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Final State Photon Bremsstrahlung in $e^+e^- \rightarrow Z^0 \rightarrow \text{hadrons}$ as a Tool for a Precise Measurement of the Weak Quark Couplings

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Abstract

We discuss for the process $e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}$ how final state photons can be used to disentangle the weak couplings of up and down type quarks. Since the flavour composition of multihadronic events with final state photons is different from that of all multihadrons, a combination of observables from the two samples allows the couplings to be determined. Both the Z^0 width and the forward-backward charge asymmetry at centre-of-mass energies around M_{Z^0} can be used to constrain the weak couplings. Taking into account the theoretical uncertainties we estimate that with about 10^6 Z^0 's the sum of the square axial and vector couplings can be determined to $\delta(v^2 + a^2) \sim 4\%$ for up and down type quarks separately. The precision from asymmetry measurements will be less significant.

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1 Introduction

One of the major goals in the study of Z^0 decays is the determination of the couplings of the fundamental fermions to the Z^0 . They can be obtained using the cross section $\sigma(Z^0 \rightarrow f\bar{f})$, the forward-backward asymmetry $A_{f\bar{f}}^{FB}$, or observables involving measurements of the polarization of either the initial or the final state. Leptons have an unambiguous signature simplifying measurements of their axial and vector coupling. Disentangling the couplings of specific quark types is much more involved and requires a detailed understanding of the structure of jets.

In this letter we discuss how the final state photon radiation from quarks [1] provides a simple way to determine the couplings of down and up type quarks. The basic idea of the method is that the samples of all multihadronic events and of those with final state photons are composed of different fractions of quark types. Since photons couple to the square of the electric charge e_i of the quark, the fraction of charge $2/3$ quarks is enriched in the sample containing final state photons. Assuming that all down type quarks (all up type quarks) are produced with the same cross section, the total multihadronic sample consists of

$$N_{q\bar{q}} \propto 3 \cdot N_d + 2 \cdot N_u \quad (1)$$

with N_d and N_u denoting the respective yields from down and up type quarks. In contrast the flavour mixture in the sample with final state radiation is, ideally,

$$N_{q\bar{q}\gamma} \propto 3 \cdot e_d^2 \cdot N_d + 2 \cdot e_u^2 \cdot N_u \propto 3 \cdot N_d + 8 \cdot N_u \quad (2)$$

Therefore observables depending on the weak couplings acquire different values for the two samples. Combining the two measurements enables us to obtain the weak quark couplings. A first application of our method can be found in [2] based on about 25,000 hadronic Z^0 decays collected at LEP. As observables we consider in the following the width of the Z^0 and the forward-backward asymmetry at a centre-of-mass energy $E_{c.m.} = M_{Z^0}$. These two yield the combinations of $(v_i^2 + a_i^2)$ and $v_i \cdot a_i$ of the vector and axial couplings separately for down and up quarks. Together they lead to tight constraints of the vector and the axial couplings for down and up type quarks.

In this letter we first show that final state photons can be identified in decays of the Z^0 produced in e^+e^- collisions. Using general, but realistic assumptions about possible photon identification we derive efficiencies and background estimations. Assuming a million multihadronic Z^0 decays we will then discuss the method and uncertainties involved in the determination of the couplings from the Z^0 width. We then address extensions of the method to the forward-backward asymmetry and to samples with a restricted flavour tag. The achievable precision of the weak couplings will be compared with the ones using other methods of quark flavour tagging.

2 Photon Sources in the Reaction $Z^0 \rightarrow$ hadrons

Photons in multihadronic events produced in e^+e^- annihilations can originate from

- the incoming electrons (fig.1a)

or can be produced within the jet development. Although uncertain in details, the latter is well described by a showering process with the original quarks cascading down in virtuality by gluon emission. Finally the partons combine into hadrons which possibly decay. Four potential sources of photons can be identified:

- primary quarks that couple directly to the Z^0 (fig.1b),
- secondary quarks that are produced within the gluon cascade by the gluon splitting $g \rightarrow q\bar{q}$ (fig.1c),
- hadrons, which electromagnetically decay, particularly $\pi^0 \rightarrow \gamma\gamma$ (fig.1d)
- hadrons emitting bremsstrahlung.

For the purpose of measuring the weak couplings only the primary photons¹ reflecting the charge of the primary quarks are of interest. Here it is irrelevant if these quarks also emit hard gluons. Photons from the other sources are background and should be eliminated by appropriate selection criteria.

In evaluating the yield and differential distributions the possible interference of the various contributions have to be taken into account. They could lead to deviations from the simple relation (2). For this analysis only the interference of initial state radiation and primary photons is of importance. Its contribution is schematically given by

$$\frac{d\sigma_{q\bar{q}\gamma}}{dx_\gamma \cdot dc_\gamma} = |C^{initial}(c_\gamma, x_\gamma)|^2 + |C^{final}(c_\gamma, x_\gamma)|^2 + 2 \cdot \text{Re}[C^{initial}(c_\gamma, x_\gamma) \cdot C^{final}(c_\gamma, x_\gamma)] \quad (3)$$

with c_γ the cosine of the photon angle with respect to the beam axis, and $x_\gamma = 2 \cdot E_\gamma / E_{cm}$ the scaled photon energy. A calculation in order α^3 is presented in [3]. We will refer several times to results of a numerical integration of these formulae. Those are given on the parton level.

Photons from the initial state tend to be aligned along the beam and are theoretically well understood. They reduce the mass of the remaining multihadronic system. Since the cross section below the Z^0 mass is much lower than for $E_{cm} = M_{Z^0}$, initial state radiation is suppressed on the pole.

Around $E_{cm} = M_{Z^0}$ the interference term is proportional to $e_f \cdot (v_f^2 + a_f^2)$ and thus depends also on the weak couplings. It is potentially significant in a kinematical region where both the initial and final state photons contribute.

The photon contribution of interest for this analysis originates from the quarks. In first order QED the differential distribution for final state photon bremsstrahlung is [3]

$$\frac{d\sigma_{q\bar{q}\gamma}}{dx_\gamma \cdot dx_q} \propto \frac{4}{3} \frac{\alpha}{2 \cdot \pi} \cdot \frac{x_\gamma^2 + x_q^2}{(1 - x_\gamma) \cdot (1 - x_q)} \cdot \sum e_f^2 \cdot (v_f^2 + a_f^2) \quad (4)$$

with $x_i = 2 \cdot E_i / E_{cm}$, $\sum x_i = 2$, α the electromagnetic coupling constant, e_f the charge of quark type f , and v_f , a_f , the vector and axial couplings of quark f . The sum goes over all flavours. Apart from the couplings it is identical to the first order QCD matrix element for gluon bremsstrahlung. Since up type quarks radiate more likely, the quark mixture in radiative events is changed relative to the total event sample.

Within the shower process photons can also be produced off mass shell and subsequently branch into fermion pairs, predominantly electrons. They are estimated to contribute about 1% of the final yield [4].

QCD effects alter the yield and the differential distributions of photons. With smaller virtual masses Q^2 of the quark photon system higher order QCD effects become more important [3]. At

¹Our notation is: primary photons are emitted from primary quarks. Secondary photons are emitted from quarks originating from a gluon branching $g \rightarrow q\bar{q}$.

$Q^2 \sim 1$ GeV hadronization takes place. This low Q^2 region is theoretically uncertain. Restricting the analysis to photons emitted with high transverse momentum relative to the quarks, retains photons produced at high Q^2 , i.e. in a region where only the lowest order QED and QCD corrections are important. With these photons other potential problems are also avoided: collinear photon emission depends on the mass of the emitting fermion. For photons at large transverse momenta mass effects become unimportant. Also the background to primary photons is significantly reduced.

To estimate the yields and distributions of the background from secondary photons and/or hadronic decays one relies on specific models. On the other hand, general arguments suggest that their properties are sufficiently distinct that they can be distinguished from the interesting primary photons.

- Photons from secondary quarks (fig. 1c) are suppressed since the branching $g \rightarrow gg$ dominates over $g \rightarrow q\bar{q}$. Moreover, the energies of these secondary photons are much smaller than those from primary photons since at least two steps in the cascade of decreasing virtuality have to precede.
- Photons from hadron decays (fig. 1d), particularly from π^0 's, produced in the fragmentation, are found mostly within jets, i.e. with other particles in their vicinity.
- The same applies for bremsstrahlung from hadrons. In addition it should be strongly suppressed because of the relatively high hadron masses. This source will be neglected in the analysis.

In the following section we will discuss cuts to select final state photons efficiently. For a precise determination of the quark couplings the selection is required to have little background, not to bias any quark flavour, and to preserve the proportionality of equation (2).

3 Selection of Final State Photons

To eliminate the background sources we apply a number of cuts. Their effect on the signal and the various competing processes is displayed in fig.2 and table 1. The numbers are obtained with a Monte Carlo simulation [5].

The efficiency of rejecting the fragmentation background due to neutral 'jets' depends on the specific properties of the detection device (longitudinal, lateral segmentation of the calorimeter, density of the showering material etc.). In general only π^0 's will pose a potential problem. In the figures and tables the following procedure is therefore adopted: Only π^0 's will be considered as fragmentation background and for all kinematical cuts they are conservatively assumed to be stable. The table shows that without cuts initial state radiation and fragmentation remnants are the most severe background sources². Secondary photons contribute only 3% of all final state photons.

In a first step initial state radiation is suppressed by selecting photons away from the beam direction: $|\cos\theta| \leq 0.8$, where θ is the angle with respect to the beam direction. Whereas final state photons are only reduced proportional to the limitation of the angular acceptance, only half of the initial state photons are retained. The energy of the remaining primary, secondary, and initial state photons, and of π^0 's are plotted in fig.2a.

The primary photons have a harder momentum spectrum than the background. Selecting photons with $x_\gamma = 2 \cdot E_\gamma / E_{cm} \geq 0.16$ ($E_\gamma \geq 7.5$ GeV) discards most of the initial state photons and reduces

² A wide preliminary cut of $c_\gamma < 0.98$ and $E_\gamma > 0.4$ GeV are used for this table.

Table 1: Contribution to Photon yield for $1 \cdot 10^6 Z^0$'s

Selection	primary γ 's	secondary γ 's	Initial state γ 's	π^0
<i>all</i>	15,000	350	29,900	11,000,000
$ \cos \theta \leq 0.8$	11,850	270	14,300	8,000,000
$x_\gamma \geq 0.16$	3,850	25	150	450,000
$p_T \geq 5 \text{ GeV}$	1,850	8	105	10,000
Isolation	1,720	8	105	800
Cluster shape	1,650	8	100	200

the fragmentation background by more than an order of magnitude. Primary photons are reduced by a factor 3.

The distributions of the retained photons as a function of the transverse momentum p_T with respect to the thrust axis (including the photon) are displayed in fig. 2b. A cut in $p_T \geq 5 \text{ GeV}$ reduces the fragmentation background by another factor 40. Primary and secondary photons are much less affected. Note that the contribution from initial state radiation is only marginally reduced since it is in general not correlated with the remaining jets.

Since in first order QCD or QED the differential distribution of jets (i.e. hard gluons) and photons is identical, hadron production dominates over photons by $O(\alpha_s/\alpha)$ even away from the jet axis. The hadron background can be strongly suppressed by requiring the photon candidate to be isolated. The remaining contribution from π^0 's is still significant and can be further reduced by exploiting details of the shape of electromagnetic clusters. Its efficiency and power of background rejection depends on the specific detection method. We refrain from discussing details. Most of the background from π^0 's is concentrated at low energies. There we assume that 75% of the π^0 's can be rejected whilst retaining 95% of genuine photons. At higher energies the rejection will be less efficient, but in total only very few π^0 's are retained.

In summary, we will base the analysis on the following cuts:

- $|\cos \theta| \leq 0.8$
- $x_\gamma = 2 \cdot E_\gamma / E_{cm} \geq 0.16$
- $p_T \geq 5 \text{ GeV}$
- the momentum sum of all charged and neutral particles within a cone of half opening angle $\alpha = 10^\circ$ around the photon must not exceed 250 MeV.

Within the shower models a sequence of emissions of gluons and photons is defined. Using the model of [5] we find that, without cuts, most photons are emitted after the first gluon bremsstrahlung. Applying our cuts 83% of the photons are emitted *before* gluon radiation. This indicates that for our selection higher order QCD corrections become less important.

With these cuts we retain 11% of the primary photons and 23% of those emitted before the first gluon. The remaining background is about 20%. Half of it is due to the well understood initial state radiation. The other half is due to the less certain fragmentation background. Several ways of estimating its contribution are possible and have been applied by previous experiments. Apart from a statistical analysis of cluster properties, isospin symmetry can be invoked to estimate the π^0 yield from

that of isolated charged pions. As a result the systematic uncertainty will be reduced with increasing statistics and is expected to be smaller than the statistical uncertainty. Note that, if necessary, the fragmentation background can be reduced further by tighter cuts.

The retained sample has not only a good signal to background ratio, but also avoids biases against specific quark flavours and retains the proportionality (2): Using the Monte Carlo simulation of [5] we evaluated the dependence of the photon yield on the quark mass. Within the shower models used, biases towards light quarks are introduced if no cuts on the photon are applied. As mentioned before, jet development is viewed as a branching process with quarks cascading down in virtuality until they reach 'masses' of about one GeV. As a result light quarks will branch more often, whereas e.g. the bottom quark stops emitting photons relatively early. Before our cuts on the photon the ratio of primary photons in $Z^0 \rightarrow b\bar{b}$ over those in $Z^0 \rightarrow d\bar{d}$ is about 0.6. This ratio is displayed in figs. 3a,b as a function of x_γ , and (after selecting $x_\gamma > 0.16$) as a function of the transverse momentum p_T with respect to the thrust axis. The results are obtained with a Monte Carlo simulation. Whereas an increase in the minimum energy hardly affects the ratio, it approaches unity for $p_T \geq 5$ GeV. This result underlines the importance of the p_T cut.

A potential distortion from the proportionality (2) is due to the interference of initial and final state photons. This contribution is not included in [5] but can be estimated by integrating the matrix element given in [3]. On the pole the interference term contributes about 0.4% to the total cross section of $q\bar{q}\gamma$ production for the cuts applied. The interference is positive for down type quarks (+1.6%) and negative for up type quarks (-0.9%). The overall contribution to $\sigma(q\bar{q}\gamma)$ is small, but the flavour dependence has to be taken into account for a high precision determination of the up and down type couplings. One should be aware that the significance both of the initial state and of the interference increases rapidly away from the pole. Whereas the cross section for the initial state contribution depends only marginally on the quark flavour, there is a large difference in the interference contribution for up and down type quarks. In fig. 4 we display the relative contribution of the initial and final state interference on the total $q\bar{q}\gamma$ cross section

$$f = \frac{2 \cdot \text{Re}[C^{\text{initial}} \cdot C^{\text{final}}]}{\sigma_{q\bar{q}\gamma}} \quad (5)$$

for up and down type quarks around the Z^0 mass. The flavour specific contribution from the interference changes rapidly with energy and is substantial even slightly away from the peak. Assuming the couplings expected in the standard model, the contributions from all flavours add up to almost zero. This delicate cancellation, however, depends on the relative contributions of up and down type quarks and thus on the true weak couplings. These distortions have to be taken into account when combining events around the Z^0 peak. To simplify the discussion we will consider only data taken at the Z^0 mass.

Before turning to the evaluation of the quark couplings let us point out that it is a new feature in e^+e^- collisions that a high fraction of the primary photons can be unambiguously identified. At lower e^+e^- center-of-mass energies evidence for final state radiation has come predominantly through the interference of initial and final state photon emission. This interference results in a forward-backward charge asymmetry [1] that has been observed by various experiments at c.m. energies around 30 GeV [6]. At these energies identification of photons radiated by a quark was prevented by the abundance of initial state radiation and by the broadness of jets. Typically only 15% of the experimentally tagged photons were due to final state radiation. On the Z^0 all of these problems are substantially reduced. The stronger collimation of jets allows prompt photon to be identified much closer to the jet axis. In addition the importance of initial state radiation is considerably reduced.

Although photon emission is suppressed by $O(\alpha)$, the expected yield after the above selection criteria is still sizeable and 0.2% of all hadronic events are expected to have an observable final

state photon. The further discussion will be based on $1 \cdot 10^6$ hadronic decays of the Z^0 or $\sim 2,000$ multihadrons with hard photons.

The aim of the analysis is to estimate to what precision the weak quark couplings can be determined using these photon events. In the standard model notation the vector and axial couplings are given by

$$v_f = 2 \cdot I_{3,f} - 4 \cdot e_f \cdot \sin^2 \theta_w \quad \text{and} \quad a_f = 2 \cdot I_{3,f} \quad (6)$$

with $I_{3,f}$ the weak isospin, e_f quark charge, and θ_w is the weak mixing angle. With the standard model expectation and $\sin^2 \theta_w = 0.23$ this leads to $v_d = 0.69$, $v_u = 0.39$, and $a_{d,u} = \mp 1$. In the following discussion we assume that the electron couplings a_e and v_e are known such that their contributions to the errors can be neglected. Note that their sign can be determined from the τ polarization or from combining measurements of $e^+e^- \rightarrow e^+e^-$ and ν scattering. In addition we assume that only the known five quarks contribute to the hadronic width. In fact, we determine the *sum* of all d quark type couplings and the *sum* of all u type quark couplings. For simplicity we quote the couplings for one quark of a certain type, implicitly assuming the weak couplings of all charge 1/3 and of all charge 2/3 quarks to be the same³.

4 Determination of $v^2 + a^2$ Using the Z^0 Width

The overall magnitude of weak couplings $c_f = v_f^2 + a_f^2$ of up and down type quarks can be determined by combining the measurement of the total hadronic width Γ_{had} with the yield of events with hard photons.

The hadronic width is given in the standard model in first order QCD as

$$\Gamma_{had} = N_c \frac{G_\mu \cdot M_Z^3}{24 \cdot \pi \sqrt{2}} \cdot \left(1 + \frac{\alpha_s}{\pi}\right) \cdot (3 \cdot c_d + 2 \cdot c_u) \quad (7)$$

where N_c is the number of colours, G_μ the muon decay constant, $M_Z = 91.18$ GeV the mass of the Z^0 and α_s the strong coupling constant at Z^0 energies. Therefore, measuring Γ_{had} fixes the relation $(3 \cdot c_d + 2 \cdot c_u)$.

The radiative width in first order QED is proportional to a novel combination of electro weak charges [7]

$$\Gamma_{q\bar{q}(\gamma)} = N_c \cdot \frac{G_\mu \cdot M_Z^3}{24 \cdot \pi \sqrt{2}} \cdot \frac{3}{4} \frac{\alpha}{\pi} \cdot \frac{1}{9} (3 \cdot c_d + 8 \cdot c_u) \quad (8)$$

To obtain this formula all photonic final states are added up including collinear and infrared photons and the vertex correction is taken into account properly. As discussed above, only hard and isolated photons are directly observable. Their yield depends on the selection criteria applied and can best be normalized with a Monte Carlo simulation including final state photons. If only primary photons are selected the yield is also proportional to $3 \cdot c_d + 8 \cdot c_u$.

These equations can be expressed in terms of combinations of the weak quark couplings

$$S_{q\bar{q}} = 3 \cdot c_d + 2 \cdot c_u = \Gamma_{had} \frac{24 \cdot \pi \sqrt{2}}{N_c \cdot G_\mu \cdot M_Z^3} \frac{1}{1 + \alpha_s/\pi} \quad (9)$$

³ Within the standard model this is true to good accuracy. The bottom quark coupling deviates from the universality due to its particular affinity to both the Higgs boson (largest mass) and the top quark (same generation). Depending on the Higgs and top masses, its partial width differs by about 1 % from that for the strange and down quark.

$$S_{q\bar{q}\gamma} = 3 \cdot c_d + 8 \cdot c_u = N_{q\bar{q}\gamma}^{obs} \cdot \frac{(3 \cdot c_d + 8 \cdot c_u)^{MC}}{N_{q\bar{q}\gamma}^{MC}} \quad (10)$$

where $N_{q\bar{q}\gamma}$ is the number of multihadronic events with hard final state photons and 'MC' denotes the Monte Carlo expectation, 'obs' the observed yield.

These two measurements provide a system of linear equations that can be solved for c_u and c_d . For the canonical event yields the statistical accuracy for c_d and c_u is ~ 0.04 or 3%. We address the systematic uncertainty next.

The error of $S_{q\bar{q}}$ is mainly due to uncertainties in Γ_{had} and α_s . Apart from statistical uncertainties, the main contributions to $\delta\Gamma_{had}$ stem from the luminosity measurement, and from the acceptance. For the $10^6 Z^0$'s the uncertainty on the hadronic width will be $\sim 1\%$. The error in the QCD corrections can be estimated from e^+e^- measurements at lower energies. From combining measurements of $\sigma(e^+e^- \rightarrow \text{hadrons})$ from various experiments at lower energies, the QCD contribution (i.e. the expansion in *all* orders of α_s) is measured to be $6.0 \pm 1.1\%$ at 34 GeV [8]. Since α_s decreases with energy this contribution is smaller at Z^0 energies. We will assign an error of 1% to $S_{q\bar{q}}$ due to the QCD contribution. In total we therefore have

$$\frac{\delta S_{q\bar{q}}}{S_{q\bar{q}}} = \sqrt{\left(\frac{\delta\Gamma_{had}}{\Gamma_{had}}\right)^2 + \left(\frac{\delta(\alpha_s/\pi)}{\delta(1 + \alpha_s\pi)}\right)^2} \sim 0.015 \quad (11)$$

We display $S_{q\bar{q}}$ in the c_d - c_u plane in fig. 5a assuming standard model values for the couplings. The band reflects the one standard deviation uncertainty.

Uncertainties in $S_{q\bar{q}\gamma}$ originate predominantly from QCD corrections. Higher order corrections in α_s are taken into account in the Monte Carlo program of [5] using a leading log approximation. An uncertainty could arise from details of their implementation. In general they should be small since our cuts suppress photons in multi-jet events. Moreover, the observed structure of multijet production is well reproduced by the JETSET Monte Carlo. The measured jet rates are in excellent agreement with the Monte Carlo expectation [9]. Also the angular correlation in 4-jet events is well reproduced [10]. Small discrepancies exist, but they hardly affect this analysis. The simulation allows also multiple photon emission from the parton shower. Since the branching $q \rightarrow qg$ and $q \rightarrow q\gamma$ have similar properties (apart from the coupling strength), the same argument as for higher order QCD corrections applies for these higher order QED - effects. We assume possible uncertainties to be less than 1%.

The treatment of initial and final state interference is missing in the simulation. Its effect in $O(\alpha)$ was discussed in the previous section. Higher order QCD corrections alter the impact of the interference term. However, as discussed before, photons in events with hard gluon emission tend to be less energetic and will be less frequently accepted by our cuts. We assign an uncertainty of 0.5% to this effect, which amounts to the total size of the interference contribution at the Z^0 mass assuming standard model couplings.

As discussed above, virtual photons can branch into fermion pairs. For our analysis this effect of about 1% has to be taken into account. However, it is not yet implemented in any simulation.

Additional uncertainties on $S_{q\bar{q}\gamma}$ are due to the modelling of hadronization. We have analyzed several potential uncertainties in the fragmentation process that could affect the number of photons retained by the cuts. To this end we varied several fragmentation parameters used in the event generation [5]. Possible uncertainties can be split into those on the parton level, leading to a different yield of produced photons, and those due to hadron distributions which could affect the efficiency of the cuts.

The fraction of photons was found to depend on the cut-off mass Q_0 and the QCD scale parameter Λ_{QCD} determining the parton distribution.

- The cut-off parameter Q_0 determines the minimum virtuality of quarks before they hadronize. Thus photons or gluons are emitted only from quarks of virtual masses of more than Q_0 . An increase of Q_0 leads to more photons within the acceptance (fig. 6a). Several different measurements, specifically sensitive to the value of Q_0 prefer its value to be 1 GeV (see [11,12]). A variation of ± 0.5 GeV changes $N_{q\bar{q}\gamma}$ by 2%.
- The QCD scale parameter Λ_{QCD} ⁴ governs the frequency of parton branchings in the shower model. The relative occurrence of photon and gluon emission depends on the ratio of the coupling constants α/α_s . The variation of the photon yield after cuts with Λ_{QCD} is shown in fig. 6b. For the preferred value of Λ_{QCD} of 300 MeV, a variation of (-10,+20) MeV [12] reflects itself in a change of 0.5% in the number of photons.

In both cases the quoted error reflects the uncertainties in Q_0 and Λ_{QCD} obtained by various experiments. Note that the variation of the event topologies with changes of these parameters are correlated: an increase of Q_0 can be compensated by a larger Λ_{QCD} [15]. As can be seen from figs 6a,b the effect on the photon yield is similar.

Uncertainties due to the hadronization mechanism are usually estimated by comparing the predictions from various simulation programs. As yet final state photons have only been incorporated into one Monte Carlo program. Moreover, the outcome of the simulation programs depends on the choice of free parameters related to details of the parton branching and/or the hadronization. The simulation used provides an excellent description of the event topologies and inclusive particle distributions over a wide range of centre-of-mass energies from $E_{cm} = 14$ GeV to 91 GeV [13,14,12]. As a result its parameterizations are already significantly constrained by the data. This will be even more true on basis of 10^6 hadronic events assumed for this analysis. Differences between fragmentation models will be either solved or washed out. The dependence of the final yield on fragmentation effects was therefore studied with model [5] by varying parameters that could affect the isolation requirement, which is the dominant source of uncertainty. In particular we

- made the jets wider by changing the average transverse momentum of the hadrons to the quark axis,
- changed the coherent gluon branching into a non-coherent one.

Neither of the variations produced any significant change in the final photon yield. Potentially critical jet properties like the particle and energy flow of soft particles, that could affect the isolation criteria, can be specifically checked with the data.

Combining the potential uncertainties, we attribute a systematic error of 2.5% to $S_{q\bar{q}\gamma}$. To go beyond that requires an improved confidence into the fragmentation models. Eventually this may be achieved, but this is beyond the current analysis. To evaluate the influence of this accuracy on the weak quark couplings we combine the systematic error with the statistical uncertainty which is of comparable size for the assumed data set. Solving the linear equation system (10), (11) taking into account the systematic and statistical errors yields a precision of

$$\frac{\delta c_d}{c_d} = 0.03 \quad ; \quad \frac{\delta c_u}{c_u} = 0.05 \quad (12)$$

⁴Note that Λ_{QCD} is specific to the shower model used and cannot be identified with Λ_{MS}

The resulting allowed range in the c_u versus c_d plane is also displayed in fig. 5a. The corresponding regions for the axial and vector couplings of up and down type quarks are displayed in figs. 5b and c. Since only the magnitude $v^2 + a^2$ can be determined, this region is a circular band in the v_f versus a_f plane for the two quark species.

5 Determination of $v \cdot a$ from the Forward-Backward Asymmetry

The couplings can be further constrained using the forward-backward asymmetry. Neglecting QED and QCD contributions, the forward-backward asymmetry is proportional to $(v_f \cdot a_f)/(v_f^2 + a_f^2)$. At $\sqrt{s} = M_Z = 91.14$ GeV one ideally expects [16]

$$A_{d\bar{d}}^{FB} = +0.0980, A_{u\bar{u}}^{FB} = +0.0671$$

(here the top and the Higgs masses are assumed as $m_t=150$ GeV and $m_H=100$ GeV). Without knowledge of the quark flavour only the charge asymmetry in events can be measured, yielding a significantly reduced observable asymmetry.

$$A_{q\bar{q}}^{charge} \propto -3 \cdot \frac{v_d^2 + a_d^2}{\Sigma(v_f^2 + a_f^2)} \cdot \frac{v_d \cdot a_d}{v_d^2 + a_d^2} + 2 \cdot \frac{v_u^2 + a_u^2}{\Sigma(v_f^2 + a_f^2)} \cdot \frac{v_u \cdot a_u}{v_u^2 + a_u^2} \quad (13)$$

$$= P_e \cdot \left[-3 \cdot \frac{v_d \cdot a_d}{\Sigma(v_f^2 + a_f^2)} + 2 \cdot \frac{v_u \cdot a_u}{\Sigma(v_f^2 + a_f^2)} \right] \sim -0.030 \quad (14)$$

with

$$P_e = 3 \cdot \frac{v_e \cdot a_e}{v_e^2 + a_e^2}$$

where v_e, a_e are the respective couplings for electrons. On the other hand, the asymmetry for events with hard photon radiation, being of a different mixture of up and down type quarks is expected to be (neglecting at the moment contributions from the interference of initial and final state radiation)

$$A_{q\bar{q}\gamma}^{charge} = P_e \cdot \left[\frac{-3}{9} \cdot \frac{v_d \cdot a_d}{\Sigma e_f^2(v_f^2 + a_f^2)} + \frac{8}{9} \cdot \frac{v_u \cdot a_u}{\Sigma e_f^2(v_f^2 + a_f^2)} \right] \sim +0.025 \quad (15)$$

Here and in what follows we define the asymmetry A^{charge} based on the number of positively charged primary quarks in the two hemispheres with respect to the beam direction. Denoting the numbers as N_1, N_2 for the intervals $[1,0]$ and $[0,-1]$ of $\cos\theta$ respectively, $A^{FB} = (N_1 - N_2)/(N_1 + N_2)$. Note that events with a hard photon may lead to topologies with both quarks in one hemisphere.

The above equations can be rewritten as

$$\frac{1}{P_e} \cdot A_{q\bar{q}}^{charge} \cdot S_{q\bar{q}} = -3 \cdot v_d \cdot a_d + 2 \cdot v_u \cdot a_u \quad (16)$$

$$\frac{1}{P_e} \cdot A_{q\bar{q}\gamma}^{charge} \cdot S_{q\bar{q}\gamma} = -3 \cdot v_d \cdot a_d + 8 \cdot v_u \cdot a_u \quad (17)$$

Once $S_{q\bar{q}}$ and $S_{q\bar{q}\gamma}$ are known from the total and radiative hadronic width, $v_d \cdot a_d$ and $v_u \cdot a_u$ can be determined from these two equations.

Initial state radiation modifies the expected asymmetry. After photon emission the resulting e^+e^- mass is lower than the nominal c.m. energy. Since, for each flavour, the electroweak asymmetry is rapidly decreasing with lower energy A^{charge} is also changed. The resulting *charge* asymmetry depends

on the variation in flavour content with energy and the detailed dependence for each flavour type. This effect is important for both the full multihadron sample and the one with final state photons. With the initial state generator of [17] as implemented in [5] we find the overall forward-backward charge asymmetry to be modified to

$$A_{q\bar{q}}^{charge} = -0.032 \quad A_{q\bar{q}\gamma}^{charge} = +0.018$$

We will now consider the theoretical and experimental uncertainties and distortions due to the experimental procedure of measuring the quark charge. Systematic uncertainties for the determination and interpretation of the quark asymmetries have been summarized in [18]. They amount to ~ 0.002 and are dominated by the uncertainty in α_s in the QCD correction of the asymmetry.

For the $q\bar{q}\gamma$ events also the interference of initial and final state bremsstrahlung has to be considered. Although on the Z^0 the total contribution of this interference is small, it has significant influence on the asymmetry: it is destructive or constructive depending on the quark flavour and the polar angle. The asymmetry due to this interference has been observed by several experiments operating around $E_{cm} = 30$ GeV [6]. On the Z^0 its effect is to reduce the magnitude of the asymmetry for down type quarks by 2.3% and to increase the asymmetry for up type quarks by 1.7%. This shift in the asymmetry depends on a mixture of terms depending on $v \cdot a$ and $v^2 + a^2$. Since here we are interested in $v \cdot a$ the shift has to be corrected for. The precision with which it is known contributes to the systematic error. To estimate its effect we have varied the isolation cut and the p_T cut within experimentally reasonable limits of $5^\circ < \alpha < 15^\circ$ and $2.5 < p_T < 7.5$ GeV. We found that the effect from the interference term changes the asymmetry by less than 0.003. Higher order QCD corrections may also alter the observable asymmetry, but as mentioned before, their effect should be rather small due to the cuts applied.

On the experimental side the largest systematic uncertainty stems from the challenge of finding the charge direction of the primary quarks from the many hadrons bundled in jets. The understanding of jet development suggests that the properties of the primary quarks are dominantly reflected in the properties of the most energetic particles in a jet. This theoretical conjecture has been supported by several experiments (see review in [11]), leading to the proposal of the weighted jet charge [19]

$$Q = \sum x_i^\alpha \cdot q_i \quad (12) \quad (18)$$

Here $x_i = 2 \cdot p_i / E_{cm}$ is the scaled momentum of particle i , q_i its charge and α is a free parameter that emphasises high momentum particles. From Monte Carlo studies the optimum is found to be ~ 0.75 . The sum goes over all particles in a jet. The reliability r for determining the charge sign can be improved by using both jets to form $Q_1 - Q_2$. It depends on the quark flavour i and is largest for charge 2/3 quarks (~ 0.83) and smallest for the bottom quark (~ 0.68). As a result the observed asymmetry is reduced to

$$A^{charge, obs} = \sum (2 \cdot r_i - 1) \cdot A_{FB,i}^{charge} \quad (13) \quad (19)$$

where the sum goes over all flavours. Here we assumed that r does not depend on the polar angle. The expected observable asymmetry is

$$A_{q\bar{q}}^{charge} \sim -0.008 \quad , \quad A_{q\bar{q}\gamma}^{charge} \sim +0.017 \quad (20)$$

The method of charge determination has been successfully applied at lower c.m. energies (e.g. [20]). The systematic error in r comes from the uncertainties in modelling the fragmentation and from the potential losses of (low energetic) particles due to acceptance cuts. Several cross checks with the data

can be made by e.g. analyzing the product or difference of the weighted jet charges. With a million hadronic Z^0 decays the statistical precision of these checks will be $\sim 10^{-3}$ or $\delta A_{q\bar{q}}^{charge} \sim 0.002$. Apart from the global reliability the relative r for specific flavours is important. With the large statistics available, fragmentation parameters like the charm and bottom fragmentation function will be known with high accuracy. In addition the assigned charge reliability can be cross checked with events of a tagged flavour. In summary we attribute an uncertainty of $\delta A_{q\bar{q}}^{charge} = 0.004$ due to the charge determination.

As seen from the discussion above, several corrections have to be applied to the data before relations (16) and (17) can be used. In summary we believe they will contribute an error of 0.006 to the asymmetry of $q\bar{q}\gamma$ events. Together with the smallness of the observable asymmetry this errors imposes severe limitations on the achievable accuracy. For a geometrical acceptance of hadronic events of $|\cos \theta_{thrust}| \leq 0.9$, the products of the axial and vector couplings will be measureable with

$$\frac{\delta(v_d \cdot a_d)}{v_d \cdot a_d} \sim 0.50 \quad ; \quad \frac{\delta(v_u \cdot a_u)}{v_u \cdot a_u} \sim 0.90 \quad (21)$$

for 10^6 Z^0 's. Note that according to the above discussion the precision will improve with larger statistics. The cross over point where statistical and systematic errors are comparable is at about the tenfold statistics.

With these errors the relative sign of the axial and vector couplings is determined. Together with the results on $v^2 + a^2$ from the width, the hyperbolas in fig. 5b and c denote the allowed regions from the asymmetry measurement.

The remaining ambiguity can be resolved using data from lower e^+e^- energies. From the analysis of $\sigma(e^+e^- \rightarrow \text{hadrons})$ and the forward-backward charge asymmetry of charm and bottom quarks, measured by various experiments between 5 and 55 GeV ⁵ Marshall [21] concludes

$$v_d = -0.35 \pm 0.95 \quad a_d = -1.02 \pm 0.15 \quad (22)$$

$$v_u = -0.03 \pm 0.75 \quad a_u = 1.13 \pm 0.10 \quad (23)$$

These results single out the correct region.

6 Applications to Partially Tagged Events

Up to now the discussion was based on all hadronic events. The method can also be applied to samples containing a different mixture of up and down type quarks. This is particularly interesting for samples containing combinations of up, down, and strange quarks, which can not so easily be tagged individually. All these considerations require a substantially larger statistics. We therefore assume $5 \cdot 10^6$ multihadrons, as expected for the first phase of LEP operation.

- A combination of up, down, and strange quarks together can be tagged by selecting events with a charged particle of high x [22]. Monte Carlo simulations suggest that for $x \geq 0.6$ one retains 4% of all u, d, s events with a marginal background from charm and bottom quarks. This leads to about 120,000 tagged events. Of those 250 should have a final state photon within our acceptance. As a result one obtains $\delta c_d/c_d \sim 0.05$, $\delta c_u/c_u \sim 0.14$, $\delta(v \cdot a)_d/(v \cdot a)_d \sim 0.3$, $\delta(v \cdot a)_u/(v \cdot a)_u \sim 1$.

⁵The weak effects at these energies are dominated by the interference of γ and Z^0 exchange. Therefore $\sigma(e^+e^- \rightarrow \text{hadrons}) \propto \sum v_f$ and $A_f \propto a_f$. These measurements are therefore directly sensitive to the *sign* of the vector and axial coupling provided the sign of the lepton couplings is known.

- More demanding is the identification of hadrons. If a high momentum $\pi^\pm$ can be identified, one can use this sample to disentangle up and down couplings (note: now the up and down quarks, not their types are considered). Depending on how well the pion can be identified, one expects $x_\pi \geq 0.6 \sim 40,000 - 80,000$ tagged up and down events and 100 - 200 with a hard photon. This implies $\delta c_d/c_d \sim 0.15$, $\delta c_u/c_u \sim 0.20$. Similar studies can be made for $K^\pm$ or $K^{*\pm}$. Even with this large statistics, the asymmetry measurement does not yield a relevant precision.

In these cases only the statistical error was considered. It is beyond this paper to discuss the systematic errors. Note that for the asymmetry measurement one does not require the weighted charge, since most of the tagged particles carry the sign of the primary quark.

The strong variation of the interference of initial and final state photons with c.m. energy, discussed in section 3, offers a way to determine the weak couplings. Its dependence on some combination of the weak couplings is more involved. A way to measure the interference is via the ratio $N_{q\bar{q}\gamma}/(N_{q\bar{q}\gamma}^{init} + N_{q\bar{q}\gamma}^{final})$ where $N_{q\bar{q}\gamma}^{init,final}$ denotes the expected number of events with hard photons expected from initial and final radiation (without interference). Since the interference term almost cancels in the sum over all quark flavours, the sensitivity to the weak couplings will be low. It would be preferable to use flavour tagged events, but the statistics required are prohibitive. To observe in the ratio a significant deviation from unity requires several million of Z^0 events. It seems unlikely that such a sample will ever be available off the Z^0 peak.

7 Conclusion and Comparison to Standard Tagging Methods

We have shown how final state photons in multihadronic Z^0 decays can be used to derive the quark couplings. With the one million events a precision of about 4% for $c_i = v_i^2 + a_i^2$ and of 50 and 90%, respectively, for $v_i \cdot a_i$ can be expected.

These numbers can be compared to the standard tagging methods for single flavours. They use completely or partially reconstructed hadrons indicative of a certain flavour: D^* 's for charmed events, ϕ 's for strange events or protons for up type events. Their efficiency is typically a few percent, with the positive exception of the bottom quark, tagged by either high p_T leptons or secondary vertices, that could reach 10-20% and the negative exception of the down quark which is elusive. These specific tags have the advantage of being in principle sensitive to possible differences of the couplings of different flavours. This is especially important for those of the bottom quark that are particularly affected by top and Higgs mass effects. However, the potential sensitivity of the standard methods of flavour tag is limited by systematic problems.

The conventional methods have a 10 - 100 times better efficiency than the photon tag, but the errors in $\Gamma_{f\bar{f}}$ or $v^2 + a^2$ will be restricted due to systematic limitations. These are due to uncertainties in the branching ratios of charmed and bottom hadrons, to the relative particle yields in the fragmentation or to the shape of the fragmentation function. As a result the systematic uncertainty will be $O(10 - 20\%)$. For large event samples the errors can be significantly reduced by tagging the flavour of both jets [23] at the expense of a much reduced efficiency. It will be about 1% for bottom quarks and between 10^{-3} and 10^{-4} for other flavours. In contrast the determination of $v^2 + a^2$ using final state photons will give a much better precision for all quark flavours with the possible exception of the bottom quark.

The situation is different for the measurement of $v \cdot a$ via asymmetry. Here the smallness of the effective charge asymmetry renders a high precision from the photon method unlikely. For asymmetry measurements using the standard tagging methods, on the other hand, many of the uncertainties

mentioned in the previous paragraph become irrelevant. In these cases one can reach $\delta(v_i \cdot a_i)/(v_i \cdot a_i) \sim 0.05 - 0.25$ depending on the flavour. This is superior to what one can hope for from the photon sample.

Due to the complexity of jets it will be difficult to unravel the weak quark couplings in Z^0 decays. Minimizing the errors requires the application of various kinds of flavour tag. Here we have discussed a new method which is completely independent from those previously suggested. It will provide additional significant constraints on the allowed range for the weak quark couplings.

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9 Figures

Figure 1: Basic contributions to photon production in $e^+e^- \rightarrow \text{hadrons}$: a. initial state radiation, b. final state radiation from primary quarks, c. final state radiation from secondary quarks, d. from π^0 decays.

Figure 2: Kinematical distributions of photons from various sources: final state photons from primary quarks (full line), from initial state photons (ISR) (dashed line), from final state photons from secondary quarks (full line), and from π^0 's (dotted line). Shown are a. the distributions in the scaled energy $x_{\gamma,\pi^0} = 2 \cdot E/E_{cm}$ for $|\cos\theta| < 0.8$, and b. the distribution in the transverse momentum with respect to the thrust axis for $x_\gamma > 0.16$.

Figure 3: Ratio of the accepted photons in $b\bar{b}$ events over those in $d\bar{d}$ events a. as a function of the minimum scaled energy of the photon, b. for $x_\gamma > 0.16$ as a function of the transverse momentum p_T of the photon to the thrust direction.

Figure 4: Relative contribution of the interference of initial and final state photons according to the matrix element of [3]. Displayed are, depending on the c.m. energy, the contributions from up and down type quarks, and of the sum over all flavours.

Figure 5: Allowed regions for the weak couplings of up and down type quarks as obtained from Γ_{had} and the number of events with hard photons (a). Shown are the one standard deviation bands for the two combinations of quark couplings. In (b) and (c) the resulting regions of vector and axial couplings are displayed for up and down type quarks, respectively. The full and dotted areas are due to the measurements of $a^2 + v^2$, the hashed area due to $v \cdot a$.

Figure 6: Variations of the accepted photon yield with QCD parameters in the parton shower model of [5]. Displayed is the expected number of photons for 10^6 hadronic decays of the Z^0 for a. a variation of Λ_{QCD} and b. for a variation of Q_0 . Also shown are the values obtained from an optimization of the Monte Carlo program to measured event shapes at c.m. energies around 91 GeV [12].

a

b

c

d

Fig. 1

Fig. 2a

Fig. 2b

Fig. 3a

Fig. 3b

Fig. 4

Fig. 5a

Fig. 5b

Fig. 5c

Fig . 6a

Fig . 6b