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Carmen Vasini

B. JOUVET

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Fermi Coupling and Mass and Charge Spectra of Bosons (*).

B. JOUVET (+)

CERN - Copenhagen

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Summary. — One demonstrates an equivalence theorem between the theory resulting from a scalar Fermi self coupling, and a Yukawa theory postulating a boson coupled with a fermion pair; mass and coupling of this meson have to satisfy equations which are conditions on the meson renormalization constants. It is then shown that the Fermi theory is renormalizable, in the whole. Analysis of the infinities in perturbation theory is given, in view of their subtraction. Effective methods for the treatment of Fermi theory are given; their practical use is exemplified. One discusses finally the magnitude of the boson renormalization constant.

It has been shown in previous works ^(1,2) that a quantum-theoretical treatment of a theory starting with Fermi-type couplings between various elementary spinor fields leads to an equivalent Yukawa-type theory, the bosons being there coupled to the Fermion pairs.

(*) This paper was presented at the International Congress on Fundamental Constants, Turin, September 1956. For this reason many points which are not essential for the understanding of the theory were only briefly mentioned. The author has the intention to develop more fully some of these points later on. Also this paper treats two connected subjects which might have been presented separately, namely the Fermi coupling treatment and the magnitude of the renormalization constants.

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⁽¹⁾ B. JOUVET: *Compt. Rend. Ac. Sci.*, **237**, 1642 (1953); *Journ. de Math.*, **33**, 201 (1954) or *Thèse de doctorat*, Paris (1954).

⁽²⁾ B. JOUVET: *Suppl. Nuovo Cimento*, **2**, 941 (1955); *Reports Pisa Conf., Suppl. Nuovo Cimento* (in print) (1956).

However, neither the masses nor the coupling constants of the bosons thus obtained were determined, as they depended on the cut-off used to eliminate the divergences which arose. Further, one was led to the apparent paradox that the Fermi theory, which is known to be non-renormalizable ⁽³⁾, was identical to a Yukawa one, which is renormalizable.

Here, we shall first determine under what conditions these two types of theory can be made equivalent. This leads us to show that

- 1) the masses and the mesic charges of the bosons are effectively determined by the Fermi theory, and
- 2) that the Fermi theory is renormalizable, though of course not in a perturbation expansion.

1. - Equivalence Theorem.

Let us consider the simplest example of a scalar Fermi self-coupling ⁽⁴⁾. Let L_F be the Fermi lagrangian, in terms of unrenormalized quantities:

$$(1) \quad L_F = L_{\text{fermion}}(\psi_0, m_0) + (g_0/2)(\bar{\psi}_0\psi_0)(\bar{\psi}_0\psi_0)$$

and let S_F be the corresponding S -matrix.

Let, on the other hand,

$$(2) \quad L_Y = L_{\text{fermion}}(\psi_0, m_0) + L_{\text{boson}}(A_0, \mu_0) + G_0 A_0 \bar{\psi}_0 \psi_0$$

a Yukawa Lagrangian, also in unrenormalized form; we assume m_0 and ψ_0 to be the same as in the Fermi theory, and we leave arbitrary the constants G_0 and μ_0 . We assume that the Matthews term is nul:

$$(3) \quad \lambda_0 G_0^4 A_0^4 = 0, \quad \text{i.e.} \quad \lambda_0 = 0.$$

Let $(S_Y)_{0m}$ be the S -matrix elements of the Yukawa theory for no real mesons.

By comparing these elements of $(S_Y)_{0m}$ proportional to G_0^{2N} with those of S_F of order g_0^N , we see that we would have

$$(4) \quad S_F = (S_Y)_{0m}$$

if

$$(5) \quad g_0 = \frac{G_0^2}{p^2 + \mu_0^2}.$$

⁽³⁾ A. SALAM: *Phys. Rev.*, **84**, 427 (1951).

⁽⁴⁾ B. JOUVET: *Nuovo Cimento*, **3**, 1133 (1956).

This relation can only hold if

$$(6) \quad G_0^2 = \text{infinite}, \quad \mu_0^2 = \text{infinite}, \quad G_0^2/\mu_0^2 = g_0.$$

Thus the contact Fermi interaction is equivalent to that resulting from the exchange of mesons with infinite mass and coupling. It must be remembered, however, that these parameters are the unrenormalized, and hence unobservable, ones.

The proof that (6) implies equality (4) requires some caution since g_0 enters in divergent expressions. Let us make a cut-off $\Lambda^2 > |p^2|$ on the fermion momenta, modifying the free propagator

$$(7) \quad S_c(p, m_0) = (i\gamma + m_0)^{-1} \quad \text{to} \quad S_A(p, m_0) = S_c(p, m_0)F(|p^2|/\Lambda^2), \quad F(0) = 1,$$

F is a sufficiently rapid cut-off function, say $\exp[-|p^2|/\Lambda^2]$. We can see that if

$$(8) \quad G_0^2(\Lambda) = \infty, \quad \mu_0^2(\Lambda) = \infty \quad \text{and} \quad G_0^2(\Lambda)/\mu_0^2(\Lambda) = g_0,$$

then

$$(9) \quad (S_Y(\Lambda))_{om} = S_F(\Lambda).$$

Let us consider this equality in terms of renormalized quantities. The renormalized mass μ and the mesic charge G are connected, respectively, with the pole and the residue which appear in the S -matrix element for two Fermions scattering in Yukawa theory. By (9) S_F has the same residue and pole, hence μ and G are functions of m_0 , g_0 and Λ . On the other hand, the renormalized constants are related to the unrenormalized ones in Yukawa theory by

$$(10) \quad G_0^2 = G^2 \cdot Z_1^2/Z_2^2 \cdot Z_3; \quad \mu_0^2 = \mu^2 + (\delta\mu^2/Z_3); \quad m_0 = m + (\delta m/Z_2); \quad \lambda_0 = \lambda Z_4.$$

(The prescription of Kroll-Ruderman has been followed for the charge renormalization).

The renormalization constants, Z_i , $\delta\mu^2$ and δm , can be expressed in terms of the renormalized quantities and the cut-off alone (G^2 , μ^2 , m , Λ). The conditions (8) are thus fulfilled if the renormalized constants μ^2 and G^2 (which are finally functions of Λ , m , and g_0), satisfy the equations

$$(11) \quad Z_3(\Lambda, G^2(\Lambda), \mu^2(\Lambda), m) = 0,$$

$$(12) \quad \delta\mu^2(\Lambda, G^2(\Lambda), \mu^2(\Lambda), m) = G^2(\Lambda)(Z_1/Z_2)^2/g_0.$$

The observable constants are the limits, if any, of the renormalized func-

tions $\mu^2(A)$ and $G^2(A)$ as $A \rightarrow \infty$. It may be recalled that the equation $Z_3 = 0$ means, in perturbation theory, that the series of counter-terms for the Lagrangian of the renormalized boson field, $L_{\text{Boson}}(A, \mu^2)$, which consists of finite terms when there is a cut-off, compensates the Lagrangian away. Then, while it may be useful to employ the infinite unrenormalized constant G_0 , it has no physical sense.

The condition $\lambda_0 = 0$, which we have imposed and which is required for the proof of (9) (because otherwise terms of the type M , containing four-vertices closed loops, would be different in the two theories), can be interpreted in the following manner. We denote by $G^4\lambda$ the value, as determinable experimentally, of the S -matrix scattering matrix element for two zero momentum and mass mesons, and let $\lambda G^4 A^4$ be the Matthews term in the renormalized Yukawa lagrangian. In calculating corrections to the meson-meson scattering, we must add to this lagrangian counter-terms of the form $(\delta(\lambda G^4)A^4)$ in each order in G^2 and λ . These terms are determined as functions of G^2 , m , μ^2 and λ , and of the cut-off of fermion and boson fields by the renormalization prescription that, after the subtraction, each subgraph and each divergent graph M vanish when the four-momenta at the vertices are null. Then the total scattering matrix element is λG^4 , under the same conditions. The sum of all the terms in A^4 in the total lagrangian can be written as $\lambda G^4 Z_4 (Z_1/Z_2)^4 \cdot A^4$, which can be expressed as a function of unrenormalized quantities such as $\lambda_0 G_0^4 A_0^4$. The condition $\lambda_0 = 0$ then means that $\lambda Z_4 = 0$, that is that the constant λ is determined by the equation

$$(13) \quad Z_4(\lambda, m, \mu^2, G^2) = 0.$$

This implies that the sum of counter-terms $\sum (\delta(\lambda G^4))A^4$ equals $(-\lambda G^4)A^4$. One can also show that this is equivalent to the omission in L_Y of the terms $\lambda G^4 A^4$ together with keeping in S_Y all the M -parts, divergent or not, whose series must sum to a finite value λG^4 .

If one assumes S_F to be unitary, the equality (4) implies that

$$(14) \quad S_Y |\text{meson}\rangle = 0.$$

This occurs if the meson is unstable, having a mass $\mu > 2m$ ⁽²⁾. Certain modifications of the Yukawa theory formalism are then required ⁽⁵⁾. The constants Z_3 and $\delta\mu^2$ must be defined, in perturbation theory, as the real

⁽⁵⁾ I am very indebted to Dr. V. GLASER for many interesting discussions on the unstable particles problem that he treated in detail in the case of Lee Model (Colloquium, October 31st, 1955, Copenhagen).

part of the quantities ordinarily calculated ⁽⁶⁾; the Bose field can then be expressed in terms of the fermion ones. There being no incoming or outgoing boson free waves at infinities, the matrix $S_Y = (S_Y)_{0m}$ can be obtained directly from a lagrangian no longer containing L_{boson} , and having instead of the coupling $G_0 A_0 J_0$, the « action-at-a-distance » ⁽⁷⁾ coupling

$$(15) \quad (G_0^2/2)J_0(X) \int \bar{D}_{\mu_0}(X - X') \cdot J_0(X') d^4 X', \quad J_0 = \bar{\psi}_0 \psi_0,$$

$\bar{D}_{\mu_0}(X - X')$ having a Fourier transform $\propto P(1/(p^2 + \mu_0^2))$.

2. - Renormalizability of Fermi Theory.

Let us now examine the problem of the renormalizability of the Fermi theory. The identity (8) or (4) shows that the divergences of S_F can be compensated. One must see, however, whether this compensation can be made by way of scale changes of the observable Fermi quantities only, namely g_F , m , ψ . To see this, we shall show that $(S_Y)_{0m}$ can be expressed as a function of these quantities only, multiplied by renormalization constants containing the divergences. Let us first define the renormalized Fermi constant, g_F , as the value of S_Y for the scattering of two zero energy fermions, under the exchange of a single meson,

$$(16) \quad ig_F = \frac{iG^2/\mu^2}{1 + (iG^2/\mu^2)K_R^{\mu^2}(p^2 = 0)},$$

$K_R^{\mu^2}(p^2)$ is the renormalized vacuum polarization operator of the boson. It is pure imaginary at $p^2 = 0$, and satisfies the renormalization conditions:

$$(17) \quad \text{Im}(K_R^{\mu^2}(-\mu^2)) = 0; \quad \left[\text{Im} \left(\frac{d}{dp^2} K_R^{\mu^2}(p^2) \right) \right]_{p^2 = -\mu^2} = 0.$$

It can then be seen ⁽⁸⁾ that $(S_Y)_{0m}$ can be expressed as a functional of the operators ψ_0 , $\bar{\psi}_0$, $S_c(m_0) = (i\gamma p + m_0)^{-1}$ and of the operator

$$(18) \quad Y_0 = iG_0^2 A'_{F0} = \frac{iG_0^2/(p^2 + \mu_0^2)}{1 + [iG_0^2/(p^2 + \mu_0^2)]K_0(p^2)},$$

⁽⁶⁾ These prescriptions were also given independently by H. UMEZAWA, Y. TOMOZAWA, M. KONUMA and S. KAMEFUCHI: *Nuovo Cimento*, **3**, 772 (1956).

⁽⁷⁾ J. A. WHEELER and R. P. FEYNMAN: *Rev. Mod. Phys.*, **21**, 425 (1949).

⁽⁸⁾ One can see that for instance in perturbation theory, by summing all the vacuum polarization effects on every internal boson lines.

(Δ'_{F_0} is the Dyson function in terms of u.r. constants), in which K_0 is the unrenormalized polarization operator; it is itself expressible as a functional of $S_c(m_0)$ and Y_0

$$(19) \quad K_0(p^2) = K_0(p^2; [S_c(m_0)]; [Y_0]) .$$

Symbolically K_0 is represented by the series of graphs

$$(20) \quad K_0(p^2) = \text{---} + \text{---} + \text{---} + \text{---} + \text{---} + \dots$$

p is the momentum of the pair; to each straight line corresponds a function $S_c(m_0)$, and to each wavy one Y_0 . One gets this symbolic series of diagrams from the known series defining $K_0(p^2)$ in terms of S_c and Δ_c (represented by fermion, respectively boson lines) by replacing every boson line by a wavy one (substituting Y_0 to $G^2\Delta_c$), and suppressing every graph with vacuum polarization effects on the internal boson lines (the latter being already included in Y_0).

Expressing the unrenormalized quantities in terms of the renormalized ones ⁽⁹⁾, we have

$$(22) \quad Y_0 = \frac{iG^2/(p^2 + \mu^2)}{1 + [iG^2/(p^2 + \mu^2)]K_R^{\mu^2}(p^2)} \cdot \left\{ \frac{Z_1}{Z_2} \right\}^2 = Y_R \cdot (Z_1/Z_2)^2 .$$

The fermion renormalization constants can be considered as functionals of Y_R , (and so of Y_0) and of m :

$$(23) \quad Z_1, Z_2, \delta m = \text{functionals of } (Y_0, m) .$$

From (16) and (22) follows

$$(24) \quad Y_0 = \frac{ig_F}{1 + ig_F(K_R^{\mu^2}(p^2) - K_R^{\mu^2}(0) + (1/iG^2)p^2)} (Z_1/Z_2)^2 ,$$

while (18) and (22) lead to

$$(25) \quad K_R^{\mu^2}(p^2) - K_R^{\mu^2}(0) + (1/iG^2)p^2 = (Z_1/Z_2)^2(K_0(p) - K_0(0)) + (Z_3/iG^2)p^2 ;$$

thus

$$(26) \quad Y_0 = \frac{ig_F(Z_1/Z_2)^2}{1 + ig_F(Z_1/Z_2)^2(K_0(p^2) - K_0(0)) + p^2 Z_3(g_F/G^2)} .$$

⁽⁹⁾ Using for instance: $\Delta'_{F_0} \equiv \langle P(A_0(x), A_0(x')) \rangle_0 = Z_3 \langle P(A(x), A(x')) \rangle_0 \equiv Z_3 \Delta'_{FR}$.

As a result we see from (19), (23) and (26) that Y_0 (and so $(S_Y)_{0m}$) is a function only of m and g_F if $Z_3 = 0$.

Under that condition (26) can be written as

$$(27) \quad Y_0 = \frac{ig_0}{1 + ig_0 K_0(p^2)},$$

with

$$(28) \quad \begin{cases} ig_0 = \frac{ig_F(Z_1/Z_2)^2}{1 - ig_F(Z_1/Z_2)^2 K_0(0)} = ig_F \cdot Z \cdot (Z_1/Z_2)^2, \\ Z = 1 + ig_0 K_0(0) = \frac{1}{1 - ig_F(Z_1/Z_2)^2 K_0(0)}. \end{cases}$$

By (19) we can regard (27) as an integral equation which determines Y_0 as a function of g_0 and m_0 . The matrix $(S_Y)_{0m}$ thus expressed in terms of g_0 and m_0 is obviously S_F . Expressing the unrenormalized fermion quantities in terms of the renormalized ones, we arrive at the renormalized Fermi Lagrangian ⁽¹⁰⁾

$$(29) \quad L_F = Z_2 \cdot L_{\text{Fermion}}(\psi, m) + \delta m \cdot \bar{\psi}\psi + (\tfrac{1}{2})g_F Z \cdot (Z_1)^2 (\bar{\psi}\psi)(\bar{\psi}\psi).$$

The renormalized function Y_R , defined by (22), can be expressed as

$$(30) \quad Y_R = iG^2 \Delta'_{FR} = \frac{ig_F}{1 + ig_F K_{RF}(p^2)},$$

where Δ'_{FR} is the renormalized Dyson function for boson, and the renormalized Fermi polarization operator K_{RF} is defined by

$$(31) \quad \begin{cases} K_{RF}(p^2) = K_{R(\psi, \Gamma)}(p^2) - K_{R(\psi, \Gamma)}(0); & K_{RF}(0) = 0 \\ K_{R(\psi, \Gamma)}(p^2) = (Z_1/Z_2)^2 K_0(p^2). \end{cases}$$

$K_{R(\psi, \Gamma)}$ is the polarization operator of the bosons in the Yukawa theory, but renormalized only with respect to the vertices and the Fermion

$$(32) \quad K_{R(\psi, \Gamma)}(p^2) \propto \int \text{Tr} \{ S'_{FR}(k) \gamma S'_{FR}(p+k) \Gamma_R(k, p+k) \} d^4k,$$

S'_{FR} denotes the renormalized Dyson function for fermions, and Γ_R the renormalized vertex function; in the scalar case γ is 1.

⁽¹⁰⁾ The constant $Z_1^2 \cdot Z$ was called Z_F in ref. ⁽¹⁾.

It may also be noted that equation (12) is equivalent to equation (16), which is simpler in form; this can be seen from (18), (22) and (28).

3. - The Infinities in S_F .

We can now compare this general proof of renormalizability of S_F with the perturbation theory.

There are three types of infinities encountered by this method:

3.1 The fictitious infinities. - These are the ones compensated when one includes higher and higher order terms in g_F ; for example, it is sufficient to insert at each vertex of the divergent graph an arbitrary number of loops (which have been rendered finite) and to sum all these graphs to notice such a compensation ⁽¹¹⁾. This compensation occurs in general, because the total series of terms has the effect of bringing in at each vertex, not g_F but instead the kernel Y_R which vanishes as $1/p^2$ at infinity, but whose development in powers of g_F would yield stronger and stronger divergences. One can connect the appearance of such infinities in the formal development of S_F in powers of g_F with the fact that g_F has dimension M^{-2} and is thereby not an expansion parameter. What is found to play such a role is the dimensionless « parameter » $ig_F K_{RF}(p^2)$, which depends on p^2 . Thus, whenever there occurs a sum over virtual states, this « parameter » becomes greater than unity, the critical value μ^2 of $-p^2$ being that for which

$$(33) \quad -ig_F K_{RF}(-\mu^2) = 1.$$

For this value, there is a pole corresponding to the boson. Beyond it, the expansion of Y_R in powers of $ig_F K_{RF}$ (or of g_F), is no longer valid.

3.2 The renormalization infinities (*) - As a result of the compensation of the fictitious infinities, the divergences of the vertex-parts (without exchange of pairs between the different Fermions) can be absorbed by the one constant Z_1 , and those carried by the Fermion lines are absorbed by δm and Z_2 .

⁽¹¹⁾ N. HU: *Phys. Rev.*, **80**, 1109 (1950); S. KAMEFUCHI: *Prog. Theor. Phys.*, **11**, 273 (1954).

It must be noted that the Fermi theories are different from the other non-renormalizable theories with regard to the objection of LEHMANN to the HU method (H. LEHMANN: *Nuovo Cimento*, **11**, 342 (1954)); indeed, contrarily to the other theories, there is no free meson propagator, since there is no given meson.

(*) For graphical illustration and syntetic view of the following discussion the reader is referred to the figure at the end of this paper.

Among the two kinds of infinities of Fermion closed loops which remain after subtraction of Z_1 , Z_2 and δm -parts from the closed loops (true divergences), the quadratically divergent terms, which in Yukawa theory are absorbed by $\delta\mu^2$, are here split off by the renormalization condition $K_{\text{FR}}(0) = 0$, and absorbed in Z .

3.3. The fundamental infinities. – The logarithmic divergence, however, which comes in, multiplied by p^2 , (as does Z_3 in Yukawa theory) can at first be separated from the finite part by specifying a renormalization condition, such as

$$(34) \quad K'_{\text{FR}}(0) = \left[\frac{d}{dp^2} K_{\text{FR}}(p^2) \right]_{p^2=0} = C,$$

where C is some given constant which is related to the value of the derivative of the Fermi interaction at $p^2 = 0$. But these infinities cannot be absorbed by any change of scale of the fermion quantities (g_{F} , m , ψ). One can, however, remove them by suitable « counter-interactions » in L_{F} . This is equivalent to the choice of a « counter-interaction »

$$(35) \quad Z(p^2, C) = \frac{1}{1 - ig_{\text{F}}[K_{R(\psi, \Gamma)}(0) + p^2(K'_{R(\psi, \Gamma)}(0) - C)]},$$

instead of the simple constant Z .

It is then an essentially new statement in the Fermi theory that *the sum of all the counter-interactions must vanish*. This means that C must be chosen such that

$$(36) \quad C_0 \equiv C + \delta C \equiv C - K'_{R(\psi, \Gamma)}(0) = 0 \quad \text{whence} \quad Z(p^2, C) \rightarrow Z.$$

Since $K_{R(\psi, \Gamma)}(0)$ is obtained in general as a series of divergent terms depending on m , g_{F} and C , equation (36) determines the constant C in the same fashion as the equation $Z_3 = 0$ determines $G = G(\mu, m)$ ⁽¹²⁾.

The M -divergences are also fundamental infinities. One can treat them in the same manner as in the Yukawa theory. The term $\lambda(g_{\text{F}})^4(J)^4$ is added to L_{F} ; the M -divergent part is separated from the convergent one by the corresponding prescriptions, and is cancelled by counter-terms of the form $\delta(\lambda g_{\text{F}}^4)(J)^4$. The total Matthews term in L_{F} is then

$$(37) \quad \lambda(g_{\text{F}})^4 \cdot Z^4 \cdot Z_4(Z_1)^4(J)^4 \quad J = \bar{\psi}\psi$$

⁽¹²⁾ One can deduce from the theorem of Gell-Mann and Low ⁽¹⁴⁾ that, if $Z_3 = 0$, one should have $(p^2 A'_{\text{FR}})_{p^2 \rightarrow \infty} = 0$, and then according to (30) $(K_{\text{FR}}(p^2)/p^2)_{p^2 \rightarrow \infty} \rightarrow 0$; this condition must be equivalent to the equations (11) and (36).

which can be expressed as $\lambda_0(g_0)^4 \cdot (J_0)^4$. The constant λ is still determined by

$$(38) \quad Z_4(\lambda, g_F, m, C(g_F, m)) = 0$$

if it is supposed that λ_0 is null, as in the lagrangian of formula (1).

4. – Practical Methods.

We conclude from this analysis that a practical method of treating S_F might be:

- 1) to avoid introducing the fictitious infinities;
- 2) to avoid treating the fundamental infinities.

The two methods which are sketched below suppress the first difficulty, but only bypass the second in the following sense. In analogy to the treatment of the M -divergences in Yukawa theory, we introduce two arbitrary constants C and λ . One can then calculate any desired effect to any order as a function of g_F , m , C and λ . But the constants C and λ are, in principle, determined as functions of g_F and m , by the condition that the infinite approximation leads to the true Fermi theory in which $C_0 = 0$ and $\lambda_0 = 0$; this is equivalent to the condition that certain series of divergent terms must have a given value.

The first method starts from L_F , written in the form:

$$(39) \quad L_F = Z_2 L_{\text{fermion}}(\psi, m) + \delta m \bar{\psi} \psi + (g_F/2)(Z_1)^2 \cdot Z \cdot J J + \lambda (g_F Z_F)^4 \cdot Z_4 \cdot Z_1^4 \cdot J^4$$

and uses for Z formula (35):

$$(40) \quad \begin{aligned} Z &= \frac{1}{1 - ig_F \{ K_{R(\psi, I)}(0) + p^2 [K'_{R(\psi, I)}(0) - C] \}} \\ &= \left(\frac{1}{1 + ig_F C p^2} \right) \cdot \left[1 + \frac{ig_F}{1 + ig_F C p^2} (K_{R(\psi, I)}^{(0)} + p^2 K_{R(\psi, I)}^{\prime(0)} + \dots) \right] \\ &= X + X \frac{ig_F X \cdot [K_{R(\psi, I)}^{(0)} + p^2 K_{R(\psi, I)}^{\prime(0)}]}{1 - ig_F X \cdot [K_{R(\psi, I)}^{(0)} + p^2 K_{R(\psi, I)}^{\prime(0)}]} = X + \delta X, \end{aligned}$$

with $X = X(p^2) = 1/(1 + ig_F C p^2)$.

The Fourier transform of L_F may be written as

$$(41) \quad \begin{aligned} L_F &= L_{\text{fermion}}(\psi, m) + (ig_F/2) J(p) X(p^2) J(-p) + \\ &\quad + \lambda (ig_F)^4 \iiint J(p_1) J(p_2) J(p_3) J(p) X(p_1^2) X(p_2^2) X(p_3^2) X(p^2) \cdot \\ &\quad \cdot \delta(p_1 + p_2 + p_3 + p) d^4 p_1 \cdot d^4 p_2 \cdot d^4 p_3 + \text{Counter-terms} . \end{aligned}$$

The interaction terms are represented by the graphs

$$(\text{---}) + 16\lambda \left(\begin{array}{c} \diagup \quad \diagdown \\ \times \end{array} \right); \quad \text{where } (\text{---}) \text{ means the interaction } (ig_F/2)X.$$

The counter-terms are such that all the infinities are subtracted to each order in « X » and λ , in such a way that the calculated observable value of m , g_F , ψ , C , and λ remain unchanged. These counter-terms have the effect to modify in L_F the preceding quantities to their unrenormalized values. But according to (36) and (38) the constants C and λ have to be chosen in such a way that in the infinite approximation the constants C_0 and λ_0 are zero.

Another possible method consists of treating (30), say by variational methods, by cutting off at some order in Y_R the series (20) which yields the kernel $K(Y_R)$. The renormalization conditions which are imposed in the course of the calculation of K_{RF} , (on the vertices, the Fermion self-energies and the M -parts and on K_{RF} and its derivative at $p^2 = 0$) are equivalent to the subtraction of counter-terms in L_F .

Of course the different methods will yield unequal results at a finite stage of approximation.

However, the practical usefulness of these methods lies in the hope that the comparison of the theory with experiment (which gives the μ and the G of the bosons) will permit one to determine a significant law for C and λ , which are known to be functions of m and g_F .

In previous attempts ^(1,2) these constants were introduced by the regularization of closed loops; further instead of suppressing the infinities by counter-terms which modify the constants, one has only «normalized» at each order the kernels K , without caring about the counter-terms this procedure implied in L_F ; apart from these differences the results are alike.

We were then able to show ⁽¹⁾ that electrodynamics follows from a neutral vector Fermi coupling between electron and neutrino pairs; there appear then two fundamental constants of type C (one for each fermion type of closed loop), which are determined by experiment; with the present method these constants are, in principle, given by equations of the type (11) and (12), and in particular α , the fine structure constant, must be the solution of the equation $Z_3(\alpha) = 0$ in the electrodynamics ⁽⁴⁾.

The case of triangular p.s. Fermi coupling

involves more constants, as the equivalent Yukawa theory contains three bosons of different masses and couplings, the lightest of which can be iden-

tified with $\pi^\pm$. The properties of $\pi^\pm$ -meson that may desintegrate into leptons and that are coupled to the nucleons, permit the deduction of a qualitative law for $C(m)$, ($C(m) \rightarrow \infty$ if $m \rightarrow 0$), in accordance with the values obtained from the photon. Taking this law into account, the properties of the pions, in particular the existence of isotopic spin, can be deduced from the given couplings ⁽²⁾:

$$(42) \quad g_F[\{(\bar{n}n) - (\bar{p}p)\}\{\text{Neutral lepton pairs}\} + (\bar{n}p)\{(\bar{e}\nu) + (\mu\nu)\}].$$

5. - The Equation $Z_3 = 0$.

The main point of our proof of the equivalence theorem rests on the hypothesis that one of the fundamental equations which determine the mesic charge and mass, namely $Z_3(\Lambda, G^2(\Lambda), \mu^2(\Lambda)) = 0$, has a solution for every value of Λ which tends to infinity. Thus it is essential to study this question in the light of the works done on the renormalization constants. We shall be mainly concerned here with the case of electrodynamics. Various arguments have been put forward, based on the formula of Källén ⁽¹³⁾

$$(43) \quad \left\{ \begin{array}{l} 1/Z_3 = 1 + \int_0^\infty \frac{\Pi(-a)}{a} da, \\ \Pi(p^2) = \lim_{v \rightarrow \infty} \frac{V}{3p^2} \sum_{p^{(2)}=p} |\langle 0 | j_\nu | z \rangle|^2 (-1)^{N_z} = \text{Sum of positive terms}, \end{array} \right.$$

in which one tried to show that the constant Z_3 is identically zero, which if true would destroy any hope of finding a solution to our equation. We therefore consider now the value of these arguments.

5.1. *In perturbation theory* ⁽¹⁴⁾. - We take a cut-off Λ on the fermion momenta and use the calculations ⁽¹⁵⁾ of the first terms of the development of the function $\Pi(p^2)$ in powers of α , the fine structure constant. We get to second order in α

$$(44) \quad I/Z_3 = I + \alpha(a_0 + b_0 L) + \alpha^2(a_1 + b_1 L + c_1 L^2); \quad L = \log(\Lambda^2/m^2)$$

(a , b , c , are geometrical constants).

⁽¹³⁾ G. KÄLLÉN: *Helv. Phys. Acta*, **25**, 417 (1952).

⁽¹⁴⁾ M. GELL-MANN and F. LOW: *Phys. Rev.*, **95**, 1300 (1954).

⁽¹⁵⁾ G. KÄLLÉN: *Dan. Mat. Fys. Medd.*, **29**, no. 17 (1955).

If we let L tend to infinity, then evidently $1/Z_3$ tends to infinity for all values of α . However we are not justified in only keeping the first few terms of the series, even as a minorant if each term of the series were positive, unless the series is convergent. But it is clear that on these first few terms the series does not behave like a convergent series; we may confirm this by showing that terms in α^n contain factors which diverge as L^n , and which are no other than the formal development of the function $1/Z_3$ in which one takes Z_3 to have the form

$$(45) \quad Z_3 = 1 - (u_0\alpha + u_1\alpha^2) - (v_0\alpha + v_1\alpha^2)L$$

u_i and v_i are geometrical constants

which have been calculated previously by the method of counter terms ⁽¹⁶⁾. This infinite value of Z_3 contradicts the lemmas of Schwinger and Lehmann ⁽¹⁷⁾ according to which $0 \leq Z_3 < 1$. But we must note that these lemmas suppose that the integral of the series (43) converges, which is not the case in perturbation theory. The study of the graphs in perturbation theory (α being the renormalized constant) shows that Z_3 would appear to be of the form

$$(46) \quad Z_3 = 1 - u(\alpha) - v(\alpha)L$$

and consequently only exceptional values of α , which give zeros of $v(\alpha)$ or singularities of $u(\alpha)$, can annihilate Z_3 .

Since the two constants v_0 and v_1 are positive, it is not possible to obtain, to second order in α , a compensation of the infinities ⁽¹⁸⁾ or an annihilation of Z_3 for non zero values of α . At present nothing is known of the behaviour of the series defining $u(\alpha)$ and $v(\alpha)$, but it is not improbable that they are asymptotic, which is suggested by the model of $\lambda\varphi^3$ coupling ⁽¹⁹⁾.

5.2. Other method. — We shall now discuss the argument of KÄLLÉN ⁽²⁰⁾.

In his words KÄLLÉN starts from «the assumption that all the quantities K , $(1 - L)^{-1}$ and $1/N$ (for notation, cfr. ref. ⁽¹³⁾) are finite or that the integ-

⁽¹⁶⁾ H. UMEZAWA and R. KAWABE: *Prog. Theor. Phys.*, **4**, 443 (1949); R. JOST and J. LUTTINGER: *Helv. Phys. Acta*, **23**, 201 (1950).

⁽¹⁷⁾ J. SCHWINGER; cf. W. PAULI: *Feldquantisierung* (Zürich, 1951); H. LEHMANN: *Op. cit.* ⁽¹¹⁾.

⁽¹⁸⁾ W. PAULI; cf. SCHWEBER *et al.*: *Mesons and Fields* (1955), Vol. I, p. 333.

⁽¹⁹⁾ C. A. HURST: *Proc. Camb. Phil. Soc.*, **48**, 625 (1952); W. THIRRING: *Helv. Phys. Acta*, **26**, 33 (1953); A. PETERMANN: *Phys. Rev.*, **89**, 1160 (1953); R. UTIYAMA and T. IMAMURA: *Prog. Theor. Phys.*, **9**, 431 (1953). I am very indebted to Dr. PETERMANN for numerous enlightening discussions on this question.

⁽²⁰⁾ G. KÄLLÉN: *Dan. Mat. Fys. Medd.*, **27**, no. 12 (1953).

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rals (1) converge. » In the notation of the preceding sections, K is the mass renormalization constant for fermions, $1 - L = Z_3$ and $N^2 = Z_1 = Z_2$. The integrals to which he refers are the integral (43) and a similar for fermion renormalization constants. KÄLLÉN's assumption implies that such functions as $\Pi(p^2)$ tend to zero as p^2 tends to $-\infty$. The function $\Pi(p^2)$ consists of a sum of positive terms. For this sum he constructs the following minorant by selecting the state $|z\rangle$ consisting of a pair of Fermions.

$$(47a) \quad \Pi(p^2) > \lim_{v \rightarrow \infty} \frac{v}{3p^2} \sum'_{a+q'=v} |\langle 0 | j_v | q, q' \rangle|^2.$$

He shows that this inequality can be written in the form

$$(47b) \quad \Pi(p^2) > (\alpha/3\pi) N^4 / Z_3^2,$$

when p^2 tends to $-\infty$.

The right hand side of this inequality is finite according to the assumption of KÄLLÉN. However, to quote KÄLLÉN again, «if all the renormalization constants K , $1/N$ and $1/(1-L)$ are finite, the function $\Pi(p^2)$ cannot approach zero for $-p^2 \rightarrow \infty$. This is an obvious contradiction and the only remaining possibility is that at least one (and probably all) of the renormalization constants is infinite. »

Before discussing this argument, let us complete this calculation by inserting (47b) into (43); we obtain

$$(48a) \quad \frac{1}{Z_3} > 1 + \left(\frac{\alpha}{3\pi}\right) \frac{N^4}{Z_3^2} \int_{m^2}^{\infty} \frac{da}{a} = 1 + \left(\frac{\alpha}{3\pi}\right) \frac{N^4}{Z_3^2} \log\left(\frac{\infty}{m^2}\right).$$

We remark that this result does not give a minorant for $1/Z_3$ but rather an inequality which Z_3 has to satisfy. This inequality does not imply that $1/Z_3$ is infinite, but rather that Z_3 must be complex, which is impossible on physical grounds. The origin of the trouble does not lie in the KÄLLÉN assumption as one might suppose, but rather, for electrodynamics, in the inequality (47a). Here what happens is that, since the photon has no mass, the function is defined by an *infinite* sum of terms for every value of $-p^2 > 4m^2$, because it is always possible to create any number of photons when the energy is greater than $2m$. The inequality (47a) is true only if the series for $\Pi(p^2)$ converges. To make the point clear consider the example $1/(1-X) = 1 + X + X^2 + \dots$; if $X > 1$, then each term of the series is positive, but a single term cannot be said to be a minorant, because the series diverges.

Now we can see that *the series (43) defining $\Pi(p^2)$ in electrodynamics is in*

fact divergent. To show this let us look at the formula (47b) from which we see that $\Pi(p^2)$ depends on the infra-red catastrophe through N^2 ⁽²⁰⁾. In perturbation theory the second member of (47a) is in fact infra-red divergent for every value of $-p^2 > 4m^2$ from the second order in α (the fine structure constant). However it is known that in perturbation series for Z_3 the sum of terms belonging to any order in α is not infra-red divergent. It follows that there cannot be any infra-red divergences in any order of the development of $\Pi(p^2)$ in powers of α ; this was seen by direct computation for the second order in α ⁽¹⁵⁾. Therefore there exist in the series (43) defining $\Pi(p^2)$ terms neglected by KÄLLÉN which cancel the infra-red catastrophes in (47a) and (47b). Since every term of the series (43) is positive, it is necessary that the compensating terms form a divergent series whose sum can be negative.

Let us now examine the case of meson theories. In that case, when p^2 is finite, $\Pi(p^2)$ is defined by a sum of a finite number of terms. As $-p^2$ tends to infinity, the number of terms increases indefinitely, while each term tends to its asymptotic value; now it is essential to remark that the asymptotic form of each term is a *function of the renormalization constants* whether this asymptotic form is finite or not. This is a consequence of the fact that some propagators then reach their asymptotic values, which are related to the values of the renormalization constants, according to the Gell-Mann and Low theorem ⁽¹³⁾; an example is the perturbative calculation of (47a). Finally, for large values of $-p^2$ the function $\Pi(p^2)$ is an explicit function of the renormalization constants. One may then separate the asymptotic part $\Pi_2(p^2)$ of $\Pi(p^2)$ from the rest $\Pi_1(p^2)$ which may also depend on the renormalization constants and which is a series of finite functions which tend to zero as p^2 tends to infinity. Inserting this decomposition into (43) one gets instead of (48a) an equation of the form

$$(48b) \quad \frac{1}{Z_3} = 1 + F_1(G, \mu, m, Z_i, \delta\mu^2, \delta m) + F_2(G, \mu, m, Z_i, \delta\mu^2, \delta m, \infty)$$

(the functions F_1 and F_2 respectively are the contributions of Π_1 and Π_2). If $\int (\Pi(-a)/a) da$ diverges at infinity, then the F_2 part is divergent, but the solution of (48b) for $1/Z_3$ may well be finite, because F_2 contains Z_3 , and may have an arbitrary value depending on the value of the other renormalization constants and of the parameters G, m, μ . As an example suppose that F_2 is of the form $\infty \cdot G_2(Z_3, \alpha)$; then the solution of (48b) is $G_2(Z_3, \alpha) = 0$. Also it may happen that for certain values of the parameters the function F_1 is singular even if the integral converges at infinity (F_2 finite).

The preceding argument applies of course also in the case of electrodynamics discussed above. In that case $Z_1 = Z_2$ and $\mu = 0$ and since there is no infra-red catastrophe in Z_3 , F_1 and F_2 do not depend on Z_1 .

From equation (48b) one also understands why the Schwinger and Lehmann lemmas (¹⁷) do not hold, as we have seen in Sect. 5.1 with perturbative method.

5.3. *With a cut-off.* — Let us now make a cut-off on the momenta of the Fermions, for example by introducing a convergence factor $V(p)$ in the interaction term of the Fermi theory

$$(49) \quad g(\bar{\psi}(p)V(p)\psi(q)V(q))(\bar{\psi}(r)V(r)\psi(s)V(s))\delta^4(p+q+r+s).$$

This implies also the modification of the coupling in the equivalent Yukawa theory

$$(50) \quad G \cdot A(p) \cdot (\bar{\psi}(q)V(q)\psi(s)V(s))\delta^4(p+q+r).$$

We note here that an arbitrary alteration of the usual Yukawa theory (which has linear equations and local coupling) may not always be considered to be a result of a modification of the Fermi theory, although the converse is true; for example we could not obtain a Yukawa theory in which the Hamiltonian of the free mesons has an arbitrary form; in such cases the changed Yukawa theories could not be identical with the Fermi theories, modified or not, except in the limit when the alteration tends towards a definite form or to zero.

With the modifications (49) and (50) the theories no longer give divergences. However a cut-off on the momenta of the fermions does not imply a cut-off on the integral (43), and the series (43) still contains an infinite number of finite terms. Indeed the states with an arbitrary number of mesons are not limited in energy, since the cut-off has no effect except on closed loops, as is seen in the graph

$$\langle 0 | \text{ [Diagram: A closed loop with four external wavy lines, each having a cross on it] } | z = 3 \text{ mesons} \rangle$$

(cross lines indicate the presence of $V(p)$ cut-off function).

On the other hand the states with a given number of pairs of fermions have an energy which is limited but which increases with the number of pairs.

$$\langle 0 | \text{ [Diagram: A triangle with two external wavy lines and two external fermion lines, each having a cross on it] } | z = 2 \text{ pairs} \rangle$$

Consequently it is not impossible, for certain values of z (in the case of electromagnetism), that on one hand the series (43) might be divergent, and on the other hand that the series, when summed, might furnish a solution which could become singular.

Let us finish by a certain remark:

Suppose that there exists a region of α and Λ^2 for which the series (43) converges, in which case (47) gives a true minorant; then the inequality (48), where Λ^2 replaces ∞^2 , implies that, since Z_3 must be real, we have

$$(51) \quad \text{Log } (\Lambda^2/m^2) < 3\pi/4\alpha N^4.$$

Several authors, using certain approximations from perturbation theory have met similar inequalities and have interpreted them as an indication of an energy limit to the validity of the theory ⁽²²⁾. We are led on the contrary to interpret this relation as determining a region of possible values of α for which the theory gives no trouble ⁽²¹⁾ when there is a cut-off; then α is not the experimental constant.

The conclusion of this discussion is that none of the arguments advanced to prove that the constant Z_3 is identically zero appear irrefutable. In addition we have seen that it is not impossible that there exist values of charge and mass of the meson for which Z_3 is zero, whether the cut-off is finite or not.

The explicit calculation of charge and mass values and their comparison with experiment for the different existing types of Fermi couplings remains an open question. As far as electrodynamics is concerned, if a vectorial Fermi coupling between electron and neutrino pairs ⁽¹⁾ really exists in Nature, the fine structure constant α , should be the solution of $Z_3 = 0$ for an infinite cut-off; then its value should be of a purely geometrical nature, because it is a zero of a function defined by a series of terms with purely geometrical coefficients. Certain aspects of the method of adiabatic switching on must be therefore revised, because it is no longer possible to consider the observable charge (which is a number) or the non renormalized charge (which is infinite) as parameters variable with time.

* * *

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⁽²¹⁾ T. D. LEE: *Phys. Rev.*, **95**, 1329 (1954); G. KÄLLÉN and W. PAULI: *Dan. Mat. Fys. Medd.*, **30**, no. 7 (1955); H. UMEZAWA, Y. TOMOZAWA, M. KONUMA and S. KAMEFUCHI: *Nuovo Cimento*, **3**, 772 (1956).

⁽²²⁾ L. D. LANDAU *et al.*: *Dokl. Akad. Nauk SSSR*, **95**, 1177 (1954); S. KAMEFUCHI and H. UMEZAWA: *Nuovo Cimento*, **3**, 1060 (1956).

TABLE I. — We call, as explained in ref. (2), « Realistic theory of meson », the Yukawa theory equivalent in which the infinities are of the only type indicated by the graphs (primitive divergences). The momentum has the indicated value; p_0 is nul, p_F is such that $i\gamma p_F + m = 0$ and p_μ such that $p_\mu^2 + \mu^2 = 0$. to (p) for open fermion lines, or with respect to p^2 for closed loops of fermion lines. We indicate only a

FERMI COUPLING THEORY				REALISTIC
	Divergent graphs	Counter terms	Unrenormaliz. quantities	Graphs or equations
Fermion part		$\delta m \bar{\psi} \psi$	$m_0 = m + \frac{\delta m}{Z_2}$
	$(Z_2 - 1)L(\psi, m)$	$\psi_0 = \psi \sqrt{Z_2}$	
Interaction part		$(Z_1^2 - 1)g_N J^2$

$g_0 = g_F Z \left(\frac{Z_1}{Z_2} \right)^2$	equations
	$(Z - 1)g_F J^2$		$\delta \mu^2 = G^2/g_N Z$ or $g_F = G^2 A'_{FR}(0)$
Fundamental part		$(\delta C)p^2 J^2$	equations $C_0 = C + \delta C$ $C_0(C, m, g_F) = 0$	$Z_3(G, \mu, m) = 0$
				$Z_4(\lambda, G, \mu, m) = 0$
		$\delta(\lambda g_F^4)J^4$	$\lambda_0 = \lambda + \delta \lambda$ $\lambda_0(\lambda, C, m, g_F) = 0$

valent to the Fermi theory, when S_F is unitary. The counter-terms as given here correspond to the notation p_0, p_F, p_μ , given in the graphs, means that one has to calculate the graph when the external The prime in certain graphs means that for these graphs one has to calculate the derivative with respect few graphs of the realistic theory since it is easy to obtain them from the usual Yukawa theory.

MESON THEORIES					
THEORY			YUKAWA THEORY		
Counter terms		Unrenormaliz. quantities	Counter terms	Divergent graphs	
$\delta n \bar{\psi} \psi$	Fermion part	$m_0 = m + \frac{\delta m}{Z_2}$	$\delta m \bar{\psi} \psi$	$\left(\text{---} \overbrace{\text{---}}^{\text{wavy}} \text{---} \right)_{P_F}$	Fermion part
$(Z_2 - 1)L(\psi, m)$		$\psi_0 = \psi \sqrt{Z_2}$	$(Z_2 - 1)L(\psi, m)$	$\left(\text{---} \overbrace{\text{---}}^{\text{wavy}} \text{---} \right)'_{P_F}$	
$(Z_1^2 - 1) \frac{G^2}{2} J \bar{D} J$	Interaction part	$G_0 = G \frac{Z_1}{Z_2 \sqrt{Z_3}}$	$(Z_1 - 1)G J A$	$\begin{array}{c} P_F \\ \diagdown \\ \text{---} \text{---} \text{---} \\ \diagup \\ P_F \end{array} \text{---} \text{---} P_0$	Interaction part
$\frac{G^2}{2} \sum_{N=1}^{\infty} (\delta \mu^2)^N J \bar{D}^{N+1} J$		$\mu_0^2 = \mu^2 + \frac{\delta \mu^2}{Z_3}$	$\delta \mu^2 A^2$	$\left(\text{---} \text{---} \text{---} , \text{---} \text{---} \text{---} \right)_{P_\mu}$	Boson part
$\frac{G^2}{2} \sum_{N=1}^{\infty} (1 - Z_3)^N J \bar{D} J$		$A_0 = A \sqrt{Z_3}$	$(Z_3 - 1)L(A, \mu^2)$	$\left(\text{---} \text{---} \text{---} , \text{---} \text{---} \text{---} \right)'_{P_F}$	
$(Z_4 - 1)\lambda G^4 \cdot \bar{D}_1 J_1 \bar{D}_2 J_2 \bar{D}_3 J_3 \bar{D} J_4$		$\lambda_0 = \lambda Z_4$	$(Z_4 - 1)\lambda G^4 A^4$	$\begin{array}{c} P_0 \text{---} \text{---} P_0 \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ P_0 \text{---} \text{---} P_0 \end{array}$	Interaction part

RIASSUNTO (*)

Si dimostra un teorema d'equivalenza tra la teoria risultante da un autoaccoppiamento di Fermi scalare e una teoria di Yukawa che postula un bosone accoppiato con una coppia di fermioni; massa e accoppiamento di questo mesone debbono soddisfare equazioni che sono condizioni imposte alle costanti di rinormalizzazione. Si dimostra poi che la teoria di Fermi è rinormalizzabile nell'insieme. Si svolge un'analisi delle grandezze infinite che s'incontrano nella teoria delle perturbazioni onde poterle sottrarre. Si indicano metodi efficaci per il trattamento della teoria di Fermi: si danno esempi del loro impiego pratico. Si discute infine la grandezza della costante di rinormalizzazione dei bosoni.

(*) *Traduzione a cura della Redazione.*

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