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Hagedorn phase transition for the closed string as the anyonic superconducting phase transition in the open topological membrane

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Abstract

The high - temperature or small compactification scales phase of closed string is studied in the 2+1 terms of the open topological membrane theory. The main feature of this phase is the breaking of the world-sheet conformal symmetry due to the Berezinsky - Kosterlitz - Thouless (BKT) phase transition. From the 2+1 point of view this breaking corresponds to the spontaneous breaking of the gauge symmetry of the topologically massive gauge theory (TMGT), which leads to the disappearing of the string's degree of freedom from the membrane boundary (string world-sheet). This spontaneous breaking of the gauge symmetry is nothing but the superconducting phase transition inside the membrane. The possible mechanism of this transition is considered and the connection with anyonic superconductivity is discussed, as well as the description of the world-sheet vortices in 2+1 terms as the charged vortices (flux tubes) in the Abelian TMGT+Higgs model.

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1 Introduction

The nature of the Hagedorn phase transition [1],[2],[3] for closed strings, or more generally the string behavior in extreme conditions (such as small compactification radii, high energy scattering, high temperatures) can help us to understand the physical picture of string theory in more detail. Contrary to the open string case, the Hagedorn temperature for the closed string is not the limiting temperature, but some critical temperature (see [4],[5] and more recent papers [6],[7]), and the new high temperature phase exists. However this phase is expected to differ strongly from the usual low-temperature (or large compactification scale) phase. As was shown in [8] and [9] the Hagedorn phase transition is the Berezinsky - Kostelitz - Thouless (BKT) phase transition on the world sheet (see [10] for more details) and there is no world-sheet conformal symmetry in the high-temperature phase due to the creation of free vortices. The attempt to understand the nature of the high-temperature phase was made by Atick and Witten [11], who discovered the strange behavior of the free energy at temperatures above the Hagedorn temperature and suggested that in the high-temperature phase the density of degrees of freedom, per unit energy, per unit spacetime volume, is much less than in the low-temperature phase. This fact may be a signal of the breakdown of the continuous world sheet picture, which indeed fails due to the free vortices (there are a lot of holes on the world sheet). Thus, this disappearance of the degrees of freedom is ultimately connected to the world-sheet conformal symmetry breaking.

Here we wish to suggest a physical picture of this high temperature phase using the open topological membrane (OTM) approach [12], where the string world sheet is the boundary of the open membrane, with topologically massive gauge fields inside (and also topologically massive gravity, see [13]). The string degrees of freedom (2d Conformal Field Theory, CFT) arise as pure gauge degrees of freedom of this 2+1 topologically massive gauge theory (TMGT). This picture is based on the connection between 2+1 Chern - Simons gauge theory (CST), which is the low energy limit of the TMGT, and 2d CFT, found by Witten [14] and Moore and Seiberg [15].

In the open topological membrane approach the string world sheet is the boundary of the world volume (really there are two boundaries, corresponding to the left- and right-moving sectors, which makes the heterotic construction quite natural, see [12] and [15]) and there is TMGT inside this world volume. The vertex operators correspond to the ends of the open Wilson lines and describe the world line of the charged particles inside the world volume, and this particles are nothing but anyons - it is wellknown that TMGT gives us the field theoretical realisation of anyons. If we consider the string in an external field, then we have a gas of the vertex operators on the world sheet and thus an anyonic gas with some density inside the world volume. We can describe this gas as some scalar field theory - really it is the Ginzburg - Landau effective Lagrangian. If the scalar field gets a non-zero vacuum expectation value, then spontaneous breaking of the gauge invariance occurs and we are in the superconducting phase. But in the case of spontaneous breaking of the gauge symmetry we lose completely the string degrees of freedom on the world sheet - because they appear

as the boundary gauge degrees of freedom in the CST or TMGT. This is the explanation of the "continuous world sheet disappearance" phenomenon above the phase transition. The relevance of the spontaneously broken gauge symmetry for this problem was understood in the paper [16], devoted to the explanation of the $R \rightarrow R'$ duality in the OTM theory.

Thus, we are led to the conclusion that the Hagedorn phase transition corresponds to the anyonic superconducting phase transition inside the open topological membrane and the string behavior at large temperatures and/or small compactification radii is described not by the string, but by the membrane - during the transition the world sheet dimensions transmutation occurs.

The organization of the paper is as follows. In the Section 2 we establish the correspondence between the string in the external field and the anyonic gas at some density, which can be effectively described in terms of the Ginzburg - Landau Lagrangian. As a by-product it gives us the 2+1 formulation of the 2d CFT non-conformal deformation. The spontaneous breaking of the gauge symmetry and the superconducting phase transition in the anyonic system will be discussed in Section 3, where we try to demonstrate that the superconducting phase transition inside the world volume and the BKT (Hagedorn) transition on the world sheet are the same phenomenon. The property of the superconducting phase and the 2+1 description of the world sheet vortices will be considered in Section 4 and Section 5 summarizes basic conclusions and outlines a program for future investigations.

2 String in external field as the anyonic system.

Let us consider the first quantised critical closed string (which means that we neglect the 2d gravity) in the external field - this is the generating function for scattering amplitudes:

$$\begin{aligned} Z &= \int DX(\xi) \exp \left[- \int d^2\xi G_{\mu\nu} \partial_a X^\mu \partial_a X^\nu + \lambda \int d^2\xi (V_+(\xi) + V_-(\xi)) \right] = \\ &= \sum_{n_+, n_-} \frac{\lambda^{n_+ + n_-}}{n_+! n_-!} \int d^2\xi \xi_1 \dots \int d^2\xi_{n_+ + n_-} (V_+(\xi_1) \dots V_-(\xi_{n_+ + n_-})), \end{aligned} \quad (1)$$

where λ is the external field amplitude, $G_{\mu\nu}$ - target space metric and $V_+(\xi), V_-(\xi)$ are the complex conjugate vertex operators (for example $\exp(i p x)$ and $\exp(-i p x)$). It is easy to see that there are equal numbers of + and - operators. The brackets (...) are the quantum averaging; it is possible to consider instead of this simplest free theory any 2d CFT (for WZW models, for example, it is necessary to introduce additional antisymmetric external field) and to consider the correlators of the vertex operators in this 2d CFT.

We see that the second line of the (1) is nothing but some statistical sum with undefined number of particles and λ being a fugacity, defines the chemical potential of the system. This gas on the world sheet is drawn on Fig.1. How to describe this picture in the 2+1 terms ? As was shown in [12],[16] we have a gas of open Wilson lines inside the membrane, ends of which are at the positions of the world sheet vertex operators. In the simplest $U(1)$ case, to which we shall restrict ourselves, each line ties a + and - operator. In such a way we

get the picture of a gas of charged particles inside the membrane, and Wilson lines describe the world lines of these particles.

Let us consider the simplest case which is relevant for us when we have one compact dimension and forget about all other noncompact dimensions (they are needed only for criticality). Thus we have the XY model on the world sheet and the corresponding TMGT is the abelian $U(1)$ theory. We can rewrite (1) (to be more precise only the relevant $U(1)$ contribution to Z) as:

$$Z = \int D\Phi(\xi) \exp \left[- \int d^2\xi \frac{1}{2} \partial_a \Phi \partial_a \Phi + \lambda \int d^2\xi \cos 2\pi(2\beta)^{1/2} \Phi \right] \quad (2)$$

where the simplest external field (tachyon) is considered. This Sine-Gordon action appears naturally as the field theoretical description of the world sheet vortices (the notation of [10] is used here) and $\beta = R^2/4\pi\alpha'$, where R is the compactification radius (for the finite temperature $T^{-1} = 2\pi R$) and as was shown by Coleman in [17] $\beta < \pi^{-1}$, or $R^2 < 4\alpha'$ - at larger values of the radius the vortices condense in dipole pairs and the restoration of the world sheet conformal symmetry occurs. It is easy to estimate the density of this world sheet gas. The statistical sum is

$$Z = \exp(-\beta\Omega)$$

where Ω is the thermodynamic potential, and the average number of particles N is

$$N = \partial\Omega/\partial\mu$$

where the chemical potential μ is determined (see [10]) as $\lambda = 2e^{-\beta\mu}/a^2$, where a is the ultraviolet cutoff. Then

$$\begin{aligned} N &= -\frac{1}{\beta Z} \frac{\partial Z}{\partial \mu} = -\frac{1}{\beta Z} \frac{\partial Z}{\partial \lambda} (-\beta\lambda) = \frac{\lambda}{Z} \frac{\partial Z}{\partial \lambda} = \\ &= \lambda \int d^2\xi (\cos 2\pi(2\beta)^{1/2} \Phi) = 2e^{-\beta\mu} \left(\frac{A}{a^2} \right)^{1-\alpha\beta} \end{aligned} \quad (3)$$

Here A is the infrared cutoff - the world sheet area. When β is changed from zero (infinite temperature) to critical value π^{-1} the average density is changed from maximal $\sim \frac{1}{a^2}$ to some finite (or zero) density. Thus we see that the parameter β indeed governs the density on the world sheet, and ,as consequence , inside the membrane. This analysis can be generalised to the case of the string in an arbitrary external field - and this is nothing but some deformation of the 2d CFT - and we shall see that parameters of the initial CFT (the compactification radius, for example) govern the density of the world sheet gas.

Thus we are led to the conclusion that we have a system of charged particles inside the membrane (anyons) and we can describe this system in terms of Ginzburg - Landau effective action. The Lagrangian we shall discuss is the TMGT + Higgs Lagrangians :

$$L = -\frac{1}{4\gamma} F_{\mu\nu}^2 + \frac{k}{8\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + |D_\mu \Phi|^2 - V(\Phi), \quad (4)$$

where the potential $V(\Phi)$ describes some possible interaction of anyons and the chemical potential of the system is included in the time covariant derivative $D_0\Phi$ as usual. The Chern-Simons coefficient k define (see [12],[16]) the compactification radius R (or temperature $T = (2\pi R)^{-1}$) via

$$k = \frac{R^2}{\alpha'} = 4\pi\beta \quad (5)$$

and the critical value $\beta_c\pi = 1$ corresponds to $k = 4$. The spontaneously broken phase of this model was investigated in [18] and [19] and its application to the OTM was considered in [16]. Here we shall not assume that gauge symmetry has been broken, contrary we try to demonstrate that such a breaking will occur in the case of small k .

3 Dynamical symmetry breaking in the TMGT+Higgs model at small k and the correspondence with the anyonic superconductivity.

Let us discuss the system (4). Due to (5) the conformal phase of the string (large radii. small temperature) exists only if $k > 4$; the value $k = 4$ is critical and corresponds to the world sheet BKT (Hagedorn) transition. At $k < 4$ we get the nonconformal phase and evidently this conformal symmetry breaking can appear in the OTM only as gauge symmetry breaking. Thus we lead to the conclusion that at $k = 4$ the spontaneous breaking of gauge symmetry occurs and our anyonic system is in the superconducting phase at $k \leq 4$. Let us note that the critical value $k = 4$ is precisely the same as was suggested by Kalmeyer and Laughlin (see [20],[21]) for the anyons (semions) in High T_c superconductors. We consider this anyonic system at zero 2+1 temperature (the non-zero temperature exists in the external world, where strings or membranes live, but inside the membrane this temperature is transformed into the CS coefficient k), so it is quite natural to suggest that at zero temperature the anyonic superconductivity exists only for $k \leq 4$ and the critical value corresponds to the semions. This suggestion coincides well with the limit $k \rightarrow \infty$, where all the specific anyonic properties disappear and we have the usual particles.

But what mechanism distinguishes the large and small k ? We wish to propose here the Coleman - Weinberg (CW)[22] mechanism of the radiative correction as the origin of spontaneous symmetry breaking. We consider here the toy model where we neglect the bare scalar field selfinteractions and consider only the kinetic term for Φ . Then we demonstrate that in the two - loop order there is a diagram, generating the CW potential and the vacuum average lies in the in the expected range of validity of our approximation only in the case of small k . Thus the proposed mechanism can generate the gauge symmetry breaking only for small k . Besides the string's phase transition applications this mechanism may be interesting itself as a new possible mechanism of High T_c superconductivity. The drawback of this mechanism is that it is macroscopic, not microscopic, but may be it is possible to understand this phenomenon in the microscopic terms.

Let us consider (4) neglecting the bare scalar selfinteracting term and suppose that there is some classical field Φ . This field will change only the gauge field propagator, the scalar field propagator being unchanged, because we neglect the scalar selfinteractions. The gauge propagator (in the Landau gauge) will be [18],[19],[16] :

$$G_{\mu\nu}(p) = \gamma[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} + i \frac{M \epsilon_{\mu\lambda} p_\lambda}{p^2 + 2\gamma\Phi^2}] \frac{p^2 + 2\gamma\Phi^2}{(p^2 - M_\pm^2)(p^2 - M_-^2)} \quad (6)$$

where

$$2M_\pm^2 = M^2 + 4\gamma\Phi^2 \pm M(M^2 + 8\gamma\Phi^2)^{1/2} \quad (7)$$

where $M = \frac{2k}{4\pi}$ is the gauge boson mass in the unbroken phase.

If the gauge symmetry breaking is large, i.e. $k \ll \phi/\sqrt{7}$, then we see from (7), that both $M_\pm^2$ are of the same value $2\gamma\Phi^2$ and this is the only scale in the theory. It is possible to show that in this case all radiation corrections are small and CW potential is not created. But if the gauge symmetry breaking is small, i.e. $k \gg \phi/\sqrt{7}$, the two masses are

$$\begin{aligned} M_+ &\approx M = \frac{\gamma k}{4\pi} \\ M_- &\approx \frac{2\gamma\Phi^2}{M} = \frac{k}{M} \end{aligned} \quad (8)$$

and there is a wide region for the momentum $M_- < p < M_+$ where the propagator behaviour is the same as in the CS + Higgs theory, and the large mass M_+ plays the role of the ultraviolet cutoff. The gauge propagator at a momentum smaller than the UV cutoff then equals

$$G_{\mu\nu}(p) = \frac{4\pi}{k} [m(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) + i\epsilon_{\mu\lambda} p_\lambda] \frac{1}{(p^2 - m^2)} \quad (9)$$

where the mass $m = M_-$ is determined by the scalar field Φ by the equation (8).

Now we wish to calculate the effective potential $\Gamma(\Phi)$. At the one-loop approximation we have (see Fig.2)

$$\begin{aligned} \Gamma^{1-loop}(\Phi) &= 1/2 \text{Tr} \ln G_{\mu\nu}(p) - 1/2 \text{Tr} \ln G_{\mu\nu}(p)|_{\Phi=0} = \\ &= 1/2 \text{Tr} \ln [i\epsilon_{\mu\lambda} p_\lambda / (p^2 - m^2)] [g_{\mu\nu} + i m \epsilon_{\mu\sigma} p_\sigma / p^2] - 1/2 \text{Tr} \ln [i\epsilon_{\mu\lambda} p_\lambda / p^2] = \\ &= 1/2 \text{Tr} \ln (g_{\mu\nu} p^2 + i m \epsilon_{\mu\sigma} p_\sigma) / (p^2 - m^2) = - \int \frac{d^3 p}{(2\pi)^3} \ln(1 + m^2/p^2) + \text{reg.} = \\ &= m^3/6\pi - \Lambda m^2/2\pi^2 + \text{reg.} \end{aligned}$$

where Tr is the total trace, including the momentum integration, and the Tr is the usual trace over the space-time indices. Λ is the UV cutoff and by the proper choosing of the counterterms we can cancel the divergent part of the m^2 terms (at the expense of terms $\sim Mm^2$ appearing). Thus we get in the one-loop approximation

$$\Gamma^{1-loop}(\Phi) = m^3/6\pi + \text{const } Mm^2 = \frac{2^8 \pi^2}{3k^3} \Phi^6 + O(\Phi^4), \quad (10)$$

Now, let us consider the two-loop contribution to the effective potential. It is possible to demonstrate that the leading contribution at small Φ comes from the diagram of Fig.3. This is the only diagram which has the IR singularities at zero Φ , and its contribution to the effective potential will be of order $\Phi^6 \ln \Phi$, which is the Coleman-Weinberg potential. To be more accurate, the contribution to the potential is

$$\Gamma^{2-loop}(\Phi) = \left(\frac{4\pi\Phi}{k}\right)^2 \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^3 q}{(2\pi)^3} \frac{m^2(1 + (pq)^2/p^2 q^2) - 2pq}{(p^2 - m^2)(q^2 - m^2)} \quad (11)$$

and the UV upper limit is determined by the mass $M_+ = M$ (see (8)). To calculate the relevant logarithmic contribution we pick up only the terms of order m^3 and after some calculations we get:

$$\Gamma^{2-loop}(\Phi) = \frac{2^{10} 3}{k^4} \Phi^6 \int_m^M \int_m^M \frac{dp}{p} \frac{dq}{q} \ln \left| \frac{p+q}{p-q} \right| = - \frac{3m^3}{\pi^2 k} \ln \frac{M}{m} \quad (12)$$

From equations (10), (12) we get the effective potential

$$V(m) = \frac{3m^3}{k\pi^2} \left(\ln \frac{m}{M} + \frac{k\pi}{18} \right) + c M m^2 \quad (13)$$

where we take into account the terms Mm^2 which arise in the $\Gamma^{2-loop}(\Phi)$ also. Neglecting these terms we get from (13) that spontaneous symmetry breaking occurs and the scalar field Φ gets the non-zero vacuum average value

$$m = M \exp\left(-\frac{1}{3} - \frac{\pi k}{18}\right)$$

and we have the dynamical symmetry breaking in the TMGT+Higgs model.

But this is not the end of the story. At large k our analysis is far from being correct, because of the next two reasons:

vacuum average value The value $\langle \Phi \rangle$ is such, that $\ln \frac{M}{\langle \Phi \rangle} = (6 + \pi k)/18$ becomes larger than unity at $k \sim 4$, thus the minimum of the potential lies far outside the expected range of validity of our two-loop approximation.

Mm^2 terms When we found the minimum we neglected the terms of order $\Phi^4 \sim Mm^2$ terms in eq.(13). This can be done only if at $m = \langle m \rangle \sim \langle \Phi \rangle^2$ we have $c M \langle m \rangle^2 \ll |V(\langle m \rangle)| = \langle m \rangle^3/\pi^2 k$. Thus we get the inequality $k \exp(\pi k/18) < O(1)$ which is correct only at small k .

Thus we see that the abovementioned mechanism of the dynamical symmetry breaking works only for small k and does not work for large k . Being unable to calculate this critical value of k at which the dynamical breaking of the gauge symmetry occurs, we suggest that this critical value coincides with that which can be obtained from the BKT phase transition. So we suggest the phase transition in parameter k , describing the anyonic superconducting phase transition, and the critical value is $k_c = 4$.

4 World sheet vortices as the charged CS vortices and the duality in the TMGT+Higgs theory.

We know that in the high temperature phase the relevant world sheet degrees of freedom are the vortices. Now we want to describe them in terms of the relevant degrees of freedom of the TMGT+Higgs model in the spontaneously broken phase. Moreover, we describe here the 2+1 analog of the $R \rightarrow \alpha'/R$ duality, which was suggested in [16].

As was shown in [23] (see also [24] and [25]) the flux tubes (Abrikosov - Nielsen - Olesen vortices), which appear as new relevant degrees of freedom in the broken phase, have nonzero charge $Q = \frac{k}{4\pi} \Phi$, where Φ is the flux. The same relation between the charge and the flux exists in the unbroken phase for charged particles. After the gauge symmetry breaking the total induced flux equals zero and the charged particles are not anyons after that, but now the vortices become anyons, because in the broken phase we have the induced charge. This duality between unbroken and broken phases demonstrates, that the correct 2+1 analog of the world-sheet vortex is nothing but the charged vortex in the TMGT+Higgs model.

It was known [8],[9],[10] that the famous string duality $R \rightarrow \alpha'/R$ (see all references in [16]) can be described as the duality in the XY model between vortices and the usual field fluctuation. Moreover, this duality can be formulated in terms of vertex operators of corresponding fields (it is the interchange between the momentum and the soliton winding numbers, roughly speaking). Because the vertex operator corresponds to the Wilson lines of the charged particles in the unbroken phase of the TMGT, the dual objects must correspond to the world lines of the charged flux tubes. It was shown in the paper [26], that after some assumptions it is possible to formulate the effective theory for these charged vortices in terms of the new scalar field Ψ , and the vortices are the usual non-topological excitations of this field Ψ . It appears that the effective lagrangian for the new field is again the TMGT+Higgs model, but with the new parameters. The connection between the old and the new ones is

$$\begin{aligned} k' &= (Ak)^{-1} \\ \gamma' &= \gamma(Ak^2) \\ \langle \Psi^2 \rangle &= \langle \Phi^2 \rangle (Ak^2)^{-1} \\ A &= 1 + 32\pi^2 \langle \Phi^2 \rangle / \gamma k^2 \end{aligned} \quad (14)$$

and it is easy to see that under this transformation both M_+ and M_- are unchanged, because $\gamma \langle \Phi^2 \rangle = \gamma' \langle \Psi^2 \rangle$ and $\gamma k = \gamma' k'$, thus spectrum (7) remains the same. We see that the deviation of the A factor from 1 is small at large k , thus this is the approximate $k \rightarrow 1/k$ duality. Let us note, that the transformations (14) can be reexpressed as the duality transformation of the complex parameter

$$\begin{aligned} \zeta &= 4\sqrt{2}\pi\Phi / \sqrt{\gamma} + ik \\ \zeta &\rightarrow 1/\zeta^* \end{aligned}$$

which is the Cardy-Rabinovici duality [27],[28].

Besides this duality, the $k \rightarrow 1/k$ duality can also be obtained as some transformation interchanging masses M_+ and M_- . This is the duality between high- and low-energy limits of the TMGT+Higgs model, or to be more precise this is the duality between TMGT and CS+Higgs models [16]. It will be extremely interesting to understand the relationship between both the duality pictures.

5 Discussion and Conclusion

Now let us briefly discuss the results which have been obtained:

- 1 In the OTM picture a string in an external field (or the 2d CFT deformation) can be described as an anyonic system; the anyons density depends both on the external field amplitude and on the anomalous dimension of the corresponding vertex operator (compactification radius in the $U(1)$ case). The field-theoretical (Ginzburg-Landau) description of such system can be expressed in terms of TMGT+Higgs models.
- 2 The Hagedorn transition (or more generally the string behaviour at high temperatures and/or small compactification radii) is the transition between conformal and nonconformal phases, or in OTM terms this is the transition between unbroken and broken gauge symmetry in the anyonic system, i.e. it is the superconducting phase transition. The critical value of the CS parameter $k_c = 4$ is the same as was suggested by Kalmeyer and Laughlin for semions (half-fermions).
- 3 There is dynamical gauge symmetry breaking (Coleman-Weinberg effective potential) in the toy TMGT+Higgs model, where the bare scalar interactions were neglected, at small k . The critical value k_c is not known, but we make a suggestion that $k_c = 4$.
- 4 The world sheet vortices can be described as the 2+1 Abrikosov-Nielsen-Olesen flux tubes (charged CS vortices). The duality between charged vortices in the broken phase and the usual anyons in the unbroken phase corresponds to the 2d duality in the XY model, and thus to the $R \rightarrow \alpha'/R$ duality.

Finally, let us mention some problems which are necessary to solve in future. First of all, it will be amusing to get $k_c = 4$ from the analysis of the effective potential itself. It is necessary to take into account higher corrections, higgs selfinteractions and to use renormalization group arguments. The second problem is to understand the connection between 2d and 2+1 dualities in more detail, in particular to understand two 2+1 dualities (see the end of the section 4). And the third one is to understand what thermodynamical ensemble we get for the OTM in the Higgs phase instead of the usual string one. These and related questions will be considered in future publications.

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Fig. 1

Fig. 2

Fig. 3