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To Jens Vigen <Jens.Vigen@cern.ch>

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Carmen Vasini

A Remark on Quantum Electrodynamics with Non-Vanishing Photon Mass and Lamb Shift Calculations.

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(ricevuto il 9 Gennaio 1956)

When treating quantum electrodynamics as the limiting case $\mu \rightarrow 0$ of a neutral vector meson theory with meson mass μ ^(1,3), it is generally assumed that the contribution of the longitudinal mesons vanishes in the limit. This statement should, however, be applied with some care, and it is the purpose of this note to show that the longitudinal mesons give a non-vanishing contribution to the cross-section for bremsstrahlung with emission of soft photons. This is of some interest in connection with the calculation of the Lamb shift ^(4,5), as the relation $\log(2k_{\min}/\mu) = \frac{5}{6}$ between the photon mass and an infrared non-covariant momentum cut off $k_{\min}$; necessary to join the covariant and non-covariant parts of the Lamb shift calculation as performed by FEYNMAN, is conveniently derived by calculating the bremsstrahlung cross section with the two methods and equating the results ^(6,7).

Incidentally, the contribution of the longitudinal mesons turns out to be the number $\frac{1}{6}$, which is rather famous in this connection; if we take only the transversal mesons into account we get the incorrect relation $\log(2k_{\min}/\mu) = 1$.

The probability for scattering of an electron by a static field with emission of photons with momenta smaller than ΔE (assumed to be small compared with the kinetic energy of the electron) is given by ⁽⁸⁾

$$(1) \quad W = \frac{e^2 |M_0|^2}{(2\pi)^3} \int_0^{\Delta E} \frac{d^3k}{2k_0} \left\{ \sum_i \left(\frac{p \cdot e_i}{p \cdot k} - \frac{p' \cdot e_i}{p' \cdot k} \right)^2 \right\},$$

⁽¹⁾ F. J. BELINFANTE: *Phys. Rev.*, **75**, 1321 (A) (1949); *Prog. Theor. Phys.*, **4**, 165 (1949).

⁽²⁾ F. COESTER: *Phys. Rev.*, **83**, 798 (1951).

⁽³⁾ R. J. GLAUBER: *Prog. Theor. Phys.*, **9**, 295 (1953).

⁽⁴⁾ R. P. FEYNMAN: *Phys. Rev.*, **74**, 1430 (1948) and **76**, 769 (1949).

⁽⁵⁾ J. B. FRENCH and V. F. WEISSKOPF: *Phys. Rev.*, **75**, 1240 (1949).

⁽⁶⁾ R. H. DALITZ: *Proc. Roy. Soc., A* **206**, 521 (1951).

⁽⁷⁾ L. R. B. ELTON and H. H. ROBERTSON: *Proc. Phys. Soc., A* **65**, 145 (L) (1952).

⁽⁸⁾ See e.g. J. M. JAUCH and F. ROHRLICH: *Helv. Phys. Acta*, **27**, 613 (1954), eqns (2-5).

M_0 is the matrix element for elastic scattering, p and p' are the initial and final four-momenta of the electron, $k_0 = \sqrt{|\vec{k}|^2 + \mu^2}$, and the summation is to be performed over three (in the case $\mu=0$ two transversal) polarization directions e_i satisfying $k \cdot e_i = 0$

As we have to do with vector mesons, that is $k^2 = -\mu^2$, this formula implies that we have neglected some terms in μ^2 in the numerator and denominator as compared with the original exact expression (eq (2) in ref ⁽⁸⁾) The omission in the numerator is trivially justified. As for the denominator, the remaining term $p \cdot k$ is of order μ at least, and a correction μ^2 will give a contribution to the final result of order $\mu \log \mu$ at most, and is thus of no consequence

It is well known (and can easily be verified by direct calculation) that because of the Lorentz condition the summation over the three (or two) polarization directions is equivalent to summation over all four directions ⁽⁹⁾, and thus the sum becomes

$$\frac{2m^2 + q^2}{(p \cdot k)(p' \cdot k)} - \frac{m^2}{(p \cdot k)^2} - \frac{m^2}{(p' \cdot k)^2}$$

where $q = p' - p$, m = electron mass

If we perform the integral, assuming $q^2/m^2 \ll 1$ and $\Delta E/\mu \gg 1$, we obtain

$$(2) \quad W = \frac{e^2 |M_0|^2}{(2\pi)^3} \frac{4\pi}{3} \frac{q^2}{m^2} \left(\log \frac{2\Delta E}{\mu} - 5/6 \right) +$$

If we instead put $\mu=0$ and integrate down to some lower limit $k_{\min}$, we have

$$(3) \quad W = \frac{e^2 |M_0|^2}{(2\pi)^3} \frac{4\pi}{3} \cdot \frac{q^2}{m^2} \log \frac{\Delta E}{k_{\min}} +$$

In this case it is easier to use the expression with summation over the two transversal directions only

We are now especially interested in the longitudinal mesons. If we write

$$W = \frac{e^2 |M_0|^2}{(2\pi)^3} \int_0^{\Delta E} \frac{d^3 k}{2k_0} \sum_i S_i,$$

the longitudinal contribution S_3 is given by

$$S_3 = \left(\frac{p \cdot e_3}{p \cdot k} - \frac{p' \cdot e_3}{p' \cdot k} \right)^2 \quad \text{with} \quad e_3 = \left(\frac{k_0}{\mu |\vec{k}|}, \vec{k}, i \frac{|\vec{k}|}{\mu} \right),$$

and is

$$(4) \quad S_3 = \frac{\mu^2}{|\vec{k}|^2} \left(\frac{p_0^2}{(p \cdot k)^2} + \frac{p_0'^2}{(p' \cdot k)^2} - \frac{2p_0 p_0'}{(p \cdot k)(p' \cdot k)} \right) = \frac{\mu^2}{|\vec{k}|^2 k_0^4} \frac{(q \cdot k)^2}{m^2} +$$

(in the last expression we have assumed $q^2/m^2 \ll 1$)

(*) Actually as we have made some approximations, we should perhaps not expect this equivalence to be valid exactly; nevertheless it turns out to be so

The approximation involved by putting $p_0 = p'_0$ can be justified by means of an argument similar to the discussion in connection with equation (1).

Integration gives the probability for emission of longitudinal mesons with momenta smaller than ΔE (still $\Delta E/\mu \gg 1$):

$$(5) \quad W_3 = \frac{e^2 |M_0|^2}{(2\pi)^3} \frac{4\pi}{3} \frac{q^2}{m^2} \frac{1}{6} +$$

A comparison of S_3 with the corresponding expression for a transversal meson shows that the probability for emission of a longitudinal meson with momentum $|\vec{k}|$ is equal to $\mu^2/(|\vec{k}|^2 + \mu^2)$ times the probability for emission of a transversal meson⁽²⁾. Though this ratio goes to zero with μ for every finite $|\vec{k}|$, it is of order one for $|\vec{k}| \leq \mu$, and, as the integral

$$\int_0^\mu \frac{|\vec{k}|^2 d|\vec{k}|}{(|\vec{k}|^2 + \mu^2)^{\frac{3}{2}}} = \int_0^1 \frac{x^2 dx}{(1+x^2)^{\frac{3}{2}}},$$

giving the contribution from transversal mesons with momenta smaller than μ , stays different from zero as $\mu \rightarrow 0$, it is obvious that also the longitudinal mesons do give a finite contribution.

It may be pointed out that S_3 is not Lorentz invariant. A vector meson with a specified polarization is, of course, not an invariant concept.

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