CP-Violation and T-odd Effects at LEP-II

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ABSTRACT

LEP-II will be a tool to study CP-violation in processes involving vector bosons, and will test in particular the CP properties of the coupling of photons and Z 's to W -pairs. While it is difficult to observe truly CP-odd effects, it is easy to measure T-odd ones. The latter can originate from CP-violation, or from radiative corrections involving the absorptive part of the scattering amplitude. T-odd effects are interesting in themselves, in that they accurately test the standard model and are sensitive to its unmeasured ingredients, such as the $WW \rightarrow WW$ scattering amplitude or the masses of the top quark and the elementary scalar. The prediction of the standard T-odd effects is a necessary stepping stone in the search for an honest-to-goodness violation of CP in the pure gauge sector. We thoroughly analyse the T-odd observables in the $e^+e^- \rightarrow W^+W^-$ process within the standard model, as well as the extra effects to be expected if the γW^+W^- and/or ZW^+W^- vertices were to violate CP in a non-standard fashion.

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1. Introduction

In their opening sentence, literally thousands of papers note that for consecutive lustrums Nature has shown a stubborn resistance to provide us with any hint of deviations from the standard model, or of physics beyond its scope. With the accumulation of data from LEP-I, that seem nigher to a text-book example than even statistical fluctuations would allow one to expect, this is even more so. We seem to be bound either to perform tests of ever-increasing accuracy, or to continue to place our hopes for a refreshing breakthrough on whatever the next more luminous or more energetic machine may be. LEP-II will give us the opportunity to do both of the above in a field that is so far only very indirectly explored: the interactions of charged and neutral vector bosons with one another.

It is well known that gauge theories, perhaps because we have not yet found the optimal way to express their perturbative series, yield "small" cross sections that result from enormous cancellations between distinct "gigantic" contributions. The scattering-angle differential cross section $d\sigma(e^+e^- \rightarrow W^+W^-)/d\cos\theta$ is an example of this behaviour. In this paper we explore subtle effects in the angular correlations of the W decay products that are more directly sensitive than $d\sigma/d\cos\theta$ to at least two unexplored domains. One is the interaction of intermediate vector bosons with themselves, as described by the standard model. The other concerns the non-standard possibility that relatively large CP-violating effects occur in γW^+W^- and/or ZW^+W^- vertices.

There exist only three known ways to investigate experimentally the interesting subject of the scattering of intermediate vector bosons with themselves, such as in the process $WW \rightarrow WW$. The first way is the detailed study of those consequences of the standard model that are sensitive to this amplitude via virtual quantum-loop effects. A future, albeit more direct, possibility is the analysis of the production of W -pairs in very high energy pp interactions, wherein the quoted process, with quasi-real ("equivalent-radiation") initial W 's, plays an important role. But, lo, the real scattering of real (on-shell) W 's can be investigated at more

modest energies at LEP-II. The most direct method involves the measurement of T-odd observables, those that change sign as the directions of spin and momenta in a given process are reversed, as we review in detail in the next Section. Suffice it to say here that upon neglect of CP-violation these observables are, to leading order of perturbation theory, linear in the absorptive part of the $e^+e^- \rightarrow W^+W^-$ amplitude. The absorptive amplitude itself is the sum of products of two amplitudes involving only real particles in their initial and final states. In the example at hand, these are $e^+e^- \rightarrow W^+W^-$ times $W^+W^- \rightarrow W^+W^-$, and $e^+e^- \rightarrow ZZ$ times $ZZ \rightarrow W^+W^-$, and a long *et coetera*.

We shall investigate two T-odd observables in the process $e^+e^- \rightarrow W^+W^-$. Let one W decay into two jets, and the other into an $l\nu$ pair, a process in which the W momenta and charges can be reconstructed. The triple product of one of the beam momenta, the momentum of a W and that of l is our first T-odd observable. The second is the triple product of the momentum of the mightiest jet, that of the other, and the momentum of l . The standard model predicts the asymmetry constructed with the first of these observables to be of the order of 1%, which is measurable but only at the expense of considerable effort. The second asymmetry is predicted to be ten times smaller and hence unobservable. We shall see that both asymmetries are quite sensitive to the value of the top-quark mass, and very insensitive to that of the fundamental scalar, in both cases within generous mass ranges.

The subject of CP-violation remains particularly mysterious. The standard model accommodates our very modest experimental information, concerning essentially only one non-null observable: the parameter ϵ in the description of the $K_0\bar{K}_0$ system. But it fails to resolve in an appealing manner the "strong CP problem", the only known theoretically satisfactory solution of which [1] would imply the existence of axions, whose characteristic scale is now cornered to domains that do not coincide with any other sensible physical scale. It is quite conceivable for CP-violation to provide us, once more, with a resounding surprise departing from the lore of the time. We have therefore investigated how our T-odd observables

may be generated by novel explicitly CP-violating three-gauge-boson couplings. Our observables, particularly the asymmetry that is predicted to be tiny within the standard model, turn out to be very sensitive to the novel trilinear vertices of gauge bosons.

The paper is organized in a straightforward manner, with much of the algebraic detail relegated to appendices.

2. T-odd effects from unitarity and T-invariance

By definition, a T-odd effect is an observable that changes sign under a flip in direction of all three-momenta and polarizations in a given process. This operation is related to, but *different* from time reversal, since the latter also involves a flip in the “arrow of time”, that is an interchange of the initial and final states. Hence, the observation of a non-zero T-odd observable does not necessarily imply T- or CP-violation, though CP-conserving T-odd effects often constitute the “background” to CP-violation studies. This is the case, for instance, for the transverse polarization of muons in $K^0 \rightarrow \pi^+ \mu^- \bar{\nu}$ decays. We recall here how T-odd observables are related to the absorptive part of a given amplitude by unitarity and T-invariance [2].

The unitarity condition $S^\dagger S = 1$ of the S-matrix describing a transition $i \rightarrow f$

$$S_{if} = \delta_{if} + i(2\pi)^4 \delta^4\left(\sum p_j\right) T_{if}, \quad (2.1)$$

may be written in terms of the absorptive part $\mathcal{A}$

$$\mathcal{A}_{if} = \sum_k T_{ik} T_{fk}^* \quad (2.2)$$

(where the sum extends over all possible on-shell intermediate states) of the scattering amplitude $\mathcal{T}$, as:

$$\mathcal{T}_{fi}^* = \mathcal{T}_{if} - i\mathcal{A}_{if}, \quad (2.3)$$

where $\mathcal{T}_{fi}^*$ is the complex conjugate of the amplitude for the transition $f \rightarrow i$. In

the example of a two-particle intermediate state k (with momenta q_1 and q_2) one has explicitly

$$\mathcal{A}_{if} = \int \frac{d^3 q_1 d^3 q_2}{(4\pi)^2 q_1^0 q_2^0} \delta^4(q_1 + q_2 - P) T_{ik} T_{fk}^*, \quad (2.4)$$

with P the total momentum of the intermediate particles. Take the absolute square of the unitarity equation (2.3) to obtain

$$|\mathcal{T}_{fi}|^2 = |\mathcal{T}_{if}|^2 + 2 \operatorname{Im}(\mathcal{A}_{if} T_{if}^*) + |\mathcal{A}_{if}|^2. \quad (2.5)$$

Now, T-invariance comes into play. If T-invariance holds we have

$$T_{fi} = T_{if}^*, \quad (2.6)$$

where T_{if} denotes the scattering amplitude with sign-flipped initial and final three-momenta and polarizations. Insert (2.6) into (2.5) to obtain

$$|\mathcal{T}_{if}|^2 - |\mathcal{T}_{if}|^2 = -2 \operatorname{Im}(\mathcal{A}_{if} T_{if}^*) - |\mathcal{A}_{if}|^2. \quad (2.7)$$

By definition, a T-odd observable changes sign under the operation $i \rightarrow \bar{i}$, $f \rightarrow \bar{f}$, with the arrow of time staying put (to be distinguished from the definition of a T-violating observable $|\mathcal{T}_{if}|^2 - |\mathcal{T}_{\bar{i}\bar{f}}|^2$, wherein the role of past and future are interchanged). Thus, up to phase-space and flux factors, the left-hand side of (2.7) is the generic form of a T-odd observable. T-odd observables need not vanish in a T-invariant theory, but they have to obey eq.(2.7), in whose derivation T-invariance and unitarity were imposed.

It is instructive to consider (2.7) in perturbation theory (for definiteness think of QED with $e^2 \sim \alpha$ as the perturbation expansion parameter). The absorptive part $\mathcal{A}_{if}$ is at least of order α^2 . Hence, the T-odd observable is at least of order α^3 . At tree-level $|\mathcal{T}_{if}|^2 \sim \alpha^2$ and no T-odd effect occurs. Furthermore, the second

term on the r.h.s of (2.7) is of higher order (α^4) than the first (α^3). Thus, to lowest non-trivial order (α^3) our equation for the T-odd observable simply reads

$$|T_{if}|^2 - |T_{if'}|^2 \simeq -2 \operatorname{Im} (A_{if} T_{if'}^*) . \quad (2.8)$$

Up to phase-space and flux factors, the l.h.s. is twice the T-odd part of the differential cross section. In the next section we will compute the r.h.s. of this equation in $e^+e^- \rightarrow W^+W^-$ to lowest non-trivial order in perturbation theory, and explicitly obtain various T-odd asymmetries.

3. T-odd observables in $e^+e^- \rightarrow W^+W^- \rightarrow l\bar{\nu}\text{jetjet}$

3.1 GENERAL CONSIDERATIONS

We investigate T-odd observables in W -pair production, in particular at LEP II. T-odd observables are necessarily odd in $\epsilon(l_1, l_2, l_3, l_4) \equiv \epsilon_{\mu\nu\rho\sigma} l_1^\mu l_2^\nu l_3^\rho l_4^\sigma$ with l_i momenta or polarizations of the external particles and $\epsilon_{\mu\nu\rho\sigma}$ the totally antisymmetric tensor, with, in our convention, $\epsilon_{0123} = -1$. We could study, for example, an observable proportional to^{*}

$$\epsilon(p_{\text{tot}}, p_{e^-}, p_{W^-}, \Sigma_{W^-}) = -\sqrt{s} \vec{\Sigma}_{W^-} \cdot (\vec{p}_{e^-} \times \vec{p}_{W^-}) , \quad (3.1)$$

where $\vec{\Sigma}_{W^-}$ denotes the polarization vector of the W^- . The quantity (3.1) is a measure of the correlation between the polarization of the W^- and the (oriented) normal to the e^-W^- reaction plane, and one could compute the corresponding

* In technical terms, this epsilon-tensor (among others) results from the computation of the r.h.s. of eq. (2.8), since the imaginary part of Dirac trace(s) selects only those terms having an odd number of epsilon-tensors.

asymmetry

$$\frac{\sigma(\vec{\Sigma}_{W^-} \cdot (\vec{p}_{e^-} \times \vec{p}_{W^-}) > 0)}{\sigma(\vec{\Sigma}_{W^-} \cdot (\vec{p}_{e^-} \times \vec{p}_{W^-}) > 0) + \sigma(\vec{\Sigma}_{W^-} \cdot (\vec{p}_{e^-} \times \vec{p}_{W^-}) < 0)} - \frac{\sigma(\vec{\Sigma}_{W^-} \cdot (\vec{p}_{e^-} \times \vec{p}_{W^-}) < 0)}{\sigma(\vec{\Sigma}_{W^-} \cdot (\vec{p}_{e^-} \times \vec{p}_{W^-}) > 0)} . \quad (3.2)$$

The polarization of W 's is only indirectly measurable, and it is more useful to compute *ab initio* observables involving only directly measurable quantities. We thus proceed to consider the decay of the W 's into fermion pairs. In order to distinguish the W^- from the W^+ we require at least one W to decay leptonically (say $W^- \rightarrow l^- \bar{\nu}$ for definiteness) since tagging the quark charge is a difficult task. So that its momentum be easily measurable, we take $l^- = e^-, \mu^-$. Furthermore, to be able to reconstruct unambiguously the W -momenta, we dispose of events with two final neutrinos by requiring that the other W (the W^+ in our instance) decay into a pair of jets. We are led to consider the T-odd observable proportional to

$$\epsilon(p_{\text{tot}}, p_{e^-}, p_{W^-}, p_{l^-}) = -\sqrt{s} \vec{p}_{l^-} \cdot (\vec{p}_{e^-} \times \vec{p}_{W^-}) , \quad (3.3)$$

with the corresponding asymmetry counting the number of (negatively) charged leptons emitted above the oriented (e^-W^-)-plane minus the number below this plane.

As discussed in the preceding section, such an asymmetry is of order $\alpha^3/\alpha^2 = \alpha \sim 1\%$. Statistics at LEP II are foreseen to be such that one can barely detect asymmetries at the 1%-level. More precisely, LEP II is expected [3] to accumulate a total integrated luminosity of about 500 pb^{-1} which corresponds, at a beam energy of $E = \sqrt{s}/2 = 100 \text{ GeV}$, to approximately 10,000 W pairs. With the branching ratios $B(W \rightarrow e\bar{\nu}, \mu\bar{\nu}) = \frac{2}{9}$ and $B(W \rightarrow \text{jetjet}) = \frac{2}{3}$ we obtain $\frac{2}{9} \times \frac{2}{3} \times 2 = 30\%$ of $e^-/\mu^- \bar{\nu}$ jet jet or jet jet $e^+/\mu^+ \nu$ events, i.e. a total of $N = 3000$ of this type of events. Hence asymmetries can be measured with a statistical significance of 1σ only if they are bigger than $N^{-\frac{1}{2}} \simeq 1.8\%$. With the combined results of four ongoing experiments this could be reduced to $\simeq 0.9\%$. It is nonetheless conceivable

that there will be some high luminosity option for LEP II. Thus, in an optimistic view, one may gain a factor of order two for N and reduce the statistical error by $\sqrt{2}$ to 0.6%. As we will see, at LEP II energies, the T-odd observables we are studying yield in the standard model asymmetries of the order of one half to one per cent, and might thus be barely observable at LEP II.

We will first consider the T-odd observable corresponding to (3.2). Our computations (and display of results) are organized in such a way that it should only be a minor effort to extract other T-odd observables. Indeed, the computation of the absorptive part of the amplitude $e^-e^+ \rightarrow W^-W^+$ for arbitrary W -polarizations, which is the hard core of our computations, is the same for all possible T-odd observables. As an important example we will also compute the T-odd observable corresponding to the following difference of cross sections

$$\sigma(\vec{p}_+ \cdot (\vec{p}_{\text{jetmax}} \times \vec{p}_{\text{jetmin}}) > 0) - \sigma(\vec{p}_+ \cdot (\vec{p}_{\text{jetmax}} \times \vec{p}_{\text{jetmin}}) < 0), \quad (3.4)$$

where we distinguish the jet with the bigger energy, thrust or momentum (p_{jetmax}) from that with the smaller one (p_{jetmin}).

3.2 KINEMATICS AND NOTATION

The kinematics of $e^-e^+ \rightarrow W^-W^+ \rightarrow l\bar{\nu}\text{jetjet}$ is shown in Figure 1. We denote by p_1, p_2, p_3, p_4 the momenta of the e^- , e^+ , W^- , W^+ , respectively, in the CM (or lab) system. We choose the e^- to be incoming along the z -axis, and the $e^-e^+ \rightarrow W^-W^+$ scattering plane to be the (xz) -plane. Let θ be the scattering angle of the W^- with respect to the e^- , and let $E = \sqrt{s}/4$ be the beam energy. Denote the W -mass simply by M , and define $\beta_W^2 = 1 - 4M^2/s$. The explicit four-momenta are:

$$\begin{aligned} p_1 &= E(1, 0, 0, \beta_e), \quad p_2 = E(1, 0, 0, -\beta_e), \\ p_3 &= E(1, \beta_W \sin \theta, 0, \beta_W \cos \theta), \quad p_4 = E(1, -\beta_W \sin \theta, 0, -\beta_W \cos \theta). \end{aligned} \quad (3.5)$$

It is useful to define the combinations

$$\begin{aligned} p &= p_1 + p_2 = p_3 + p_4 = 2E(1, 0, 0, 0), \\ p_- &= p_1 - p_2 = 2E(0, 0, 0, \beta_e), \\ k_- &= p_3 - p_4 = 2E(0, \beta_W \sin \theta, 0, \beta_W \cos \theta). \end{aligned} \quad (3.6)$$

Besides the customary Mandelstam variables s, t and u , we introduce

$$v = p_- \cdot k_- = -s \beta_e \beta_W \cos \theta. \quad (3.7)$$

We may neglect the electron mass, except on those rare occasions (diagram $d3$ below) in which it serves to regularize infra-red divergences at intermediate steps of the calculation. It will also prove handy to introduce

$$\zeta = s - 4M^2, \quad \xi = 1 - \frac{s}{2M^2}. \quad (3.8)$$

As illustrated in Figure 1, the momentum of the negatively (positively) charged fermion produced by the decay of the W^- (W^+) will be labeled k_3 (k_4).

All conventions for γ -matrices and epsilon-tensors are those of ref. 4, in particular $\epsilon_{0123} = -1$.

3.3 FROM THE ABSORPTIVE PART OF THE $e^-e^+ \rightarrow W^-W^+$ AMPLITUDE TO THE T-ODD OBSERVABLES

The computation of T-odd observables in $e^-e^+ \rightarrow W^-W^+$ proceeds in two steps.

The first step

To begin, we compute the absorptive part of the $e^-e^+ \rightarrow W^-W^+$ amplitude for arbitrary W -polarizations. This part of the calculation is universal and it underlies the predictions for all T-odd observables in $e^-e^+ \rightarrow W^-W^+$ for unpolarized electron beams. As (2.7) and (2.2) dictate, and to lowest non-trivial order,

one has to evaluate the interference between the Born approximation to the amplitude and that for all cut one-loop diagrams with on-shell intermediate states. As we work in the frame of the standard model, the possible diagrams merely reduce to those of Figure 2. The contribution of each diagram (in unitary gauge) is given analytically in Appendix A. Note that some ways to cut a one-loop diagram, as illustrated in Figure 3, do not lead to possible on-shell intermediate states. These diagrams do not contribute to T-odd effects, and are omitted from Figure 2. Other cut diagrams (see Figure 4) only give contributions when the finite width of the W is taken into account. These diagrams are suppressed^{*} by an extra factor of $\Gamma_W/M = O(\alpha)$. They are also omitted from Fig. 2, since *sensu stricto* they are of higher order in perturbation theory.

The tree-level amplitude is given by the standard formula [5]

$$T_{if} = -e^2 \bar{v}(p_2) \left(\not{G}(a + b\gamma_5) + c \not{f}_4^*(\not{p}_1 - \not{p}_3) \not{f}_3^*(1 - \gamma_5) \right) u(p_1), \quad (3.9)$$

where

$$a = \frac{1}{s} - \frac{1}{s - M_Z^2} \left(1 - \frac{1}{4 \sin^2 \theta_w} \right), \quad b = -\frac{1}{s - M_Z^2} \frac{1}{4 \sin^2 \theta_w}, \quad c = \frac{1}{t} \frac{1}{4 \sin^2 \theta_w}. \quad (3.10)$$

ϵ_3, ϵ_4 are the polarization vectors of the W^- and W^+ and the γWW -vertex is $-ie\not{G}$ with

$$G^\mu = k_-^\mu \epsilon_3^* \cdot \epsilon_4^* - 2 \left(\epsilon_3^{*\mu} p \cdot \epsilon_4^* - \epsilon_4^{*\mu} p \cdot \epsilon_3^* \right). \quad (3.11)$$

^{*} Dealing with these diagrams requires some degree of care. To treat them properly we give a small mass to the photon and a mass $\tilde{M}$ to the final W . Upon convolution with the Breit-Wigner formula, we see that $\tilde{M} - M$ is of order Γ_W . Though we find for certain diagrams that the absorptive part of the amplitude is $O((\Gamma_W/M)^0)$ or diverges as we take the photon mass to zero, upon combining these diagrams two by two the $O((\Gamma_W/M)^0)$ or divergent part is proportional to the tree-amplitude. As a consequence it does not contribute to the imaginary part in (2.8). The only part that does contribute remains finite and can indeed be seen to be suppressed by a factor $\Gamma_W/M \sim O(\alpha)$. Cf. our general discussion on the absence of infra-red divergences in Sect. 3.4.

The absorptive part of the amplitude $e^- e^+ \rightarrow W^- W^+$ has the general form:

$$A_{if} = \sum_{\text{diagrams}} \int \frac{d^3 q_1 d^3 q_2}{(4\pi)^2 q_1^0 q_2^0} T_{ik} T_{jk}^* \delta^4(q_1 + q_2 - P_{\text{cut}}) = -\frac{1}{32\pi^2} \bar{v}(p_2) \mathcal{M} u(p_1), \quad (3.12)$$

with^{*}

$$\begin{aligned} \mathcal{M} = & \not{G} (c_1 + d_1 \gamma_5) + \not{k}_- \epsilon_3^* \cdot \epsilon_4^* (c_2 + d_2 \gamma_5) + \not{k}_- p \cdot \epsilon_3^* p \cdot \epsilon_4^* (c_3 + d_3 \gamma_5) \\ & + (\not{f}_4^* p_- \cdot \epsilon_3^* + \not{f}_3^* p_- \cdot \epsilon_4^*) (c_4 + d_4 \gamma_5) \\ & + \not{k}_- (p_- \cdot \epsilon_3^* p \cdot \epsilon_4^* - p_- \cdot \epsilon_4^* p \cdot \epsilon_3^*) (c_5 + d_5 \gamma_5) \\ & + \not{k}_- p_- \cdot \epsilon_3^* p_- \cdot \epsilon_4^* (c_6 + d_6 \gamma_5) + \not{f}_4^* \not{k}_- \not{f}_3^* (c_7 + d_7 \gamma_5) + \not{f}_3^* \not{k}_- \not{f}_4^* (c_8 + d_8 \gamma_5) \\ & + (\not{f}_3^* p \cdot \epsilon_4^* - \not{f}_4^* p \cdot \epsilon_3^*) (c_9 + d_9 \gamma_5) \\ & + i\gamma_\rho \epsilon (\rho, k_-, \epsilon_3^*, \epsilon_4^*) (c_{10} + d_{10} \gamma_5) \\ & + i\gamma_\rho (\epsilon (p, k_-, \rho, \epsilon_3^*) p \cdot \epsilon_4^* - \epsilon (p, k_-, \rho, \epsilon_4^*) p \cdot \epsilon_3^*) (c_{11} + d_{11} \gamma_5) \\ & + i\gamma_\rho \epsilon (\rho, p, \epsilon_3^*, \epsilon_4^*) (c_{12} + d_{12} \gamma_5) \\ & + i\gamma_\rho (\epsilon (p, k_-, \rho, \epsilon_3^*) p \cdot \epsilon_4^* + \epsilon (p, k_-, \rho, \epsilon_4^*) p \cdot \epsilon_3^*) (c_{13} + d_{13} \gamma_5). \end{aligned} \quad (3.13)$$

We have evaluated the complete absorptive part of the amplitude to order α^2 , i.e. the contributions of all diagrams of Figure 2. They are given analytically in Appendix A. All computations were done in the unitary gauge. The absorptive part of the amplitude to order α^2 does not involve loops, and all currents are conserved[†], so that unphysical states, such as extra polarizations of the gauge bosons or Faddeev-Popov ghosts do not contribute: the answer is obviously gauge invariant. In Appendix A, the results are presented in such a way that one can immediately read off the contributions of any of the diagrams to the coefficients $c_1 \dots c_{13}$ and $d_1 \dots d_{13}$.

^{*} Note that not all terms in $\mathcal{M}$ are independent, as one can prove several identities between the various pieces of $\bar{v}(p_2) \mathcal{M} u(p_1)$. (This can also be seen on the explicit contributions to the functions $\mathcal{B}_i$ in eq. (3.17) below.) Yet, it is convenient to keep all of the 13 terms in $\mathcal{M}$. Note also that we introduced a minus-sign in the defining equation (3.12) so that $\bar{u} \mathcal{M} u$ be directly given by the Feynman diagrams.

[†] To achieve current conservation one has to combine certain diagrams, e.g. c_5 and d_3 .

The terms in $\mathcal{M}$ proportional to c_{12} , d_{12} or c_{13} , d_{13} violate CP explicitly. In the standard model CP-violation is only due to non-trivial phases of the Cabibbo-Kobayashi-Maskawa mixing matrix. At the order in perturbation theory we are working, the standard model prediction for c_{12} , d_{12} and c_{13} , d_{13} is that they vanish. To study later on how our T-odd observables would be affected by (non-standard) explicitly CP-violating three-gauge-boson couplings, we keep the general form of (3.13) with possible non-zero c_{12} , d_{12} and c_{13} , d_{13} .

The second step

The second step is to exploit the results on the absorptive part and convert them into statements about measurable T-odd observables and the corresponding asymmetries.

We have to compute $-2 \text{Im} \mathcal{A}_{if} \mathcal{T}_{if}^*$ with use of equations (3.9) and (3.12,13). In general this is a sum of five terms, each proportional to one of the five independent epsilon-tensors one can form out of $p, p_-, k_-, \epsilon_3, \epsilon_4, \epsilon_3^*, \epsilon_4^*$ or equivalently, with the momenta p, p_-, k_-, k_4 . To simplify the discussion, we will first only consider the T-odd observable proportional to (3.3), and discuss the one corresponding to (3.4) later on in this subsection.

Concentrate on one W^- , say the W^- , decaying into $l^- \bar{\nu}$ while the other decays into two quarks, i.e. jet-jet. Measuring the momenta carried by the charged lepton and each of the jets allows one to reconstruct both W 's and their momenta. Since we do not (even indirectly) observe the polarization of the W^+ we must average over ϵ_4 :

$$\sum \epsilon_4^\rho \epsilon_4^{\sigma*} \rightarrow -g^{\rho\sigma} + \frac{p_4^\rho p_4^\sigma}{M^2}. \quad (3.14)$$

The decay of the W^- into $l^- \bar{\nu}$ is implemented by the substitution

$$\epsilon_3^\nu \epsilon_3^\mu \rightarrow \frac{3}{8\pi M^2} L_3^{\nu\mu} B_{W \rightarrow l^- \bar{\nu}} d\Omega^*, \quad (3.15a)$$

$$L_3^{\nu\mu} = \text{tr } \not{k}_3 \gamma^\nu (\not{p}_3 - \not{k}_3) \gamma^\mu (1 - \gamma_5)/2, \quad (3.15b)$$

where Ω^* denotes the solid angle in the rest frame of the decaying W^- , M its mass and $B_{W \rightarrow l^- \bar{\nu}}$ is the branching ratio^{*}. Note that, since we are computing asymmetries, the branching ratio will drop out from the final result. Its only effect is on the available statistics.

Now that we are left with only four independent vectors our T-odd observable $\sim -2 \text{Im} \mathcal{A}_{if} \mathcal{T}_{if}^*$ is necessarily proportional to $\epsilon(p, p_-, k_-, k_3)$. Let us recall again that the sign of this epsilon tensor distinguishes leptons l^- above the $e^- W^-$ scattering plane from those below. With use of (3.9) and (3.12) to (3.15) we obtain

$$\begin{aligned} \mathcal{B}_{if} &\equiv -2 \text{Im} \mathcal{A}_{if} \mathcal{T}_{if}^* \Big|_{W \rightarrow l^- \bar{\nu}} \\ &= -\frac{6e^2}{(8\pi)^3 M^4} \epsilon(p, p_-, p_3, k_3) (\mathcal{B}_1 + \mathcal{B}_2 p \cdot k_3 + \mathcal{B}_3 p_- \cdot k_3). \end{aligned} \quad (3.16)$$

The $\mathcal{B}_i$ are functions only of the CM energy and scattering angle θ . They are given in terms of the coefficients c_j , d_j by

$$\begin{aligned} \mathcal{B}_1 &= 2R_2 \zeta v + R_3 s \zeta v - R_4 s(s + 8M^2) + R_5 \zeta(2s^2 - s\zeta + v^2) - R_6 s^2 v \\ &\quad - R_7 \zeta(s + 4M^2 + v) - R_8 \zeta(v - s - 4M^2) + R_9 \zeta v - (\zeta R_{10} + s\zeta R_{11} + sR_{12} - s^2 R_{13})v \\ &\quad + (-2I_2 - \zeta I_3 - vI_5 + I_7 + I_8 - I_9)s^2 + (\zeta I_{10} + s\zeta I_{11} + sI_{12} + s\zeta I_{13})(s + 4M^2) \\ &\quad + ((s + 4M^2)I_4 - s\zeta I_5 + svI_6 + \zeta I_7 - \zeta I_8)v \\ &\quad + (J_1 - \frac{1}{2}J_9)4M^2(s + 2M^2 + v) - J_7 4M^2(s - 2M^2 + v) - (sJ_3 + 3J_4)2M^2(s + v) \\ &\quad + \frac{1}{2}(2J_2 + sJ_3 - J_4 - 2J_7 + J_9)(s^2 - 2\zeta M^2 + v^2 + 2sv - 2M^2 v) \\ &\quad + \frac{1}{2}J_5(s^3 + 2s^2 M^2 - 16sM^4 + v^3 + 3sv^2 - 6M^2 v^2 + 3s^2 v - 4sM^2 v) \\ &\quad - J_6(4sM^4 + \frac{1}{2}v^3 + sv^2 + M^2 v^2 + \frac{1}{2}s^2 v + sM^2 v) + 4(J_4 - 4J_8)M^4 \\ &\quad - 8J_{12}M^4 - \frac{1}{2}(J_{10} + J_{12})(s^2 + 2sM^2 - 16M^4 + v^2 + 2M^2 v + 2sv) \end{aligned}$$

^{*} Equation (3.15a) means that integration of its r.h.s. over $d\Omega^*$ is equivalent to summing the l.h.s over the three physical polarizations of the W^- ; the branching ratio $B_{W \rightarrow l^- \bar{\nu}}$ takes care of the yield of the process we have selected.

$$-\frac{1}{2}sJ_{13}(s+v)(s+v-6M^2), \quad (3.17a)$$

$$B_2 = 16R_{13}vM^2 + 4(2I_2 - \zeta I_3 + vI_5 - I_7 - I_8 + I_9)s$$

$$+ 2(-2J_2 + sJ_3 - vJ_5 + J_7 + J_8 - J_9)(s+v) + 4(J_{10} + sJ_{11} - J_{12})M^2 \\ + 4((2s+v)J_3 + J_4 - sJ_5 + J_7 - J_8 + (2v-s)J_{13})M^2, \quad (3.17b)$$

$$B_3 = 4(\zeta R_{10} + s\zeta R_{11} + sR_{12} + s\zeta R_{13}) - 4((s+4M^2)I_4 - s\zeta I_5 + svI_6 + \zeta I_7 - \zeta I_8) \\ + 2(J_4 - sJ_5 + vJ_6 + J_7 - J_8 + J_{12})(s+v) + 2(J_{10} + sJ_{11} + sJ_{13})(\zeta+v) \\ + 4(-4J_1 - 2J_2 + 2J_4 + (2s+v)J_5 + sJ_6 + J_7 + 5J_8 + J_9)M^2, \quad (3.17c)$$

where

$$R_i = ac_i + bd_i, \quad I_i = bc_i + ad_i, \quad J_i = c(c_i - d_i), \quad (3.18)$$

with a, b, c as in eq. (3.10).

From equation (2.8) of Section 2, we know that $B_{if} \equiv -2\text{Im}A_{if}T_{if}^*|_{W+l-\bar{\nu}}$ is, up to the phase-space and flux factors, twice the T-odd part of the fourfold differential cross section :

$$d\sigma_{T\text{-odd}}^{W+l-\bar{\nu}} = \frac{\beta_W}{2s} B_{W+l-\bar{\nu}} \frac{d\Omega}{32\pi^2} \frac{B_{if}}{2} d\Omega^* \quad (3.19)$$

We wish to give results also in terms of asymmetries of the type

$$AS^{W+l-\bar{\nu}} = \frac{\sigma(l^- \text{ above the } e^-W^- \text{ plane}) - \sigma(l^- \text{ below the } e^-W^- \text{ plane})}{\sigma(l^- \text{ above the } e^-W^- \text{ plane}) + \sigma(l^- \text{ below the } e^-W^- \text{ plane})} \quad (3.20)$$

Numerator and denominator might be integrated over all CM scattering angles θ or only over a certain range of θ . We parametrize the momentum of the lepton l^- in the rest frame of the W^- (denoted by a^*) as

$$k_3^* = \frac{M^2}{2} (1, \sin\psi \cos\alpha, \cos\psi, \sin\psi \sin\alpha). \quad (3.21)$$

Then, $0 \leq \psi \leq \pi/2$ corresponds to l^- 's above the plane and $\pi/2 \leq \psi \leq \pi$ to

the l^- 's below^{*}. Since $\epsilon(p_-, p_-, p_3, k_3) = -2E^3 M \beta_W \sin\theta \cos\psi$ we obtain from equation (3.16)

$$\frac{d\sigma_{T\text{-odd}}^{W+l-\bar{\nu}}}{d\Omega} = \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{2}}^{\pi} \right) d\cos\psi \int_0^{2\pi} d\alpha \frac{d\sigma_{T\text{-odd}}^{W+l-\bar{\nu}}}{d\Omega d\Omega^*} \\ = \frac{3\alpha}{4(16\pi)^3} B_{W+l-\bar{\nu}} \frac{\sqrt{s}}{M^3} \beta_W^2 \sin\theta \left(B_1(\theta) + \frac{s}{4} B_2(\theta) - \frac{s}{4} \beta_W B_3(\theta) \cos\theta \right). \quad (3.22)$$

Integrating next over a given range of the CM scattering angle θ , we define

$$\sigma_{T\text{-odd}}^{W+l-\bar{\nu}}(\theta_{\min}, \theta_{\max}) = 2\pi \int_{\theta_{\min}}^{\theta_{\max}} \sin\theta d\theta \frac{d\sigma_{T\text{-odd}}^{W+l-\bar{\nu}}}{d\Omega}, \quad (3.23)$$

and the asymmetries

$$AS^{W+l-\bar{\nu}}(\theta_{\min}, \theta_{\max}) = \frac{\sigma_{T\text{-odd}}^{W+l-\bar{\nu}}(\theta_{\min}, \theta_{\max})}{\sigma_{W+l-\bar{\nu}}(\theta_{\min}, \theta_{\max})}, \quad (3.24)$$

where $\sigma_{W+l-\bar{\nu}}(\theta_{\min}, \theta_{\max})$ denotes the usual tree-level cross section for $e^-e^+ \rightarrow W^-W^+$ with cuts for $\theta \leq \theta_{\min}$ and $\theta \geq \theta_{\max}$ times the branching ratio for $W \rightarrow l\bar{\nu}$. Note that the branching ratio drops out from the asymmetry. We shall simply write AS for $AS(0, \pi)$.

Other T-odd observables

We could consider T-odd observables other than the one computed in the preceding subsection. The “first step” is exactly the same and only the second one is modified. Instead of defining the plane with respect to which we measure the asymmetry to be the e^-W^- scattering plane, we could use the plane defined by the two jets (one of the W 's, say the W^- , is still supposed to decay leptonically).

* Note that this statement is Lorentz invariant, i.e. it is true in the W^- rest frame as well as in the CM frame, since the boost relating both frames is orthogonal to the y -axis.

This plane can be oriented by distinguishing the jet with the bigger total three-momentum (jetmax) from the other (jetmin). Since we now have five different momenta at our disposal (the two jet momenta k_4 and $p_4 - k_4$ or equivalently k_4 and k_- , and as before k_3 , p_- and p) the analogue of equation (3.16) for $\mathcal{B}_{if}$ contains five different epsilon-tensors (although only three of them are independent). This makes the “second step” slightly more complicated, but the generalization to this case is still fairly straightforward.

The asymmetry we now wish to compute is

$$\mathcal{AS}^{\text{jetmax/jetmin}} = \frac{\sigma(l^- \text{ above the jetmax/jetmin plane}) - \sigma(l^- \text{ below})}{\sigma(l^- \text{ above the jetmax/jetmin plane}) + \sigma(l^- \text{ below})}. \quad (3.25)$$

Instead of summing over the polarizations of the W^+ as we did before, we now make the substitutions (3.15a,b) and their analogue for the decay of W^+ , to wit:

$$\epsilon_4^{\rho} \epsilon_4^{*\sigma} \rightarrow \frac{3}{8\pi M^2} L_4^{\rho\sigma} B_{W \rightarrow \text{jet jet}} d\Omega^{**}, \quad (3.26a)$$

$$L_3^{\rho\sigma} = \text{tr } \not{k}_4 \gamma^{\rho} (\not{p}_4 - \not{k}_4) \gamma^{\sigma} (1 - \gamma_5)/2, \quad (3.26b)$$

where Ω^{**} denotes the solid angle in the W^+ rest frame. Experimentally, one cannot in general tell the charge of the initial quark of a jet, but this does not prevent us from fixing our notation at an intermediate stage of the computation: call k_4 the momentum of the jet generated by the quark (originating in the decay of the W^+ with momentum p_4) with the bigger positive charge. The final asymmetry will be expressed only in terms of p_{jetmax} and p_{jetmin} . Let k_3 and k_4 be respectively given in the rest frames of the W^- and W^+ by

$$k_3^* = \frac{M}{2}(1, \hat{v}), \quad k_4^{**} = \frac{M}{2}(1, \hat{w}), \quad (3.27)$$

where $\hat{v}$ and $\hat{w}$ are unit (three-)vectors. Denote further by $\hat{u} = \vec{k}_-/(2E\beta_W)$ the direction of the W^- and by $\hat{e}_z = \vec{p}_-/(2E)$ the direction of the incoming e^- in the

CM frame. Transforming k_3 and k_4 into the CM frame, we have

$$k_3 = \frac{E}{2} \left(1 + \beta_W \hat{u} \cdot \hat{v}, \quad (\beta_W + (1 - M/E)\hat{u} \cdot \hat{v})\hat{u} + M/E \hat{v} \right) \quad (3.28a)$$

$$k_4 = \frac{E}{2} \left(1 - \beta_W \hat{u} \cdot \hat{w}, \quad (-\beta_W + (1 - M/E)\hat{u} \cdot \hat{w})\hat{u} + M/E \hat{w} \right) \quad (3.28b)$$

$$p_4 - k_4 = \frac{E}{2} \left(1 + \beta_W \hat{u} \cdot \hat{w}, \quad (-\beta_W - (1 - M/E)\hat{u} \cdot \hat{w})\hat{u} - M/E \hat{w} \right) \quad (3.28c)$$

A convenient parametrization is

$$\begin{aligned} \hat{u} &= (0, 0, 1), \quad \hat{v} = (\sin \psi \cos \alpha, \cos \psi, \sin \psi \sin \alpha), \\ \hat{w} &= (\sin \chi, 0, \cos \chi), \quad \hat{e}_z = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta). \end{aligned} \quad (3.29)$$

As before, θ is the CM scattering angle, but we have performed a rotation of the spatial reference frame with respect to eqs.(3.5). Comparing the last two equations (3.28) we see that the condition $|\vec{k}_4| > |\vec{p}_4 - \vec{k}_4|$ (resp. $|\vec{p}_4 - \vec{k}_4| > |\vec{k}_4|$), i.e. for the first (resp. second) jet to be the most impulsive, is equivalent to $\hat{u} \cdot \hat{w} < 0$ (resp. > 0). In view of eq. (3.29) this is in turn tantamount to $\cos \chi < 0$ (resp. > 0).

Suppose first $\cos \chi < 0$. Then $|\vec{k}_4| > |\vec{p}_4 - \vec{k}_4|$ and

$$\begin{aligned} \vec{k}_3 \cdot (\vec{p}_{\text{jetmax}} \times \vec{p}_{\text{jetmin}}) &= \vec{k}_3 \cdot (\vec{k}_4 \times \vec{p}_4) \\ &= -M^2 \beta_W \hat{v} \cdot (\hat{w} \times \hat{u}) = M^2 \beta_W \sin \chi \cos \psi. \end{aligned} \quad (3.30)$$

An event will be counted with a “+”-sign (resp. “-”-sign) in the numerator of the asymmetry $\mathcal{AS}^{\text{jetmax/jetmin}}$ if this expression is positive (resp. negative) i.e. if $\cos \psi > 0$ (resp. $\cos \psi < 0$).

Suppose next $\cos \chi > 0$. Then $|\vec{p}_4 - \vec{k}_4| > |\vec{k}_4|$ and

$$\vec{k}_3 \cdot (\vec{p}_{\text{jetmax}} \times \vec{p}_{\text{jetmin}}) = \vec{k}_3 \cdot (\vec{p}_4 \times \vec{k}_4) = -M^2 \beta_W \sin \chi \cos \psi. \quad (3.31)$$

An event will be counted with a “+”-sign (resp. “-”-sign) in the numerator of the asymmetry $\mathcal{AS}^{\text{jetmax/jetmin}}$ if this expression is positive (resp. negative) i.e. if $\cos \psi < 0$ (resp. $\cos \psi > 0$).

The analogue of the definition (3.22) is

$$\frac{d\sigma_{T-\text{odd}}^{l^+ \bar{\nu} \text{jetmax jetmin}}}{2\pi d\cos\theta} = \left(\int_{\frac{\pi}{2}}^{\pi} d\cos\chi \left(\int_0^{\frac{\pi}{2}} - \int_{\frac{\pi}{2}}^{\pi} \right) d\cos\psi + \int_0^{\frac{\pi}{2}} d\cos\chi \left(\int_{\frac{\pi}{2}}^{\pi} - \int_0^{\frac{\pi}{2}} \right) d\cos\psi \right) \times \int_0^{2\pi} d\phi \int_0^{2\pi} d\alpha \frac{d\sigma_{T-\text{odd}}^{l^+ \bar{\nu} \text{jetmax jetmin}}}{d\Omega d\Omega^* d\Omega^{**}}, \quad (3.32)$$

where in complete analogy with (3.19) we have

$$\frac{d\sigma_{T-\text{odd}}^{l^+ \bar{\nu} \text{jetmax jetmin}}}{d\Omega d\Omega^* d\Omega^{**}} = -\frac{\beta_W}{2s} B_{W \rightarrow l^+ \bar{\nu} B_W \rightarrow \text{jet jet}} \frac{1}{32\pi^2} \text{Im} \mathcal{A}_{if} T_{if}^* \Big|_{l^+ \bar{\nu} \text{jetmax jetmin}}. \quad (3.33)$$

It is straightforward to compute the r.h.s of this expression. The result is much more complex than (3.16,17), due to the five independent momenta involved, and we do not report it explicitly. Once all integrations in (3.32) are performed, the final result is once again a relatively simple function of the CM scattering angle :

$$\frac{1}{2\pi} \frac{d\sigma_{T-\text{odd}}^{l^+ \bar{\nu} \text{jetmax jetmin}}}{d\cos\theta} = \frac{3\alpha s^2 \beta_W^2}{4(16\pi)^3 M^2} B_{W \rightarrow l^+ \bar{\nu} B_W \rightarrow \text{jet jet}} (\sin^2\theta \mathcal{B}_4 + \mathcal{B}_5), \quad (3.34)$$

where (here and for the rest of this section, we simply write β for β_W)

$$\begin{aligned} \mathcal{B}_4 &= 8R_2\beta + 2R_3\zeta - 2R_4(1-\beta^2)\cos\theta - 4R_5\zeta\cos\theta \\ &\quad - R_6s\beta(2 + (1-\beta^2)\sin^2\theta) - 4(R_7 + R_8 - R_9)\beta \\ &\quad + (J_1 - J_4)(1-\beta^2)\cos\theta + J_2(4(\beta - \cos\theta) + (1-\beta^2)\cos\theta) + J_3\zeta(\beta - \cos\theta) \\ &\quad - 2J_5(\zeta\cos\theta - s\beta\cos^2\theta) - J_6s(\beta - \cos\theta + \frac{1}{2}\beta(1-\beta^2)\sin^2\theta) \\ &\quad + (J_7 + J_8)(-2\beta + (1+\beta^2)\cos\theta) + 2J_9(\beta - \cos\theta), \\ \mathcal{B}_5 &= \frac{1}{\beta} (J_{12} + s\beta^2 J_{13}) (2 + 2\beta^2 - 4\beta\cos\theta - (3 + \beta^2)\sin^2\theta) \\ &\quad + 8(I_{12} + s\beta^2 I_{13}) \cos\theta. \end{aligned} \quad (3.35a) \quad (3.35b)$$

Note that most of the terms $\sim I_i$ do not contribute to the integrated result. The same can be seen to be true for the other T-odd observable in eq. (3.22): there are important cancellations between $\mathcal{B}_1$, $\mathcal{B}_2$ and $\mathcal{B}_3$.

Integrating once more over a given range of the CM scattering angle θ we define

$$\sigma_{T-\text{odd}}^{l^+ \bar{\nu} \text{jetmax jetmin}}(\theta_{\min}, \theta_{\max}) = \int_{\theta_{\min}}^{\theta_{\max}} \sin\theta d\theta \frac{d\sigma_{T-\text{odd}}^{l^+ \bar{\nu} \text{jetmax jetmin}}}{d\cos\theta}, \quad (3.36)$$

and the asymmetries

$$\mathcal{A}_{S^{\text{jetmax/jetmin}}}(\theta_{\min}, \theta_{\max}) = \frac{\sigma_{T-\text{odd}}^{l^+ \bar{\nu} \text{jetmax jetmin}}(\theta_{\min}, \theta_{\max})}{\sigma_{T-\text{odd}}^{l^+ \bar{\nu} \text{jet jet}}(\theta_{\min}, \theta_{\max})}, \quad (3.37)$$

where $\sigma^{l^+ \bar{\nu} \text{jet jet}}(\theta_{\min}, \theta_{\max})$ denotes the tree-level cross section for $e^- e^+ \rightarrow W^- W^+$ with cuts for $\theta \leq \theta_{\min}$ and $\theta \geq \theta_{\max}$ times the branching ratios for $W \rightarrow l\bar{\nu}$ and $W \rightarrow \text{jet jet}$. As before, the branching ratios cancel between the numerator and denominator of the asymmetry.

3.4 INFRA-RED DIVERGENCES

Several individual diagrams of Figure 2 give contributions to the absorptive part of the amplitude that diverge as the photon mass tends to zero (cf. Appendix A). This is the case for diagrams $b4$, $c6$, $d1$, $d4$, $g4$ and $g6$. However, there should not be an infra-red divergence in the T-odd observables, and the imaginary part $\text{Im} \mathcal{A}_{if} T_{if}^*$ has to be infra-red finite. Indeed, this is the case as one can explicitly check on the results of Appendices A and B. Let us briefly discuss why this is so.

In the case of diagrams $g4$ and $g6$ the potentially divergent terms simply add up to zero in the absorptive part. This is clearly a consequence of the gauge-invariant nature of the electromagnetic coupling.

The case of the diagrams $b4$, $c6$, $d1$, $d4$ is different. The IR divergent part of their contribution to the absorptive part is of the form $\mathcal{A}_{if} \sim \log(m_\gamma^2/s) \times \mathcal{T}_{if}$. Thus $\text{Im } \mathcal{A}_{if} \mathcal{T}_{if}^* \sim \text{Im } |\mathcal{T}_{if}|^2 = 0$. This can be easily understood without actually computing these diagrams. Consider for example diagrams $b4$ and $d1$, and let k , as in (2.2), denote the intermediate WW state. The dangerous contribution arises when the momentum carried by the photon vanishes. In this limit, the amplitude associated with $(W^- W^+)_k \rightarrow W^- W^+$ in these diagrams reduces to a numerical multiple of $\log(m_\gamma^2/s)$, whereas the sum of the amplitudes corresponding to $e^- e^+ \rightarrow (W^- W^+)_k$ exactly reproduces the tree amplitude $\mathcal{T}_{if}$. The same applies for diagrams $c6$ and $d4$. For our numerical computations (see next section) we have not extracted the IR divergent parts explicitly, but simply computed the appropriate sums of these diagrams with small but finite photon masses (10^{-2} to 10^{-3} GeV) and checked that the result stays put (i.e. the asymmetry varies by less than $10^{-3}\%$) as the tiny regulator photon mass is reduced by a factor of ten.

3.5 THE HIGH-ENERGY BEHAVIOUR OF THE T-ODD OBSERVABLES

The dominating contributions at very high energy to our T-odd observables originate from the diagrams $a3, b7, c1, c3$ which yield asymmetries $\mathcal{A}^{W^+ l^- \nu}$ growing like $s^{5/2}$ and from the diagrams $b4 + d1, b5, d2$ growing like $s^{3/2}$ (up to $\log s$ corrections). The corresponding asymmetries $\mathcal{A}^{S^{\text{etmax}}/j^{\text{etmin}}}$ grow like s^2 and s , respectively. These diagrams involve intermediate W 's and the sum over their polarizations gives rise to terms $\sim s/M^2$ or even $\sim (s/M^2)^2$. The leading $s^{5/2}$ -behaviour cancels between $a3$ and $c1$, and between $b7$ and $c3$, for reasons similar to those discussed in the previous subsection. It is less trivial to check analytically (but easy to read off the numerical results of the next section) that all contributions to the asymmetry that are $\sim s^{3/2}$ and even $\sim s^{1/2}$ cancel between these 8 diagrams. The resulting asymmetry does not violate the unitarity-bound $\mathcal{A}S \leq 1$. These cancellations are rather subtle, a wee change in the coupling constant of the four- W -vertex, for instance, would lead to a completely unacceptable high-energy behaviour of the asymmetries: not an unheard of behaviour for a gauge theory.

3.6 EFFECTS OF EXPLICITLY CP-VIOLATING VERTICES ON THE T-ODD OBSERVABLES

So far, we have computed the T-odd observables in the standard model. Let us emphasize again that the CP-violation inherent in the standard model is totally negligible as a source of the T-odd effects at hand. CP-conserving T-odd effects constitute in our case, as in many others, the “background” to a search for novel sources of CP-violation. In this subsection, we will study the effect on our T-odd observables of explicitly CP-violating three-gauge-boson vertices.

Two possible CP-violating (i.e. CP-odd) γWW or ZWW couplings exist:

$$ie \lambda_V \epsilon(\epsilon_V, p, \epsilon_3^*, \epsilon_4^*) \quad (3.38a)$$

and

$$i \frac{e \lambda'_V}{M^2} [\epsilon(p, k_-, \epsilon_V, \epsilon_3^*) p \cdot \epsilon_4^* + \epsilon(p, k_-, \epsilon_V, \epsilon_4^*) p \cdot \epsilon_3^*] \quad (3.38b)$$

where V denotes either the photon or the Z boson, ϵ_V its polarization vector, and λ_V and λ'_V are (small) dimensionless parameters. These couplings are the momentum space forms of the lowest-dimension CP-odd effective interactions of γ 's and Z 's with W pairs, that describe the electric dipole and magnetic quadrupole moments of W 's and their generalizations to weak neutral currents. These exist in the standard model only at the level of multiloop radiative corrections involving all three quark generations and they are numerically negligible. In theories, such as supersymmetry, in which there are sources of CP-violation other than the phase in the Cabibbo-Kobayashi-Maskawa matrix, the coefficients of the CP-odd trilinear vector boson vertices may be bigger than in the standard model. It is convenient [6] to extract explicitly the coupling strengths of γ 's and Z 's to W 's and to use the mass of the Z as the parameter setting the relative scale between couplings of different dimensionality to write the CP-odd effective Lagrangian from which

(3.38a,b) stem in the form^{*}:

$$\delta\mathcal{L}_{\text{CP}}^\gamma = -ie\delta^\gamma W_\mu W_\nu^\dagger \tilde{F}_\gamma^{\mu\nu} + ie\frac{\delta^\gamma}{M_Z^2} W_\mu^\rho W_\nu^\dagger \tilde{F}_\gamma^{\mu\nu}, \quad (3.39a)$$

$$\delta\mathcal{L}_{\text{CP}}^Z = -ie\frac{\delta^Z}{\sin\theta_w \cos\theta_w} W_\mu W_\nu^\dagger \tilde{F}_Z^{\mu\nu} + ie\frac{\delta^Z}{\sin\theta_w \cos\theta_w M_Z^2} W_\mu^\rho W_\nu^\dagger \tilde{F}_Z^{\mu\nu}. \quad (3.39b)$$

where $\delta^{\gamma,Z}$ and $\tilde{F}^{\gamma,Z}$ are dimensionless parameters, W_μ is the W -field and $W_{\mu\nu}$ the associated field strength, while $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ is the dual field strength of the photon or Z . The parameters $\lambda_{\gamma,Z}$ and $\lambda'_{\gamma,Z}$ are related to $\delta^{\gamma,Z}$ and $\tilde{F}^{\gamma,Z}$ by

$$\begin{aligned} \lambda_\gamma &= \delta^\gamma + \left(\frac{s}{2M^2} - 1\right) \cos^2\theta_w \tilde{\delta}^\gamma \\ \lambda_Z &= \frac{1}{\sin\theta_w \cos\theta_w} \delta^Z + \left(\frac{s}{2M^2} - 1\right) \frac{\cos\theta_w}{\sin\theta_w} \tilde{\delta}^Z \\ \lambda'_\gamma &= -\frac{1}{2} \cos^2\theta_w \tilde{\delta}^\gamma \\ \lambda'_Z &= -\frac{1}{2} \frac{\cos\theta_w}{\sin\theta_w} \tilde{\delta}^Z. \end{aligned} \quad (3.40)$$

The above discussion is devoid of theoretical prejudice other than the expectation that in a theory wherein CP-violation originates from dynamics at a scale similar to or larger than the standard weak scale, the low energy CP-odd effective interactions of gauge bosons will be describable by the couplings of lowest dimension. A more thorough analysis of the consequences of gauge invariance [6] adds extra information on the coefficients appearing in (3.39) and (3.40). The standard model is $SU(2) \times U(1)$ gauge invariant above the weak symmetry-breaking scale, a condition playing a crucial role in its consistency, renormalizability and predictive power. It is difficult to imagine how any hypothetical theory mimicking the predictions of the standard model at the currently explored energies could violate

* Note that, with respect to the convention of ref. 6, we have to write an extra minus sign since we use the opposite sign convention for the ϵ -tensor.

$SU(2) \times U(1)$ gauge invariance beyond the weak scale. This is particularly true in connection with those observables that are phenomenologically understood at the level of radiative corrections, though in particular cases (such as Veltman's “ ρ -parameter”) it is possible to impose ad-hoc remedial or “custodial” symmetries as a device to avert evil. The consequences of the imposition of the $SU(2) \times U(1)$ local symmetry on the CP-violating effective Lagrangians in (3.39) were analysed in detail in ref. 6. It was shown that gauge invariance puts all the dimension 4 and dimension 6 CP-violating operators on the same footing: they all “descend” from dimension 6 operators. Moreover, the $SU(2) \times U(1)$ gauge invariance implies the following relations between the dimensionless parameters δ^γ , $\tilde{\delta}^\gamma$ and δ^Z , $\tilde{\delta}^Z$:

$$\begin{aligned} \delta^Z &= -\sin^2\theta_w \delta^\gamma, \\ \tilde{\delta}^Z &= \cos^2\theta_w \tilde{\delta}^\gamma. \end{aligned} \quad (3.41)$$

We shall analyse the possible observability of the CP-odd three-gauge-boson couplings with and without the constraints (3.41). When these are imposed, we parametrize all results in terms of $\delta^\gamma \equiv \delta$ and $\tilde{\delta}^\gamma \equiv \tilde{\delta}$ in which case the equations (3.40) are to be substituted by:

$$\begin{aligned} \lambda_\gamma &= \delta + \left(\frac{s}{2M^2} - 1\right) \cos^2\theta_w \tilde{\delta} \\ \lambda_Z &= -\frac{\sin\theta_w}{\cos\theta_w} \delta + \left(\frac{s}{2M^2} - 1\right) \frac{\cos^3\theta_w}{\sin\theta_w} \tilde{\delta} \\ \lambda'_\gamma &= -\frac{1}{2} \cos^2\theta_w \tilde{\delta} \\ \lambda'_Z &= -\frac{1}{2} \frac{\cos^3\theta_w}{\sin\theta_w} \tilde{\delta}. \end{aligned} \quad (3.42)$$

To first order in the parameters λ , λ' the vertices (3.38a,b) contribute to the square of the total tree-level amplitude

$$|\mathcal{T}_{if}|^2 = |\mathcal{T}_{if}^{\text{SM}}|^2 + 2 \text{Im}(\mathcal{T}_{if}^{\text{SM}*} i\mathcal{T}_{if}^{\text{CP}}), \quad (3.43)$$

where $\mathcal{T}_{if}^{\text{SM}}$ and $\mathcal{T}_{if}^{\text{CP}}$ denote the tree-level contributions to the amplitude of the

standard model (i.e. eq. (3.9)) and of the CP-violating vertices (3.38) :

$$iT_{if}^{\text{CP}} = \frac{\lambda_\gamma e^2}{s} \bar{v} i\gamma^\mu \epsilon(\mu, p, \epsilon_3^*, \epsilon_4^*) u + \frac{\lambda'_\gamma e^2}{sM^2} \bar{v} i\gamma^\mu [\epsilon(p, k_-, \mu, \epsilon_3^*) p \cdot \epsilon_4^* + \epsilon(p, k_-, \mu, \epsilon_4^*) p \cdot \epsilon_3^*] u \quad (3.44)$$

for the γWW coupling, and similarly for the ZWW coupling. The second term on the right-hand-side of (3.43) is T-odd, and hence

$$|T_{if}|^2 - |\bar{T}_{if}|^2 = 4 \text{Im} (T_{if}^{\text{SM}*} i T_{if}^{\text{CP}}) \quad (3.45)$$

Compare with equations (2.8), (3.12) and (3.13) to conclude that the contribution to the T-odd observables is simply found by adopting the following values for the coefficients c_{12} , c_{13} and d_{12} , d_{13}

$$\begin{aligned} c_{12} &= 64\pi^2 \lambda_\gamma e^2 \frac{1}{s} & , & \quad d_{12} = 0 & (\gamma WW \text{ coupling}) \\ d_{12} &= -\frac{64\pi^2 \lambda_Z e^2}{4 \sin \theta_w \cos \theta_w} \frac{1}{s - M_Z^2} & , & \quad c_{12} = (4 \sin^2 \theta_w - 1) d_{12} & (ZWW \text{ coupling}) \\ c_{13} &= 64\pi^2 \lambda'_\gamma e^2 \frac{1}{sM^2} & , & \quad d_{13} = 0 & (\gamma WW \text{ coupling}) \\ d_{13} &= -\frac{64\pi^2 \lambda'_Z e^2}{4 \sin \theta_w \cos \theta_w} \frac{1}{(s - M_Z^2)M^2} & , & \quad c_{13} = (4 \sin^2 \theta_w - 1) d_{13} & (ZWW \text{ coupling}) \end{aligned} \quad (3.46)$$

with all other c_i, d_i equal to zero.

4. Results

The formulae of the preceding section and of the appendices A and B permit a numerical evaluation of the various T-odd parts of the cross sections $\sigma_{\text{T-odd}}$ that we have considered. These T-odd effects are of order α^3 . The asymmetries are obtained by dividing by the total cross section σ_{tot} which is $\sim \alpha^2 + \text{higher order}$. The higher order terms in σ_{tot} should be consistently neglected since they contribute to the asymmetries at order α^2 (as would α^4 -contributions in $\sigma_{\text{T-odd}}$ - which were neglected). One might argue that some of the one-loop corrections to σ_{tot} , although $O(\alpha^3)$, multiply large logarithms $\log \frac{\Delta E}{E}$ [7,8]. Here ΔE is a soft photon energy cut-off due to the inability of the detector to distinguish $e^+e^- \rightarrow W^+W^-$ from $e^+e^- \rightarrow W^+W^- \gamma$ with a photon energy less than ΔE . For ΔE as small as $0.05E$ this could lead to corrections of about 10% to σ_{tot} (at $E = 100\text{GeV}$) [7]. This would still affect our asymmetries only by an unmeasurable amount, but nevertheless, we give below the total cross sections σ_{tot} (without cuts) and $\sigma_{\text{tot}}^{\text{cut}}$ (with cuts for $\theta < 0.15 \pi$ and $\theta > 0.85 \pi$) we used for computing the asymmetries. If for some mysterious reason, the measured total cross section were found to deviate considerably from the tree-level one, one could easily correct our numerical results for the asymmetries by multiplying them with $\sigma_{\text{tot}}/\sigma_{\text{tot}}^{\text{measured}}$.

Table 1 : The total cross sections σ_{tot} (without cuts) and $\sigma_{\text{tot}}^{\text{cut}}$ (with cuts for $\theta < 0.15 \pi$ and $\theta > 0.85 \pi$) we use for computing the asymmetries

$E [\text{GeV}]$	92	100	120	150	200	400
$\sigma_{\text{tot}} [\text{pb}]$	19.7	20.1	17.7	14.0	9.9	3.8
$\sigma_{\text{tot}}^{\text{cut}} [\text{pb}]$	16.1	15.5	11.6	7.3	3.9	0.88

All of our calculations, analytical and numerical, were performed twice and independently, using two different computers and different algebraic manipulation programs (REDUCE and FORM) where appropriate. We required numerical agreement for the asymmetries better than $10^{-3}\%$.

For the numerical computations we used the following set of parameters :

$$\begin{aligned}
 \alpha(M_Z) &= 1/128 \\
 \sin^2 \theta_w &= 0.227 \\
 M_Z &= 91.17 \text{ GeV} \\
 M &= 80.2 \text{ GeV}
 \end{aligned}
 \tag{4.1}$$

All quark and lepton masses were set to zero, except for the top quark. (Taking, for example, the bottom quark mass to be $m_b = 4.5 \text{ GeV}$, would change the contribution of the diagram $a1$ to the asymmetry by less than a negligible 0.001%.) We kept variable the masses of the top quark as well as that of the Higgs boson.

4.1 THE ASYMMETRIES $\mathcal{AS}^{W^+l^-\bar{\nu}}$

The various contributions at different energies

The contributions of the various diagrams (in unitary gauge) to the asymmetry $\mathcal{AS}^{W^+l^-\bar{\nu}}$ are given in tables 2 to 5. We considered different beam energies $E = \sqrt{s}/2$: first, $E = 100 \text{ GeV}$ is approximately the energy for which the $e^+e^- \rightarrow W^+W^-$ cross section is maximum, and hence LEP II will probably be operated a fair fraction of the time at this energy. There is a good chance that LEP II will also be run at somewhat higher energies, in order to achieve a better sensitivity to possible non-standard gauge couplings (e.g. anomalous magnetic dipole and electric quadrupole moments of the W 's). Hence we have included other energies, $E = 120, 150 \text{ GeV}$, and also, in view of possible future machines, $E = 200$ and 400 GeV . Table 2 shows the contributions of all diagrams involving neither the top quark nor the Higgs boson * . Table 3 displays our results for those contributions (diagrams $a1$ and $b2$) that depend on the mass of the top quark for various m_{top} and beam energies E . We only give the sum of the contributions of

* Strictly speaking, diagrams $e1$ to $e3$ would depend on the top quark mass if the top were light enough for a W to decay into a top-bottom pair. This is barely excluded experimentally by now, and hence, the diagrams $e1$ to $e3$ should be independent of m_{top} .

diagrams $a1$ and $b2$ (as an example for $E = 100 \text{ GeV}$ and $m_{\text{top}} = 150 \text{ GeV}$, the individual contributions are 2.519% for $a1$ and -1.709% for $b2$). Table 4 shows the individual Higgs-boson-dependent contributions of the diagrams $a5, b5, b6$ and $d2$ for various masses M_H and fixed energy $E = 100 \text{ GeV}$, while Table 5 displays the sum of these four contributions for different Higgs boson masses and energies.

Table 2 : The asymmetries $\mathcal{AS}^{W^+l^-\bar{\nu}}$ in per cent
for those diagrams not involving m_{top} and M_H ,
at beam energies $E = 100, 120, 150, 200$ and 400 GeV .

diagram	100 GeV	120 GeV	150 GeV	200 GeV	400 GeV
$a3$	2.297	8.375	24.881	81.401	1379.285
$b1$	-0.486	-0.608	-0.688	-0.757	-0.970
$b3$	-0.304	-0.358	-0.390	-0.419	-0.530
$b4 + d1$	-0.064	0.412	0.991	1.669	3.645
$b7$	-0.881	-3.870	-13.771	-53.791	-1159.723
$c1$	-2.583	-9.274	-26.879	-85.486	-1397.582
$c3$	1.205	4.613	15.272	56.897	1175.395
$c5 + d3$	-0.367	-0.385	-0.334	-0.259	-0.133
$c6 + d4$	0.041	0.031	0.018	0.007	-0.005
$c7$	-0.680	-1.047	-1.501	-2.191	-4.702
$d5$	0.751	1.141	1.611	2.286	4.765
$e1 + e2 + e3$	0.516	0.493	0.395	0.271	0.100
$f4$	0.077	0.094	0.100	0.098	0.082
$g4 + g6$	0.168	0.132	0.071	-0.003	-0.094
$g5$	-0.186	-0.204	-0.188	-0.149	-0.072
subtotal	-0.496	-0.455	-0.412	-0.426	-0.539

Table 3 : The asymmetries $\mathcal{A}^{W^+l^-\bar{\nu}}$ in per cent for those diagrams (*a1* and *b2*) involving m_{top} , at beam energies $E = 100, 120, 150, 200$ and 400 GeV.

m_{top}	100 GeV	120 GeV	150 GeV	200 GeV	400 GeV
0	0.548	0.678	0.812	0.976	1.419
20	0.556	0.687	0.821	0.983	1.423
40	0.573	0.710	0.844	1.001	1.432
60	0.588	0.736	0.871	1.069	1.444
80	0.552	0.735	0.889	1.046	1.458
90	0.531	0.726	0.893	1.056	1.466
100	0.740	0.736	0.902	1.066	1.481
120	0.772	0.913	0.925	1.088	1.488
150	0.810	0.958	1.069	1.118	1.513
200	0.854	1.014	1.131	1.245	1.554
250	0.882	1.052	1.177	1.295	1.591
300	0.899	1.078	1.211	1.336	1.624
500	0.930	1.126	1.277	1.433	1.801
1000	0.945	1.152	1.322	1.503	1.974
∞	0.950	1.161	1.338	1.534	2.104

As usual in gauge theories, there are enormous cancellations. At the favoured LEP II energy, $E = 100$ GeV, although several individual contributions are bigger than 2%, the total result is below 1%. These cancellations become more and more dramatic as one increases the energy. As discussed in the preceding section, the dominant contributions at high energy come from diagrams involving intermediate W 's (*a3, b7, c1, c3* increasing as E^5 and *b4 + d1* increasing as E^3). We observe a perfect cancellation between these leading contributions (each on the order of 1000% at beam energy $E = 400$ GeV), with the total asymmetry staying at the 1% level.

The sum of the contributions of the diagrams involving a four- W -coupling (*b7* and *c3*) grows as E^3 . It equals 200% at 1000 GeV. Changing the strength of this

four- W -coupling by a tiny amount would lead to a violation of the unitarity bound $\mathcal{A}S \leq 1$ at high energies, and to easily measurable deviations from the standard model prediction already at moderately high energies. This contribution to the asymmetry of the diagrams involving a four- W -coupling is 0.324% at $E = 100$ GeV, 1.50% at $E = 150$ GeV, 15.67% at $E = 400$ GeV and 201% at $E = 1000$ GeV. Thus, if the four- W -coupling were modified by a factor $1 + \epsilon$, one would observe a 1% deviation of the total asymmetry for the following values of ϵ : $\epsilon = 3.1$ ($E = 100$ GeV), $\epsilon = 0.67$ ($E = 150$ GeV), $\epsilon = 0.064$ ($E = 400$ GeV) and $\epsilon = 0.005$ ($E = 1000$ GeV).

Concerning the dependence on the top quark mass (Table 3), although there are again important cancellations between diagrams *a1* and *b2*, their sum is rather sensitive to m_{top} . If, again for $E = 100$ GeV, we consider m_{top} ranging from 90 GeV (which is the present lower limit as given by CDF [9]) with a contribution to the asymmetry of $\mathcal{A}S_{a1+b2}^{W^+l^-\bar{\nu}} = 0.53\%$ to $m_{\text{top}} = 500$ GeV with $\mathcal{A}S_{a1+b2}^{W^+l^-\bar{\nu}} = 0.93\%$, we see that varying the top quark mass within the present limits affects the total asymmetry by about 0.4%. This variation is approximately one half of the foreseen statistical error (1σ) at LEP II (or about two thirds if one is more optimistic about the expected luminosity, see section 3.1). In any case the top quark mass dependence of the T-odd asymmetries is too small to be observed at LEP II to the level required to allow a direct determination of the top quark mass. Yet these measurement will give valuable extra input to global fits to the top quark mass.

Table 4 : The asymmetries $\mathcal{AS}^{W^+l^-\bar{\nu}}$ in per cent for those diagrams involving M_H , at a beam energy $E = 100$ GeV.

M_H	$a5$	$b5$	$b6$	$d2$	subtotal
10	0.124	-0.254	-0.261	0.017	-0.374
50	0.104	0.079	-0.235	-0.235	-0.287
100	0.038	0.078	-0.108	-0.163	-0.156
150	0	0.052	0	-0.104	-0.052
200	0	0.035	0	-0.069	-0.034
500	0	0.007	0	-0.013	-0.006
1000	0	0.002	0	-0.003	-0.001

Table 5 : The asymmetries $\mathcal{AS}^{W^+l^-\bar{\nu}}$ in per cent for those diagrams ($a5 + b5 + b6 + d2$) involving M_H , at beam energies $E = 100, 120, 150, 200$ and 400 GeV.

M_H	100 GeV	120 GeV	150 GeV	200 GeV	400 GeV
10	-0.375	-0.339	-0.280	-0.218	-0.146
50	-0.287	-0.286	-0.251	-0.204	-0.144
100	-0.156	-0.214	-0.210	-0.183	-0.139
150	-0.052	-0.071	-0.164	-0.162	-0.135
200	-0.034	-0.049	-0.094	-0.138	-0.130
300	-0.016	-0.027	-0.037	-0.066	-0.120
500	-0.006	-0.011	-0.017	-0.025	-0.091
1000	-0.001	-0.003	-0.005	-0.008	-0.026

We see that the contributions to the asymmetry of diagrams involving the Higgs boson are rather small, not exceeding 0.2% for a Higgs boson mass above 100 GeV, with a somewhat bigger contribution for a lighter Higgs boson.

Optimizing the asymmetry

So far we have only considered the total integrated asymmetry, i.e. the ratio between the T-odd part of the cross section integrated over all CM-scattering angles θ and the total cross section. Figure 5 illustrates the θ -dependence of the T-odd part of the differential cross section. Its shape is largely due to the factor $\sin \theta$ in the ϵ -tensor. The regions $\theta \sim 0, \pi$ contribute little to the numerator of the asymmetry but much to the denominator and their elimination yields a bigger asymmetry. Figure 6 shows the asymmetry $\mathcal{AS}(\theta_{min}, \theta_{max})$ (see eq. (3.24)) with cuts for $\theta < \theta_{min}$ and $\theta > \theta_{max} = \pi - \theta_{min}$ as a function of θ_{min} , for beam energies $E = 100$ and 200 GeV. Increasing the cut-angle θ_{min} augments the asymmetry, but decreases the total cross section and the statistics.

In order to find the optimal integration region for the scattering angle θ , we have to consider the ratio of the asymmetries $\mathcal{AS}(\theta_{min}, \theta_{max})$ with cuts and of the (1σ) statistical error $N^{-1/2}$, where N is the number of available events: $N = L \sigma_{tot}(\theta_{min}, \theta_{max})$, L being the effective luminosity (luminosity times the branching coefficient 30% and corresponding to four LEP experiments). As discussed in section 3.1 we shall take $L = 1200 \text{ pb}^{-1}$ (corresponding to the high-luminosity option). For an "efficiency" $\mathcal{AS}\sqrt{N} = n$ the asymmetries can be measured with a statistical significance of $n\sigma$. We proceed to find the angular cuts yielding a maximal efficiency.

Since the ratio $(d\sigma_{T-odd}/d\theta)(d\sigma_{tot}/d\theta)^{-1}$ is maximum for $\theta = \frac{\pi}{2}$ and an almost symmetric function with respect to its maximum, we have chosen to compute the efficiency $\mathcal{AS}(\theta_{min}, \theta_{max}) \sqrt{N}$ for $\theta_{max} = \pi - \theta_{min}$. The result is given in Figure 7 for two energies, $E = 100$ and 200 GeV, and with $m_{top} = 150$ GeV, $M_H = 200$ GeV. The efficiency is maximal at $\theta_{min} = 0.2\pi$ for $E = 100$ GeV and at $\theta_{min} = 0.15\pi$ for $E = 200$ GeV. The dependence of the best θ_{min} -value on

the beam energy is rather weak and we have also checked that there is almost no dependence on the top quark or Higgs boson masses. We conclude that it is best to work with cuts such that $0.15 \pi \leq \theta \leq 0.85 \pi$.

Choosing $\theta_{\min} = 0.15 \pi$ instead of 0, increases the efficiency from 0.44 to 0.47 for $E = 100$ GeV ($m_{\text{top}} = 150$ GeV, $M_H = 200$ GeV), which is rather negligible, but from 0.60 to 0.74 for $E = 200$ GeV which might be appreciable. This gain in precision is more and more pronounced as one increases the energy further.

The energy dependence

In Figure 8a we have plotted the total asymmetry (without any cut) as a function of the beam energy for three different top quark masses $m_{\text{top}} = 90, 150, 500$ GeV and for fixed Higgs boson mass $M_H = 200$ GeV. The same asymmetry, but with a cut $\theta_{\min} = 0.15 \pi$, $\theta_{\max} = 0.85 \pi$ is shown in Figure 8b, whereas Figure 8c shows the corresponding efficiency (with cuts) in order to determine the “ideal” beam energy. One observes two facts: first, as we already know from Table 3, the bigger the top quark mass the bigger the asymmetry. For a heavy top quark, the efficiency reaches 1, meaning that at LEP II the asymmetry can only reach the meager statistical significance of 1σ . Second, for all top quark masses, it is clearly much better to work at 120 GeV beam energy rather than at 100 GeV. Except in the case of a “light” top quark (90 GeV), the efficiency reaches its maximum at 140 GeV beam energy (0.8 and 1.1 for $m_{\text{top}} = 150$, resp 500 GeV) which thus appears as the ideal beam energy. However, the efficiency at 120 GeV beam energy is only slightly less (0.7 and 1.0), and one might just as well work at this energy. The case of the “light” top quark (90 GeV) is different: the efficiency continues to grow with increasing energy. It reaches 0.8 for $E = 300$ GeV.

4.2 THE ASYMMETRIES $\mathcal{A}^{\text{jetmax/jetmin}}$

Table 6 shows the various contributions to the asymmetry $\mathcal{A}^{\text{jetmax/jetmin}}$ for $E = 100, 200$ and 400 GeV, with fixed $m_{\text{top}} = 150$ GeV and $M_H = 200$ GeV.

Table 6 : The asymmetries $\mathcal{A}^{\text{jetmax/jetmin}}$ in per cent with $m_{\text{top}} = 150$ GeV and $M_H = 200$ GeV, at beam energies $E = 100, 200$ and 400 GeV.

diagram	100 GeV	200 GeV	400 GeV
$a1$	0.138	0.175	0.125
$a3$	0.126	3.330	29.430
$a5$	0.000	0.002	0.002
$b1$	-0.019	-0.026	-0.020
$b2$	-0.073	-0.152	-0.099
$b3$	-0.013	-0.015	-0.011
$b4 + d1$	0.058	0.372	0.776
$b5$	0.000	-0.016	-0.014
$b6$	0.000	-0.004	-0.002
$b7$	0.008	0.599	10.240
$c1$	-0.141	-3.497	-29.820
$c3$	0.034	-0.630	-10.479
$c5 + d3$	-0.011	-0.003	-0.001
$c6 + d4$	-0.013	-0.031	-0.027
$c7$	-0.037	-0.090	-0.100
$d2$	-0.004	0.015	0.014
$d5$	0.038	0.115	0.133
$e1 + e2 + e3$	-0.005	-0.009	-0.002
$f4$	0.005	0.007	0.003
$g4 + g6$	0.007	0.007	0.005
$g5$	-0.009	-0.017	-0.010
total	0.073	0.132	0.143

These asymmetries are much smaller than those of Tables 2 to 5, and the total asymmetry $\mathcal{A}S^{\text{jetmax/jetmin}}$ of 0.073% at $E = 100$ GeV is absolutely unobservable at LEP II. The asymmetry increases slightly with energy, but still remains unobservable (0.14% at $E = 400$ GeV). It is smaller than $\mathcal{A}S^{W^+l^-\bar{\nu}}$ by one order of magnitude. We will not examine in detail variations of this asymmetry with m_{top} or M_H . We have checked the effects of varying these parameters within any reasonable range: they do not increase the asymmetry $\mathcal{A}S^{\text{jetmax/jetmin}}$ anywhere close to observability. We have considered the effect of imposing cuts : they do not substantially increase the asymmetries $\mathcal{A}S^{\text{jetmax/jetmin}}$ either.

4.3 EFFECTS OF CP-VIOLATING VERTICES

In section 3.6, we computed the effect on our T-odd observables of γWW and ZWW vertices that explicitly violate CP. We saw that the requirement of an unbroken $SU(2) \times U(1)$ local symmetry above the standard symmetry breaking scale results in explicit relations between the coupling strengths of the various CP-odd effective operators describing the three-gauge-boson vertices. Rather than imposing these conditions *ab initio*, in this section we present the most “open-minded” results, followed by an analysis of the impact of the requirement of gauge invariance.

To first order in the couplings $\lambda_{\gamma,Z}$ or $\lambda'_{\gamma,Z}$ of (3.38) all results are linear in the coupling strength and we give the numerical results for $\lambda_{\gamma,Z} = 1$ and $\lambda'_{\gamma,Z} = 1$.

Table 7 shows the asymmetries $\mathcal{A}S^{W^+l^-\bar{\nu}}$ and $\mathcal{A}S^{\text{jetmax/jetmin}}$ due to the CP-violating γWW and ZWW vertices (3.38a) with $\lambda_{\gamma,Z} = 1$ (and $\lambda'_{\gamma,Z} = 0$) without any angular cuts, as functions of the beam energy E .

Table 8 shows the corresponding asymmetries $\mathcal{A}S^{W^+l^-\bar{\nu}}$ and $\mathcal{A}S^{\text{jetmax/jetmin}}$ due to the CP-violating γWW and ZWW vertices (3.38b) with $\lambda'_{\gamma,Z} = 1$ (and $\lambda_{\gamma,Z} = 0$) again without any angular cuts, as functions of the beam energy E .

Table 7 : The asymmetries $\mathcal{A}S^{W^+l^-\bar{\nu}}$ and $\mathcal{A}S^{\text{jetmax/jetmin}}$ in per cent due to the CP-violating γWW and ZWW vertices (3.38a) for $\lambda_{\gamma,Z} = 1$ at various beam energies E .

$E[\text{GeV}]$	$\mathcal{A}S^{W^+l^-\bar{\nu}}_{\gamma WW}$	$\mathcal{A}S^{W^+l^-\bar{\nu}}_{ZWW}$	$\mathcal{A}S^{\text{jetmax/jetmin}}_{\gamma WW}$	$\mathcal{A}S^{\text{jetmax/jetmin}}_{ZWW}$
85	2.64	3.99	6.45	5.90
90	4.02	5.57	14.09	12.35
95	5.17	6.68	22.55	19.09
100	6.18	7.54	31.70	26.08
110	7.88	8.83	51.77	40.74
120	9.26	9.77	73.75	56.17
150	12.19	11.57	148.6	106.7
200	15.19	13.34	298.5	205.2
400	22.32	18.28	1171	773.6
1000	49.01	39.47	6986	4562

As these Tables show, the asymmetry $\mathcal{A}S^{\text{jetmax/jetmin}}$ is much more sensitive (about 5 times at $E = 100$ GeV and more than 100 times at $E = 1000$ GeV) to the CP-violating vertices than the asymmetry $\mathcal{A}S^{W^+l^-\bar{\nu}}$ is. This is in a sense fortunate, since, as we have seen above, the standard model background to the former asymmetry is completely negligible (not exceeding 0.2% at $E = 100$ GeV), while it may not be negligible for the latter one. We also observe that the asymmetries increase with the beam energy E , and that those of Table 8 due to the vertices (3.38b) increase much faster with energy than those of Table 7 (vertices (3.38a)). This is as expected from the momentum dependence of the vertices (3.38).

Table 8 : The asymmetries $\mathcal{A}^{W^+l^- \bar{\nu}}$ and $\mathcal{A}^{\text{jetmax/jetmin}}$ in per cent, due to the CP-violating γWW and ZWW vertices (3.38b) for $\lambda'_{\gamma, Z} = 1$ at various beam energies E .

$E[\text{GeV}]$	$\mathcal{A}^{W^+l^- \bar{\nu}}_{\gamma WW}$	$\mathcal{A}^{W^+l^- \bar{\nu}}_{ZWW}$	$\mathcal{A}^{\text{jetmax/jetmin}}_{\gamma WW}$	$\mathcal{A}^{\text{jetmax/jetmin}}_{ZWW}$
85	1.25	1.56	3.18	2.91
90	3.97	4.67	14.62	12.81
95	7.92	8.87	36.36	30.78
100	13.00	14.02	70.35	57.87
110	26.37	26.82	182.5	143.6
120	43.74	42.63	365.5	278.3
150	117.6	106.3	1485	1066
200	310.3	265.5	6231	4283
400	2119	1724	111900	73880
1000	30250	24340	4317000	2819000

We have studied the angular dependence of the T-odd part of the cross sections $d\sigma_{T\text{-odd}}^{W^+l^- \bar{\nu}}/d\theta$ and $d\sigma_{T\text{-odd}}^{\text{jetmax/jetmin}}/d\theta$, respectively. For some of the cases studied, this T-odd part of the cross sections changes sign at some angle θ . In these cases, one could slightly increase the asymmetries, and hence the sensitivity to CP-violations, if instead of integrating $d\sigma_{T\text{-odd}}/d\theta$ over all θ from 0 to π we compute $\int_0^{\theta_0} d\sigma_{T\text{-odd}} - \int_{\theta_0}^{\pi} d\sigma_{T\text{-odd}}$, where θ_0 is the angle where the sign change occurs. However, in all cases, the subtracted second term is much smaller than the first one, and the gain in sensitivity is rather small. We thus refrain from displaying these “subtracted” asymmetries.

We proceed to display results for the case in which the CP-odd γW^+W^- and ZW^+W^- couplings of (3.39) are constrained by the $SU(2) \times U(1)$ conditions (3.41). The results for the asymmetries are presented in Table 9, for $\delta = 1$ (with $\hat{\delta} = 0$) and for $\hat{\delta} = 1$ (with $\delta = 0$). These large values of the CP-odd gauge-symmetry constrained γW^+W^- and ZW^+W^- couplings are excluded by limits

on the neutron’s electric dipole moment, to which these couplings contribute via radiative corrections [6]. Should the condition of gauge invariance be lifted, there is no relation between the Z and the γ couplings to W ’s and no useful limit on the ZW^+W^- vertex. We take in here the healthy attitude that there is no entirely satisfactory alternative to a direct observation: theoretical preconceptions are to be challenged.

Table 9 : The asymmetries $\mathcal{A}^{W^+l^- \bar{\nu}}$ and $\mathcal{A}^{\text{jetmax/jetmin}}$ in per cent due to the CP-violating γWW and ZWW vertices (3.38a,b) for $\delta = 1$ and for $\hat{\delta} = 1$, at various beam energies E .

$E[\text{GeV}]$	$\mathcal{A}^{W^+l^- \bar{\nu}}_{\delta=1}$	$\mathcal{A}^{W^+l^- \bar{\nu}}_{\hat{\delta}=1}$	$\mathcal{A}^{\text{jetmax/jetmin}}_{\delta=1}$	$\mathcal{A}^{\text{jetmax/jetmin}}_{\hat{\delta}=1}$
85	0.48	8.04	3.25	13.40
90	1.00	11.92	7.40	28.51
95	1.55	15.04	12.21	44.67
100	2.09	17.74	17.57	61.70
110	3.10	22.30	29.69	98.13
120	3.97	26.05	43.31	137.1
150	5.92	34.20	90.78	267.2
200	7.96	42.67	187.3	524.0
400	12.41	63.73	751.8	1983
1000	27.62	141.0	4514	11590

We find that the imposition of gauge invariance leads to a more moderate growth of the asymmetries at high energies.* For the favoured LEP II energy $E = 100$ GeV, however, we still find the asymmetry $\mathcal{A}^{W^+l^- \bar{\nu}}$ to be of the order of 10% and the asymmetry $\mathcal{A}^{\text{jetmax/jetmin}}$ of the order of 100%, for $\hat{\delta} = 1$. For $\delta = 1$, the gauge invariance constraint leads to a slight loss of sensitivity. Again

* In fact, this moderation is due partly to the parametrization (3.40) and partly to the imposition of gauge invariance (3.41).

we observe the asymmetry $\mathcal{A}S^{\text{jetmax}/\text{jetmin}}$ to be much more sensitive to the CP-violating vertices than the asymmetry $\mathcal{A}S^{W^+l^- \bar{\nu}}$ is (about 5 to 10 times at $E = 100$ GeV and more than 100 times at $E = 1000$ GeV).

Recall from the discussion in section 3.1 that, in an optimistic, but not unrealistic view (with four LEP experiments and for a total operation time of three years), the statistical error (1σ) at LEP II ($E = 100$ GeV) on any asymmetry is expected to be 0.6%. Thus, using the asymmetry $\mathcal{A}S^{\text{jetmax}/\text{jetmin}}$ one should be able to measure δ with a precision of 0.03 at 1σ and $\hat{\delta}$ with a precision of 0.01 (at 1σ). Relaxing the gauge-invariance constraint, the asymmetry $\mathcal{A}S^{\text{jetmax}/\text{jetmin}}$ at $E = 100$ GeV is

$$\mathcal{A}S^{\text{jetmax}/\text{jetmin}} = 31\% \times \delta^\gamma + 62\% \times \delta^Z + 24\% \times \hat{\delta}^\gamma + 48\% \times \hat{\delta}^Z$$

and the achievable precision on the various $\hat{\delta}$'s is again of the order of 0.01. Increasing the beam energy to $E = 120$ GeV would allow a gain of a factor of two in precision.

5. Conclusions

We have analysed in considerable detail the two exemplary T-odd observables defined in (3.20) and (3.25) for the process $e^+e^- \rightarrow W^+W^-$. In the standard model, wherein the radiatively induced CP-odd trilinear couplings of gauge bosons are tiny, these T-odd effects arise solely from the relative phase-shifts induced by “final state interactions” or, more precisely, by the different possible intermediate states in the absorptive part of the scattering amplitude. We find the predicted standard results to be $\mathcal{O}(1\%)$, that is, barely measurable at LEP-II. Somewhat discouragingly, the effects are only marginally sensitive to the top quark mass, and almost blind to the assumed mass of the neutral elementary scalar(s).

Even rather small deviations from the standard model would induce large and measurable corrections to our T-odd asymmetries. A first example is a (non-gauge-invariant) multiplicative correction $(1+\epsilon)$ to the quartic self-coupling of W 's.

Although at a LEP-II beam energy $E = 100$ GeV such a correction would induce an observable 1% asymmetry of the type (3.20) only for $\epsilon \simeq 3$, at higher energies the sensitivity to a non-zero ϵ rapidly increases (at fixed statistics) reaching $\epsilon = 0.67$ at $E = 150$ GeV and $\epsilon = 0.005$ at $E = 1000$ GeV, perhaps obtainable at a future linear collider of very high luminosity.

Non-standard CP-odd γW^+W^- or $Z W^+W^-$ couplings would also have large effects on our asymmetries, particularly on the one defined in (3.25), that is constructed in terms of jet momenta in W -decay. The effects are large even if gauge invariance is imposed, in the sense of forcing the novel CP-violating couplings to be low-energy effective interactions in models that *do* respect the standard $SU(2) \times U(1)$ local symmetry above the electroweak symmetry-breaking scale. As an example, the asymmetry (3.25) reaches an accurately measurable 3% level at $E = 100$ GeV for a mere $\lambda_Z = 0.1$ or $\lambda'_Z = 0.05$ value of the dimensionless CP-violating coefficients defined in (3.39) and (3.40).

T-odd effects afford sensitive tests of the standard electroweak theory, while CP-violation in its pure gauge-sector is experimentally unexplored. We have shown how to address these two topics at LEP-II. Both a surprise in this realm, or the by now consuetudinary obedience of Nature to the forecasts of frayed text-books, would constitute a significant advance in our understanding.

APPENDIX A

The absorptive part of the $e^+e^- \rightarrow W^+W^-$ amplitude

The core of our calculations is the evaluation of the absorptive part of the amplitude, $\mathcal{A}_{if} = \sum_k \mathcal{T}_{ik} \mathcal{T}_{fk}^*$ with the sum extending over all possible on-shell intermediate states. We compute it to order e^4 , i.e. each of the $\mathcal{T}$'s is taken at lowest order. We are computing hence cut one-loop integrals. All relevant diagrams are given in Figure 2. In all cases one has two cut propagators and two intermediate particles with momenta q_1 and q_2 . Explicitly we have (cf. eq. (3.12))

$$\mathcal{A}_{if} = \sum_{\text{diagrams}} \int \frac{d^3 q_1 d^3 q_2}{(4\pi)^2 q_1^0 q_2^0} \delta^4(q_1 + q_2 - P_{\text{cut}}) \mathcal{T}_{ik} \mathcal{T}_{fk}^* \equiv -\frac{1}{32\pi^2} \bar{v}(p_2) \mathcal{M} u(p_1) \quad (A.1)$$

where P_{cut} denotes the total cut momentum, i.e. p for the diagrams $a-d$, p_3 for the diagrams e^- , f^- , g^- and p_4 for the diagrams e^+ , f^+ , g^+ .

In this Appendix we give the contributions of the different diagrams to $\mathcal{M}$, for any polarizations ϵ_3 and ϵ_4 of the W^- and W^+ . (We average over the initial $e^\pm$ helicities.) For the sake of compactness, we introduce the following notation: for the functions f_i, g_i given in Appendix B we define the combinations

$$\begin{aligned} f_{12} &= f_1 - f_2, & f_{32} &= f_3 - 2f_2, & f_{41} &= \frac{1}{s} f_4 - 2f_1, \\ g_{12} &= g_1 - g_2, & g_{32} &= g_3 - 2g_2, & g_{41} &= \frac{1}{s} g_4 - 2g_1. \end{aligned} \quad (A.2)$$

P_L will denote the projector $(1 - \gamma_5)/2$, and we define $s_w^2 = \sin^2 \theta_w$, $c_w^2 = \cos^2 \theta_w$. As throughout the paper, M denotes the W mass and ζ stands for $s - 4M^2$. Furthermore, β_{M_1, M_2} denotes the velocity of the intermediate particles:

$$\beta_{X,Y}^2 = \frac{1}{s^2} ((s - M_X^2 - M_Y^2)^2 - 4M_X^2 M_Y^2), \quad \beta_X^2 \equiv \beta_{X,X}^2. \quad (A.3)$$

In the following, $\beta_{X,Y}$ denotes $\sqrt{\beta_{X,Y}^2}$ if $s > (M_X + M_Y)^2$ and is set to vanish otherwise.

A1 CUT VACUUM-POLARIZATION DIAGRAMS : TYPE A

For the diagram $a1$ of Fig. 2 we obtain:

$$\begin{aligned} \mathcal{M}_{a1} &= -\frac{16\pi}{3} e^4 \mathcal{G} \left\{ \frac{20}{3} \frac{1}{s} \phi_t + \frac{15 - 40s_w^2}{6s_w^2} \frac{1}{s - M_Z^2} \phi_t \left(1 - \frac{s_w^2}{c_w^2} + \frac{P_L}{2c_w^2} \right) \right. \\ &\quad \left. - \frac{s}{(s - M_Z^2)^2} \left(\frac{160s_w^4 - 120s_w^2 + 63}{24s_w^2 c_w^2} \phi_t + \beta_t \frac{45 - 57\varphi_t}{80s_w^2 c_w^2} \right) \left(1 - \frac{P_L}{2s_w^2} \right) \right\} \quad (A.4a) \end{aligned}$$

where the subscript t refers to the top quark and

$$\varphi_t = 1 + \frac{2m_t^2}{s}, \quad \phi_t = \left(1 + \frac{\beta_t \varphi_t}{5} \right). \quad (A.4b)$$

All other quarks are taken to be massless. Recall that G_μ was defined in (3.11).

Next, we obtain

$$\mathcal{M}_{a2} = 0, \quad (A.5)$$

$$\begin{aligned} \mathcal{M}_{a3} &= -\frac{4\pi}{3} e^4 \beta_W \zeta \left(3 + \frac{5s}{M^2} + \frac{s^2}{4M^4} \right) \left(\frac{1}{s} + \frac{c_w^2}{s_w^2} \frac{1}{s - M_Z^2} \right) \times \\ &\quad \times \mathcal{G} \left(\frac{1}{s} - \frac{1}{s - M_Z^2} \left(1 - \frac{P_L}{2s_w^2} \right) \right), \end{aligned} \quad (A.6)$$

$$\mathcal{M}_{a4} = 0, \quad (A.7)$$

$$\mathcal{M}_{a5} = \frac{4\pi e^4 \beta_{Z,H}}{s_w^2 c_w^2} \left(M_Z^2 - \frac{s \beta_{Z,H}^2}{12} \right) \frac{1}{(s - M_Z^2)^2} \mathcal{G} \left(1 - \frac{P_L}{2s_w^2} \right). \quad (A.8)$$

A2 CUT THREE-GAUGE-BOSON VERTEX DIAGRAMS : TYPE B

To express $\mathcal{M}_{b1}$ to $\mathcal{M}_{b3}$ it is useful to introduce the following functions (M_1 is the mass of the intermediate fermion and m that of the t -channel fermion).

$$\begin{aligned}
B_1^R(s, M_1, m) &= 16(\not{f}_3^* p \cdot \not{e}_4^* - \not{f}_4^* p \cdot \not{e}_3^*) \\
&\quad \times (s(g_{32} - g_{41} - 3g_{12} - g_{12}/\beta_{M_1}^2) + 8(M^2 - M_1^2)g_1/\beta_{M_1}^2) \\
&\quad + 16\not{k}_- \not{e}_3^* \cdot \not{e}_4^* s (g_{32} - g_{41} - g_{12} + g_{12}/\beta_{M_1}^2) \\
&\quad + 16\not{k}_- p \cdot \not{e}_3^* p \cdot \not{e}_4^* \left(\frac{s}{\zeta} (8g_{12} - 6g_{32} + 5g_{41}) - 16 \frac{(M^2 - M_1^2)}{\zeta \beta_{M_1}^2} g_1 + g_{41} \right) \\
B_1^I(s, M_1, m) &= 8i\gamma_\rho \epsilon(\rho, k_-, \epsilon_3^*, \epsilon_4^*) s (-2g_{12}(1 + 1/\beta_{M_1}^2) + g_{32}) \\
&\quad + 8i\gamma_\rho (\epsilon(p, k_-, \rho, \epsilon_3^*) p \cdot \not{e}_4^* - \epsilon(p, k_-, \rho, \epsilon_4^*) p \cdot \not{e}_3^*) \times \\
&\quad \times \left(\frac{s}{\zeta} (-4g_{12} + 3g_{32}) + 16 \frac{(M^2 - M_1^2)}{\zeta \beta_{M_1}^2} g_1 \right), \\
B_2^R(s, M_1, m) &= -64 \frac{M_1^2}{\beta_{M_1}^2} g_{12} ((\epsilon_3^* p \cdot \not{e}_4^* - \not{e}_4^* p \cdot \not{e}_3^*) - \not{k}_- \not{e}_3^* \cdot \not{e}_4^*), \\
B_2^I(s, M_1, m) &= -64 \frac{M_1^2}{\beta_{M_1}^2} g_{12} i\gamma_\rho \epsilon(\rho, k_-, \epsilon_3^*, \epsilon_4^*),
\end{aligned} \tag{A.9}$$

where $g_i = g_i(s, M_1^2, M_1^2, m^2)$ etc. Recall that $\zeta = s - 4M^2$. Then we have

$$\begin{aligned}
\mathcal{M}_{b1} &= \frac{3\pi e^4}{32s_w^2} (B_1^R(s, 0, 0) + B_1^I(s, 0, 0)) \left(\frac{1}{s} + \frac{1}{s - M_2^2} \frac{2s_w^2 - 1}{2c_w^2} \left(1 - \frac{P_L}{2s_w^2} \right) \right), \\
\mathcal{M}_{b2} &= \frac{\pi e^4}{32s_w^2} \left\{ \frac{1}{s} \left[(6B_1^R - 2B_1^I)(s, 0, 0) + \beta_6^3 (B_1^R + B_2^R + B_1^I + B_2^I)(s, m_b, m_t) \right. \right. \\
&\quad \left. \left. + 2\beta_6^3 (B_1^R + B_2^R - B_1^I - B_2^I)(s, m_t, m_b) \right] \right. \\
&\quad \left. + \frac{1}{c_w^2} \frac{1}{s - M_2^2} \left[-2s_w^2 B_1^I(s, 0, 0) - 6c_w^2 B_1^R(s, 0, 0) \right. \right. \\
&\quad \left. \left. + \beta_6^3 \left((s_w^2 - \frac{3}{2})(B_1^R + B_1^I) + s_w^2 (B_2^R + B_2^I) \right) (s, m_b, m_t) \right] \right\}
\end{aligned} \tag{A.10}$$

$$\begin{aligned}
&+ \beta_t^3 \left(2s_w^2 - \frac{3}{2} \right) (B_1^R - B_1^I) + 2s_w^2 (B_2^R - B_2^I)(s, m_t, m_b) \Big] \\
&\quad \times \left(1 - \frac{P_L}{2s_w^2} \right) \Big\},
\end{aligned} \tag{A.11}$$

$$\mathcal{M}_{b3} = \frac{3\pi e^4}{64s_w^2 c_w^2} \frac{1}{s - M_2^2} \left(-B_1^R(s, 0, 0) + B_1^I(s, 0, 0) \right) \left(1 - \frac{P_L}{2s_w^2} \right). \tag{A.12}$$

Note that if all quarks and leptons were degenerate in mass, the total coefficient of B_1^I (i.e. of the ϵ -tensors) in the sum of the three triangle diagrams would vanish. This is the usual cancellation of the axial triangle anomaly. Note also that any effects of a non-diagonal Kobayashi-Maskawa mixing matrix K are solely due to the mass differences between the top and the other quarks. These effects are $\sim |K_{ts}|^2, |K_{ts}|^2$ and can be safely neglected.

$$\begin{aligned}
\mathcal{M}_{b4} &= 4\pi e^4 \beta_W \left(B_4(s, M^2, 0) + \frac{c_w^2}{s_w^2} B_4(s, M^2, \frac{M_2^2}{M^2}) \right) \\
&\quad \times \left(\frac{1}{s} - \frac{1}{s - M_2^2} \left(1 - \frac{P_L}{2s_w^2} \right) \right),
\end{aligned} \tag{A.13a}$$

where, abbreviating $\xi = 1 - s/(2M^2)$:

$$\begin{aligned}
B_4(s, M^2, z) &= \not{k}_- p \cdot \not{e}_3^* p \cdot \not{e}_4^* \left(1 + \frac{\xi z}{8} \right) (3g_{32} - 4g_{12} + (2M^2/s - 3)g_{41}) \\
&\quad + \not{k}_- \not{e}_3^* \cdot \not{e}_4^* \left\{ \frac{2s}{M^2} - \frac{\zeta \xi}{6M^2} + 4M^2 g_1 \left(\xi + \frac{z}{4} \left(3 - z + \frac{(z-4)s}{2M^2} \right) \right) \right. \\
&\quad \left. + \frac{\zeta}{2} \left(1 + \frac{\xi z}{8} \right) g_{41} + 2szg_2 \right\} \\
&\quad + (\not{f}_3^* p \cdot \not{e}_4^* - \not{f}_4^* p \cdot \not{e}_3^*) \left\{ \left(2 - \frac{5s}{2M^2} \right) (1 + zM^2 g_2) + \frac{\xi \zeta}{12M^2} \right. \\
&\quad \left. - \frac{\zeta}{2} \left(1 + \frac{\xi z}{8} \right) g_{32} + 2\zeta g_2 + 2s \left(1 + \frac{z}{4} \right) g_1 + \frac{\zeta}{2} \left(1 + \frac{\xi z}{8} \right) g_{41} \right\}
\end{aligned} \tag{A.13b}$$

with $g_i = g_i(s, M^2, M^2, zM^2)$, etc.

$$\mathcal{M}_{b5} = -\frac{4\pi e^4}{s_w^2} \beta_W B5(s, M^2, M_H^2) \left(\frac{1}{s} - \frac{1}{s - M_Z^2} \left(1 - \frac{P_L}{2s_w^2} \right) \right), \quad (\text{A.14a})$$

where

$$\begin{aligned} B5(s, M^2, M_H^2) = & \not{k} \cdot p \cdot \epsilon_3^* p \cdot \epsilon_4^* \xi \left(-\frac{\xi M^2}{4s} g_{41} + \frac{1}{4s} g_4 - \frac{1}{8} (3g_3 - 2g_2) \right) \\ & + \not{k} \cdot \epsilon_3^* \cdot \epsilon_4^* \left(M^2 g_1 - \frac{\xi \zeta}{16} g_{41} \right) \\ & + (\not{k}_3^* p \cdot \epsilon_4^* - \not{k}_4^* p \cdot \epsilon_3^*) \left(\frac{\zeta}{2} g_2 - \frac{s}{2} g_1 + \frac{\xi \zeta}{16} (g_{32} - g_{41}) \right) \end{aligned} \quad (\text{A.14b})$$

with $g_i = g_i(s, M^2, M^2, M_H^2)$, etc.

$$\mathcal{M}_{b6} = \frac{\pi e^4}{2s_w^2 c_w^2} \beta_{Z,H} B6(s, M_Z^2, M_H^2, M^2) \frac{1}{s - M_Z^2} \left(1 - \frac{P_L}{2s_w^2} \right), \quad (\text{A.15a})$$

where

$$\begin{aligned} B6(s, M_Z^2, M_H^2, M^2) = & \not{k} \cdot p \cdot \epsilon_3^* p \cdot \epsilon_4^* \frac{1}{\zeta} \left(2\beta_{Z,H}^2 (s - M^2) g_{41} + \beta_{Z,H}^2 g_4 \right. \\ & \left. - s\beta_{Z,H}^2 (3g_3 - 2g_2) + 2\omega(2 - \omega)\zeta g_1 \right) \\ & + \not{k} \cdot \epsilon_3^* \cdot \epsilon_4^* \left(-\frac{s\beta_{Z,H}^2}{2} g_{41} + s\frac{M^2}{M_Z^2} (2b_{Z,H}^2 g_3 - 4\omega g_1) + 16M^2 g_2 \right) \\ & + (\not{k}_3^* p \cdot \epsilon_4^* - \not{k}_4^* p \cdot \epsilon_3^*) \left(\frac{s\beta_{Z,H}^2}{2} (-g_{41} + g_{32}) - 16\omega M^2 g_2 \right. \\ & \left. + 4M_Z^2 (g_1 + (\omega - 2)g_2) \right) \end{aligned} \quad (\text{A.15b})$$

with $g_i = g_i(s, M_Z^2, M_H^2, M^2)$, etc. and $\omega = (s + M_Z^2 - M_H^2)/s$.

$$\mathcal{M}_{b7} = -\frac{\pi e^4}{s_w^2} \beta_W \zeta \frac{s + 10M^2}{2M^4} (\not{k}_3^* p \cdot \epsilon_4^* - \not{k}_4^* p \cdot \epsilon_3^*) \left(\frac{1}{s} - \frac{1}{s - M_Z^2} \left(1 - \frac{P_L}{2s_w^2} \right) \right). \quad (\text{A.16})$$

A3 CUT $e^\pm$ - GAUGE-BOSON VERTEX DIAGRAMS : TYPE C

The results for the diagrams c1 to c4 of Fig. 2 are:

$$\mathcal{M}_{c1} = \frac{2\pi e^4}{s_w^2} \beta_W \left(-\frac{\xi}{6M^2} (\zeta + 24M^2) + \frac{\zeta}{4} f_{32} + 4M^2 f_2 \right) \times \left(\frac{1}{s} + \frac{c_w^2/s_w^2}{s - M_Z^2} \right) \not{G} P_L, \quad (\text{A.17})$$

$$\mathcal{M}_{c2} = 0, \quad (\text{A.18})$$

$$\begin{aligned} \mathcal{M}_{c3} = & -\frac{\pi e^4}{s_w^4} \beta_W \left\{ (\not{k}_4^* p \cdot \epsilon_3^* + \not{k}_3^* p \cdot \epsilon_4^*) (-f_{12}) \right. \\ & \left. + (\not{k}_3^* p \cdot \epsilon_4^* - \not{k}_4^* p \cdot \epsilon_3^*) \left(3f_{12} - \frac{s + 8M^2}{4M^4} \right) \right\} P_L, \quad (\text{A.19}) \end{aligned}$$

$$\mathcal{M}_{c4} = 0, \quad (\text{A.20})$$

where $f_i = f_i(s, M^2, M^2, 0)$, etc.

It is convenient to combine diagram c5 and the cut box diagram d3, since it is only then that the intermediate photons or Z's are coupled to conserved currents on the left and on the right of the diagrams.

$$\begin{aligned} \mathcal{M}_{c5} + \mathcal{M}_{d3} = & e^4 \left\{ D3(0, 0) - 2\beta_{\gamma,Z} D3(0, M_Z^2) \left(1 - \frac{P_L}{2s_w^2} \right) \right. \\ & \left. + \beta_Z D3(M_Z^2, m_Z^2) \left(1 + \frac{1 - 4s_w^2}{4s_w^4} P_L \right) \right\}, \end{aligned} \quad (\text{A.21})$$

where

$$\begin{aligned} D3(M_1^2, M_2^2) = & \not{k} \cdot \epsilon_3^* \cdot \epsilon_4^* (F^- + s(I^- - \alpha^-) + v(I^+ - \alpha^+) - 2M^2 \beta^-) \\ & + \not{k} \cdot p \cdot \epsilon_3^* p \cdot \epsilon_4^* (-\omega(2 - \omega)\beta^- + 2C^- + 2D^- - F^-/s - L^-) \\ & + (\not{k}_4^* p \cdot \epsilon_3^* + \not{k}_3^* p \cdot \epsilon_4^*) \left((s - M_1^2 - M_2^2 + \frac{M_1^2 M_2^2}{2M^2})(\alpha^+ - I^+) \right. \end{aligned}$$

$$+ s(\alpha^+ - B^+) + vD^- + E^+)$$

$$\begin{aligned}
& + \not{k}_- (p_- \cdot \epsilon_3^* p \cdot \epsilon_4^* - p_- \cdot \epsilon_4^* p \cdot \epsilon_3^*) (\omega(\omega - 2)I^+ + 2\alpha^+ + 2\beta^+ \\
& \quad - A^+ / s - B^+ - C^+ - D^+ + J^+) \\
& + \not{k}_- p_- \cdot \epsilon_3^* p_- \cdot \epsilon_4^* (-2D^- + H^-) \\
& + \not{k}_4 \not{k}_- \not{k}_3^* \left((s - M_1^2 - M_2^2 + \frac{M_1^2 M_2^2}{2M^2}) \beta(s, \cos \theta_{\text{CM}}) - A^+ \right) \\
& + \not{k}_3^* \not{k}_- \not{k}_4^* \left(-(s - M_1^2 - M_2^2 + \frac{M_1^2 M_2^2}{2M^2}) \beta(s, -\cos \theta_{\text{CM}}) + A^+ \right) \\
& + (\not{k}_3^* p \cdot \epsilon_4^* - \not{k}_4^* p \cdot \epsilon_3^*) \left\{ -s(I^- - \alpha^- - \beta^- + D^-) \right. \\
& \quad \left. - (M_1^2 + M_2^2 - \frac{M_1^2 M_2^2}{2M^2})(I^- - \alpha^-) - \zeta\beta^- - A^- + F^- \right. \\
& \quad \left. + v(-I^+ + \alpha^+ - (1 - \omega)^2 I^+ - 2\beta^+ + A^+ / s + C^+) \right\}. \quad (\text{A.22})
\end{aligned}$$

Here $\alpha^\pm = \alpha(s, \cos \theta_{\text{CM}}) \pm \alpha(s, -\cos \theta_{\text{CM}})$, etc. and the functions $I, \alpha, \beta, \dots, L \equiv I(s, \cos \theta_{\text{CM}}), \dots$ are those appropriate for the diagram d3 as given in Appendix B by the equations (B.8-10). (The function β should not be confused with the velocities β_X , the latter always carry a subscript labelling the particle X.) Also, $\omega = (s + M_1^2 - M_2^2)/s$ and $v = p_- \cdot k_-$ as usual.

$$\begin{aligned}
\mathcal{M}_{c6} = & \pi e^4 \mathcal{G} \left\{ (4f_1 + 2f_2 + f_3)(s, 0, m_\gamma^2) \right. \\
& + (4f_1 + 2f_2 + f_3)(s, 0, M_Z^2) \frac{s_w^2}{c_w^2} \left(1 + \frac{1 - 4s_w^2}{4s_w^4} P_L \right) \left. \right\} \\
& \times \left(1 - \frac{s}{s - M_Z^2} \left(1 - \frac{P_L}{2s_w^2} \right) \right) \quad (\text{A.23})
\end{aligned}$$

$$\mathcal{M}_{c7} = -\frac{\pi e^4}{4s_w^4} (4f_1 + 2f_2 + f_3)(s, 0, M_Z^2) \frac{s}{s - M_Z^2} \mathcal{G} P_L. \quad (\text{A.24})$$

A4 CUT BOX-DIAGRAMS : TYPE D

We proceed to give the results for the diagrams d1 to d5 of Fig. 2.

$$\mathcal{M}_{d1} = \frac{e^4}{2s_w^2} \beta_W \left(D1(s, M^2, m_\gamma^2) + \frac{c_w^2}{s_w^2} D1(s, M^2, M_Z^2) \right) P_L, \quad (\text{A.25})$$

where

$$\begin{aligned}
D1(s, M^2, m^2) &= \not{k}_- \epsilon_3^* \cdot \epsilon_4^* \left((s + v)(I - \alpha) - 2M^2 \beta + F - \frac{8\pi}{M^2} - \frac{\pi\zeta}{3M^2} \right. \\
&\quad \left. + \frac{\pi m^2 \zeta}{8M^4} g_{41} - 2\pi(2 - \frac{m^2}{M^2})^2 g_1 + 8\pi(2 - \frac{m^2}{M^2}) g_2 \right) \\
&\quad + \not{k}_- p \cdot \epsilon_3^* p \cdot \epsilon_4^* \left(-\beta + 2C + 2D - F/s - L + \frac{\pi m^2}{8M^4} (6g_{32} - 8g_{12} - (5 + \beta_W^2) g_{41}) \right) \\
&\quad + (\not{k}_4^* p_- \cdot \epsilon_3^* + \not{k}_3^* p_- \cdot \epsilon_4^*) \left((s - 2M^2 + m^2/2)(\alpha - I) + s(\alpha - B) \right. \\
&\quad \left. + vD + E + 2\pi f_{12} \right) \\
&\quad + \not{k}_- (p_- \cdot \epsilon_3^* p \cdot \epsilon_4^* - p_- \cdot \epsilon_4^* p \cdot \epsilon_3^*) (-I + 2\alpha + 2\beta - A/s - B - C - D + J) \\
&\quad + \not{k}_- p_- \cdot \epsilon_3^* p_- \cdot \epsilon_4^* (-2D + H) + \not{k}_4^* \not{k}_- \not{k}_3^* ((s - 2M^2 + m^2/2)\beta - A) + \not{k}_3^* \not{k}_- \not{k}_4^* A \\
&\quad + (\not{k}_3^* p \cdot \epsilon_4^* - \not{k}_4^* p \cdot \epsilon_3^*) \left(-s(I - \alpha - \beta + D) - \zeta\beta - A + F \right. \\
&\quad \left. - (2M^2 - m^2/2)(I - \alpha) + v(-I + \alpha - 2\beta + A/s + C) + \frac{10\pi}{M^2} + \frac{\pi\zeta}{6M^4} \right. \\
&\quad \left. - 2\pi f_{12} + \frac{\pi\zeta m^2}{8M^4} (g_{41} - g_{32}) + 2\pi(2 - \frac{m^2}{M^2})(g_1 - 5g_2) + 4\pi g_{12} \right), \quad (\text{A.26})
\end{aligned}$$

with $f_i = f_i(s, M^2, M^2, 0)$, $g_i = g_i(s, M^2, M^2, m^2)$, etc.

$$\begin{aligned} \mathcal{M}_{d2} = & -\frac{2\pi e^4}{s_W^4} \beta_W \left\{ \not{k}_- \cdot \epsilon_3^* \cdot \epsilon_4^* \left(-\frac{\zeta}{32M^2} g_{41} \right) \right. \\ & + \not{k}_- \cdot p \cdot \epsilon_3^* \cdot p \cdot \epsilon_4^* \frac{1}{32M^2} (8g_{12} - 6g_{32} + (5 + \beta_W^2)g_{41}) \\ & + (\not{f}_3 p \cdot \epsilon_4^* - \not{f}_4 p \cdot \epsilon_3^*) \left(\frac{M^2}{8\pi} (\alpha - I) + \frac{1}{2} g_{12} + \frac{\zeta}{32M^2} (g_{32} - g_{41}) \right) \\ & \left. + (\not{f}_4 p_- \cdot \epsilon_3^* + \not{f}_3 p_- \cdot \epsilon_4^*) \frac{M^2}{8\pi} (I - \alpha) - \not{f}_4 \not{k}_- \not{f}_3 \frac{M^2}{8\pi} \beta \right\} P_L \end{aligned} \quad (A.27)$$

with $g_i = g_i(s, M^2, M^2, M_H^2)$, etc.

$$\mathcal{M}_{d4} = -\frac{e^4}{8} \left(\frac{1}{s_W^2} D4(s, m_\tau^2, 0) + \frac{1}{c_W^2} \left(1 - \frac{1}{2s_W^2} \right)^2 D4(s, M_Z^2, 0) \right) P_L \quad (A.28)$$

$$\mathcal{M}_{d5} = -\frac{e^4}{16s_W^4} D5(s, M^2, 0) P_L,$$

where

$$\begin{aligned} D4(s, m_1^2, m_2^2) &= 4 \not{k}_- \cdot p \cdot \epsilon_3^* \cdot p \cdot \epsilon_4^* (\beta - 2C + D + F/s + L) \\ &+ \not{k}_- \cdot (p_- \cdot \epsilon_3^* \cdot p \cdot \epsilon_4^* - p_- \cdot \epsilon_4^* \cdot p \cdot \epsilon_3^*) (-2I - 4\alpha + 4\beta - 2A/s \\ &\quad - 2B - 2C + 4D - 4J) \\ &+ (\not{f}_4 p_- \cdot \epsilon_3^* + \not{f}_3 p_- \cdot \epsilon_4^*) (-s(\alpha + 2B + G) + E + 2v(D + H) - \zeta J) \\ &+ \not{k}_- \cdot p_- \cdot \epsilon_3^* \cdot p_- \cdot \epsilon_4^* (-4D - 4H) + \not{f}_4 \not{k}_- \not{f}_3 (2F - 2A) \\ &+ \not{f}_3 \not{k}_- \not{f}_4 (s(I + 2\alpha - \beta + B + H) + A - 3F - \zeta(C - L) - 2vJ) \\ &+ (\not{f}_3 p \cdot \epsilon_4^* - \not{f}_4 p \cdot \epsilon_3^*) (s(\alpha + 2B - 2D + G - 2H) - 4A - 5E + 2F \\ &\quad + \zeta(-2\beta + 4C - 2D + J - 2L) \\ &\quad + 2v(I + 2\alpha - 2\beta + A/s + B + C - 3D - H + 2J)) \end{aligned} \quad (A.29)$$

$$D5 = D4(\epsilon_3^* \leftrightarrow \epsilon_4^*, k_- \rightarrow -k_-, \cos \theta \rightarrow -\cos \theta). \quad (A.30)$$

A5 CUT THREE-GAUGE-BOSON VERTEX DIAGRAMS : TYPE E

Next, we give the contributions of the triangle diagrams cut as shown in diagrams $e1^\pm, e2^\pm, e3^\pm$. We denote by $\mathcal{M}_{e_{1,2,3}^\pm}$ the sum of the contributions of these diagrams. Since a W cannot decay into an on-shell top-bottom pair, only leptons and u, d, c, s quarks are to be taken into account. We take all fermions in the cut triangle to be effectively massless.

$$\mathcal{M}_{e_{1,2,3}^\pm} = \frac{\pi e^4}{4s_W^2} \left(\frac{1}{s} (9ER + EI) + \frac{1}{s - M_Z^2} \left(\frac{s_W^2}{c_W^2} EI - 9ER \right) \left(1 - \frac{P_L}{2s_W^2} \right) \right) \quad (A.31)$$

where

$$\begin{aligned} ER &= \not{k}_- \cdot \epsilon_3^* \cdot \epsilon_4^* (-6sg_1^* - 4M^2 g_2^* - 2M^2 g_3^* + sg_4^*/M^2) \\ &+ 4 \not{k}_- \cdot p \cdot \epsilon_3^* \cdot p \cdot \epsilon_4^* \left(4g_1^* + \frac{6M^2}{\zeta} g_1^* - \frac{2}{M^2} g_4^* - \frac{5}{\zeta} g_4^* - \frac{2M^4}{s\zeta} (3g_3^* - 2g_2^*) \right) \\ &+ (\not{f}_3 p \cdot \epsilon_4^* - \not{f}_4 p \cdot \epsilon_3^*) (2sg_1^* + 12M^2 g_2^* - 2M^2 g_3^* + sg_4^*/M^2) \\ EI &= i\gamma_\rho \epsilon(\rho, k_-, \epsilon_3^*, \epsilon_4^*) \left(4sg_1^* - 2M^2 g_2^* - M^2 g_3^* + \frac{s}{2M^2} g_4^* \right) \\ &+ i\gamma_\rho \left(\epsilon(p, k_-, \rho, \epsilon_3^*) p \cdot \epsilon_4^* - \epsilon(p, k_-, \rho, \epsilon_4^*) p \cdot \epsilon_3^* \right) \times \\ &\quad \times \left(\left(\frac{M^2}{s} + \frac{2M^2}{\zeta} \right) (3g_3^* - 2g_2^*) - \frac{g_4^*}{2M^2} \right) \end{aligned} \quad (A.32)$$

The functions g_i^* are given in Appendix B by equations (B.20).

A6 CUT CROSSED BOX-DIAGRAMS : TYPE F

For the diagrams $f_4^\pm$ we have

$$\mathcal{M}_{f_4^-} + \mathcal{M}_{f_4^+} = \frac{\pi e^4 (1 - 2s_W^2) M^2}{4s_W^4 c_W^2 t} F4(M_Z^2) P_L \quad (A.33)$$

where t is the Mandelstam variable and

$$F_4(m_1^2) = \left((\not{f}_3 p \cdot \epsilon_4^* - \not{f}_4 p \cdot \epsilon_3^*) - (\not{f}_4 p_- \cdot \epsilon_3^* + \not{f}_3 p_- \cdot \epsilon_4^*) \right) \times \quad (A.34)$$

$$\times \left(f_1^* + 2f_2^* - f_3^* + \frac{M^2}{4p_1 \cdot p_3} (3f_3^* - 2f_2^*) \right)$$

$$+ \not{f}_4 \not{k}_- \not{f}_3^* \left(\frac{2p_1 \cdot p_3}{M^2} f_1^* + f_2^* + \frac{1}{2} f_3^* \right).$$

The functions $f_i^* = f_i^*(m_1^2)$ are given by equation (B.18) of Appendix B.

A7 STILL MORE CUT BOX DIAGRAMS : TYPE G

Finally we give the contributions of the diagrams $g_{4,5,6}^\pm$:

$$\mathcal{M}_{g_4^-} + \mathcal{M}_{g_4^+} = \frac{e^4}{8s_w^2} \left(G_4(m_\gamma^2) + \frac{(2s_w^2 - 1)^2}{4c_w^2 s_w^2} G_4(M_2^2) \right) P_L \quad (A.35)$$

$$\mathcal{M}_{g_5^-} + \mathcal{M}_{g_5^+} = \frac{e^4}{16s_w^4} G_5(M^2) P_L$$

with (Abbreviating $z = p_1 \cdot p_3/M^2$ and $x = s/M^2$)

$$G_4(m_1^2)$$

$$= \not{k}_- p \cdot \epsilon_3^* p \cdot \epsilon_4^* \left\{ 2z^2(G + 2H) + 2xz(H + 2J) + z(2B - G - 4H - 4J) \right.$$

$$\left. + x^2(J/2 + L) + x(D - H/2 - 2J - 2L) - 2(E + 2F)/M^2 \right.$$

$$\left. + B + 4(C + D - \alpha - \beta) \right\}$$

$$+ (\not{f}_4 p_- \cdot \epsilon_3^* + \not{f}_3 p_- \cdot \epsilon_4^*) M^2 \left\{ z^2(4B + G) + xz(-2B + 2D + H) - E/M^2 \right.$$

$$\left. - (4z - 2x + 1)\alpha - x^2(D - J/4) + x(2B + 2D - H - J) \right\}$$

$$+ \not{k}_- (p_- \cdot \epsilon_3^* p \cdot \epsilon_4^* - p_- \cdot \epsilon_4^* p \cdot \epsilon_3^*) \left\{ z^2 G + xzH + 2z(B + 2D) + x^2 J/4 \right.$$

$$\left. + x(2C + D) + 5\alpha + 4\beta - 4C - 4D - E/M^2 \right\}$$

$$+ \not{k}_- p_- \cdot \epsilon_3^* p_- \cdot \epsilon_4^* \left\{ -z(2B + G) - x(D + H/2) + 2\alpha + \beta \right\}$$

$$+ \not{f}_4 \not{k}_- \not{f}_3^* (2zE + xF + 2A)$$

$$+ \not{f}_3 \not{k}_- \not{f}_4^* M^2 \left\{ z^3 G + 3xz^2 H/2 - 3z^2 B + 3x^2 zJ/4 - xz(3D + H + J) \right.$$

$$\left. + \frac{x^3}{8} L - \frac{x^2}{4} (3C + 2J + 2L) - \frac{3xF}{2M^2} + x \left(3C + 3D - 4\alpha - \frac{5}{2}\beta \right) \right.$$

$$\left. - \frac{3z}{M^2} + 3z\alpha - I + \frac{3A}{M^2} \right\}$$

$$+ (\not{f}_3 p \cdot \epsilon_4^* - \not{f}_4 p \cdot \epsilon_3^*) M^2 \left\{ -2z^3 G - 3xz^2 H - z^2(10B + 16D + G) - 3x^2 zJ/2 \right.$$

$$\left. + xz(2B - 8C + H + 2J) + 2z \frac{E}{M^2} + 2z(-9\alpha - 8\beta + 2B + 8C + 8D) \right.$$

$$\left. - \frac{x^3}{4} L + \frac{x^2}{4} (10C + 4D + 3J + 4L) + x \frac{F}{M^2} \right.$$

$$\left. + x(6\alpha + 5\beta - 2B - 6C - 6D + J) - 3\alpha - 2I + (2A + 5E)/M^2 \right\}, \quad (A.36)$$

$$G_5 = G_4(\epsilon_3^* \leftrightarrow \epsilon_4^*, k_- \rightarrow -k_-, \cos\theta \rightarrow -\cos\theta). \quad (A.37)$$

The functions $I, \alpha, \dots, L$ are those given in equations (B.22-24) with the appropriate m_1^2 . We should have denoted them $I^*, \alpha^*, \dots, L^*$ but dropped the $*$ in eq. (A.35) in order not to clutter the notation even more. The same remark holds for the contribution of diagrams $g_6^\pm$ which we give next, where all $I, \alpha, \dots, L$ should be read $I^{**}, \alpha^{**}, \dots, L^{**}$. They are given by eqs. (B.25-27). We also introduce $y = p_1 \cdot p_4/M^2$ and x, z as before, and obtain:

$$\mathcal{M}_{g_6^-} + \mathcal{M}_{g_6^+} = -\frac{e^4}{4s_w^2} \left(G_6(m_\gamma^2) + \frac{1 - 2s_w^2}{2s_w^2} G_6(M_2^2) \right) P_L \quad (A.38)$$

with

$$G_6(m_1^2) = \not{k}_- \epsilon_3^* \cdot \epsilon_4^* (2zE - 2yF)$$

$$+ \not{k}_- p \cdot \epsilon_3^* p \cdot \epsilon_4^* \left\{ 2\alpha - 2\beta - \left(\frac{1}{2} + z \right) B + \left(\frac{1}{2} + y \right) C + (y - z) D + \frac{E - F}{M^2} \right.$$

$$\left. + \left(\frac{z}{2} - z^2 \right) G + (2yz + z^2 - \frac{y}{2} - z) H - (2yz + y^2 - \frac{z}{2} - y) J - \left(\frac{y}{2} - y^2 \right) L \right\}$$

$$+ (\not{f}_4 p_- \cdot \epsilon_3^* + \not{f}_3 p_- \cdot \epsilon_4^*) M^2 \left\{ I + \left(\frac{3}{2} - 2y \right) \alpha + \left(\frac{3}{2} - 2z \right) \beta + (2yz - z^2 - z - 2y) B \right.$$

$$\begin{aligned}
& + (2yz - y^2 - y - 2z)C + (2yz - 2y^2 - 2z^2 - 3y - 3z + x)D - \frac{z^2}{2}G \\
& + (yz - \frac{z^2}{2} - \frac{x}{2})H + (yz - \frac{y^2}{2} - \frac{x}{2})J - \frac{y^2}{2}L - \frac{2A - E - F}{2M^2} \Big\} \\
& + \not{p}_- \cdot \epsilon_3^* p_- \cdot \epsilon_4^* - p_- \cdot \epsilon_4^* p_- \cdot \epsilon_3^* \Big\{ \frac{3}{2}\alpha + \frac{3}{2}\beta - zB - yC + (y + z - 3)D \\
& + \frac{E + F}{2M^2} - \frac{z^2}{2}G + (yz - \frac{z^2}{2})H + (yz - \frac{y^2}{2})J - \frac{y^2}{2}L \Big\} \\
& + \not{p}_- \cdot \epsilon_3^* p_- \cdot \epsilon_4^* \Big\{ -\alpha + \beta - (\frac{1}{2} - z)B + (\frac{1}{2} - y)C + (z - y)D \\
& + \frac{z}{2}G + (z - \frac{y}{2})H - (y - \frac{z}{2})J - \frac{y}{2}L \Big\} \\
& + \not{f}_4^* \not{f}_3^* M^2 \{ -I + 2(z + 2y)\alpha + 2y\beta - z^2B - y^2C + (2yz - x)D \} \\
& + \not{f}_3^* \not{f}_4^* M^2 \{ I - 2z\alpha - 2(y + 2z)\beta + z^2B + y^2C - (2yz - x)D \} \\
& + (\not{f}_3^* p_- \cdot \epsilon_4^* - \not{f}_4^* p_- \cdot \epsilon_3^*) M^2 \Big\{ (\frac{3}{2} - 5z + 2y)\alpha - (\frac{3}{2} - 5y + 2z)\beta \\
& + (-2yz + 4z^2 - z + 2y)B - (-2yz + 4y^2 - y + 2z)C \\
& + (2y^2 - 2z^2 - 3y + 3z)D + (\frac{5}{2} - 3z)\frac{E}{M^2} - (\frac{5}{2} - 3y)\frac{F}{M^2} + z^2(z - \frac{1}{2})G \\
& + (yz + \frac{z^2}{2} - 3yz^2 - \frac{x}{2} + xz)H - (yz + \frac{y^2}{2} - 3zy^2 - \frac{x}{2} + xy)J - y^2(y - \frac{1}{2})L \Big\} \\
& \quad \quad \quad (A.39)
\end{aligned}$$

This concludes our enumeration of the different contributions to the absorptive part of the amplitude to order e^4 .

APPENDIX B

The cut “one-loop” integrals

The evaluation of the absorptive part of an amplitude involves the sum over on-shell intermediate states. In all cases at hand this gives rise to an integration $\int d\Omega$ over the angular distribution of two intermediate particles with momenta q_1 , q_2 and masses $q_1^2 = M_1^2$, $q_2^2 = M_2^2$. The angular integration is conveniently carried out in the reference frame where $\vec{q}_1 + \vec{q}_2 = 0$. In the cases of the diagrams a) to d) this coincides with the centre-of-mass frame of $e^+e^- \rightarrow W^+W^-$, whereas for the diagrams e) to g) this is the rest frame either of the W^- or of the W^+ .

B1 INTEGRALS FOR THE DIAGRAMS A) TO D)

All angular integrations are in the CM frame. We denote $E_i = q_i^0$ and define $\omega = E_1/E = (s + M_1^2 - M_2^2)/s$ (in most cases where $M_1 = M_2$, $\omega = 1$). It is useful to introduce

$$\begin{aligned}
q = 2q_1 - \omega p & \Rightarrow q_1 = \frac{1}{2}(\omega p + q) \quad , \quad q_2 = \frac{1}{2}((2 - \omega)p - q) \quad , \quad \text{if } M_1^2 \neq M_2^2 \\
q = q_1 - q_2 & \Rightarrow q_1 = \frac{1}{2}(p + q) \quad , \quad q_2 = \frac{1}{2}(p - q) \quad , \quad \text{if } M_1^2 = M_2^2 \quad .
\end{aligned} \tag{B.1}$$

In any case, $q = (0, \vec{q})$ has no time-component. One has

$$q^2 \equiv -\vec{q}^2 \equiv -4\vec{q}_1^2 = -((s - M_1^2 - M_2^2)^2 - 4M_1^2 M_2^2)/s \quad . \tag{B.2}$$

We also introduce the quantities

$$s_1 = s - M_1^2 - M_2^2 - 2m_e^2 + 2m_1^2 \quad , \quad s_2 = s - M_1^2 - M_2^2 - 2M_2^2 + 2m_2^2 \quad , \tag{B.3}$$

where m_1 and m_2 will denote in the following the masses of the exchanged particles.

Integrals with no denominator

Integrals without denominator arise from the cut vacuum-polarization diagrams a). They are trivial:

$$\int d\Omega [1, q^\mu, q^\mu q^\nu] = 4\pi \times [1, 0, \frac{1}{3}q^2(g^{\mu\nu} - \frac{p^\mu p^\nu}{s})]. \quad (B.4)$$

Integrals with one denominator

These arise when a triangle diagram is cut. There are essentially two types, corresponding to the diagrams b) and c). For the integrals corresponding to c) one has

$$\begin{aligned} \int \frac{d\Omega [1, q^\mu]}{(q_1 - p_1)^2 - m_1^2} &= 4\pi \times [f_2, f_1 p_-^\mu] \\ \int \frac{d\Omega q^\mu q^\nu}{(q_1 - p_1)^2 - m_1^2} &= -\pi q^2 \left((g^{\mu\nu} - \frac{p^\mu p^\nu}{s})(f_3 - 2f_2) + \frac{p_-^\mu p_-^\nu}{s\beta_e^2} (3f_3 - 2f_2) \right) \\ \int \frac{d\Omega q^\mu q^\nu q^\rho}{(q_1 - p_1)^2 - m_1^2} &= -\pi \frac{q^2}{s} \left\{ 2f_4 \frac{p_-^\mu p_-^\nu p_-^\rho}{s\beta_e^2} \right. \\ &\quad \left. + (2sf_1 - f_4) \left(\left(\frac{p^\mu p^\nu}{s} - g^{\mu\nu} - \frac{p_-^\mu p_-^\nu}{s\beta_e^2} \right) p_-^\rho + \text{cyclic perm.} \right) \right\} \end{aligned} \quad (B.5)$$

where the function $f_1, \dots, f_4$ are given by the following expressions

$$\begin{aligned} f_2 \equiv f_2(s, M_1^2, M_2^2, m_1^2) &= \frac{1}{4|\vec{p}_1||\vec{q}_1|} \log \left(\frac{s - M_1^2 - M_2^2 - 4|\vec{p}_1||\vec{q}_1| + 2m_1^2 - 2m_e^2}{s - M_1^2 - M_2^2 + 4|\vec{p}_1||\vec{q}_1| + 2m_1^2 - 2m_e^2} \right) \\ f_1 &= \frac{1}{s\beta_e^2} (2 + s_1 f_2), \quad f_3 = \frac{s_1}{2\vec{q}_1^2} f_1, \quad f_4 = \frac{1}{\beta_e^2} \left(\frac{4}{3} + \frac{s_1^2}{2\vec{q}_1^2} f_1 \right). \end{aligned} \quad (B.6)$$

The corresponding integrals for the diagrams b) are given by the same equations merely modified by the following substitutions:

$$f_i \rightarrow g_i, \quad m_1 \rightarrow m_2, \quad m_e \rightarrow M_W, \quad \beta_e \rightarrow \beta_W, \quad s_1 \rightarrow s_2, \quad p_- \rightarrow k_- . \quad (B.7)$$

Integrals with two denominators

These arise from cut box diagrams. For the case of the diagrams d) one has

$$[I, I^\mu, I^{\mu\nu}, I^{\mu\nu\rho}] = \int \frac{d\Omega [1, q^\mu, q^\mu q^\nu, q^\mu q^\nu q^\rho]}{[(q_1 - p_1)^2 - m_1^2][(q_1 - p_3)^2 - m_2^2]}. \quad (B.8)$$

It is straightforward to express all integrals in terms of I and the f_i, g_i :

$$\begin{aligned} I^\mu &= \alpha p_-^\mu + \beta k_-^\mu, \\ I^{\mu\nu} &= A(g^{\mu\nu} - \frac{p^\mu p^\nu}{s}) + B p_-^\mu p_-^\nu + C k_-^\mu k_-^\nu + D(p_-^\mu k_-^\nu + k_-^\mu p_-^\nu), \\ I^{\mu\nu\rho} &= [(g^{\mu\nu} - \frac{p^\mu p^\nu}{s})(E p_-^\rho + F k_-^\rho) + \text{cyclic perm.}] + G p_-^\mu p_-^\nu p_-^\rho \\ &\quad + H[p_-^\mu p_-^\nu k_-^\rho + \text{cyclic perm.}] + J[p_-^\mu k_-^\nu k_-^\rho + \text{cyclic perm.}] + L k_-^\mu k_-^\nu k_-^\rho, \end{aligned} \quad (B.9)$$

where the functions $\alpha, \dots, L$ are given by (recall that $\zeta = s\beta_W^2$, $v = p_- \cdot k_-$)

$$\begin{aligned} \alpha &= \frac{(s_1 \zeta + s_2 v)I + 8\pi(\zeta g_2 + v f_2)}{s\zeta - v^2}, \\ \beta &= \frac{(s_1 v + s_2 s)I + 8\pi(v g_2 + s f_2)}{s\zeta - v^2}, \\ A &= q^2 I + s_1 \alpha + s_2 \beta, \\ B &= \frac{(s_1 \zeta + s_2 v)\alpha + \zeta A + 8\pi v f_1}{s\zeta - v^2}, \\ C &= \frac{-s_1 \alpha + s_2 \beta + s B}{\zeta}, \\ D &= \frac{(s_1 v + s_2 s)\alpha + v A + 8\pi s f_1}{s\zeta - v^2}, \\ E &= \frac{(s_1 \zeta + s_2 v)A + 8\pi \vec{q}_1^2 (\zeta(g_3 - 2g_2) + v(f_3 - 2f_2))}{s\zeta - v^2}, \\ F &= \frac{(s_1 v + s_2 s)A + 8\pi \vec{q}_1^2 (v(g_3 - 2g_2) + s(f_3 - 2f_2))}{s\zeta - v^2}, \\ G &= \frac{(s_1 \zeta + s_2 v)B + 2\zeta E + 8\pi \frac{\vec{q}_1^2}{s} v(3f_3 - 2f_2)}{s\zeta - v^2}, \\ H &= \frac{(s_1 \zeta + s_2 v)D + vE + \zeta F}{s\zeta - v^2}, \end{aligned}$$

$$J = \frac{(s_1 v + s_2 s)D + sE + vF}{s\zeta - v^2},$$

$$L = \frac{(s_1 v + s_2 s)C + 2sF + 8\pi \frac{t^2}{\zeta} v(3g_3 - 2g_2)}{s\zeta - v^2}. \quad (B.10)$$

We are left with the evaluation of the integral I only. Introducing a Feynman parameter x the angular integration is easily done with the result

$$I = 4\pi \int_0^1 dx (\bar{A}x^2 + 2\bar{B}x + \bar{C})^{-1} \quad (B.11)$$

with

$$\bar{A} = \frac{1}{4}(s_1 - s_2)^2 - q^2(p_3 - p_1)^2,$$

$$\bar{B} = \frac{1}{4}s_1(s_1 - s_2) - q^2(p_1 \cdot p_3 - m_e^2), \quad (B.12)$$

$$\bar{C} = \frac{1}{4}s_1^2 + q^2(s/4 - m_e^2).$$

The x -integration is also trivial, and we obtain

$$I = \frac{2\pi}{\Delta} \log \left(\frac{\bar{A} + \bar{B} - \Delta}{\bar{A} + \bar{B} + \Delta} \frac{\bar{B} + \Delta}{\bar{B} - \Delta} \right), \quad \Delta = \sqrt{\bar{B}^2 - \bar{A}\bar{C}}. \quad (B.13)$$

B2 INTEGRALS FOR THE DIAGRAMS E) TO G)

For the diagrams e^- to g^- (e^+ to g^+) the integration over the angular dependence of the momenta of the intermediate particles may be carried out in the rest frame of the W^- (W^+). Since all results for e^+ to g^+ can be deduced from those for e^- to g^- by the substitutions $p_1 \leftrightarrow p_2$, $p_3 \leftrightarrow p_4$, $\epsilon_3 \leftrightarrow \epsilon_4$, we will consider the former case only.

In all cases of interest, the intermediate particles are massless: $q_1^2 = q_2^2 = 0$. We denote the angular integration in the rest frame of the W^- ($\vec{p}_3 = 0$) by $d\Omega^*$. In this frame the momenta are explicitly given by ($\hat{v}$ and $\hat{w}$ denote unit vectors):

$$p_3 = (M, 0), \quad p_4 = \left(\frac{s}{2M} - M, \frac{s\beta_W}{2M}\hat{v}\right), \quad p = \frac{s}{2M}(1, \beta_W\hat{v}), \quad p_1 = \frac{p_1 \cdot p_3}{M}(1, \hat{w})$$

$$p_2 = \left(\frac{p_1 \cdot p_4}{M}, \frac{s\beta_W}{2M}\hat{v} - \frac{p_1 \cdot p_3}{M}\hat{w}\right), \quad \text{with } \hat{v} \cdot \hat{w} = \frac{p_1 \cdot p_3 - M^2}{\beta_W p_1 \cdot p_3}. \quad (B.14)$$

We introduce

$$q = q_1 - q_2 \Rightarrow q_1 = \frac{1}{2}(p_3 + q), \quad q_2 = \frac{1}{2}(p_3 - q), \quad q^2 = -M^2. \quad (B.15)$$

The tensorial decomposition of the integrals is done in terms of the vectors n , l , $\tilde{l}$ orthogonal to p_3 :

$$n = p_1 - zp_3, \quad l = p - \hat{s}p_3, \quad \tilde{l} = -p_2 + yp_3$$

$$z = \frac{p_1 \cdot p_3}{M^2}, \quad \hat{s} = \frac{s}{2M^2}, \quad y = \frac{p_1 \cdot p_4}{M^2} \quad (B.16)$$

$$n^2 = -M^2 z^2, \quad l^2 = -\frac{s^2 \beta_W^2}{4M^2}, \quad \tilde{l}^2 = -M^2 y^2.$$

Integrals with no denominator

Integrals without denominator are trivial. They are immediately deduced from equation (B.4) with the obvious substitution $p^\mu p^\nu / s \rightarrow p_3^\mu p_3^\nu / M^2$.

Integrals with one denominator

They are essentially deduced from (B.5) by substituting $p \rightarrow p_3$, $p_- \rightarrow$

$n, p_-^2 \rightarrow n^2$, i.e. $s\beta_e^2 \rightarrow -n^2$. Explicitly one has:

$$\begin{aligned} \int \frac{d\Omega^* [1, q^\mu]}{(q_1 - p_1)^2 - m_1^2} &= 4\pi \times [f_2^*, f_1^* n^\mu] \\ \int \frac{d\Omega^* q^\mu q^\nu}{(q_1 - p_1)^2 - m_1^2} &= -\pi q^2 \left((g^{\mu\nu} - \frac{p_3^\mu p_3^\nu}{M^2})(f_3^* - 2f_2^*) - \frac{n^\mu n^\nu}{n^2} (3f_3^* - 2f_2^*) \right) \\ \int \frac{d\Omega^* q^\mu q^\nu q^\rho}{(q_1 - p_1)^2 - m_1^2} &= -\pi \frac{q^2}{M^2} \left\{ -2f_4^* \frac{n^\mu n^\nu n^\rho}{n^2} \right. \\ &\quad \left. + (2M^2 f_1^* - f_4^*) \left(\left(\frac{p_3^\mu p_3^\nu}{M^2} - g^{\mu\nu} + \frac{n^\mu n^\nu}{n^2} \right) n^\rho + \text{cyc. perm.} \right) \right\} \end{aligned} \quad (B.17)$$

and the functions f_i^* are given by ($s_1 \rightarrow s_1^* = 2(p_1 \cdot p_3 - m_e^2 + m_1^2)$)

$$\begin{aligned} f_2^* &= \frac{1}{2p_1 \cdot p_3} \log \left(\frac{m_1^2 - m_e^2}{2p_1 \cdot p_3 + m_1^2 - m_e^2} \right) \\ f_1^* &= -\frac{1}{2n^2} (2 + s_1^* f_2^*), \quad f_3^* = \frac{s_1^*}{M^2} f_1^*, \quad f_4^* = -\frac{M^2}{2n^2} \left(\frac{4}{3} + \frac{s_1^*}{M^2} f_1^* \right). \end{aligned} \quad (B.18)$$

Next, consider the integrals

$$\int \frac{d\Omega^* [1, q^\mu, q^\mu q^\nu, q^\mu q^\nu q^\rho]}{(q_1 - p)^2}. \quad (B.19)$$

To deduce the corresponding formula from the preceding ones substitute

$$\begin{aligned} n^\mu &\rightarrow l^\mu, \quad n^2 \rightarrow l^2, \quad f_i^* \rightarrow g_i^*, \quad s_1^* \rightarrow -s \\ g_2^* &= \frac{1}{s\beta_W} \log \frac{1 + \beta_W}{1 - \beta_W}, \quad g_1^* = -\frac{1}{2l^2} (2 - sg_2^*), \\ g_3^* &= -\frac{s}{M^2} g_1^*, \quad g_4^* = -\frac{M^2}{2l^2} \left(\frac{4}{3} + \frac{s^2}{M^2} g_1^* \right). \end{aligned} \quad (B.20)$$

We also need the integrals (for the diagrams $g6^\pm$)

$$\int \frac{d\Omega^* [1, q^\mu, q^\mu q^\nu, q^\mu q^\nu q^\rho]}{(q_1 - (p_1 - p_4))^2 - M^2}. \quad (B.21)$$

They are obtained from equations (B.17,18) by the substitutions $n \rightarrow \tilde{l}, m_1^2 \rightarrow$

$M^2, p_1 \cdot p_3 \rightarrow p_1 \cdot p_4$, hence $s_1^* \rightarrow 2(p_1 \cdot p_4 + M^2 - m_e^2)$, and with the functions f_i^* becoming g_i^{**} .

Integrals with two denominators

There are two types of them, those for diagrams $g4^\pm, g5^\pm$ and those for $g6^\pm$. Consider the case of $g4^\pm, g5^\pm$ first:

$$[I^*, I^{*\mu}, I^{*\mu\nu}, I^{*\mu\nu\rho}] = \int \frac{d\Omega^* [1, q^\mu, q^\mu q^\nu, q^\mu q^\nu q^\rho]}{[(q_1 - p_1)^2 - m_1^2][(q_1 - p)^2]}. \quad (B.22)$$

The tensorial decomposition is the same as in (B.9) with the replacements $\alpha, \dots, L \rightarrow \alpha^*, \dots, L^*, p_-^\mu \rightarrow n^\mu, k_-^\mu \rightarrow l^\mu, p^\mu p^\nu / s \rightarrow p_3^\mu p_3^\nu / M^2$. The functions $\alpha^*, \dots, L^*$ are given by (B.10) with the following substitutions:

$$\begin{aligned} f_i &\rightarrow \frac{1}{2} f_i^*, \quad g_i \rightarrow \frac{1}{2} g_i^*, \quad v \rightarrow n \cdot l, \quad s \rightarrow -n^2, \quad \zeta \rightarrow -l^2, \\ s_1 &\rightarrow p_1 \cdot p_3 + m_1^2, \quad s_2 \rightarrow -\frac{s}{2}, \quad \vec{q}_1^2 = \frac{1}{4} M^2, \quad q^2 = -M^2. \end{aligned} \quad (B.23)$$

I^* is given by eq. (B.13) with

$$\begin{aligned} \bar{A} &= (s + m_1^2)(m_1^2 + 2p_1 \cdot p_3), \\ \bar{B} &= -(s + 2m_1^2)p_1 \cdot p_3 + \frac{1}{2}s(M^2 - m_1^2) - m_1^4, \\ \bar{C} &= m_1^2(m_1^2 + 2p_1 \cdot p_3). \end{aligned} \quad (B.24)$$

The integrals for $g6^\pm$ are

$$[I^{**}, I^{**\mu}, I^{**\mu\nu}, I^{**\mu\nu\rho}] = \int \frac{d\Omega^* [1, q^\mu, q^\mu q^\nu, q^\mu q^\nu q^\rho]}{[(q_1 - p_1)^2 - m_1^2][(q_1 - (p_1 - p_4))^2 - M^2]} \quad (B.25)$$

and are derived from (B.9) and (B.10) by $\alpha, \dots, L \rightarrow \alpha^{**}, \dots, L^{**}$, and

$$f_i \rightarrow \frac{1}{2} f_i^{**}, \quad g_i \rightarrow \frac{1}{2} g_i^{**}, \quad p_-^\mu \rightarrow n^\mu, \quad k_-^\mu \rightarrow \tilde{l}^\mu, \quad v \rightarrow n \cdot \tilde{l}, \quad s \rightarrow -n^2, \quad \zeta \rightarrow -\tilde{l}^2,$$

$$s_1 \rightarrow p_1 \cdot p_3 + m_1^2, \quad s_2 \rightarrow p_1 \cdot p_4 + M^2, \quad \vec{q}_1^2 = \frac{1}{4} M^2, \quad q^2 = -M^2. \quad (B.26)$$

Note that f_i^{**} is the same as f_i^* given by (B.18) whereas g_i^{**} is obtained as described

below (B.21). I^{**} is given by eq. (B.13) with

$$\begin{aligned}\bar{A} &= (u + m_1^2 - M^2)(m_1^2 - t - M^2) + M^4, \\ \bar{B} &= -(u + m_1^2 - M^2)(m_1^2 + p_1 \cdot p_3) + m_1^2(M^2 - \frac{s}{2}) - M^2 p_1 \cdot p_4, \\ \bar{C} &= m_1^2(m_1^2 + 2p_1 \cdot p_3).\end{aligned}\tag{B.27}$$

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FIGURE CAPTIONS

- Fig. 1. Kinematics of the process $e^+e^- \rightarrow W^+W^- \rightarrow l^-\bar{\nu}$ jet jet.
- Fig. 2. Diagrams contributing to the absorptive part of the $e^+e^- \rightarrow W^+W^-$ amplitude at lowest non-trivial order in the standard model.
- Fig. 3. Some ways to cut a one-loop diagram do not lead to *on-shell* intermediate states.
- Fig. 4. Diagrams that give a non-zero contribution to the absorptive part of the amplitude only when the finite width of the W is taken into account.
- Fig. 5. The angular distribution of the T-odd part of the cross section as a function of the scattering angle.
- Fig. 6. The asymmetry $\mathcal{AS}^{W^+l^-\bar{\nu}}(\theta_{\min}, \pi - \theta_{\min})$ with cuts on the scattering angle $\theta_{\min} \leq \theta \leq \pi - \theta_{\min}$ as a function of $\theta_{\min}$ for two different beam energies E .
- Fig. 7. The efficiency $\mathcal{AS}^{W^+l^-\bar{\nu}}(\theta_{\min}, \pi - \theta_{\min}) \times \sqrt{N}$ as a function of $\theta_{\min}$ for two different beam energies E .
- Fig. 8. The asymmetry $\mathcal{AS}^{W^+l^-\bar{\nu}}$ without cuts (a), with cuts $0.15\pi \leq \theta \leq 0.85\pi$ (b) and the corresponding efficiency (with cuts) (c) for three different top quark masses. All these quantities are displayed as functions of the beam energy E .

Fig. 1

a1

a2

a3

a4

a5

Figure 2

Figure 2

Figure 2

Figure 2

Figure 2

Figure 2

Figure 2

Figure 2

Fig. 4a

Fig. 4b

Fig. 4c

Fig. 3a

Fig. 3b

Figure 5

Figure 6

Figure 7