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TOTAL CROSS SECTION FOR ANOMALOUS FERMION-NUMBER VIOLATION VIA DISPERSION RELATION

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ABSTRACT

We investigate the behaviour of the total cross section for electroweak baryon and lepton number violation (in association with multi-W and multi-Higgs production) at low energies ($\sqrt{s} < \mathcal{O}(10)$ TeV) by means of a dispersion relation. The imaginary part of the relevant forward elastic scattering amplitude is calculated in a saddle point approximation, systematically introduced by the so-called valley method. Taking into account the first correction to the basic instanton result (and neglecting the higher order corrections) we find that the total cross section does not get exponentially large. The question if the cross section becomes of observable size remains still open, however.

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1. Introduction

It has recently become clear that nonperturbative electroweak baryon and lepton number (B+L) violating processes [1] which are associated with the nontrivial θ -vacuum structure [2] of the nonabelian SU(2) gauge theory, while being exponentially suppressed by $\exp(-4\pi/\alpha_w)$ at low energies [1], may be unsuppressed at high temperatures [3–5] or high energies $E > M_{sp} \simeq \pi m_w/\alpha_w = \mathcal{O}(10)$ TeV [6–9]. The subscript “sp” refers to the “sphaleron”, a classical static saddle point solution of the electroweak theory corresponding to the energy barrier between topologically nontrivial vacua [10]. The sphaleron energy, M_{sp} , is the natural scale for processes connected to the θ -vacuum structure.

It was shown by one of the authors (A.R.) [7] (and later in ref. [8]) that instantons [11, 1, 12] induce S -matrix elements for (B+L)-violating processes like

$$q + q \rightarrow (3n_g - 2)\bar{q} + n_g\bar{l} + n_h H + n_w W, \quad (1.1)$$

(n_g is the number of generations, H denotes the neutral Higgs boson, and W stands generically for $W^\pm$ and Z^0 bosons) which are essentially concentrated in the s -wave and have, in zeroth order of the semiclassical approximation, *no form factors* on the mass shell (see also [13]). This leads to the fact that the cross sections for the exclusive processes (1.1) grow rapidly with the cm energy $\sqrt{s}$ of the colliding partons, leading to a violation of partial wave unitarity at high energies. Summing the exclusive cross sections over the Higgses and W 's yields an exponentially rising total cross section for (B+L)-violation [9] which is dominated by multi- W production [14–16].

After an inspection of the higher order corrections to the basic instanton result [15, 17–19] it was argued that the inclusive cross section for the processes (1.1) has the generic form, for $\alpha_w \rightarrow 0$, $\sqrt{s}/M_{sp}$ fixed,

$$\sigma_{\Delta(B+L)} \sim \exp \left[-\frac{4\pi}{\alpha_w} F \left(\frac{\sqrt{s}}{M_{sp}} \right) \{1 + \mathcal{O}(\alpha_w)\} \right]. \quad (1.2)$$

For energies much less than the sphaleron energy the function F may be computed in perturbation theory, which is equivalent to a low energy expansion. At low energies one finds [14, 15], for $\sqrt{s}/M_{sp} \ll 1$,

$$F \simeq 1 - \left(\frac{9}{16\sqrt{8}} \right)^{2/3} \left(\frac{\sqrt{s}}{M_{sp}} \right)^{4/3}. \quad (1.3)$$

Here we have defined *

$$M_{sp} = \pi \frac{m_w}{\alpha_w} = 7.2 \text{ TeV}. \quad (1.4)$$

Unfortunately, the function F is not known beyond the zeroth order contribution. Only if F has a zero is there a chance that electroweak (B+L)-violation in association with multi- W production becomes relevant at future colliders. It was argued [15, 17–19] that at $\sqrt{s} \simeq M_{sp}$ all tree graphs in the background of the instanton contribute substantially to F , indicating a breakdown of perturbation theory around the instanton at this energy. This may signal that it is inappropriate to expand around the instanton and that a better zeroth order approximation has to be found (the so-called *distorted* instanton [9, 22]).

An alternative approach to this problem is to consider dispersion relations [14, 16], i.e. to calculate the total cross section for (B+L)-violation via the imaginary part of the forward elastic scattering amplitude. The main reason why dispersion relations are particularly convenient for our purpose is that phase space and quantum corrections due to W 's and H 's are automatically taken into account. Also, unitarity is manifestly built in.

In this paper we will elaborate the dispersion relation approach by means of the valley method [20, 21]. The distorted instanton [9, 22] will appear in this approach automatically due to instanton/anti-instanton ($I\bar{I}$) interactions. By means of the saddle point approximation, systematically incorporated by the valley method, we will explicitly demonstrate the validity of the “perturbation” expansion of F , eq. (1.3), calculate the next-to-leading order term in eq. (1.3) and find the form of all the omitted corrections (see eq. (3.17)). Then the leading contribution to the pre-exponent of the fermion-number violating cross section will be calculated in a toy electroweak model.

We will consider a model with the Euclidean action

$$S_E = \int d^4x \left\{ \frac{1}{2} \text{tr}(F_{\mu\nu} F_{\mu\nu}) + |D_\mu \Phi|^2 + \lambda(|\Phi|^2 - \frac{v^2}{2})^2 + \sum_{k=1}^2 \Psi_k^\dagger (i\sigma_\mu D_\mu) \Psi_k \right\}, \quad (1.5)$$

where $D_\mu = \partial_\mu - igW_\mu$, $W_\mu = W_\mu^a \frac{\sigma_a}{2}$, $\sigma_\mu^{\alpha\dot{\alpha}} = (i, \vec{\sigma})$. This model corresponds to the electroweak theory with two massless fermion flavours in the limit of vanishing Weinberg

* The mass (1.4) is of the order but not identical to the sphaleron mass [10] which depends slightly on the Higgs mass and varies between 7 and 14 TeV if one varies the Higgs mass between zero and infinity.

angle, $\theta_w = 0$. The masses of the W boson and the Higgs boson are given by $m_w = \frac{1}{2}gv$ and $m_h = \sqrt{2}\lambda v$, respectively. The relevant fermion-number violating processes are

$$f_1 + f_2 \rightarrow n_w W + n_h H, \quad (1.6)$$

where f_i denote fermions.

The dispersion relation for the total cross section for fermion-number violating processes (sum over all W's and H's in eq. (1.6)) reads

$$\sigma_{\Delta F} = \frac{1}{4s} \cdot \text{Im} \mathcal{T}(p_1, p_2, p_1, p_2) \quad (1.7)$$

$$= \frac{1}{4s} \cdot \text{Im} \left\{ (\text{LSZ}) \left(\langle T[\Psi_1(x_1)\Psi_2(x_2)\mathcal{P}\Psi_1^\dagger(x_3)\Psi_2^\dagger(x_4)] \rangle \right) \right\}$$

Here $\mathcal{T}$ denotes the elastic scattering amplitude and "LSZ" stands for the usual LSZ procedure. The factor $1/4$ in eq. (1.7) is due to averaging over *isospins*. We should, by means of a projection operator $\mathcal{P}$, select only those intermediate states which contain no fermions (see eq. (1.6)) [16].

Generally speaking what we need to do is to: 1) evaluate the Euclidean functional integral for the 4-point Green's function

$$G_E = N^{-1} \int DW D\Phi \prod_{k=1}^2 [\mathcal{D}\Psi_k \mathcal{D}\Psi_k^\dagger] \Psi_1 \Psi_2 \mathcal{P} \Psi_1^\dagger \Psi_2^\dagger \exp(-S_E[W, \Phi, \Psi]) \quad (1.8)$$

in the zero topological charge sector; 2) Fourier transform it; 3) amputate the external legs, multiply by the external wave functions and perform the summation over isospins; 4) continue to Minkowski space; 5) go on mass-shell; 6) take the imaginary part.

Unfortunately, already point 1) is too complicated! What we will really do instead is the following. In sect. 2 we will recall the valley method [20, 21] for finding the dominant contribution to the Euclidean Green's function (1.8). The dominant configuration, at low energies ($\sqrt{s} \ll M_{sp}$), is an instanton/anti-instanton *valley* in functional space. As the final result of sect. 2 we will represent G_E , eq. (1.8), in terms of an integral over the collective coordinates of the valley, eq. (2.22). In sect. 3 we will continue the Fourier transform of G_E , eq. (2.22), to Minkowski space. The dominant configuration in Minkowski space will be a *saddle point* - one point of the valley bottom - with one negative mode, which

will give rise to the imaginary part of the forward scattering amplitude. We will evaluate $\sigma_{\Delta F}$ in the saddle point approximation, eq. (3.30). Our conclusions and discussions are contained in sect. 4.

2. The Valley Approach

We are going to evaluate the functional integral (1.8) by expanding around the dominant contribution in the *zero* topological charge sector. It is clear that the expansion around the perturbative vacuum will give no contribution to the fermion-number violating process. Therefore we should expand (1.8) around nontrivial fields. In the model (1.5) there exist approximate classical solutions (constrained instantons) [1, 12] in the *unit* topological charge sector which are given by

$$W_\mu^I(x - x_I; \rho_I, U_I) = \begin{cases} \frac{i}{g} \frac{\rho_I^2}{(x - x_I)^2} \frac{U_I [\sigma_\mu \vec{\sigma}_\nu - \delta_{\mu\nu}] U_I^\dagger (x - x_I)_\nu}{(x - x_I)^2 + \rho_I^2}, & \text{for } |x - x_I| < m_w^{-1}; \\ -\frac{2i\pi^2 \rho_I^2}{g} U_I [\sigma_\mu \vec{\sigma}_\nu - \delta_{\mu\nu}] U_I^\dagger G_{m_w}(x - x_I), & \text{for } |x - x_I| \gg m_w^{-1}, \end{cases} \quad (2.1)$$

$$\Phi^I(x - x_I; \rho_I) = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{cases} \left(\frac{(x - x_I)^2}{[1 - 2\pi^2 \rho_I^2 G_{m_h}(x - x_I)]} \right)^{1/2}, & \text{for } |x - x_I| < m_h^{-1}; \\ [1 - 2\pi^2 \rho_I^2 G_{m_h}(x - x_I)], & \text{for } |x - x_I| \gg m_h^{-1}. \end{cases} \quad (2.2)$$

Here $\vec{\sigma}_{aa} = (-i, \vec{\sigma})$ and $G_m(x) = 1/(4\pi^2)(m/r)K_1(mr)$. The $SU(2)$ matrix U_I gives the instanton orientation in isospin space. The anti-instanton solution is identical to (2.1)-(2.2) with the substitution $\sigma_\mu \vec{\sigma}_\nu \rightarrow \bar{\sigma}_\mu \sigma_\nu$ and index $I \rightarrow \bar{I}$.

Because of the qualitatively different behaviour of the constrained instanton for small and large distances, one is supposed to consider two different cases, namely when the distance $R = x_I - x_{\bar{I}}$ is much smaller (larger) than m^{-1} . Actually we shall deal with the first situation and we can check its validity afterwards*.

There is exactly one fermion zero mode of the Euclidean Dirac equation in the background of the constrained (anti-)instanton [1]. It is given by

$$\psi_{I\alpha}^k(x - x_I; \rho_I, U_I, \theta_k) = \frac{\rho_I}{\pi} \frac{(x - x_I)_\mu}{[(x - x_I)^2]^{1/2} [(x - x_I)^2 + \rho_I^2]^{3/2}} [\bar{\sigma}_\mu U_I^\dagger \epsilon]_\alpha^k \theta_k, \quad (2.3a)$$

* The introduction of a constraint [12], necessary in order to get a nontrivial solution of the classical equations of motion, can be avoided if one extends the valley approach, described below, to the single-instanton sector. In this case one can work from the beginning with the small distance fields shown in eqs. (2.1),(2.2). [21, 23].

where $\alpha = 1, 2$ and $\alpha = 1, 2$ are spin and weak isospin indices, respectively, and the θ_k are Grassmann collective coordinates. It should be remembered that the two-component spinors Ψ and $\Psi^\dagger$ are independent field variables; we have

$$\psi_I^{\dagger\dagger} = 0. \quad (2.3b)$$

The fermion zero mode in the anti-instanton background reads

$$\psi_I^\dagger = 0, \quad (2.3c)$$

$$\psi_{I\alpha}^{\dagger\dagger} (x - x_I, \rho_I, U_I, \theta_k^\dagger) = \frac{\rho_I}{\pi} \frac{(x - x_I)_\mu}{[(x - x_I)^2]^{1/2} [(x - x_I)^2 + \rho_I^2]^{3/2}} [\epsilon U_I \bar{\sigma}_\mu]_{\alpha\alpha} \theta_k^\dagger. \quad (2.3d)$$

The approximate instanton solution ϕ_I , (2.1)-(2.3), is characterized by a set of collective coordinates $\tau_I = \{\rho_I, x_I, U_I, \theta_1, \theta_2\}$ which correspond to zero and approximate zero modes. Due to the existence of the approximate solution of the Euclidean equations of motion there is a *valley* in the zero topological charge sector of the functional space [20, 21]. It means that there are some almost *flat* directions in the functional space corresponding to *quasizero* modes of the operator $\Delta(\phi) = \delta^2 S_E[\phi]/\delta\phi^2$.

Let us now consider a pair of such (instanton and anti-instanton) solutions, $\phi_I(x - x_I)$ and $\phi_{\bar{I}}(x - x_{\bar{I}})$, at large separations $R_\mu = x_{I\mu} - x_{\bar{I}\mu}$. This pair is not, generally speaking, a new solution of the equations of motion; it represents a valley configuration. Indeed, if solutions ϕ_I and $\phi_{\bar{I}}$ vanish sufficiently rapidly at large x , their interaction at large R is small and converts some of the zero modes of each solution ϕ_I and $\phi_{\bar{I}}$ into quasizero ones. The corresponding eigenvalues are parametrically small at large R .

In order to integrate over this segment of the functional space the following steps should be performed [20]:

- (1) Find the valley trajectory (streamline) $\phi_r = \phi(x; \tau_I, \tau_{\bar{I}})$ ($\phi_r = \{W_{\mu r}, \Phi_r, \Psi_{kr}, \Psi_{kr}^\dagger\}$) as a function of a double set of collective coordinates. It must satisfy the boundary condition*

$$\phi(x; \tau_I, \tau_{\bar{I}}) |_{R \rightarrow \infty} = \phi_I(x; \tau_I) + \phi_{\bar{I}}(x; \tau_{\bar{I}}). \quad (2.4)$$

* The boundary condition (2.1) defines the simplest case, namely the single-instanton/single-anti-instanton valley. Multi-instanton/multi-anti-instanton valleys will give a much more suppressed contribution at low energies.

Note that (2.4) is only the boundary condition and the real valley trajectory need not coincide with (2.4) for arbitrary R .

- (2) Integrate over non-quasizero modes near the valley trajectory $\phi(x; \tau_I, \tau_{\bar{I}})$. These are integrations in the directions orthogonal to the valley bottom. Since the walls of the valley are steep (non-quasizero modes $\gg 1$) these integrations are almost Gaussian.
- (3) Perform the final integrations along the bottom of the valley, i.e. the integrations on the set of collective coordinates $\tau = \{\tau_I, \tau_{\bar{I}}\}$.

To find the valley trajectory ϕ_r , note that $\phi_r = \phi(x; \tau)$ for fixed τ is a minimal point of $S_E[\phi]$ within the subset of functions ϕ which satisfy

$$\left\langle (\phi - \phi_r), \frac{\partial \phi_r}{\partial \tau} \right\rangle_w = 0, \quad (2.5)$$

where the scalar product reads $\langle f, g \rangle_w = \int d^4x g_r(x) w(x; \tau) f_r(x)$. In eq. (2.5), w means some weight function.

The action minimization gives the following equation for the valley trajectory ϕ_r [21]

$$\epsilon^2(\tau) w(x; \tau) \frac{\partial \phi_r}{\partial \tau} = \frac{\delta S_E[\phi]}{\delta \phi} \Big|_{\phi=\phi_r}, \quad (2.6)$$

where $\epsilon^2(\tau)$ are Lagrange multipliers.

By means of the Faddeev-Popov technique we can express the Green's function (1.8) in terms of collective coordinates. In the Gaussian approximation we have

$$G_E = N^{-1} \int \prod_i d\tau_i^\dagger \left| \det_{ij} \left(\frac{\partial \phi_r^\dagger}{\partial \tau_i^\dagger}, \frac{\partial \phi_r^\dagger}{\partial \tau_j^\dagger} \right) \right| \int \mathcal{D}\phi^\dagger \prod_i \delta \left((\phi^\dagger - \phi_r^\dagger), \frac{\partial \phi_r^\dagger}{\partial \tau_i^\dagger} \right) \quad (2.7)$$

$$\times \exp \left[-S_E[\phi_r^\dagger] - \frac{1}{2} \langle (\phi^\dagger - \phi_r^\dagger), \Delta(\phi_r^\dagger)(\phi^\dagger - \phi_r^\dagger) \rangle \right] \cdot G_f,$$

where

$$\begin{aligned} G_f = & \int \prod_{k=1}^2 d\theta_k d\theta_k^\dagger \mathcal{D}\xi_k \mathcal{D}\xi_k^\dagger \cdot \delta \left(\frac{\partial \Psi_{kr}^\dagger}{\partial \theta_k^\dagger}, \xi_k \right) \cdot \delta \left(\xi_k^\dagger, \frac{\partial \Psi_{kr}}{\partial \theta_k} \right) \\ & \times (\Psi_{1r} + \xi_1)(\Psi_{2r} + \xi_2) \mathcal{P}(\Psi_{1r}^\dagger + \xi_1^\dagger)(\Psi_{2r}^\dagger + \xi_2^\dagger) \\ & \times \exp \left[- \int d^4x \sum_{k=1}^2 (\Psi_{kr}^\dagger + \xi_k^\dagger)(i\sigma_\mu D_\mu^{(r)})(\Psi_{kr} + \xi_k) \right]. \end{aligned} \quad (2.8)$$

In eq. (2.7) the superscript "b" refers to the bosonic part of the variables (W and H). The bosonic collective coordinates are $r^b = \{\rho_I, \rho_T, x_I, x_T, U_I, U_T\}$. In the fermionic contribution to the Green's function, G_f (2.8), the new field variables $\xi_k = \psi_k - \bar{\psi}_k$, $\xi_k^\dagger = \bar{\psi}_k^\dagger - \psi_k^\dagger$ stand for the deviation from the fermionic part of the valley bottom. The symbol $D_\mu^{(\tau)}$ means the covariant derivative in the valley trajectory background. Note that there are no linear terms in the exponents due to the presence of the δ -functions, provided ϕ_r satisfies the valley equation (2.6).

Evaluating the Gaussian integral one has for (2.7), (2.8)

$$G_E = \int d\mu F(\phi_r) \exp(-SE[\phi_r]) [1 + \mathcal{O}(\epsilon^2)], \quad (2.9)$$

where F is the appropriate polynomial in the fields. The measure is given by

$$d\mu = N^{-1} \prod_{i=1}^{2p} d\tau_i \det_{ij}^{1/2} \left(\frac{\partial \phi_r}{\partial \tau_i}, \frac{\partial \phi_r}{\partial \tau_j} \right) \left[\det^{(2p)} \Delta(\phi_r) \right]^{1/2}, \quad (2.10)$$

where $\det^{(2p)} \Delta(\phi_r)$ means the determinant with the double set of lowest modes excluded. Note that for the sake of simplicity we wrote eqs. (2.9), (2.10) for the pure bosonic case.

Now let us discuss the following question [21]: how does one make the proper choice of the solution of (2.6) (the pair of the functions ϕ_r and w) to represent the valley trajectory? It is clear that (2.6) has an infinite number of solutions satisfying the boundary condition (2.4). The different choices of the functions ϕ_r and w correspond to a redistribution of classical ($SE[\phi_r]$) and quantum (determinant) effects. The physics, of course, would be the same. If ϕ_r is chosen close to the valley bottom, then the linear term $\delta SE/\delta \phi_r$ is parametrically small ($\epsilon^2(\tau) \ll 1$ at large R in (2.6))* . Then the $\bar{I}\bar{I}$ interaction is also small and (2.10) factorizes into a product of two instanton measures

$$d\mu|_{R \rightarrow \infty} = d\mu_1 \cdot d\mu_2. \quad (2.11)$$

Due to this factorization it is easy to take into account quantum fluctuations near the trajectory ϕ_r . Thus the strategy is to minimize the linear term in eq. (2.6) in order to use eqs. (2.9)-(2.11) for the calculation of quantum effects.

* For the gauge boson valley equation, $\epsilon_w^2 \sim \rho_I^2 \rho_T^2 / R^4$, for the Higgs boson equation, $\epsilon_h^2 \sim \lambda v^2 \rho_I^2 \rho_T^2 / R^2$.

In the *leading order approximation* (this means for very large R) the valley trajectory ϕ_r will be simply

$$W_r = W^I + W^{\bar{I}}, \quad (2.12)$$

for the gauge boson part,

$$\Phi_r = \Phi^I + \Phi^{\bar{I}}, \quad (2.13)$$

for the Higgs boson part, and

$$\begin{aligned} \Psi_r &= \psi_I + \psi_{\bar{I}} = \psi_I, \\ \psi_r^\dagger &= \psi_I^\dagger + \psi_{\bar{I}}^\dagger = \psi_I^\dagger, \end{aligned} \quad (2.14)$$

for the fermion parts. Note, that the action of the fermion part, $S_f^{(\tau)}$, of the valley trajectory will not appear in our formulae due to the integration over θ 's, as will be explained below.

The proper solution of the gauge boson valley equation which minimizes the linear term (see eq. (2.6)) was found in ref. [21] in the *next-to-leading order* (in terms of ρ/R) approximation*. The action of the valley trajectory ($\bar{I}\bar{I}$ configuration) reads [21] (see also ref. [24])

$$S^{(\tau)} = S_w^{(\tau)} + S_h^{(\tau)} + S_f^{(\tau)}, \quad (2.15)$$

where

$$\begin{aligned} S_w^{(\tau)} &= \frac{16\pi^2}{g^2} - \frac{32\pi^2}{g^2} \left\{ 4 \frac{(v \cdot R)^2}{R^2} - 1 \right\} \frac{\rho_I^2 \rho_T^2}{R^4} \\ &\quad + \frac{64\pi^2}{g^2} \left\{ 4 \frac{(v \cdot R)^2}{R^2} - 1 \right\} \frac{(\rho_I^2 + \rho_T^2) \rho_I^2 \rho_T^2}{R^6} + \mathcal{O} \left(\frac{1}{g^2} \frac{\rho_I^4 \rho_T^4}{R^8} \right), \end{aligned} \quad (2.16)$$

$$S_h^{(\tau)} = \pi^2 v^2 (\rho_I^2 + \rho_T^2) + \mathcal{O}(\lambda v^4 (\rho_I^4 + \rho_T^4)) + 2\pi^2 v^2 \frac{\rho_I^2 \rho_T^2}{R^2} \left\{ 2 \frac{(v \cdot R)^2}{R^2} - 1 \right\} + \mathcal{O} \left(v^2 \frac{\rho_I^3 \rho_T^3}{R^6} \right), \quad (2.17)$$

$$S_f^{(\tau)} = \int d^4x \sum_{k=1}^{4n_f} \psi_I^\dagger (i\sigma_\mu D_\mu^{\bar{I}\bar{I}}) \psi_I^\dagger \left[1 + \mathcal{O} \left(\frac{\rho}{R} \right) \right]. \quad (2.18)$$

All explicitly indicated corrections in eqs. (2.16), (2.17) will give contributions of the same order to the exponent of the total cross section, see eq. (3.17) below. In eqs. (2.16), (2.17)

* It is possible to consider it as a sum of distorted instanton and distorted anti-instanton. This can be shown by using eq. (3.21) in ref. [21].

a unit four vector v_μ was introduced corresponding to the relative orientation of instanton and anti-instanton,

$$v_\mu \bar{\sigma}_\mu \equiv U_I^\dagger U_I. \quad (2.19)$$

Note that we have, in the Gaussian approximation,

$$(\Psi_{1r} + \xi_1)(\Psi_{2r} + \xi_2) \mathcal{P}(\Psi_{1r}^\dagger + \xi_1^\dagger)(\Psi_{1r}^\dagger + \xi_2^\dagger) \simeq \Psi_{1r} \cdot \Psi_{2r} \cdot \Psi_{1r}^\dagger \cdot \Psi_{1r}^\dagger, \quad (2.20)$$

because the ξ 's are small fluctuations near the valley trajectory. The projector $\mathcal{P}$ is absent on the rhs of this formula. To calculate the leading order contribution to the pre-exponent in eq. (1.2) we will substitute eq. (2.14) into the rhs of eq. (2.20).

Due to the integrations over θ_1 and θ_2 only the following part of the exponent in eq. (2.8) will give a nontrivial contribution

$$- \int d^4x \sum_{k=1}^2 \xi_k^\dagger (i\sigma_\mu D_I^\dagger) \xi_k. \quad (2.21)$$

After integration over the fluctuations ξ_k this will lead to a product of fermion determinants in eq. (2.8). Thus our final formula for the Green's function is

$$G_E(x_1, x_2, x_3, x_4) = \int d\rho_I d\rho_I^\dagger d^4x_I d^4x_I^\dagger dU_I dU_I^\dagger n(\rho_I) n(\rho_I^\dagger) \times \psi_I^\dagger(x_1 - x_I) \psi_I^2(x_2 - x_I) \psi_I^{\dagger 1}(x_3 - x_I) \psi_I^{\dagger 2}(x_4 - x_I) \exp(-S_b^{(\dagger)}). \quad (2.22)$$

Here, $S_b^{(\dagger)} = S_w^{(\dagger)} + S_h^{(\dagger)}$, and $n(\rho_{I(\bar{I})})$ stands for the instanton (anti-instanton) density [1]

$$n(\rho) = \frac{1}{4\pi^2} \left(\frac{8\pi^2}{g^2} \right)^4 \exp[-\alpha(1) + (n_f - 1)\alpha(1/2)] \frac{(\mu\rho)^{\frac{43}{3} - \frac{7}{2}}}{\rho^{5 - \frac{7}{2}}}, \quad (2.23a)$$

$$\alpha(1) = 0.443307, \quad \alpha(1/2) = 0.145873. \quad (2.23b)$$

The number of fermion flavours, n_f , is equal to two in our model. In eq. (2.16) g^2 should be replaced by the renormalization point dependent quantity $g^2(\mu)$.

3. The Total Cross Section

The next step in order to compute the total cross section for fermion-number violation is to continue analytically the Fourier transform of the 4-point Green's function to Minkowski space. The Fourier transform of eq. (2.22) reads

$$(2\pi)^4 \delta^{(4)}(0) \tilde{G}_{E \alpha\alpha\beta\beta}(p_1, p_2, p_1, p_2) = \int d\rho_I d\rho_I^\dagger d^4x_I d^4x_I^\dagger dU_I dU_I^\dagger n(\rho_I) n(\rho_I^\dagger)$$

$$\times \psi_{I\alpha}^{\dagger a}(p_1) \psi_{I\alpha}^{\dagger b}(p_1) \bar{\psi}_{I\beta}^{2\dagger}(p_2) \bar{\psi}_{I\beta}^{2\dagger}(p_2) \exp[iR \cdot (p_1 + p_2) - S_b^{(\dagger)}]. \quad (3.1)$$

Here we have already specialized to the relevant part for the forward elastic scattering amplitude. Using the explicit expressions for the fermion zero modes and the fact that the integrand only depends on the relative orientation v we obtain for (3.1)

$$(2\pi)^4 \delta^{(4)}(0) \tilde{G}_{E \alpha\alpha\beta\beta}(p_1, p_2, p_1, p_2) = \int d\rho_I d\rho_I^\dagger d^4x_I d^4x_I^\dagger \int \frac{dS^3(v)}{2\pi^2} n(\rho_I) n(\rho_I^\dagger) \times h_{\alpha\alpha}^\lambda h_{\beta\beta}^\lambda v_\lambda v_\nu \exp\left[iR \cdot (p_1 + p_2) - S_b^{(\dagger)}\left(\frac{(v \cdot R)^2}{R^2}, R^2, \rho_I, \rho_I^\dagger\right)\right], \quad (3.2)$$

where

$$h_{\gamma\gamma}^\mu = f(p, \rho_I) f(p, \rho_I^\dagger) \frac{1}{p^\lambda} \left[\bar{\sigma}_\lambda p_\lambda \sigma_\mu \bar{\sigma}_\nu p_\nu \right]_{\gamma\gamma}, \quad (3.3a)$$

$$f(p, \rho) = 4\pi\rho \int_0^\infty dr J_2(pr) \frac{r^2}{(r^2 + \rho^2)^{3/2}}. \quad (3.3b)$$

For the integral over the relative orientation we need the the following expression,

$$\int \frac{dS^3(v)}{2\pi^2} v_\lambda v_\nu F\left(\frac{(v \cdot R)^2}{R^2}\right) = \frac{1}{2\pi} \delta_{\lambda\nu} \int_0^\pi d\gamma \sin^2 \gamma F(\cos^2 \gamma), \quad (3.4)$$

where γ is the angle between the four vectors R and v . With the help of (3.4) we can express eq. (3.2) as

$$\tilde{G}_{E \alpha\alpha\beta\beta}(p_1, p_2, p_1, p_2) = \frac{1}{2\pi} \int d\rho_I d\rho_I^\dagger d^4R \int_0^\pi d\gamma \sin^2 \gamma n(\rho_I) n(\rho_I^\dagger) \times h_{\alpha\alpha}^\lambda h_{\beta\beta}^\lambda \exp\left[iR \cdot (p_1 + p_2) - S_b^{(\dagger)}(\cos^2 \gamma, R^2, \rho_I, \rho_I^\dagger)\right]. \quad (3.5)$$

Performing the integration over angular variables one finds that $\tilde{G}_E$ reduces to

$$\tilde{G}_{E \alpha\alpha\beta\beta}(p_1, p_2, p_1, p_2) = \frac{2\pi}{E} \int d\rho_I d\rho_I^\dagger dR R^2 J_1(ER) \int_0^\pi d\gamma \sin^2 \gamma n(\rho_I) n(\rho_I^\dagger) \times h_{\alpha\alpha}^\lambda h_{\beta\beta}^\lambda \exp\left[-S_b^{(\dagger)}(\cos^2 \gamma, R^2, \rho_I, \rho_I^\dagger)\right], \quad (3.6)$$

where $E = |p_1 + p_2|$.

In order to come to the forward elastic scattering (FES) amplitude $\mathcal{T}$ we should continue eq. (3.6) to Minkowski space, go on the mass-shell, amputate the external legs,

multiply by the external wave functions, and sum over isospins. Performing the continuation of eq. (3.6) to Minkowski space momenta, we make the substitution $E \rightarrow iE$ in the exponent. After that procedure, E corresponds to the cm energy of the colliding fermions, $E = \sqrt{s}$. Taking into account the asymptotic behaviour of the Bessel function I_1 one has for the region $ER \gg 1^*$ the following general expression for the FES amplitude

$$T(p_1, p_2, p_1, p_2) = 4(p_1 \cdot p_2) \frac{(2\pi)^{9/2}}{E^{3/2}} \int d\rho_I d\varphi_I dR \cdot R^{3/2} \int_0^\pi d\gamma \sin^2 \gamma \times n(\rho_I) n(\rho_I) \rho_I^2 \rho_I^2 \exp \left[ER - S_b^{(\gamma)}(\cos^2 \gamma, R^2, \rho_I, \rho_I) \right]. \quad (3.7)$$

Due to the factor ER in the exponent in eq. (3.7) we can now make a saddle point approximation for the integral over collective coordinates**. The exponent in eq. (3.7) reads

$$\Gamma = ER - \frac{16\pi^2}{g^2} + \frac{32\pi^2}{g^2} [4 \cos^2 \gamma - 1] \frac{\rho_I^2 \rho_I^2}{R^4} - \frac{64\pi^2}{g^2} [4 \cos^2 \gamma - 1] \frac{(\rho_I^2 + \rho_I^2) \rho_I^2 \rho_I^2}{R^6} + \mathcal{O} \left(\frac{1}{g^2} \frac{\rho_I^4}{R^8} \right) - \pi^2 v^2 (\rho_I^2 + \rho_I^2) + \mathcal{O}(\lambda v^4 (\rho_I^4 + \rho_I^4)) - 2\pi^2 v^2 \frac{\rho_I^2 \rho_I^2}{R^2} [2 \cos^2 \gamma - 1] + \mathcal{O} \left(v^2 \frac{\rho_I^3 \rho_I^3}{R^6} \right). \quad (3.8)$$

In the saddle point approximation we have $\cos^2 \gamma = 1$, which corresponds to the maximal attraction between instanton and anti-instanton, and $\rho_I = \rho_I = \rho$, due to symmetry.

Defining

$$F = -\frac{g^2}{16\pi^2} \Gamma, \quad (3.9)$$

$$c = \frac{g^2}{16\pi^2} \frac{E}{m_w}, \quad (3.10)$$

$$\chi = \frac{\rho}{R}, \quad \xi = m_w \rho, \quad (3.11)$$

we can write F as

$$F = 1 - c \frac{\xi}{\chi} + \frac{\xi^2}{2} (1 + \chi^2) - 6\chi^4 (1 - 4\chi^2) + \mathcal{O} \left(\frac{\lambda}{g^2} \xi^4 \right) + \mathcal{O}(\chi^8) + \mathcal{O}(\xi^2 \chi^4). \quad (3.12)$$

* This corresponds to $E \gg m_w$, see eq. (3.26b).

** Note that before the analytic continuation to Minkowski space the integral in eq. (3.6) is dominated by the whole valley trajectory.

Solving the saddle point equations in the next-to-leading order approximation we obtain

$$\chi_*^2 = \frac{c^{2/3}}{24^{1/3}} + 2 \frac{c^{4/3}}{24^{2/3}} + \dots, \quad (3.13)$$

$$\xi_*^2 = 24^{1/3} c^{4/3} - 4c^2 + \dots, \quad (3.14)$$

$$F_* = 1 - 24^{1/3} \frac{3}{4} c^{4/3} + \frac{3}{2} c^2 + \mathcal{O} \left(\frac{\lambda}{g^2} 24^{2/3} c^{8/3} \right) + \mathcal{O} \left(\frac{c^{8/3}}{24^{4/3}} \right) + \mathcal{O} \left(\frac{c^{8/3}}{24^{1/3}} \right). \quad (3.15)$$

The dots in eqs. (3.13)-(3.14) stand for terms which will contribute only to the higher order corrections in F_* (see the last three terms in eq. (3.15)).

In order to discuss the physical meaning of eq. (3.15) let us express F_* in terms of M_{sp} , (1.4). Thus instead of eq. (3.10) one has

$$c = \frac{1}{4} \frac{E}{M_{sp}} \quad (3.16)$$

Taking into account eq. (3.16) one has for eq. (3.15) the following expression

$$F_* = 1 - \left(\frac{9}{16\sqrt{8}} \right)^{2/3} \left(\frac{E}{M_{sp}} \right)^{4/3} + \frac{3}{32} \left(\frac{E}{M_{sp}} \right)^2 + \mathcal{O} \left(\frac{\lambda}{g^2} \left(\frac{3}{32} \right)^{2/3} \left(\frac{E}{M_{sp}} \right)^{8/3} \right) + \mathcal{O} \left(\frac{1}{24 \cdot 2^6 \cdot 6^{1/3}} \left(\frac{E}{M_{sp}} \right)^{8/3} \right) + \mathcal{O} \left(\frac{1}{2^6 \cdot 6^{1/3}} \left(\frac{E}{M_{sp}} \right)^{8/3} \right). \quad (3.17)$$

The first term in eq. (3.17) is the well known 't Hooft factor [1]. The second term, which is the leading order term, is negative and coincides with previous results [14, 15]. It also coincides with ref. [16], up to some misprints in notations. The second term comes from W boson production. The third term is the next-to-leading order term, it has a positive sign and it comes from W and Higgs production. This term is one of our main results. Note that one cannot separate in this term the W and Higgs contribution as was done in ref. [16]. This term is not simply the sum of the pure Higgs and pure vector boson (- in the next-to-leading order -) inclusive contributions. The W 's contribute to this term because of the instanton/anti-instanton distortion.

Note that we can control the order of the corrections (see the last three terms in eq. (3.17)).

It is worthwhile to make here the following comment. We consider in this paper a toy model with only two fermion flavours. However, the exponent of the fermion-number

violating total cross section will be the same for the real case of twelve fermions and is given by eq. (3.17). This is due to the fact that the fermion contribution appears only in the pre-exponent.

In the rest of this section we evaluate the leading order contribution to the pre-exponent of $\sigma_{\Delta F}$. Let us expand the exponent in eq. (3.7) near the saddle point and integrate over Gaussian fluctuations. This expansion reads

$$\Gamma(\tau) \simeq \Gamma(\tau_*) + \frac{1}{2} \sum_{i,j} \frac{\partial^2 \Gamma}{\partial \tau_i \partial \tau_j} \Big|_{\tau_*} \delta \tau_i \delta \tau_j, \quad (3.18)$$

where τ stands for the set of collective coordinates, τ_* means the saddle point, and $\delta \tau$ gives the fluctuations near the saddle point. To perform the Gaussian integration we need to evaluate the second term in eq. (3.18) in the leading order approximation, because it will appear in the pre-exponent only. In the leading order approximation one has

$$\Gamma = -\frac{16\pi^2}{g^2} + ER - \frac{4\pi^2}{g^2} m_w^2 (\rho_I^2 + \rho_T^2) + \frac{32\pi^2}{g^2} (4 \cos^2 \gamma - 1) \frac{\rho_I^2 \rho_T^2}{R^4}. \quad (3.19)$$

The main contribution to the functional integral (3.7) comes from the region of maximal instanton-anti-instanton attraction, which corresponds to $\cos^2 \gamma = 1$. The integral over the relative group orientation reads

$$\int_0^\pi d\gamma \sin^2 \gamma \exp[t \cdot (4 \cos^2 \gamma - 1)], \quad (3.20)$$

where

$$t = 32 \frac{\pi^2 \rho_I^2 \rho_T^2}{g^2 R^4}. \quad (3.21)$$

Now one can perform the Gaussian integration over a small fluctuation γ . In the Gaussian approximation one has for the integral (3.21)

$$\exp(3t) \int_{-\infty}^{+\infty} d\gamma \cdot \gamma^2 \cdot \exp[-4 \cdot t \cdot \gamma^2] = \frac{\sqrt{\pi}}{16} \frac{1}{t\sqrt{t}} \cdot \exp(3t). \quad (3.22)$$

This means that the integration over the Gaussian fluctuations around the maximal attraction point gives the following factor in the pre-exponent,

$$\frac{\sqrt{\pi}}{16} \frac{1}{t\sqrt{t}}. \quad (3.23)$$

To proceed let us define the new variables

$$x = m_w R, \quad y = m_w (\rho_I - \rho_T), \quad z = \frac{\rho_I \rho_T}{R^2}. \quad (3.24)$$

Equation (3.19) gives, in terms of these variables and for the case of maximal attraction,

$$\Gamma = -\frac{16\pi^2}{g^2} + \frac{Ex}{m_w} - \frac{4\pi^2}{g^2} (y^2 + 2x^2 z - 24z^2). \quad (3.25)$$

The saddle point solution in the leading order (cf. eqs. (3.13), (3.14)) reads

$$\tau_* = \{x_*, \simeq (24c)^{1/3}, y_* \simeq 0, z_* \simeq \frac{c^{2/3}}{24^{1/3}}\}. \quad (3.26a)$$

This means, in terms of physical variables,

$$R_* \simeq (24c)^{1/3} / m_w, \quad \rho_{I*} = \rho_{T*} \simeq 24^{1/6} c^{1/3} / m_w. \quad (3.26b)$$

Collecting all results, we have the following expression for the second term in eq. (3.18),

$$\frac{1}{2} \sum_{i,j} \frac{\partial^2 \Gamma}{\partial \tau_i \partial \tau_j} \Big|_{\tau_*} \delta \tau_i \delta \tau_j = -\frac{4\pi^2}{g^2} \left(y^2 + \frac{3c^{2/3}}{24^{1/3}} x^2 - 24 \left(z - \frac{c^{1/3}}{24^{2/3}} x \right)^2 \right). \quad (3.27)$$

There is one negative mode in eq. (3.27) which will give rise to the imaginary part of the FES amplitude. Evaluating the integral over the Gaussian fluctuations, we have

$$\begin{aligned} & \int d\rho_I d\rho_T dR \phi(\rho_I, \rho_T, R) \exp \left[-\frac{1}{2} \sum_{i,j} \frac{\partial^2 \Gamma}{\partial \tau_i \partial \tau_j} \Big|_{\tau_*} \delta \tau_i \delta \tau_j \right] \\ &= i g^3 \frac{1}{8\sqrt{3}\pi^{3/2}(24c)^{1/3}} \frac{1}{m_w^2 (\rho_{I*} + \rho_{T*})} \phi(\rho_{I*}, \rho_{T*}, R_*), \end{aligned} \quad (3.28)$$

where ϕ is the general notation for the pre-exponent in eq. (3.7). For ϕ we have the following expression (see eqs. (3.7), (3.23), (3.21))

$$\phi = g^3 (p_1 \cdot p_2) \frac{\pi^2}{2^5 E^{3/2}} \frac{n(\rho_I) n(\rho_T)}{\rho_I \rho_T} R_*^{15/2}. \quad (3.29)$$

Collecting all our results (see eqs. (1.7), (3.28), (3.29), (2.23), (3.26b), (3.9), (3.17)) and taking into account the formulae $(p_1 \cdot p_2) = s/2$ and $E = \sqrt{s}$, we have for the

fermion-number violating total cross section the following expression, valid in the region $m_w \ll \sqrt{s} \ll M_{sp}$,

$$\sigma_{\Delta F} = \frac{1}{m_w^2} \frac{1}{7/2} \frac{3^{8/3}}{\alpha_w} \pi^6 e^{-2[\alpha(1)-\alpha(1/2)]} \left(\frac{\mu}{m_w}\right)^{13} \left(\frac{\sqrt{s}}{M_{sp}}\right)^{8/3} \times \exp \left[-\frac{4\pi}{\alpha_w(\mu)} \left(1 - \left(\frac{9}{16\sqrt{8}} \right)^{2/3} \left(\frac{\sqrt{s}}{M_{sp}} \right)^{4/3} + \frac{3}{32} \left(\frac{\sqrt{s}}{M_{sp}} \right)^2 + \mathcal{O} \left(\left(\frac{\lambda}{g^2} + 1 \right) \left(\frac{\sqrt{s}}{M_{sp}} \right)^{8/3} \right) \right) \right] \quad (3.30)$$

4. Discussion

We have considered the total cross section for fermion-number violating processes in an electro-weak type model via the method of dispersion relations.

Generally speaking, this method is manifestly unitary and does not specify the concrete fermion-number violating process. Indeed, in order to find the dominant configuration (valley) for the partition function in the zero topological charge sector one should solve the equation (2.6) with the general, multi-instanton/multi-anti-instanton, boundary conditions instead of eq. (2.4).

For sufficiently small energies the main contribution certainly comes from the single-instanton/single-anti-instanton valley. For infinite values of the valley parameter this valley trajectory is simply the sum of an infinitely separated instanton and anti-instanton pair (in that case the valley parameter is nothing other than the instanton anti-instanton separation), and for small values it approaches the perturbative vacuum (see ref. [21]), reflecting the instanton/anti-instanton collapse. The dominant contribution to the functional integral for the Green's function in *Minkowski* space comes from a *saddle point*, i.e. one point of the valley trajectory. This means that the valley parameter is fixed at the saddle point. It depends on $x = \sqrt{s}/M_{sp}$.

In this paper we considered the case of small x . In that case the valley parameter of the saddle point is sufficiently large ($R/\rho \gg 1$). It was argued that in this region the valley trajectory is the sum of distorted (due to the interaction) instanton and distorted anti-instanton. This means that for energies below M_{sp} the main contribution to $\sigma_{\Delta F}$ is induced by a single distorted instanton. This distortion appears in next-to-leading order in the expansion in ρ/R . The corrections to the next-to-leading order term were not evaluated in this paper but their form is explicitly shown in our main result (see also eq. (3.17))

$$\sigma_{\Delta F} = \frac{1}{m_w^2} \frac{1}{7/2} \frac{3^{8/3}}{\alpha_w} \pi^6 e^{-2[\alpha(1)-\alpha(1/2)]} \left(\frac{\mu}{m_w}\right)^{13} \left(\frac{\sqrt{s}}{M_{sp}}\right)^{8/3} \quad (4.1)$$

$$\times \exp \left[-\frac{4\pi}{\alpha_w(\mu)} \left(1 - \left(\frac{9}{16\sqrt{8}} \right)^{2/3} \left(\frac{\sqrt{s}}{M_{sp}} \right)^{4/3} + \frac{3}{32} \left(\frac{\sqrt{s}}{M_{sp}} \right)^2 + \mathcal{O} \left(\left(\frac{\lambda}{g^2} + 1 \right) \left(\frac{\sqrt{s}}{M_{sp}} \right)^{8/3} \right) \right) \right],$$

which is valid at $m_w \ll \sqrt{s} \ll M_{sp}$. If we naively omit the unknown corrections (the last term in the exponent in eq. (4.1)) we find that the exponent in (4.1) has a maximum at $\sqrt{s}/M_{sp} = 8\sqrt{2}/3$, where the cross section is still exponentially small, $\sigma_{\Delta F} \sim \exp[-(4\pi/\alpha_w) \cdot (1/3)]$. This maximum is of course outside the region of validity of our formula. However, it demonstrates that there is the possibility that the total cross section, even in the single- (distorted-)instanton sector, will not violate the unitarity bound. In this case one does not need to invoke multi-instanton induced processes [25] in order to save unitarity.

Nevertheless, the question of the suppression of eq. (4.1) for high energies remains still open. To clarify the situation one should investigate the general expression for the function F and the question of its zero. One possible way is to investigate the valley trajectory without any assumptions, perhaps on qualitative grounds.

During the preparation of this work we received a paper by Yung [24], where instanton-induced processes in the $SU(2)$ gauge-Higgs model without fermions are studied via the valley approach. In contrast to our method, Yung tried to evaluate Green's functions of the type (3.1) in *Euchidean* space (before analytic continuation to Minkowski space). The integral over R in eq. (3.1) is badly divergent in the region of $R \rightarrow 0$ if one substitutes the approximate expression for the valley trajectory, received in the limit of large R . In order to overcome this difficulty, the author of ref. [24] performed the continuation $g^2 \rightarrow -g^2$, obtained a convergent integral, made the opposite continuation $-g^2 \rightarrow g^2$ after integration, and obtained as the final result a *complex* expression for the Euclidean Green's function. Yung assumed that only the real part has physical sense. This procedure was called "subtracting the perturbative part" of the answer. Strictly speaking, this procedure means that the so-called perturbative part is *defined* to be the unsuppressed part of the answer (the rest, the "nonperturbative part", is defined to be proportional to $\exp(-16\pi^2/g^2)$). Therefore it is not surprising that a suppressed result for the "non-perturbative" part is obtained. The justification for Yung's procedure remains unclear to us. Moreover, in the model with fermions we have no perturbative part at all.

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