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To Jens Vigen <Jens.Vigen@cern.ch>

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Carmen Vasini

A. KIND and L JESS

1956, Settembre

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On the Real Part of the Complex Potential Well of the Nucleus.

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Summary — Starting from a well defined assumption about nuclear forces, the real part of the complex potential which enters into the description of the neutron nucleus interaction is determined for energies of the incoming neutron ranging from 0 to 400 MeV. The assumption is made that nuclear matter remains unperturbed as the neutron passes through it. The exchange character of nuclear forces is essential for the results obtained.

In a note by KIND and VILLI ⁽¹⁾ (later indicated as I) some results have been given of a calculation, the purpose of which was to determine the real part of the complex potential of the nucleus starting from well defined assumptions about the interaction between free nucleons.

The purpose of the present note is to give the results of a more complete calculation on the same subject, and to discuss these results.

1 - Experimental data.

The real part V of the complex potential, which enters into the optical model description of the neutron-nucleus interaction, can be determined experi-

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(1) A. KIND and C. VILLI: *Nuovo Cimento*, 1, 749 (1955)

mentally from the analysis of the total neutron cross-sections as functions of the energy E of the incoming neutron. For the limiting case $E = 0$, the value of V has been determined by ADAIR ⁽²⁾ and by FESHBACH, PORTER, and WEISSKOPF ⁽³⁾ to be nearly 42 MeV.

For $50 < E < 100$ MeV the potential V has been analyzed by T. B. TAYLOR ⁽⁴⁾

2 - Interaction between Nucleons

Very little can be said to-day about the real features of nuclear forces and especially about the relation between the forces acting between two free nucleons and those acting in the interior of the nucleus.

In any case it must, however, be admitted that n -body forces may play a role in the nucleus. They might, at least partly, be responsible for the saturation of the binding energy.

About the n -body forces it can, for instance, be said that for different n 's they will be affected by the exclusion principle in a different way. One can therefore foresee that for the interaction of one nucleon with the nucleus they will depend in different ways on the velocity of the incident particle.

Our knowledge of this and other questions about the nuclear forces are, however, to-day so poor that the best thing we can do is to start with the simplest possible assumption. On the basis of the results obtained we might then try to draw some conclusions on more detailed questions. To do this we assume that nuclear forces in the nucleus are the same as those acting between two free nucleons, and that they are given by the interaction Hamiltonian

$$(1) \quad H_{\text{int}}(1, 2) = - \frac{\exp[-r/\beta]}{r/\beta} W(1 + \alpha P^x),$$

where r is the distance between the two nucleons, and P^x is the space coordinates exchange operator. We choose W and β so as to fit correctly the experimental low energy behaviour of the two nucleons systems:

$$(2) \quad W(1 + \alpha) = 57.9 \text{ MeV}$$

$$(3) \quad \beta = 1.25 \cdot 10^{-13} \text{ cm} \quad (5)$$

⁽²⁾ R. K. ADAIR: *Phys. Rev.*, **94**, 737 (1954)

⁽³⁾ H. FESHBACH, C. E. PORTER and V. F. WEISSKOPF: *Phys. Rev.*, **96**, 448 (1954)

⁽⁴⁾ T. B. TAYLOR: *Phys. Rev.*, **92**, 831 (1953)

⁽⁵⁾ W. WILD and K. WILDERMUTH: *Zeits. f. Naturfor.*, **9a**, 799 (1954)

Thereby the ratio α between the Majorana and Wigner parts of the potential is not determined

We shall see that this ratio plays an essential role in the description of the neutron-nucleus interaction

3 - The Weak Incoherent Interaction Approximation

The assumption that the interaction Hamiltonian H_{int} can be written in the form (1) makes it possible to calculate the potential $V(E)$ using the weak incoherent interaction approximation as described in a note of one of the authors ⁽⁶⁾ We shall stop here at the zero order approximation

For low energies the validity of such an approach is essentially supported by the effect of the exclusion principle which strongly inhibits transitions in the nucleus ⁽⁷⁾

If the energy E of the incoming nucleon is increased, the exclusion principle becomes rapidly less effective and correlation effects become more important The convergence of the method becomes worse

However, if the energy E is increased to such a value that the interaction between two nucleons can be correctly treated in the Born approximation, it is again correct to determine $V(E)$ in terms of the weak incoherent interaction approximation, i.e. by assuming that nuclear matter remains nearly unchanged when the nucleon passes through it

4 - Calculation of $V(E)$

The main lines of the calculation of $V(E)$ have been given in I In that paper, however, the assumption has been made of no neutron excess in the nucleus Furthermore it must be pointed out that, because of a numerical error, the quantity $V(k')$ calculated in I is too big This error has been corrected in the present calculation Here, as in I, we assume that the nuclear radius is

$$(4) \quad R = r_0 A^{\frac{1}{3}} \quad \text{with} \quad r_0 = 1.43 \cdot 10^{-13} \text{ cm},$$

and we take into account a neutron excess by putting

$$(5) \quad \frac{2Z}{A} = 0.82,$$

where Z and A are the numbers of protons and nucleons, respectively

⁽⁶⁾ A. KIND: *Nuovo Cimento*, **2**, 443 (1955)

⁽⁷⁾ V. F. WEISSKOPF: *Helv. Phys. Acta*, **23**, 187 (1950)

Nuclear matter is described by means of a product of plane wave functions q_j , with wave vector k_j , antisymmetrized with the plane wave function q' describing the incident neutron, whose wave vector inside the nucleus is k' .

In the approximation which we have chosen, the depth V of the potential well felt by the neutron inside the nucleus is

$$(6) \quad V(k') = \sum_j \int q_j^*(q_1) q'^*(q_2) H_{\text{int}}(q_1, q_2) [q_j(q_1) q'(q_2) - q_j(q_2) q'(q_1)] dq_1 dq_2,$$

where the summation is extended over all states q_j . The q_i represent the space, spin, and isotopic spin coordinates of the nucleons.

By dividing the integral on the right hand side of (6) in its direct and exchange parts we get

$$(7) \quad V(k') = I_d + I_{\text{ex}}$$

with

$$(8) \quad I_d = -W \left[\frac{2Z}{A} (\alpha - 4) + \frac{2(A - 2Z)}{A} (\alpha - 2) \right] \frac{2}{3} \frac{k^3}{\pi^0},$$

and

$$(9) \quad I_{\text{ex}} = W \left[\frac{2Z}{A} \frac{16\alpha - 4}{4} + \frac{2(A - 2Z)}{A} (2\alpha - 1) \right] \frac{1}{\pi} G(k_0, k'),$$

where k_0 is the radius of the Fermi sphere in units β^{-1} , and $G(k_0, k')$ is the function given by (6) in I.

Finally from (7) and

$$(10) \quad E[(k'/k)^2 - 1] = V(k')$$

(see formulae (2) and (7) of I), we get V as a function of the kinetic energy of the neutron outside the nucleus.

5 - Results and Discussion

In Fig. 1 the empirical data are indicated by the cross (+) and the dotted curve. The theoretical results are reproduced by the three full curves obtained by putting $\alpha = 1, 2.3$, and 4. The value $\alpha = 2.3$ has been chosen to fit the empirical data at high energy ($E = 400$ MeV). At this value of E and with the H_{int} given by (1), one has $W(1+\alpha)\beta/\hbar v = 0.4$ (v is the velocity of the incident nucleon). This means that in this case it begins to be reasonable to treat the two nucleons interaction in the Born approximation and, therefore, also our expression (6) for $V(k')$ is significant. The value $\alpha = 2.3$ is, however, not in accord with the high energy neutron-proton scattering, which requires

a value of α near to 1. On the other hand, the curve for $\alpha = 1$ lies much too high with respect to the empirical data.

Fig. 1

So, although there is a qualitative agreement between theoretical and empirical data (and we emphasize that this essentially depends on the exchange term (9)), we must conclude that there are some essential discrepancies between the theoretical and the empirical results which cannot depend only on the approximation (6) which we used, but must essentially depend on the Hamiltonian (1) which is not the correct one.

We know that, to fit the two-body experimental data, it is necessary to take into account a tensor force and to introduce also a repulsive core in the interaction between two nucleons. Both of these corrections are such as to modify our theoretical results toward a better accord with the empirical data.

In fact, with the introduction of a tensor force, it is possible to fit correctly the two-body experimental data by reducing the strength W of the central force ⁽⁸⁾. In our expression (6), which would continue to be valid for the

⁽⁸⁾ For example, with the potential mentioned in the book of BLATT and WEISSKOPF: *Theoretical Nuclear Physics*, at p. 103, the strength of the central force is reduced to about $\frac{1}{3}$ of the value we have used here.

limiting case of high energies, the tensor force would give a zero contribution, and the curve for $\alpha = 1$ would get deeper.

The presence of a repulsive core would diminish the interaction between nucleons essentially in case of higher energies and would therefore deform the curve for $\alpha = 1$ so as to fit the empirical data better ⁽⁹⁾.

The correlation effects are not contained in the approximation (6). These would modify the curve $V(E)$ essentially in the middle energy region. However, it has to be emphasized that, as shown in I, $V(E)$ is not equal to the total work done by nuclear forces during the entrance of the neutron into the nucleus, but only equal to the part of this work which effectively serves to increase the mean kinetic energy of the incoming neutron. The rest of this work is elastically absorbed by the virtual excitation of nuclear matter. Therefore, although correlations imply an augmentation of the interaction energy between neutron and nucleus, it is, nevertheless, not possible to foresee their effect on $V(E)$ without the performance of an exact analysis of the problem.

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⁽⁹⁾ See also R. JASNOW: *Phys. Rev.*, **82**, 261 (1951).

RIASSUNTO

Partendo da una data interazione nucleone nucleone si determina la parte reale del potenziale complesso che descrive l'interazione neutrone nucleo, per energie comprese fra 0 e 400 MeV. Il calcolo è effettuato nell'ipotesi che la materia nucleare si possa considerare imperturbata durante l'interazione col neutrone incidente. Il carattere di scambio delle forze nucleari è essenziale per i risultati che si ottengono.