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DOES STRING THEORY LEAD TO EXTENDED INFLATION ?

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ABSTRACT

We consider the relationship between string theory and currently proposed models of extended inflation. In doing so, we discuss the conformal actions in string theory and in Jordan-Brans-Dicke gravity. We show explicitly the equivalence of pictures in which either gauge or gravitational couplings are changing with time. We demonstrate that the existence of the dilaton in string theory does not naturally lead to extended inflation as currently discussed. We also discuss the resolution of the graceful exit problem of old inflation in Einstein gravity using either power-law inflation, or exponential inflation with a changing bubble formation rate.

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Although the final form of inflationary cosmology is not yet fixed, most cosmologists believe that an inflationary phase must have occurred in the early Universe in order to solve a host of cosmological problems¹⁾. The old inflationary Universe of Guth²⁾ was based on the assumption that the early Universe begins in a high temperature state in which gauge symmetries (in particular GUT symmetries) were restored. As the Universe cools, the Higgs field, responsible for the gauge symmetry breaking phase transition, becomes trapped in the symmetric state due to the presence of a large potential barrier in the effective potential for the Higgs field. The Universe supercools and it was hoped that the phase transition would end by the percolation of vacuum bubbles formed during the tunneling of the Higgs field to the true vacuum state. The fact that the requirements of sufficient inflation (slow bubble formation rate) and percolation (fast bubble formation rate) are not compatible is known as the graceful exit problem³⁾.

Of course it is well known that the graceful exit problem can be avoided by either new inflation⁴⁾ or by chaotic inflation⁵⁾ in which the phase transition is completed not by bubble formation but rather by the rollover of a classical scalar field. Our purpose, here however is to examine the newly proposed solution to the graceful exit problem, namely extended inflation⁶⁾. In this model based on Jordan-Brans-Dicke⁷⁾ (JBD) gravity, the effective gravitational constant changes in time. In the beginning the gravitational constant is big, expansion is rapid and bubbles do not percolate. Later the gravitational constant becomes small, expansion is more slow, bubbles percolate and the phase transition from the false vacuum state completes*. Thus, extended inflation gives us the hope that the cosmological problems may be solved using ordinary GUT phase transitions after some modifications of Einstein gravity. One should note, that chaotic inflation may solve the cosmological problems even without any phase transitions and modification of Einstein gravity. Still the new possibilities offered by extended inflation deserve investigation.

It would be most important to obtain the necessary modification of Einstein gravity in the context of a realistic theory of all fundamental interactions. The best candidate at present is string theory⁸⁾. It is often claimed that string theory may be cast in the JBD form useful for extended inflation. It is this connection between string theory and extended inflation that we wish to explore. We make extensive use of conformal transformations in describing both JBD theories and string theories. In light of our result, i.e. that the form in which the dilaton enters string theory is not that which is required for the realization of extended inflation, we will also consider other alternatives to solving the graceful exit problem of old inflation in Einstein gravity (non-string theory). Two solutions are available : the first is based upon inflationary models in which the expansion rate considerably changes during the evolution of the Universe.

* Jordan-Brans-Dicke gravity was considered in the context of new inflation in ref.8.

This condition is satisfied practically in all models of chaotic inflation, and in particular in the models with exponentially growing effective potential, which lead to power-law inflation^{10,12)}. The models of power-law inflation are completely equivalent to JBD gravity up to a conformal transformation. The second solution still uses exponential inflation, but makes use of a bubble formation rate which is time dependent allowing for percolation^{13,14)}.

We begin our discussion by writing down the actions for JBD gravity and string theory as well as their conformal equivalent in Einstein gravity. For JBD, one can write⁷⁾

$$S_{\text{JBD}} = \int d^4x \sqrt{g} \left[\phi R - \frac{\omega}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \partial_\mu y \partial^\mu y - m^2 y^2 - V(y) \right] \quad (1)$$

where ϕ is the JBD scalar (dilaton), y represents any other massive scalar field as an example, and $V(y)$ represents the self interactions of y . ($V(y)$ is ϕ -independent). In the action (1), all gauge couplings and Yukawa couplings are ϕ - and hence time independent (though they are not explicitly displayed), instead it is the gravitational constant which is completely determined by ϕ .

Upon performing a conformal transformation given by^{15,16,17)}

$$\tilde{g}_{\mu\nu} = 2\kappa^2 \phi g_{\mu\nu} \equiv \Omega^2 g_{\mu\nu} \quad (2)$$

where $\kappa^2 = 8\pi G_N$, S_{JBD} may be written as

$$S_{\text{JBD}} = \int d^4x \sqrt{\tilde{g}} \left[\frac{\tilde{R}}{2\kappa^2} - \left(\omega + \frac{3}{2} \right) \frac{\partial_\mu \phi \partial^\mu \phi}{2\kappa^2 \phi^2} - \frac{\partial_\mu y \partial^\mu y}{2\kappa^2 \phi} - \frac{m^2 y^2}{4\kappa^4 \phi^2} - \frac{V(y)}{4\kappa^4 \phi^2} \right]. \quad (3)$$

Then with the field redefinition

$$\kappa \sigma = \left(\omega + \frac{3}{2} \right)^{1/2} \ln 2\kappa^2 \phi \quad (4)$$

we have

$$S_{\text{JBD}} = \int d^4x \sqrt{g} \left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} e^{-\kappa\sigma/(\omega+3/2)^{1/2}} \partial_{\mu\nu} \partial^{\mu\nu} y \right. \\ \left. - e^{-2\kappa\sigma/(\omega+3/2)^{1/2}} \frac{2}{m^2} y^2 - e^{-2\kappa\sigma/(\omega+3/2)^{1/2}} V(y) \right]. \quad (5)$$

Note that now, the theory is expressed in terms of Einstein gravity, in which the gravitational constant is fixed but all gauge and Yukawa couplings are σ -dependent and evolve with time¹⁵. Solutions to the action (5) have been written down for a variety of circumstances. For extended inflation, we need to consider the epoch when S_{JBD} is dominated by a constant vacuum energy density V_0 . In this case S_{JBD} takes the form characteristic of theories of power law inflation. To discuss the solutions to the equations of motion generated by (5), we define the coordinate systems by the metrics (in their Friedmann-Robertson-Walker form)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) dx^i{}^2 \quad (6a)$$

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = \Omega^2 g_{\mu\nu} dx^\mu dx^\nu \\ = \Omega^2 (dt^2 - a^2(t) dx^i{}^2) \equiv d\eta^2 - b^2(\eta) dx^i{}^2. \quad (6b)$$

(Hereafter η will always refer to the time coordinate in the Einstein frame.)

The solutions for metrics $\tilde{g}_{\mu\nu}$ based on the general form

$$S = \int d^4x \sqrt{\tilde{g}} \left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - e^{\lambda\sigma} V_0 \right] \quad (7)$$

are obtained from the equations of motion

$$\tilde{\sigma}'' + 3H\tilde{\sigma}' + \partial V/\partial\sigma = 0 \quad (8a)$$

$$\frac{b'}{b} = \frac{\kappa^2}{3} \left[\frac{1}{2} \sigma'^2 + V \right] \quad (8b)$$

where prime denotes a derivative with respect to η . The solutions to these equations are^{16,19}

$$b(\eta) \sim \eta^p \quad (9a)$$

$$\sigma(\eta) \sim c \ln \eta/\eta_0 \quad (9b)$$

with

$$p = \kappa^2 c^2/2; \quad c = -2\Lambda$$

$$\kappa^2 V_0 \eta_0^2 = p(3p-1). \quad (10)$$

For the action (5), the solutions using $\lambda = -2\kappa/((\omega+3/2)^{1/2})$, $c = (\omega+3/2)^{1/2}/\kappa$ and $p = \frac{\omega}{2} + \frac{3}{4}$, become^{15,16}

$$b(\eta) \sim \eta^{\omega/2 + 3/4} \quad (11a)$$

$$\kappa\sigma(\eta) \sim (\omega + \frac{3}{2})^{1/2} \ln \eta/\eta_0 + \text{const.} \quad (11b)$$

The solutions for $g_{\mu\nu}$ are read off from (6b)

$$a(t) \sim t^{\omega+1/2} \quad (12a)$$

$$\varphi(t) \sim t^2/2\kappa^2 t_0^2 \quad (12b)$$

In this case, eq.(12a) describes unambiguously the expansion of the Universe as all other quantities with respect to which we measure are held fixed, e.g. the Compton wavelength of y_0 . Since (5) equally well describes dynamics with a new metric $\tilde{g}_{\mu\nu}$, we can interpret (9a) as a

solution. But, in this case, our units of measure are σ dependent. For example, the y-Compton wavelength evolves as

$$\lambda_c \sim e^{\kappa\sigma/2(\omega+3/2)^{1/2}} \sim \eta^{1/2} \quad (13)$$

(note that the kinetic term of y in (5) also has a scaling with σ). The ratio

$$b/\lambda_c \sim \eta^{\omega/2+3/4}/\eta^{1/2} \sim \eta^{\omega/2+1/4} \sim \eta^{\omega+1/2} \sim a/\lambda_c \quad (14)$$

is the same for both solutions using $\tilde{g}_{\mu\nu}$ and $g_{\mu\nu}$ as it must be. Therefore it becomes simply a matter of choice as to whether one uses the action (1) or the action (5). It merely amounts to field redefinitions of $g_{\mu\nu}$, ϕ , and y . In this sense power law inflation and inflation in JBD theory are equivalent.

We now turn to the case of string theory. We wish to represent the dynamics of string theory at distance scales greater than the Planck scale in terms of an effective action describing the dynamics of light (relative to the Planck mass) modes. Among the light scalar modes which appear in four dimensional string constructions, is a dilaton, which in spacetime supersymmetric string constructions appears in the gravitational multiplet of the theory. As well as the dilaton, there will in general appear numbers of both gauge singlet, and gauge non-singlet, scalar fields among the light particles of the theory. A complete survey of inflationary dynamics in string theory would require that we consider, for all phenomenologically viable string ground state constructions, the dynamical evolution of all the scalar fields present in that construction, to determine the possibility of inflationary solutions. Clearly this is a task beyond the scope of this paper; indeed at the present time we are very far even from an exhaustive classification of phenomenologically viable string constructions.

Given that we cannot do a complete survey of inflation in string theory, we may ask a more limited question. Since string theory ground state constructions generally give a light dilaton field, we may ask if the dynamics of this dilaton alone, in conjunction with a source of vacuum energy, could result in useful inflationary dynamics. This question is motivated by comparison with theories of extended inflation, where the dynamics is treated by freezing out all the scalar degrees of freedom except the dilaton, whose evolution is driven by a source of vacuum energy. Also, since both string theory and Jordan-Brans-Dicke theory involve dilatons it has been suggested that string theory may be useful for Jordan-Brans-Dicke extended inflation.

We examine these issues in sequence : first we consider the known string solutions with only dilaton dynamics in both the Einstein and conformal pictures; then we discuss the differences between string gravity, and gravity of the Jordan-Brans-Dicke form, which produce the differences between the forms of the solutions; finally we generalize to the most general form of dilaton dynamics arising perturbatively in string theory, discuss cosmological solutions, and the issue of scale units with which to discuss expansion.

We turn now to the case of string theory. We begin with the string effective action²⁰⁾

$$S_{\text{String}} = \int d^4x \sqrt{g} e^{-\sqrt{2}\kappa\phi} \left[\frac{R}{2\kappa^2} + \partial_\mu\phi \partial^\mu\phi - \frac{1}{2} \partial_\mu y \partial^\mu y - m^2 y^2 - V_0 \right] \quad (15)$$

In writing (15), we have ignored the 3-form field strength contribution from the gravitational multiplet as it will not enter into our subsequent discussion. The dilaton scaling of the potential term in equation (15) is that appropriate to classical (genus-0) potential terms, as in the solutions of references [22-24]. We discuss the origin and scaling of potential terms in more detail below. Assuming that mass and potential terms appear as above, we proceed to examine the solutions and conformal properties of the action (15). We choose a conformal transformation,

$$\tilde{g}_{\mu\nu} = e^{-\sqrt{2}\kappa\phi} g_{\mu\nu} \equiv \Omega^2 g_{\mu\nu} \quad (16)$$

to find²¹⁾ (with $\sigma = \phi$)

$$S_{\text{String}} = \int d^4x \sqrt{\tilde{g}} \left[\frac{R}{2\kappa^2} - \frac{1}{2} \partial_\mu\sigma \partial^\mu\sigma - \frac{1}{2} \partial_\mu y \partial^\mu y - e^{-\sqrt{2}\kappa\sigma} \frac{m^2 y^2}{2} - e^{-\sqrt{2}\kappa\sigma} V_0 \right] \quad (17)$$

We see once again that the two actions differ only in interpretation, i.e. in one case the gravitational constant is fixed (17) while in (15) it is the gauge and Yukawa couplings which are fixed. The solutions based on (17) are again easily found^{22,23,24)}. In this case, $\lambda = \sqrt{2} \kappa$, $c = -\sqrt{2}/\kappa$, $p=1$,

$$b(\eta) \sim \eta \quad (18a)$$

$$\kappa\sigma(\eta) \sim -\sqrt{2} \ln \eta/\eta_0 + \text{constant} \quad (18b)$$

for $\tilde{g}_{\mu\nu}$.

In ref. 23 an additional solution was found when the 3-form was included. For $\tilde{g}_{\mu\nu}$,

we have

$$a(t) \sim \text{constant} \quad (19)$$

The apparent non-expansion of the Universe, in string units, in this solution was discussed in ref. 25. Again for the action (17) the Compton wavelength of y is evolving as

$$\lambda_c \sim \exp(-\kappa\sigma/\sqrt{2}) \sim \eta \quad (20)$$

so that the ratio

$$b/\lambda_c \sim \text{constant} \quad (21)$$

as is obviously the case for the action (15) using $g_{\mu\nu}$.

If the dynamics of the dilaton were such that its value was fixed by minimization of a potential, then the two conformal pictures would be related by a constant rescaling, and any cosmological expansion that might occur with such dynamics would look similar in both conformal frames. Such a dilaton potential is expected to arise from supersymmetry breaking²⁶ in string theories, perhaps at an epoch early enough to influence inflationary dynamics²⁴.

We now consider the generation of potential and mass terms in string theory, comparing it to Jordan-Brans-Dicke theories, and considering more generally the prospects for dilaton induced extended inflation in string theory. Where do these potentials arise? First they may be effective Lagrangian descriptions of non perturbative dynamics²⁶ (gaugino condensation, Kalb-Ramond axion condensation, etc.). To give a vacuum energy contribution they must break supersymmetry dynamically; a non zero constant V_0 gives a dilaton potential as discussed in ref.9.

Second they may arise as properties of the direct construction of classical string ground states^{22,23,27}, by assigning conformal theories on the world sheet that cancel the Virasoro anomaly and are modular invariant.

For example, Antoniadis et al^{23,27} construct a model with the world sheet conformal field theory carrying a GKO conformal coset theory for $O(4,1)/O(3,1)$, and a free boson with a background charge (Dotsenko-Fatveev/Feigen-Fuchs). The background charge is associated with a "central charge deficit" of the coset piece, and acts as a potential for the four dimensional effective action :

$$S = \int d^4x \sqrt{g} \left\{ \frac{R}{2\kappa^2} - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{6} e^{\frac{1}{2}\kappa\sigma} \delta c \right\} \quad (22)$$

where the "central charge deficit" δc functions as a tree level (genus 0) potential $V_0^{(0)}$.

This construction is not spacetime supersymmetric even for world sheet theories of the heterotic or type II form. Spontaneous breaking of the spacetime supersymmetry is associated with a tree level (genus zero) vacuum energy, which drives the expansion of the solutions which we discuss below. These solutions then, are of the classical string dynamics, and might be modified, or extended, by quantum mechanical effects such as effective potentials generated by gaugino condensation. The important thing for our consideration is that whatever the origin of an effective potential term might be, its scaling with the dilaton is governed by the genus at which it appears; we return to this point shortly.

To compare string theory to Jordan-Brans-Dicke gravity, let us consider them both in the Einstein conformal frame, c.f. Equations (5) and (17). From comparison of the forms of the actions it is clear that there is no choice of the parameter ω for which these actions are of the same form. Simply put, the full string theory effective action is not of Jordan-Brans-Dicke form. If we were to truncate the string effective action by retaining only the graviton and dilaton kinetic terms and the potential $V_0^{(0)}$, then this truncation of the action could be considered to be of Jordan-Brans-Dicke form with $\omega = \frac{1}{2}$. If we further truncated by dropping the potential, and retained only the graviton and dilaton kinetic terms, we would have removed all ω dependence. This truncation could then be claimed to be of Jordan-Brans-Dicke form with any value of ω which the reader desired*, and by conformal transformation and field redefinition could be

* This is the origin of claims in the literature that the truncated string effective action is of Jordan-Brans-Dicke form, with a variety of choices for the value of the parameter ω .

made to so appear in the Jordan conformal frame. But in both cases (with $\omega = \frac{1}{2}$ or with an arbitrary ω but without potential terms) there is no inflation in JBD theory.

The absence of inflationary solutions of the "extended inflation" form, with the dynamics arising solely from dilaton evolution, can be directly traced to the form of the dilaton coupling to matter. So it is important to ask what other dilaton-matter couplings might arise beyond the string tree level. With our convention for the dilaton, $e^{\sqrt{2}\kappa\phi}$ is the string loop expansion parameter^{28,9)}. All the light fields in our effective action will pick up kinetic terms at string tree level, and potentials may appear classically (genus 0) or at higher genus (as will corrections to kinetic terms which we ignore). In the conformal picture

$$S_{\text{string}} = \int d^4x \sqrt{g} \left\{ e^{-\sqrt{2}\kappa\phi} (\text{genus } 0) + e^0 (\text{genus } 1) + \dots \right. \\ \left. + e^{\sqrt{2}\kappa\phi(G-1)} (\text{genus } G) + \dots \right\} \quad (23)$$

where :

$$(\text{genus } 0) \equiv \frac{R}{2\kappa^2} + \partial_\mu \phi \partial^\mu \phi + \dots - V^{(0)} + \dots$$

$$(\text{genus } G) \equiv -V^{(G)} + \text{corrections to kinetic terms} + \dots$$

Making a conformal transformation to the Einstein picture

$$S_{\text{string}} = \int d^4x \sqrt{g} \left\{ \frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - e^{-\sqrt{2}\kappa\sigma} V^{(0)} + \dots \right. \\ \left. - e^{-\sqrt{2}\kappa\sigma(G+1)} V^{(G)} + \dots \right\} \quad (24)$$

where now the kinetic terms are in canonical form, and the contributions to the potential scale with the dilaton according to the genus of their origin.

Now let us consider what would happen if the string potential V was only generated as a higher genus string correction first appearing in genus G ($V^{(G)}$).

Going to the Einstein picture this would give

$$S_{\text{string}} = \int d^4x \sqrt{g} \left\{ \frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \dots - e^{-\sqrt{2}\kappa(G+1)\sigma} V^{(G)} + \dots \right\} \quad (25)$$

In the truncation where we retain only the graviton, dilaton and potential terms shown, this may be considered to be of the general JBD form (in Einstein picture)

$$S = \int d^4x \sqrt{g} \left\{ \frac{R}{2\kappa^2} - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - e^{\lambda\sigma} V \right\} \quad (26)$$

with $\lambda = \sqrt{2} \kappa(G+1)$ ($= 2\kappa/\sqrt{\omega+3/2}$ (JBD) $\Rightarrow \omega = \frac{2}{(G+1)^2} - \frac{3}{2}$). The power law expansion (but not inflation) solution now gives $b(\eta) \sim \eta^p$ with $p = 2\kappa^2/\lambda^2 = 1/(G+1)^2$ so that

$$b(\eta) \sim \eta^{1/(G+1)^2} \quad (27)$$

and $\sigma(\eta) \sim (-2\lambda) \ln \eta/\eta_0$ giving

$$\sigma(\eta) \sim \frac{-\sqrt{2}}{\kappa(G+1)} \ln \eta/\eta_0 \quad (28)$$

for vacuum energy density from potentials $V_0^{(G)}$ arising at genus G . For the case of tree level vacuum potential ($G=0$) we find the linear solution $b(\eta) \sim \eta$ of refs. 22-24. For vacuum energy from higher genus the expansion is even slower. To decide whether time dependence of b truly constitutes expansion we must measure lengths in some physical units.

First let us consider string units. These are set by the string tension (or length) and will appear at the classical level (genus 0) in setting the mass of string excited states. The effective Lagrangian description of the tree level dynamics of a massive scalar string mode at the first level will be (in Einstein picture)

$$S_{\text{massive scalar mode}} = \int d^4x \sqrt{g} \left(-\frac{1}{2} \partial_\mu y \partial^\mu y - e^{\sqrt{2}\kappa\sigma} m^2 y^2 \right) \quad (29)$$

But we know that σ is evolving as : $\kappa\sigma \sim -\frac{\sqrt{2}}{(G+1)} \ln \eta$, so the mass term evolves as

$$(e^{\sqrt{2}\kappa\sigma})^{2/2} = e^{\kappa\sigma/\sqrt{2}} \sim m \sim m e^{-\ln \eta/(G+1)} \sim \eta^{-1/(G+1)} \quad m \quad (30)$$

The Compton wavelength for this mode (and string scale) go as

$$\lambda_c \sim \frac{1}{\text{Mass}} \sim \eta^{1/(G+1)} \quad (31)$$

So the scale factor in string units goes as

$$\frac{b(\eta)}{\lambda_c(\eta)} \sim \eta^{1/(G+1)^2} \eta^{-1/(G+1)} \sim \eta^{-G/(G+1)^2} \quad (32)$$

For the case of a genus zero generated potential this results in a static universe, in string units²⁵⁾, while for potentials generated at genus $G > 0$, the universe actually contracts, in string units. The expansion (or not) of the universe in string units however, may not be directly relevant for astrophysical comparison. Rather we should be comparing to the physical measuring units (scales of physical objects) which we use*, such as, for example, the radius of the proton.

Let us consider, then, the growth of the scale factor as measured in units of proton size. This is determined by the QCD confinement scale Λ (for simplicity we ignore quark mass effects). The QCD scale Λ is determined by dimensional transmutation (as we expect in general for dynamical breaking of scale invariance)

$$\alpha_3(M_{\text{PL}}^2) \equiv \frac{g_3^2(M_{\text{PL}}^2)}{4\pi} \approx \frac{12\pi}{(33-2N_f) \ln(M_{\text{PL}}^2/\Lambda^2)} \quad (33)$$

where we use the form of the RGE running for N_f flavors of color SU(3) quark triplets. This gives (for $N_f = 4$)

$$\ln(\Lambda^2/M_{\text{PL}}^2) \approx -\frac{48\pi^2}{25} \frac{1}{g_3^2(M_{\text{PL}}^2)} \quad (34)$$

* Physical measures of cosmological expansion are also being considered by other authors²⁹⁾.

But the unification scale ($\sim$ Planck scale) gauge coupling constant depends on the vev of the dilaton as

$$\frac{1}{g_3^2(M_{\text{PL}}^2)} = \frac{\alpha' \text{Re}(S)}{2\kappa} = \frac{\alpha' e^{-2\kappa\sigma}}{2\kappa} \quad (35)$$

so that

$$\Lambda^2 \sim (M_{\text{PL}}^2) \exp\left(-\frac{48\pi^2}{25} \frac{\alpha'}{2\kappa} e^{-2\kappa\sigma}\right) \quad (36)$$

For our solution $\sigma \sim \frac{-\sqrt{2}}{\kappa(G+1)} \ln(\eta)$ so that

$$\Lambda \sim M_{\text{PL}} \exp\left(-\frac{24\pi^2}{25} \frac{\alpha'}{2\kappa} \eta^{2/(G+1)}\right) \quad (37)$$

This means that the QCD confinement length scale ($r_{\text{QCD}} \sim 1/\Lambda$), and hence the proton radius, evolve in our solution as

$$r_{\text{QCD}} \sim M_{\text{PL}}^{-1} \exp\left(\frac{12\pi^2}{25} \frac{\alpha'}{\kappa} \eta^{2/(G+1)}\right) \quad (38)$$

Thus, in units of the QCD length scale (proton radius) the universe shrinks as the exponential of a power. Clearly a similar result will hold for any length scale set by the dimensional transmutation of a (dilaton dependent) gauge coupling constant.

As this example illustrates, in order to discuss the dynamical evolution of the scale factor of the universe in a physically meaningful way, one must take care to use measuring rods appropriate to the physical definition of the length scale of interest. In discussing processes in early universe cosmology, one would be interested in the expansion of the universe relative to the natural length scale of that process. The necessity of specifying the scale of measuring rods

is particularly acute for scales set by dimensional transmutation, as the dilaton dependence of the coupling constant is amplified exponentially when converted to a scale factor. It is also important to note that the rapid evolution of the scales of our measuring rods through dimensional transmutation has occurred precisely because we assumed that it was the time evolution of the dilaton that governed the expansion of the universe. In alternate scenarios where, for example the dilaton was fixed by a potential, and it was some other scalar field(s) that underwent dynamical evolution, then the QCD length scale would remain fixed, and our conclusions about cosmological expansion in units of the proton radius would be substantially different.

In sum, we see that the full effective gravity theory produced by string theory is not of the Jordan-Brans-Dicke form. Also, if we restrict our scalar dynamics to the evolution of the dilaton field alone, as done in models of "extended inflation", we do not find useful inflationary solutions of this type in string theory, for vacuum energy from effective potentials generated at any order in the string loop expansion. So "extended inflation" solutions associated with dilaton dynamics alone do not appear to occur in string theory. The full dynamics of string theory may lead to inflation in a realistic four dimensional string construction; a more general study of this issue is necessary. What our considerations do indicate is that such inflation, if it occurs, will not be due solely to dilaton evolution as in "extended inflation" models.

The use of non JBD couplings of the dilaton to matter for extended inflation has been attempted. As discussed above, if such couplings are to arise from a string theory they must be of the form of the effective action (17). Recently it has been pointed out³⁰ that such couplings may have a profound effect on the bubble formation rate. For the string effective action, the results of reference 30 imply that the nucleation rate is given by (for $G = 0$)

$$\tilde{\lambda} \sim A \exp [-B(e^{-2\kappa\sigma})] \sim A \exp [-B(\eta/\eta_0)^2] \quad (39)$$

which evolves as

$$\lambda \sim \Omega^4 \tilde{\lambda} \sim e^{4U_0} A \exp [-B e^{2U_0}] \quad (40)$$

Thus the effective action from string theory would lead to a non-constant nucleation rate of a form which might not solve the graceful exit problem, even if inflation in this theory were possible. However, as we argued above, extended inflation does not appear in this theory at all.

Given that extended inflation does not arise from the dynamics of the dilaton in string theory, we can still ask what sort of models give rise to a solution of the graceful exit problem of old inflation. From the equivalence of the actions (1) and (5) it is easy to surmise that any action in which the "dilaton" is coupled exponentially to matter will be capable in principle of solving the graceful exit problem. In short there is no need to go to a theory with non-minimal gravity. The reason JBD extended inflation works, is that it leads to power-law inflation, not because it is a fundamentally different theory. As power-law inflation has been well studied^{10, 12, 16, 18, 19} it is an easy task to pick out the key features.

As we saw earlier JBD gravity is equivalent to Einstein gravity, in which the kinetic and potential terms of the matter sector take on specific couplings to the dilaton (5). This in an unnecessarily complicated theory. What is sufficient for solving the graceful exit problem is just the type of models studied for power-law inflation^{11, 18, 19}. The action can be written as

$$S = \int d^4x \sqrt{g} \left[\frac{R}{2\kappa^2} - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} e^{\alpha\sigma} \partial_\mu \Phi \partial^\mu \Phi - e^{\beta\sigma} V(\Phi) \right], \quad (41)$$

where we display explicitly the dependence of the dilaton/inflaton couplings. In (41) σ is the dilaton/inflaton and Φ is another scalar field (perhaps the SU(5)-adjoint if we have GUT symmetry-breaking in mind as in old inflation). As it is in fashion to claim no small parameters, one can do the same here. One should take into account two things though. First of all, since we are not using superstring theory (or even just ordinary supersymmetry) here (as well as in the extended inflation models), small parameters inevitably occur in the theory (remember the gauge hierarchy problem). One such parameter (and not the smallest one!) is the ratio $\frac{V_0}{M_{Pl}^4}$, where M_{Pl} is the Planck mass, $M_{Pl} = \frac{\sqrt{8\pi}}{\kappa}$. In order to get $\frac{\delta\rho}{\rho} \sim 10^{-5}$ one must have $\frac{V_0}{M_{Pl}^4} \sim 10^{-4}$, just as in the extended inflation scenario.

In our opinion, the theory (41), with only the inflaton terms scaled by the dilaton (matter terms not displayed do not need to be scaled), offers a more compact and less obtrusive solution to the graceful exit problem than having to modify gravity/all other matter couplings.

For this model the results of ref. 30 imply an Einstein frame bubble nucleation rate which scales as

$$\tilde{\lambda} \sim A \exp [-B e^{(2\alpha-\beta)\sigma}] \quad (42)$$

which using equations (9b,10) gives

$$\tilde{\lambda} \sim A \exp [-B (\eta/\eta_0)^{2-4\alpha/\beta}] \quad (43)$$

so that for $\alpha/\beta \geq 1/2$ the nucleation rate is non-decreasing, and the phase transition completes. In a specific model one would have to insure that the bubble formation rate was not too fast.

But the power-law inflation in the model (41) is not the most general way of solving the graceful exit problem. If one is permitted to consider models with a nonpolynomial interaction of several fields, as is the case with all models discussed above, one may consider a more general theory than (41),

$$S = \int d^4x \sqrt{g} \left[\frac{R}{2} - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - U(\sigma) V(\Phi) \right]. \quad (44)$$

For example, one may take $V(\Phi) = M^4 + \alpha M^2 \Phi^2 - \beta M \Phi^3 + \gamma \Phi^4$, $U(\sigma) = \frac{\delta}{2} \kappa^2 \sigma^2$. Then at $\Phi=0$ one has a standard realization of the chaotic inflation scenario in a theory of a massive field σ with a mass squared $m^2(\sigma) = \delta \kappa^2 M^4$. Since the Hubble parameter in this model is also time-dependent (it changes by a factor ~ 20 during the last 60 e-foldings of inflation), it is possible to solve the graceful exit problem for the field Φ in this model as well. Density perturbations produced in this model are of the required magnitude $\frac{\delta \rho}{\rho} \sim 10^{-5}$ for $m \sim 10^{12}$ GeV which occurs for $\lambda=O(1)$, $M \sim 10^{15}$ GeV.

There is also a complementary solution to the graceful exit problem. That is, instead of considering a time-dependent rate of expansion during inflation one can consider models in which the bubble formation rate increases with time due to the coupling of two scalar fields^{13,14}. In these models, the phase transition with respect to one of these fields is controlled by the value of another field, which changes in time. Similar models were considered earlier in the theory of formation of cosmic strings^{31,32} and of exponentially big bubbles³² during inflation. The simplest example of such a model¹³ has an effective potential of the form

$$V(\sigma, \Phi) = V(\Phi) - \left(\frac{1}{2} (c\beta) M_{Pl}^2 - \beta \Phi^2 \right) \sigma^2 + \frac{C\beta^2}{64\pi^2} \sigma^4 \left(\ln \frac{\beta^2 \sigma^2}{m^2} + O(1) \right) \quad (45)$$

The last term appears due to radiative corrections. This theory with $c, C = O(1)$ and $\beta \sim 10^{-6}$ leads to a natural realization of the chaotic inflation scenario. $V(\Phi)$ is again your favorite GUT potential which can be taken in the form $V(\Phi) \sim M^4 + \alpha M^2 \Phi^2 - \gamma M \Phi^3 + \delta \Phi^4$. It is clear that the curvature of the effective potential $V(\sigma, \Phi)$ at $\Phi=0$ depends on the value of the field σ ,

$$\frac{\partial^2 V(\sigma, \Phi)}{\partial \Phi^2} \Big|_{\Phi=0} = 2(\alpha M^2 + \beta \sigma^2) \quad (46)$$

Initially the field σ in this model is big as in the ordinary chaotic inflation scenario. At this time the only minimum of $V(\sigma, \Phi)$ with respect to the field Φ is located at $\Phi=0$ and this field rapidly rolls down to this minimum. Later the field σ decreases, another minimum of $V(\sigma, \Phi)$ appears at $\Phi \neq 0$ and the field Φ may tunnel there. The probability of tunneling increases exponentially with a decrease of the field σ . This also provides a solution of the graceful exit problem. In particular, for $\alpha < 0$ the phase transition occurs at $\sigma^2 = \frac{|\alpha|}{\beta} M^2$ without any bubble formation, as the usual second order phase transition.

In summary, we have considered the possibilities for extended inflation in string theory. We have compared in detail the actions in string theory and in JBD gravity. Only under some extreme assumptions do they agree (such as throwing out the entire matter sector of the theory). In general, string theory is not a theory of JBD gravity. When applied to extended inflation, string theory allows only a linearly "expanding" solution with an exponentially decreasing bubble formation rate. We conclude that the existence of a dilaton in string theory by itself does not provide a natural basis for a realization of the extended inflation scenario.

We have also shown that the graceful exit problem in old inflation can be solved in models based on minimal gravity using two coupled scalar fields. Time-dependence of the expansion rate in such models leads to the same effects as in extended inflation. Alternatively, the decay rate of the false vacuum in such models may strongly depend on the value of one of the scalar fields and hence on time. In both cases the graceful exit problem can be solved without any modifications of Einstein gravity.

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