}.$$

$$(1a) \quad R_0 = \sqrt{\frac{L_k}{C_k}};$$

where

$$(1b) \quad \omega_c = 2\pi f_c = \frac{2}{\sqrt{L_k C_k}}$$

is the cut-off frequency,

$$(1c) \quad \omega_r = 2\pi f_r = \frac{1}{\sqrt{L_1 C}}$$

is the resonance frequency of the whole coil.

⁽⁴⁾ H. E. KALLMANN: *P.I.R.E.*, **34**, 646 (1946).

Fig. 2 shows the behaviour of Z for several values of ω_r/ω_c . For simplicity we put $n = \omega_r/\omega_c$ and we consider only cases such that $n \geq 1$. Z is real for ω varying in the intervals $0 - \omega_c$ and $\omega_r - \infty$. For $\omega_c \leq \omega \leq \omega_r$ Z is imaginary and the frequencies in this interval are attenuated. The characteristic impedance tends to the constant value $\sqrt{L_k/C_k}$ when n approaches 1. Thus it is possible to obtain good termination of a line without employing terminal half sections. For $n = \infty$ we have the case of the M-derived filter; for $n = 1$ the case of the bridged T filter suggested by GINZTON *et al.*

The expression for the phase is:

$$(2) \quad \theta = 2 \operatorname{arctg} \left\{ m \frac{\omega}{\omega_c} [1 - (\omega/\omega_c)^2]^{-\frac{1}{2}} [1 - (\omega/\omega_r)^2]^{-\frac{1}{2}} \right\}.$$

Fig. 3 shows θ for various values of m and n . The usual value $m = 1.27$ is close to the value 1.3 which we have calculated. The best linearity is ob-

Fig. 2.

Fig. 3.

tained, in this case, when $n = \infty$, i.e. when the capacity of the coil is entirely neglected. However, this linearity is not as good as that for $m = 1.5$, $n = 1.2$. It should be noted that the bridging capacity, for the case $m = 1$, cannot be considered as a self capacity.

The delay, as a function of frequency, is given by the slope of the phase curve. For $\omega = 0$, one finds that:

$$(3) \quad \tau_0 = \left(\frac{d\theta}{d\omega} \right)_{\omega=0} = 2 \frac{m}{\omega_c}.$$

This formula is useful, as it allows a determination of the parameter m experimentally, from a measurement of the phase curve.

The gain of an amplifier section, when the grid and anode lines are identical is given by:

$$(4) \quad -g_m \frac{R_0}{2} \sqrt{\frac{1 - (\omega/\omega_r)^2}{1 - (\omega/\omega_c)^2}} \frac{\exp[-j\theta]}{\{1 - (\omega/\omega_r)^2\} \{1 - (\omega/\omega_c)^2\} + (m(\omega/\omega_c))^2},$$

where g_m is the mutual conductance of the tube.

If the lines are different, the gain becomes:

$$(5) \quad -g_m \frac{R_{01}R_{02}}{2Z_1} \frac{\sin(\theta_1/2)}{m_1(\omega/\omega_{c1})} \frac{\sin(\theta_2/2)}{m_2(\omega/\omega_{c2})} \exp\left[-j\frac{\theta_1 + \theta_2}{2}\right],$$

where the suffix 1 refers to the grid line and 2 to the anode line.

Fig. 4.

Fig. 4 shows the expression (5) for different values of m and n , including the extreme values $n = 1$ and $n = \infty$.

In the above the effects of grid damping and mistermiation of the lines have been neglected, as both have been treated by GINZTON *et al.*

3. - Grid and anode lines with different propagation speeds.

It is shown in Fig. 4 that if the case $n = 1$ is excluded the gain of a section goes to infinity at $\omega = \omega_c$. To avoid the increase in the gain produced by the K constant section GINZTON *et al.* proposed the use of the M-derived or bridged T with $n = 1$ filters.

However the only advantage gained by employing the m derived and also bridge T with $n \neq 1$, over the K constant, is a more uniform gain and a more linear phase shift over a wider frequency band. The case $n = 1$, which is the only case where the gain does not rise to a peak, is not easy to realize practically, as it involves the tuning of two circuits to the same frequency. This is difficult, as the adjustment must be carried out for each section of the amplifier, with the additional complication that ω_c (grid) is a sensitive function of the anode current.

It has been found possible to control the shape of the frequency response curve, to a certain extent, by properly adjusting the speeds of anode and grid lines; for example the peaking of the gain at the upper end of the band may be avoided.

The gain of a single section (5) may be written:

$$(6) \quad A(\omega) \exp \left[-j \left(\frac{\theta_1 + \theta_2}{2} - \pi \right) \right],$$

where

$$(7) \quad A(\omega) = \frac{g_m}{2} \frac{R_{01}R_{02}}{Z_1} \frac{\sin(\theta_1/2)}{m_1(\omega/\omega_{c1})} \frac{\sin(\theta_2/2)}{m_2(\omega/\omega_{c2})}.$$

The maximum of $A(\omega)$, which in practice is limited by the losses, occurs at $\omega = \omega_{c1}$.

If the amplifier is composed of s sections the contribution of the q -th section is given by:

$$(8) \quad A(\omega) \exp \left[-j \left\{ \frac{\theta_1 + \theta_2}{2} - \pi + (q-1)\theta_1 + (s-q)\theta_2 \right\} \right]$$

and the total gain by:

$$(9) \quad A(\omega) \frac{\sin(s(\theta_1 - \theta_2)/2)}{\sin((\theta_1 - \theta_2)/2)} \exp \left[-j \left(s \frac{\theta_1 + \theta_2}{2} - \pi \right) \right].$$

It is seen from (9) that if $\omega_{c1} < \omega_{c2}$ the maximum in $A(\omega)$ is annulled if

$$(10) \quad s\{\pi - \theta_2(\omega_{c1})\} = 2\pi.$$

For $\omega_{c1} > \omega_{c2}$ the attenuation introduced by the fact that θ_2 is imaginary at $\omega = \omega_{c1}$ merely diminishes the maximum. This attenuation can be controlled by varying ω_{c2} .

The formula (10) shows that when the number of tubes s is increased the relation remains satisfied for a smaller phase difference.

If the propagation speeds are altered, only just sufficiently to compensate for the peak in the gain, the frequency response in the working band is not

Fig. 5.

very much affected, as may be seen from (9). Fig. 5 shows the experimental response of a compensated amplifier, in which the lines were empirically adjusted to give a good frequency response. The continuous curve in the figure is obtained from (9) using phase shifts as described in section 4. This curve was normalized to the experimental gain at low frequencies.

The rise time corresponds to 2.5 nps.

It is possible to examine the transient response in a little more detail. For simplicity we examine the case

$$A(\omega) = 1; \quad \theta_1 = \frac{\pi}{\omega_c} \omega = \omega \tau_1; \quad \theta_2 = \frac{\pi}{\omega_c(s/(s-2))} \omega = \omega \tau_2.$$

The response of the amplifier to a unit step function is given by an integral of the form.

$$(11) \quad V(t) = \frac{1}{2\pi} \int \frac{\exp[+j\{\omega t - s((\theta_1 + \theta_2)/2) + \pi\}]}{j\omega} \frac{\sin(s(\theta_1 - \theta_2)/2)}{\sin((\theta_1 - \theta_2)/2)} d\omega,$$

which corresponds to a signal of the form shown in Fig. 6, delayed in time by $s(\tau_1 + \tau_2)/2$. The expression $(s-1)|\tau_1 - \tau_2|$ corresponds to the rise time.

In a practical case, in which $A(\omega)$, θ_1 and θ_2 are functions of ω corresponding to actual line characteristics, it is inconvenient to calculate the integral (11). It is clear that the response will still be composed of s steps, each having now a finite rise time. The sharp

corners shown in Fig. 6 will be smoothed out if $|\tau_1 - \tau_2|$ is much smaller than the rise time of a section. This result is obvious, as an actual amplifier will

Fig. 6.

not pass the high frequency components belonging to the sharp corners. Of course, the total effective rise time of an entire amplifier is not much larger than $(s-1)|\tau_1 - \tau_2|$.

In practical cases the transient response may be calculated, instead, by approximate methods, once the response and phase shift curves have been determined experimentally.

It should be noticed that as the phase function of a complete amplifier depends on $(\theta_1 + \theta_2)/2$ it is possible to adjust the characteristics of the grid and anode lines to compensate for non linearities introduced by the lines separately.

Obviously a peak in a response curve is associated with the possibility of oscillations. Instability has been observed at the upper extreme of the frequency band of a 12 tube amplifier, but a reduction of gain at these high frequencies, by the method suggested, has resulted in complete stability.

4. - The determination of the characteristics of lines.

For the determination of the phase characteristics of lines the arrangement shown in Fig. 7, with the line short circuited, has been used. Here, (1) and (2) are high frequency voltmeters, and the procedure was to keep the output of the signal generator, as measured by (1), constant and note the reading of (2) at different frequencies. The resonance frequencies were detected as minima in the reading of (2). If $\theta(\omega)$ is the phase shift per section, and the line is composed of s sections, the resonance condition is:

$$(12) \quad s\theta = k\pi \quad (1 \leq k \leq s).$$

Fig. 7.

Fig. 8.

There are thus s resonances, excluding $\omega = 0$. From (12) we have that a frequency ω_r corresponds to a phase shift $\pi r/s$. When $r = s$, the corresponding frequency is the cut-off frequency.

In Fig. 8 a series of measurements made with the arrangement of Fig. 7

is shown. The two curves refer to the grid line of a 12 section amplifier using 6AK5 tubes. The curve (a) was taken with cold tubes and (b) with the tubes

Fig. 9.

under operating conditions. The change in capacity and the increase in the losses at high frequencies can be seen in b).

The curve (2) in Fig. 9 has been calculated to fit the resonances of Fig. 8-b) using formula (2). Of the three necessary parameters, ω_r and ω_c were measured directly, and m was determined from the initial slope of the experimental phase shift curve according to formula (3). The curves (3) and (1) of Fig. 9

refer to anode line resonances before and after loading with condensers to reduce the propagation speed. As an example, we present the parameters of the three curves in the following table:

	m	f_c	f_r
1	1.44	292 MHz	425 MHz
2	1.46	342	445
3	1.43	368	425

The experimental points are shown in Fig. 9 for comparison with the calculated curves. It was also found by direct measurement that the lines transmitted again above the frequency ω_r in agreement with formula (1).

From curves (1) and (2) the response curve shown in Fig. 5 has been calculated.

The same arrangement (Fig. 7) can be used to measure the characteristic impedance, if the phase curve is known. The input impedance of a short-circuited line and condenser in series is:

$$(13) \quad Z_{\text{tot}} = \frac{1}{j\omega c} + jZ \operatorname{tg} \Xi(\omega),$$

where

$$\Xi(\omega) = s \cdot \theta(\omega)$$

is the phase shift of the complete line. At resonance, since Z_{tot} is zero, it is sufficient to know C to determine Z at that frequency.

5. – The practical design.

The first step in a practical design is to select the parameters m , ω_c , ω_r for the two lines. Figs. 2, 3 and 4 will help in the choice of m and n , from the points of view of good frequency response and phase linearity.

However, the final values of m and n , as well as the values of ω_{c1} and ω_{c2} , will depend on a compromise between a number of points, some of which are listed briefly in the following.

The choice of the characteristic impedance Z_2 of the anode line, determines not only the gain, but also ω_{c2} . This is due to the fact that, for a given tube and given m , ω_c and Z are not independent, as they are related, in the working band, by the formula:

$$(14) \quad f_c Z \cong f_c R_0 = \frac{m}{\pi c_t},$$

where R_0 is the low frequency impedance (1a) defined in section 2. ω_{c1} is of the same order of magnitude as ω_{c2} , even if the lines have different speeds, so that once ω_{c2} is decided, the grid line impedance is no longer completely arbitrary. In general, it will be smaller than the anode line impedance, as the input capacity of a tube is larger than the output capacity. The reduction of gain in cascading stages when $Z_1 < Z_2$, and the impedance of connecting cables, both place a lower limit on the practical size of the grid impedance.

In choosing the number of tubes and number of stages the rule that each stage should have a gain of about $e = 2.7$ is not of much practical importance, since other things should be taken into account. For example, the number of tubes in an output stage depends on the maximum signal amplitude desired; the number of stages is even or odd, according to the polarity of the pulses.

A method of changing the speed of the lines, without altering the coils, is to place small fixed condensers in parallel with the tube capacity. When the optimum value of the padding condensers has been found experimentally, it is possible to design new coils, so that the same phase curve is obtained, but with higher characteristic impedance.

It should be noted that the grid line phase curve is quite sensitive to changes in the anode current of the tube.

Having selected the three parameters m , ω_c and ω_r for both lines, the sizes of the coils must be calculated.

From (1a), (1b) and Fig. 1 of section 2 the total inductance is:

$$(15) \quad L_1 = mL_k = \frac{R_0 m}{\pi f_c}.$$

The inductance of a half coil is:

$$(16) \quad L = \left(\frac{1+m^2}{4m} \right) \cdot L_k - L_t,$$

and the numerical value of the mutual inductance

$$(17) \quad M = \left(\frac{m^2-1}{4m} \right) \cdot L_k + L_t,$$

where L_t is the parasitic inductance. Thus the total inductance $L_1 = 2(L+M)$ does not depend on L_t . However the presence of L_t should be taken into account in deciding the correct amount of coupling between the two half coils.

It can be shown that when the Nagaoka formula ⁽⁵⁾ for coils, is valid, the quantity

$$(18) \quad \frac{L+M}{L} = \frac{mL_k/2}{((1+m^2)/4m) \cdot L_k - L_t} = F\left(\frac{D}{l}\right),$$

where

$$\frac{D}{l} = \frac{\text{Diameter of the coil}}{\text{Length of the coil}}.$$

This function can be computed from a table of the numerical factors entering into the Nagaoka formula.

A plot of the relation (18) has been presented by KELLEY ⁽⁶⁾.

When D/l is known, for the required total inductance L_1 , one obtains from the Nagaoka formula: $D \cdot (\text{total number of turns})^2$. The number of turns and D can be adjusted within this product, to obtain a coil of reasonable dimensions.

This procedure, for the determination of the size of the coils, is only a rough approximation, if the number of turns is very small.

The next step should be to measure the total inductance of the coil, for example with a Q meter, and alter the size to obtain the correct inductance. Following this, to adjust the phase curve, and therefore m , one should make a model amplifier with a reasonable number of tubes and use the method described in section 4.

By proceeding in this way it is not necessary to know in advance, with great accuracy the stray capacity and inductance of the tube, so that rough values of C_t and L_t can be used for the provisional design. From the experi-

⁽⁵⁾ F. E. TERMAN: *Radio Engineers' Handbook* (New York, 1943), pp. 53 to 55.

⁽⁶⁾ G. G. KELLY: *Rev. Scient. Instr.*, **21**, 75 (1950).

mentally determined ω_c and m , together with the experimental value of R_0 (or the value of the total inductance L_1 determined by Q -meter) it is possible to get C_t . On the contrary, no information is obtained by this method for L_t . We have obtained for a 6AK5 tube C_t grid = 7.1 pF and C_t anode = 4.3 pF. These values of course refer to our own arrangement of tubes and coils, but can be taken as a guide for other designs. The grid capacity corresponds to an anode current of about 10 mA.

The distributed capacity of the coil depends on the diameter of the wire, the dielectric constant of the former and the depth of the groove in which the wire lies, if the coil is not close wound.

A paper by PALERMO⁽⁷⁾ gives very rough information on the distributed capacity of a coil. However a direct measurement of the resonance frequency is much more reliable as it gives ω_r . It is thus possible to adjust the size of the wire, or the other two variables, to get the desired ω_r .

In conclusion of this section we would like to remark that direct measurement of the characteristics of the lumped lines is useful as it gives the information which is required for understanding the behaviour of an amplifier, including the variation of the components with frequency, the effect of the half coil distributed capacity, and other possible effects, which cannot be calculated conveniently.

* * *

The progress of our work was aided by using a set of coils suggested by Professor W. L. KRAUSHAAR of M.I.T. We would like to thank Dr. B. COLLINGE for helpful comments. One of us (G.F.) would like to thank Professor H. W. B. SKINNER for his kind hospitality.

APPENDIX

On completion of the present paper, an article by G. D. SARMA, in *Proc. Inst. Elec. Eng.*, part B, September, 1955, was brought to our notice by Dr. B. COLLINGE, of this laboratory.

This article is concerned with the effects of «staggering» the anode and grid lines, treated here in Section 3.

In the present paper, «staggering» was suggested mainly to avoid the peak in the gain at the upper end of the band, without affecting very much the working band, as can be seen on consideration of the explicit sine terms in (9).

(7) A. J. PALERMO: *P.I.R.E.*, **22**, 897 (1934).

In our discussion suitable adjustment of the parameters m , ω_c , ω_r , for each line, leads to a flat response curve and a more linear phase shift characteristic. The Figs. 2, 3 and 4, may be used as a guide in judging the correct direction for variation of these parameters.

In SARMA's paper all the cases treated have an anode line cut-off at a frequency lower than grid line cut-off. If this is so, the vectors sum on the anode line in phase at $\omega = \omega_{c1}$, so that the peak in gain which occurs just at $\omega = \omega_{c1}$, is not removed, although, of course, the attenuation in the anode line decreases it.

The experimental response curve, in Fig. 16 of SARMA's paper, should have a peak just above the working band, but unfortunately no experimental data is presented for this region.

By increasing the grid line m , the peak, finite in height because of the losses, moves towards higher frequencies and decreases in size. In Sarma's amplifier the grid losses should be small, as the frequency does not go much above 1 MHz; it is thus the Q 's of the line sections which play the main role.

The improvement in damping of the after pulse oscillations, shown in Fig. 15 of SARMA's paper, may be interpreted as being due more to a decrease in the peak, than to an improvement in the shape of the frequency response curve.

RIASSUNTO

Viene presentata un'analisi del comportamento di un amplificatore distribuito, tenendo conto della capacità propria delle bobine, e l'effetto della differente velocità di propagazione dei segnali nelle due linee, di placca e di griglia. Quest'ultimo effetto può essere utilizzato per evitare il forte aumento di guadagno che si verifica all'estremo superiore della banda di un amplificatore con sezioni del tipo M-derivato. Si suggerisce un possibile metodo per il progetto pratico di un amplificatore ed una disposizione sperimentale per la determinazione delle caratteristiche delle due linee di trasmissione.

<p>G. FIDECARO and A. M. WETHERELL 1956, Febbraio <i>Il Nuovo Cimento</i> Serie X, Vol. 3, pag. 359-370</p>
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