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TETRAQUARKS, PENTAQUARKS and HEXAQUARKS^{*}

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ABSTRACT

We review the theoretical speculations on stable multiquark states bound by either chromoelectirc or chromomagnetic forces.

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1 Introduction

The simple quark model describes astonishingly well the spectrum of ordinary mesons and baryons. A first non-trivial achievement is to account for the quantum numbers of the hadrons. The well-established states have *non exotic* quantum numbers, which correspond to $q_1\bar{q}_2$ or $q_1q_2q_3$ configurations with all possible spin and angular momentum wave-functions. Quantitative models (bags, potentials ...) have even produced remarkable quantitative estimates of the mass spectrum. A crucial ingredient is *flavour independence* of the confining interaction which allows one, for instance, to describe simultaneously the $c\bar{c}$ and $b\bar{b}$ excitation spectra [1].

One should first try to understand the success of the simple model from the underlying QCD. Already, lattice calculations [2] have provided mass spectra which exhibit the same regularities as the spectra arising from simple wave-equations, which are thus plausible candidates for being decent approximations. The static potential relevant for heavy quark bound states has also been calculated by lattice simulations [2]. The result looks similar to the empirical potentials used in phenomenological models.

A second type of activity consists of improving the simple model. Many refinements have been elaborated along the years: relativistic corrections, spin-dependent forces, mixing of configurations, coupling to virtual or real decay channels, etc. ...

A third problem is whether or not there exist other types of hadrons. In our current understanding, ordinary $q_1\bar{q}_2$ or $q_1q_2q_3$ hadrons correspond to states where the gluonic field is frozen to its lowest configuration [3]. Hybrid (or hermaphrodite) states can be viewed as excitations of the gluons surrounding the quarks. Even more extreme are the gluonia, which do not contain any quark at all, in the first approximation before any mixing of configurations. The spectroscopy of these states will be discussed by other speakers [4].

Molecules have also received much attention during the last years [5]. There are good candidates near the $K\bar{K}$ threshold and, perhaps, in the dibaryon sector.

In this review, we shall concentrate on another category of states, those which are stable under strong interactions. If they exist, they will offer clear experimental signatures, and for theorists, an ideal tool for studying the coherences of the interquark forces, as well as the weak decay of flavoured quarks in new environments.

2 A simple model

To show how it is difficult to obtain stable multiquarks, let us consider two quarks and two antiquarks interacting through the simple hamiltonian

$$H = \sum_i \frac{\vec{p}_i^2}{2m_i} - \frac{3}{16} \sum_{i < j} \tilde{\lambda}_i \cdot \tilde{\lambda}_j V(r_{ij}) \quad (1)$$

which contains pairwise, colour-octet exchange potentials. The sum of the strengths, $\sum \tilde{\lambda}_i \cdot \tilde{\lambda}_j$, is the same for any colour singlet of two quarks and two antiquarks. In the equal mass case, the threshold, with labelling $(q\bar{q} + q\bar{q} = 1, 2 + 3, 4)$ and any possible bound state with frozen colour structure and labelling $(qq\bar{q}\bar{q} = 1, 2, 3, 4)$, are obtained from the same hamiltonian

$$H(x) = \sum_i \frac{\vec{p}_i^2}{2m} + \frac{1+2x}{3}(V_{12} + V_{34}) + \frac{1-x}{3}(V_{13} + V_{14} + V_{23} + V_{24}) \quad (2)$$

Using the variational principle, one easily shows that the lowest binding energy, $E_0(x)$, is a convex function of x , with a maximum at $x = 0$. The function $E_0(x)$ is also likely to be almost symmetric around $x = 0$. Thus, the dissociation threshold $(q\bar{q}) + (q\bar{q})$, which has the largest asymmetry parameter $x = 1$ lies below the “true” baryonium configuration, with colour structure $(qq\bar{q}\bar{q} = \bar{3} - 3)$ and asymmetry coefficient $x = 1/4$, and very probably below the “mock baryonium” with sextet-antisextet structure and coefficient $x = -7/8$ [6]. This is confirmed by numerical calculations [7].

In other words, there is no proliferation of multiquarks, because the binding is deeper when the colour forces are asymmetrically concentrated on small clusters $q\bar{q}$ or qqq than when spread over many quark pairs.

To get a stable multiquark, one should rely on new forces: this is the way chosen by the *Hexaquark* (also known as *H*) and the *Pentaquark* *P*, whose tentative binding is due to chromomagnetic forces. Alternatively, one can use another asymmetry, namely in the constituent masses, to compensate and overcome the colour asymmetry measured by the coefficient x . This is the reason why the *Tetraquark* *T* involves different flavours.

3 Tetraquark

It is well known that a heavy particle enjoys more binding than a light one in a given (flavour independent) potential. This is why the Tetraquark $QQ\bar{q}\bar{q}$ survives against dissociation into two flavoured mesons, $Q\bar{q} + Q\bar{q}$, provided

the quark mass ratio M/m is large enough. The critical value $(M/m)_0$ above which binding occurs depends on the particular potential which is used. With current models, a state $cc\bar{u}\bar{d}$ might be slightly unstable, whereas $bb\bar{u}\bar{d}$ has better chances [7].

There are several nice features associated to the tetraquark. All theorists agree on its existence, so far [6,7,8]. Apart from when M/m is close to the critical value $(M/m)_0$, in which case a molecular structure $(Q\bar{q} - Q\bar{q})$ is obtained, the tetraquark consists of a well-located flavoured diquark QQ with colour $\bar{3}$, surrounded by two antiquarks $\bar{q}$ of colour $\bar{3}$, to form an overall singlet. Thus it uses combinations of the very same spatial and colour wave-functions which exist in ordinary baryons such as QQq or $\bar{Q}\bar{q}\bar{q}$ [7,8]. This contrasts with the speculations on hidden colour and diquark-antidiquark clustering of "mock" baryonia in colour chemistry [9].

4 Pentaquark and Hexaquark

Spin-dependent forces between quarks, already included in early works on hadron spectroscopy, have received more attention once they have been associated to one-gluon-exchange [10,11]. Even if this perturbative interpretation is too naive, it remains true that a chromomagnetic hamiltonian of the type

$$V_{SS} = -C \sum_{i < j} \frac{\tilde{\lambda}_i \cdot \tilde{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} \delta^{(3)}(r_{ij}) \quad (3)$$

accounts fairly well for the hyperfine splittings which are observed in mesons and baryons.

The coherences of the chromomagnetic operator $\sum \tilde{\lambda}_i \cdot \tilde{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$ were first noticed by Jaffe [12], who showed that the Hexaquark configuration $H = (uuddss)$ with $I = J = 0$ receives more chromomagnetic attraction than its dissociation threshold $(\Lambda + \Lambda)$. It was observed more recently [13] that the same effect occur for the Pentaquark $P = (\bar{Q}qqqq)$, where $\bar{Q}$ is a heavy antiquark and $qqqq$ a set of light quarks u, d or s , in a spin $J = 0$ state and flavour 3 representation.

The H and the P have been studied extensively in a variety of models [14]. The large binding of 150 Mev predicted in the simplest chromomagnetic calculation is reduced by many effects: breaking of the $SU(3)$ flavour symmetry, finite mass for the heavy quark, careful accounting for the kinetic energy and spin-independent forces, etc.

If one uses the additive potential (1) and a regularized version of the chromomagnetic term (3), one does not get stability for both H and P . The results however depend on the models, and, in particular, on the extrapolation of the confining potential from the meson and baryon sector to the multiquark configurations. The simple choice of eq. (1) is far from being satisfactory and improvements based on non-perturbative QCD are badly needed. Already some lattice calculations suggest that the H might be very tightly bound [15].

It remains true that we do not expect a proliferation of stable multiquarks and that H and P are among the most likely exceptions.

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