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Carmen Vasini

A System of General Relativistic Equations of Low Type.

R. OMNES (*)

CERN - Geneva

(ricevuto il 22 Gennaio 1957)

Summary. — Using Bogoliubov's methods ⁽³⁾, we give here a new demonstration of the Low equation, thus displaying its general character. The method has been extended to give a set of equations of Low type for the productions of mesons by mesons and one nucleon.

1. - Introduction.

It has been known for some time that the Low equation ⁽¹⁾ is a general property of quantized systems. In particular, LEHMANN, SYMANZIK and ZIMMERMANN ⁽²⁾ were able to derive it by general methods not involving the concept of renormalization. Recently, a new formulation of quantum theory of fields by N. N. BOGOLIUBOV and his collaborators ⁽³⁾, formulated in terms of a few mathematical and physical postulates, allowed to establish in a rigorous way the dispersion relations for scattering.

In this work we have used these new methods to derive the Low equation thus exhibiting its general character. The method is quite easily extended to derive a complete system of Low equations for the processes of creation of any number of mesons by a nucleon and some incident mesons. It is still

(*) On leave of absence from C.E.A. Saclay.

⁽¹⁾ F. E. LOW: *Phys. Rev.*, **97**, 1392 (1955).

⁽²⁾ H. LEHMANN, K. SYMANZICK and W. ZIMMERMANN: *Nuovo Cimento*, **1**, 205 (1955).

⁽³⁾ N. N. BOGOLIUBOV (*Phys. Rev.* to be published). Reported at the International Conference for Theoretical Physics in Seattle, September 1956.

not evident, however, how one might extend the Low equation to include nucleon-nucleon scattering and production of particles.

2. - Summary of Bogoliubov's Hypothesis.

Let us recall some important features of Bogoliubov's methods.

2.1. General features.

- 1) There exist asymptotic states $\rangle$ without mutual interaction.
- 2) There exists a group G of some transformations L (including Lorentz transformations) unitarily represented by operators U_L over the states $\rangle$.
- 3) If $|p\rangle$ is an eigenstate of energy-momentum with eigenvalue p

$$(1) \quad U_{L_a}|p\rangle = \exp[-ipa]|p\rangle,$$

for the translation $x \rightarrow x + a$. There exists in particular a unique vacuum $|0\rangle$ such that

$$(2) \quad U_{L_a}|0\rangle = |0\rangle.$$

A result of conditions 1), 2), 3) is the additivity of momentum and energy of asymptotic states.

- 4) There exists a complete set of eigenstates of energy and momentum with *positive* eigenvalues of energy. Thus one may write

$$(3) \quad \langle\alpha|AB|\beta\rangle = \langle\alpha|A|0\rangle\langle 0|B|\beta\rangle + \sum_n \frac{1}{(2\pi)^3} \int d\bar{k} \langle\alpha|A|n\bar{k}\rangle \langle n\bar{k}|B|\beta\rangle.$$

- 5) There exists a unitary matrix S characterizing transition probabilities.
- 6) One may group asymptotic states into three classes:

- a) one particle states,
- b) bound states,
- c) direct product of some states from classes a) and b).

One assumes stability of one-particle and bound states (and, of course, of the vacuum) i.e. for these states

$$(4) \quad S|\alpha\rangle = |\alpha\rangle.$$

2.2. Physical properties.

1) Elementary particles are characterized by creation operators $a_e^{(+)}(\bar{q})$ and annihilation operators $a_e^{(-)}(\bar{q})$ which can act on asymptotic states and verify the known commutation relations

$$(5) \quad \begin{cases} [a_\alpha^{(-)}(\bar{q}'), a_{\alpha'}^{(+)}(\bar{q}')] = \delta_{\alpha\alpha'} \delta(\bar{q} - \bar{q}'), \\ [a_\alpha^{(-)}(\bar{q}), a_{\alpha'}^{(-)}(\bar{q}')] = [a_\alpha^{(+)}(\bar{q}), a_{\alpha'}^{(+)}(\bar{q}')] = 0. \end{cases}$$

(We have considered the case of bosons which is the only one to have interesting consequences for us.)

One may define operators $\varphi_e(x)$ in space-time configuration by

$$(6) \quad \varphi_e(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{d\bar{k}}{\sqrt{2k^0}} \{ \exp[ikx] a_e^{(+)}(\bar{k}) + \exp[-ikx] a_e^{(-)}(\bar{k}) \}.$$

Note that these operators verify the Klein-Gordon equation where enters the real mass of mesons.

2) One may define S -matrix as

$$(7) \quad S = \sum_{l,m=0}^{\infty} \int d\bar{k}_1' \dots d\bar{k}_l' d\bar{k}_1 \dots d\bar{k}_m f^{lm}(k_1' \dots k_m') a_{(\bar{k}_1')}^{(-)} \dots a_{(\bar{k}_l')}^{(-)} a_{(\bar{k}_1)}^{(+)} \dots a_{(\bar{k}_m)}^{(+)},$$

or, from (5) and (6), by introducing Wick's normal product,

$$(8) \quad S = \sum_{n=0}^{\infty} \int dx_1 \dots dx_n f^n(x_1 \dots x_n) : \varphi(x_1) \dots \varphi(x_n) :.$$

The functional derivative $\delta S / \delta \varphi(x)$ will be defined from (8) by subtracting an operator $\varphi(x_i)$ in every possible manner and replacing it by a $\delta(x - x_i)$. So for the S matrix element between a state α with one nucleon p and r mesons $q_1 \dots q_r$, and a state ω with one nucleon p' and s mesons $q_1' \dots q_s'$, it is possible to write,

$$(9) \quad S(\alpha\omega) = \langle p' q_1' \dots q_s' | S | p q_1 \dots q_r \rangle = \\ = \langle p' | a_{e_1'}^{(-)}(\bar{q}_1') \dots a_{e_s'}^{(-)}(\bar{q}_s') S a_{e_1}^{(+)}(\bar{q}_1) \dots a_{e_r}^{(+)}(\bar{q}_r) | p \rangle.$$

From (5) and (6) result the commutation relations

$$(10) \quad \begin{cases} [a_e^{(+)}(\bar{p}), \varphi_{e'}(x)] = \frac{\delta_{ee'}}{(2\pi)^{\frac{3}{2}}} \frac{\exp[-ipx]}{\sqrt{2p^0}}, \\ [a_e^{(-)}(\bar{p}), \varphi_{e'}(x)] = \frac{\delta_{ee'}}{(2\pi)^{\frac{3}{2}}} \frac{\exp[ipx]}{\sqrt{2p^0}}, \end{cases}$$

from which follows

$$(11.1) \quad [a_e^{(+)}(\bar{p}), S] = \frac{1}{(2\pi)^{\frac{3}{2}}} \int dx \frac{\delta S}{\delta \varphi(x)} \frac{1}{\sqrt{2p^0}} \exp[-ipx],$$

$$(11.2) \quad [a_e^{(-)}(\bar{p}), S] = \frac{1}{(2\pi)^{\frac{3}{2}}} \int dx \frac{\delta S}{\delta \varphi(x)} \frac{1}{\sqrt{2p^0}} \exp[ipx].$$

After introduction in (9), (11) gives

$$(12) \quad S(\alpha\omega) = \frac{(-1)^s}{(2\pi)^{3(r+s)/2}} \cdot \int dx'_1 \dots dx'_s dx_1 \dots dx_s \frac{\exp[i \sum q'_i x'_i - i \sum q_i x_i]}{2^{(r+s)/2} (q_1'^0 \dots q_s'^0 q_1^0 \dots q_r^0)^{\frac{1}{2}}} \left\langle p' \left| \frac{\delta^{r+s} S}{\delta \varphi(x'_1) \dots \delta \varphi(x_r)} \right| p \right\rangle.$$

3) One has the causality condition in the form (*)

$$(13) \quad \frac{\delta}{\delta \varphi(x)} \left[\frac{\delta S}{\delta \varphi(y)} S^+ \right] = 0 \quad \text{for } x \lesssim y.$$

(the notation meaning: x anterior to y , or $x - y$ space-like, the coincidence $x = y$ being excluded) With the aid of these assumptions which have been used by BOGOLIUBOV, MEDVEDEV and POLIVANOV to give a rigorous demonstration of the dispersion relations, we shall establish a complete set of integral equations of the type of Low.

3. - Derivation of the Low Equation.

First, let us derive the classical Low equation.

For the matrix element of S between an initial state α with a nucleon of momentum p and a meson of momentum q to a final state ω , p' , q' , (12) becomes

$$(14) \quad S(\alpha p q, \omega p' q') = \frac{-1}{(2\pi)^3} \int dx dy \frac{\exp[iq'x - i q y]}{\sqrt{4q^0 q'^0}} \left\langle p' \left| \frac{\delta^2 S}{\delta \varphi_e(x) \delta \varphi_e(y)} S^+ \right| p \right\rangle,$$

or, from the causality condition (13)

$$(15) \quad \frac{1}{(2\pi)^3} \int dx dy \frac{\exp[iq'x - i q y]}{\sqrt{4q^0 q'^0}} \langle p' | T(j_e(x), j_e(y)) | p \rangle,$$

(*) One may show that a hamiltonian theory always verifies (13).

with

$$(16) \quad j_e(x) = i \frac{\delta S}{\delta \varphi_e(x)} S^+.$$

One may write

$$T(j_e(x') j_e(y)) = \theta(x - y) j_e(x) j_e(y) + \theta(y - x) j_e(y) j_e(x)$$

and, for instance, using the complete set of states $|nk\rangle$

$$(17) \quad \langle p' | j_e(x) j_e(y) | p \rangle = \langle p' | j_e(x) | 0 \rangle \langle 0 | j_e(y) | p \rangle + \\ + \frac{1}{(2\pi)^3} \sum_n \int d\bar{k} \langle p' | j_e(x) | n\bar{k} \rangle \langle n\bar{k} | j_e(y) | p \rangle.$$

Noting that

$$(18) \quad \langle \alpha | j(x) | \beta \rangle = \exp[i(p_\alpha - p_\beta)x] \langle \alpha | j(0) | \beta \rangle,$$

one may group (15), (16), (17), (18) to obtain, after performing the integrations

$$(19) \quad S(\alpha p q, \omega p' q') = \frac{(2\pi)^4}{(2\pi)^3} \frac{\delta(q' + p' - q - p)}{\sqrt{4q^0 q'^0}} \cdot \\ \cdot \sum_n \left[\frac{\langle p' | j_e(0) | n\bar{k} \rangle \langle n\bar{k} | j_e(0) | p \rangle}{E - E_n + i\varepsilon} + \frac{\langle p' | j_e(0) | n\bar{k} \rangle \langle n\bar{k} | j_e(0) | p \rangle}{-E + p'^0 - p^0 - E_n + i\varepsilon} \right].$$

The last point in the demonstration of Low's equation is to interpret the quantities $\langle p' | j(0) | n \rangle$ in terms of S matrix-elements, when n is on the energy-shell.

In this case one has

$$S(\alpha p q, n) = \langle n | S | \alpha p q \rangle \\ = \langle n | S a_e^{(+)}(q) | p \rangle,$$

but

$$(20) \quad S a_e^{(+)}(\bar{q}) = [S, a_e^{(+)}(\bar{q})] + a_e^{(+)}(\bar{q}) S.$$

For the second term of (20) one may write

$$a_e^{(+)}(\bar{q}) S | p \rangle = a_e^{(+)}(q) | p \rangle = | p q \alpha \rangle,$$

where the first equality follows from (4) and the second one is self-evident.

From (11), the first term of (20) reads

$$-\frac{1}{(2\pi)^{\frac{3}{2}}} \int dx \frac{\delta S}{\delta p_e(x)} \frac{1}{\sqrt{2q''}} \exp[-iqx],$$

from which follows

$$S(\alpha pq, n) = \delta n, \alpha pq - \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{dx}{\sqrt{2q''}} \exp[-iqx] \left\langle n \left| \frac{\delta S}{\delta p_e(x)} S^+ \right| p\alpha \right\rangle,$$

where we have used $|p\rangle = S|p\rangle$ and $S^+S = 1$ to write

$$|p\rangle = S^+|p\rangle.$$

We have thus obtained

$$(21) \quad S(\alpha pq, n) = \delta qp\alpha, n - \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{dx}{\sqrt{2q''}} \exp[-iqx] \langle n | j_e(x) | p \rangle.$$

Using (14), (11) and putting

$$S(if) = \delta if + i \delta(p^i - p^f) T if,$$

one thus has

$$(22) \quad T_{\alpha p\alpha, n} = \frac{i}{(2\pi)^{\frac{3}{2}}} \frac{\langle n | j_e(0) | p \rangle}{\sqrt{2q''}},$$

and this equation may be used as a definition of the T matrix element out of the energy-shell. We have thus obtained the Low equation without explicit appearance of the renormalization local term (λ term) which appeared in the first formulation of Low, (*) and which contains only matrix elements of scattering (note that as ever these elements are not on the energy-shell).

One may note that there is a flaw in the demonstration given here: one has multiplied a distribution $\theta(x-y)$ by a T -product which is strongly singular for $x=y$ (note that the causality condition (13) leads to a commutability of $j_e(x)$ and $j_e(y)$ when $x-y$ is space-like so that the only difficulty is for $x=y$). Thus, strictly speaking, one has demonstrated nothing at all (just as in the current derivations). However, it is very easy, by using Bogoliubov's techniques, to pass from the causal form of T to its retarded form and justify (14) rigorously.

(*) This term is potentially included in $j(x)$ and would appear by explicit calculation of the one-nucleon term of (19).

4. – Extension to the Transition from One to Two Mesons. General Case.

We wish now to derive, by the same methods as above, a Low equation for the transition from a state containing one nucleon of momentum p and one meson of momentum q_1 to a state with one nucleon of momentum p' and two mesons of momenta q_2 and q_3 . The total energy of these two states will be designed by E .

From (12), one has

$$(23) \quad S(pq_1, p'q_2q_3) = -\frac{1}{(2\pi)^{\frac{3}{2}}} \int dx_1 dx_2 dx_3 \frac{\exp[i(q_2x_2 + q_3x_3 - q_1x_1)]}{\sqrt{8q_1^0q_2^0q_3^0}} \cdot \left\langle p' \left| \frac{\delta^3 S}{\delta\varphi_{e_1}(x_1)\delta\varphi_{e_2}(x_2)\delta\varphi_{e_3}(x_3)} S^+ \right| p \right\rangle.$$

Let us study

$$\frac{\delta^3 S}{\delta\varphi(x_1)\delta\varphi(x_2)\delta\varphi(x_3)} S^+,$$

in the case where

$$(24) \quad x_1 \lesssim x_2, \quad x_2 \lesssim x_3.$$

One may write

$$(25) \quad \begin{aligned} \frac{\delta^3 S}{\delta\varphi(x_1)\delta\varphi(x_2)\delta\varphi(x_3)} S^+ &= \frac{\delta}{\delta\varphi(x_2)} \frac{\delta}{\delta\varphi(x_1)} \left(\frac{\delta S}{\delta\varphi(x_3)} S^+ \right) - \\ &- \left[\frac{\delta}{\delta\varphi(x_1)} \left(\frac{\delta S}{\delta\varphi(x_3)} S^+ \right) \right] S \frac{\delta S^+}{\delta\varphi(x_2)} - \\ &- \left(\frac{\delta S}{\delta\varphi(x_3)} S^+ \right) \frac{\delta}{\delta\varphi(x_1)} \left(S \frac{\delta S^+}{\delta\varphi(x_2)} \right) - \frac{\delta^2 S}{\delta\varphi(x_2)\delta\varphi(x_3)} \frac{\delta S^+}{\delta\varphi(x_1)}. \end{aligned}$$

Remembering the causality condition (6), one sees that the three first terms of (18) are zero. Thus one can write

$$(26) \quad \begin{aligned} \frac{\delta^3 S}{\delta\varphi(x_1)\delta\varphi(x_2)\delta\varphi(x_3)} S^+ &= -\theta(x_2 - x_1)\theta(x_3 - x_1) \frac{\delta^2 S}{\delta\varphi(x_2)\delta\varphi(x_3)} \frac{\delta S^+}{\delta\varphi(x_1)} - \\ &- \theta(x_3 - x_2)\theta(x_2 - x_2) \frac{\delta^2 S}{\delta\varphi(x_3)\delta\varphi(x_1)} \frac{\delta S^+}{\delta\varphi(x_2)} - \\ &- \theta(x_1 - x_3)\theta(x_2 - x_3) \frac{\delta^2 S}{\delta\varphi(x_1)\delta\varphi(x_2)} \frac{\delta S^+}{\delta\varphi(x_3)}. \end{aligned}$$

Introducing (26) in (23) and putting, for instance

$$(27) \quad \left\langle p' \left| \frac{\delta^2 S}{\delta \varphi(x_1) \delta \varphi(x_2)} \frac{\delta S^+}{\delta \varphi(x_3)} \right| p \right\rangle = \left\langle p' \left| \frac{\delta^2 S}{\delta \varphi(x_1) \delta \varphi(x)} \right| 0 \right\rangle \left\langle 0 \left| \frac{\delta S^+}{\delta \varphi(x_3)} \right| p \right\rangle + \\ + \sum_n \frac{1}{(2\pi)^3} \int d\bar{k} \left\langle p' \left| \frac{\delta^2 S}{\delta \varphi(x_1) \delta \varphi(x_2)} \right| n\bar{k} \right\rangle \left\langle n\bar{k} \left| \frac{\delta S^+}{\delta \varphi(x_3)} \right| p \right\rangle,$$

defining

$$(28) \quad \left\langle p \left| \frac{\delta^2 S}{\delta \varphi(x_1) \delta \varphi(x_2)} \right| n\bar{k} \right\rangle = (2\pi)^3 \exp \left[i \frac{p-k}{2} (x+y) \right] F_{p\varrho_1\varrho_2, n\bar{k}}(x-y),$$

$$(29) \quad \left\langle p \left| \frac{\delta S}{\delta \varphi(x)} \right| n\bar{k} \right\rangle = \frac{i}{(2\pi)^4} T_{p\varrho, n\bar{k}} \exp [i(p-k)x],$$

and introducing the Fourier transform

$$(30) \quad F_{p\varrho_1\varrho_2, n\bar{k}}(x) = \frac{1}{(2\pi)^8} \int dk' \exp [-ik'x] T_{p\varrho_1\varrho_2, n\bar{k}}(k').$$

One obtains the equation

$$(31) \quad \langle p' q_2 q_3 | S | p q_1 \rangle = 2\pi \frac{\delta(q_2 + q_3 + p' - p - q_1)}{\sqrt{8q_1^0 q_2^0 q_3^0}} (A_1 + A_2 + A_3),$$

where

$$(32.1) \quad A_1 = \sum_n \int \frac{T_{p\varrho_2\varrho_3, n\bar{k}}(k') T_{p\varrho_1, n\bar{k}}^* \left| \begin{array}{l} \bar{k} = \bar{p}' + \bar{q}_2 + \bar{q}_3 \\ \bar{k}' = (\bar{q}_2 + \bar{q}_3)/2 \end{array} \right.}{\left(q_2^0 + \frac{p'^0 - E_n(\bar{k})}{2} - k'^0 + i\varepsilon \right) \left(q_3^0 + \frac{p'^0 - E_n(\bar{k})}{2} + k'^0 + i\varepsilon \right)} dk'^0,$$

$$(32.2) \quad A_2 = \sum_n \int \frac{T_{p\varrho_3\varrho_1, n\bar{k}}(k') T_{p\varrho_2, n\bar{k}}^* \left| \begin{array}{l} \bar{k} = \bar{p}' + \bar{q}_1 - \bar{q}_3 \\ \bar{k}' = (\bar{q}_1 + \bar{q}_3)/2 \end{array} \right.}{\left(-q_1^0 + \frac{p'^0 - E_n(\bar{k})}{2} + k'^0 + i\varepsilon \right) \left(q_3^0 + \frac{p'^0 - E_n(\bar{k})}{2} - k'^0 + i\varepsilon \right)} dk'^0,$$

And A_3 is deduced from A_2 by exchange of q_2 and q_3 .

One may note that, as in the first Low equation, the different coefficients of the θ -functions are identical when the arguments are space-like, the only singularity being for $x_1 = x_2$ or $x_1 = x_3$, etc., which could result in the addition of polynomials in equation (32).

The remaining task is to interpret the T coefficient by means of elements of the scattering matrix, i.e. to relate the first members of (28) and (29) directly to S matrix when the intermediate states are on the energy-shell.

Let us study the S matrix element between an arbitrary state n and a state consisting of one nucleon p and one meson q . One has, from (11)

$$\begin{aligned}\langle pq|S|n\bar{k}\rangle &= \langle p|a_e^{(-)}(\bar{q})S|n\bar{k}\rangle = \\ &= \delta pq, n\bar{k} + \frac{1}{(2\pi)^{\frac{3}{2}}} \int dx \frac{\exp[iqx]}{\sqrt{2q^0}} \left\langle p \left| \frac{\delta S}{\delta \varphi(x)} \right| n\bar{k} \right\rangle,\end{aligned}$$

so

$$(33) \quad \langle pq|S|n\bar{k}\rangle = \delta pq, n\bar{k} + \frac{i}{(2\pi)^{\frac{3}{2}}} \frac{\delta(p+q-k)}{\sqrt{2q^0}} T_{pq, n\bar{k}}.$$

For the S matrix element between a state n and a state with one nucleon p and two mesons $q_1 q_2$ one has

$$\langle pq_1 q_2|S|n\bar{k}\rangle = \langle p|a_{e_1}^{(-)}(\bar{q}_1)a_{e_2}^{(-)}(\bar{q}_2)S|n\bar{k}\rangle$$

by using equations (4) and (11), one obtains

$$\begin{aligned}(34) \quad \langle pq_1 q_2|S|n\bar{k}\rangle &= \frac{1}{(2\pi)^3} \int \left\langle p \left| \frac{\delta^2 S}{\delta \varphi_{e_1}(x) \delta \varphi_{e_2}(y)} \right| n\bar{k} \right\rangle \frac{\exp[i(q_1 x + q_2 y)]}{\sqrt{4q_1^0 q_2^0}} dx dy + \\ &+ \left[\frac{1}{(2\pi)^{\frac{3}{2}}} \sum_{\lambda=1}^n \int \left\langle p \left| \frac{\delta S}{\delta \varphi(x_2)} \right| n\lambda \right\rangle \delta_{a, p\lambda} \frac{\exp[iq_2 x_2]}{\sqrt{2q_2^0}} dx + \text{Permuted term (+2)} \right] + \delta_{pq_1 q_2, n\bar{k}}.\end{aligned}$$

This equation has been obtained by commuting the operators $a^{(-)}$ with S . The term $\delta_{a, p\lambda}|n\lambda\rangle$ is provided by action of an operator $a_{e_1}^{(-)}(q_1)$ over the state n when there exists in n one meson (p_λ) identical to q_1 . In this case $|n\lambda\rangle$ represents the incident state composed of the same particles as n with the exception of the meson p_λ .

Retaining the principal part of $\langle pq_1 q_2|S|n\bar{k}\rangle$ which consists of the first term of (34) (it is the only one which appears in the general case where n does not contain q_1 nor q_2) one has

$$\langle pq_1 q_2|S|n\bar{k}\rangle = \frac{1}{(2\pi)^3} \int \left\langle p \left| \frac{\delta^2 S}{\delta \varphi_{e_1}(x) \delta \varphi_{e_2}(y)} \right| n\bar{k} \right\rangle \frac{\exp[i(q_1 x + q_2 y)]}{\sqrt{4q_1^0 q_2^0}} dx dy,$$

which, following (28) and (30) is identical with

$$(35) \quad \frac{i}{(2\pi)^3} \delta(p + q_1 + q_2 - k) \frac{T_{pq_1 q_2, n\bar{k}}((q_1 - q_2)/2)}{\sqrt{4q_1^0 q_2^0}}.$$

Again we shall use (30) and (29) as the definition of the T matrix-elements

out of the energy-shell so that the only matrix-elements appearing in eq. (31) are those of the reaction matrix T .

It is now clear how one can derive a complete set of relativistic Low equations. Every transition matrix element between a state with one nucleon and r mesons and another one with one nucleon and s mesons will be expressible in a form analogous to (16) where the variational derivative of order $r+s$ will be expressible as the ordered product of two variational derivatives of order $(r+s-1)$ and 1. By the same process as outlined will be obtained an expression for this element consisting in a sum of terms, each one having the form of the inverse product of $(r+s-1)$ terms of the form $(E' - En + i\epsilon)$ multiplied by the product of two matrix elements of T . One of these matrix elements will be between the arbitrary state n and the initial nucleon plus one meson from the set of the ingoing and outgoing ones, the other element being between n and the final nucleon plus all the other mesons.

There are no analogous formulae for transition between states containing nucleon-antinucleon pairs, or hyperons, if any, although all these states must appear in the sums $\sum_n$. So the system is not quite complete in this respect.

A delicate feature of these equations is that they contain matrix elements from an arbitrary state n to a known state in which we are interested and not, as it can be done in the classical Low equation, between the studied state and an arbitrary one. On the whole this does not eliminate the possibility of doing effective calculations and approximations.

Finally, let us signal that the crossing relations

$$\begin{aligned}
 (36) \quad T_{q_2 q_2, q_3 q_3; q_1 q_1} &= T_{-q_1 q_1; -q_2 q_2, -q_3 q_3} \\
 &= T_{q_2 q_2, -q_1 q_1; -q_3 q_3} \\
 &= T_{-q_1 q_1, q_3 q_3; -q_2 q_2}
 \end{aligned}$$

are fairly evident on equation (24).

One could use (32) to study numerically the production of mesons by one meson bypassing to the non-relativistic limit, but this work has already been done directly in the Chew and Low model ⁽⁴⁾.

Finally let us note that (31) (32) is not the only possible form of Low equations obtainable by this method.

* * *

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⁽⁴⁾ R. OMNES: To be published.

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RIASSUNTO (*)

Servendoci dei metodi di Bogoliubov diamo qui una nuova dimostrazione dell'equazione di Low, mettendo così in evidenza il suo carattere generale. Il metodo è stato esteso per ottenere una serie di equazioni del tipo di quella di Low per la produzione di mesoni da parte di mesoni interagenti con un nucleone.

(*) *Traduzione a cura della Redazione.*