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On the Relation between the Lee Model and Ordinary Meson Theory

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On the Relation between the Lee Model and Ordinary Meson Theory

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(ricevuto il 28 Marzo 1957)

Summary — Some features of a possible generalization of the Lee model are examined. It is shown that it can be considered as an approximation of the neutral scalar meson theory with fixed sources, obtained by introducing a cut off on the total energy of the mesons.

1 - Introduction

The simple model of a renormalizable field theory proposed by T. D. LEE ⁽¹⁾ has raised much discussion. In particular, by analyzing its mathematical aspects PAULI and KÄLLÉN ⁽²⁾ have emphasized the existence of a critical value g_{cr} of the renormalized coupling constant g in connection with the shape of the cut off function. If $g > g_{cr}$, one obtains typical consequences such as non-hermiticity of the Hamiltonian and the appearance of «ghosts», and not even the introduction of an indefinite metric makes it possible to construct a satisfactory theory (for example the S -matrix is not unitary in this case). When there is no cut off, then $g_{cr} = 0$, i.e. for any value of $g \neq 0$ one gets

(*) On leave of absence from the Istituto di Scienze Fisiche dell'Università, Milano

⁽¹⁾ T. D. LEE: *Phys. Rev.*, **95**, 1329 (1954)

⁽²⁾ G. KÄLLÉN and W. PAULI: *Dan. Mat. Fys. Medd.*, **30**, no. 7 (1955)

a theory with these unpleasant features. On the other hand, it is well known that a scalar neutral meson theory with fixed nucleons is renormalizable and exactly soluble, both with and without cut off.

The aim of this paper is to examine whether the Lee model can be regarded as a kind of approximation of the neutral scalar theory, to look for the possibility of further approximations, and to study their limits of applicability.

First of all we notice that the one-nucleon problem in the Tamm-Dancoff approximation (0-meson and 1 meson states taken into account) is very similar to the one V-particle problem of the Lee model. The difference consists in the fact that, in the TD case, the nucleon in a 1-meson state is regarded as identical with the nucleon in a 0-meson state, whereas in the Lee case they are looked upon as different particles (V and N); consequently the renormalization proceeds differently in the two cases. In both cases, however, one may regard these theories as derived from the neutral scalar theory, with a cut-off on the allowed numbers of mesons, independent of a possible « normal » cut-off on meson frequencies. Now, we can look for the next approximations, i.e. for extended Lee models related to the original one as the higher TD approximations are to the first.

We would like to remark in this connection that these « Lee approximations » of the one nucleon problem may have a better chance than TD approximations of having a physical meaning (we use this word in a rather vague sense, principally due to the vague hope that a neutral scalar theory may somehow give useful suggestions on the behaviour of a « true » meson theory). Namely we suppose a nucleon which « cannot » emit mesons to be of a nature somewhat different from a nucleon which « can ».

There are two essential possibilities for these approximations:

- a) an introduction of a cut-off on frequencies, and a cut-off on the meson number, independent of each other;
- b) introduction of a cut-off of a more general type which acts both on frequencies and on the meson number.

HABER SCHAIM and THIRRING ⁽³⁾ have investigated an approach of the a) type which, already in the 2-meson approximation, gives very complicated formal equations. However, they worked with a symmetrical pseudo-scalar theory.

A simple theory of the b) type follows from the introduction of an energy cut-off. This theory is essentially an approximation of the neutral scalar theory with fixed sources, obtained cutting-off the states in which the total

⁽³⁾ U. HABER SCHAIM and W. THIRRING: *Nuovo Cimento*, **1**, 100 (1955).

energy of the mesons is greater than a fixed value. In such a way we impose an upper limit to the frequencies of each meson and at the same time, due to the finiteness of the rest mass of the mesons, to their number.

The cut-off functions will therefore depend not only on the frequency of the meson which performs the transition (emission or absorption), but also on the total energy of the other mesons in the state.

2 - General formalism

Let us consider the Hamiltonian

$$(1) \quad \begin{cases} H = H_0 + H_{\text{int}}, \\ H_0 = \sum_i M_i \sum_{\mathbf{p}_i} a_i^*(\mathbf{p}_i) a_i(\mathbf{p}_i) + \sum_{\mathbf{k}} b^*(\mathbf{k}) b(\mathbf{k}), \\ H_{\text{int}} = -\frac{1}{\sqrt{v}} \sum_i g_{0i} \sum_{\mathbf{p}_i = \mathbf{p}_{i+1} + \mathbf{k}} [2\omega(\mathbf{k})]^{-\frac{1}{2}} \{f(\Omega) a_i(\mathbf{p}_i) a_{i+1}^*(\mathbf{p}_{i+1}) b^*(\mathbf{k}) + \text{c.c.}\}, \end{cases}$$

where $b(\mathbf{k})$ and $b^*(\mathbf{k})$ are absorption and emission operators of mesons (π) with momentum $\mathbf{k}$ and mass μ , $\omega(\mathbf{k}) = (\mu^2 + \mathbf{k}^2)^{\frac{1}{2}}$. Everywhere we put $\hbar = c = 1$. $a_i(\mathbf{p})$ and $a_i^*(\mathbf{p})$ are absorption and emission operators of « i nucleons » N_i , with mass M_i . They satisfy the commutation relations

$$(2) \quad \begin{cases} [b(\mathbf{k}), b^*(\mathbf{k}')] = \delta_{\mathbf{k}\mathbf{k}'}, & \{a_i(\mathbf{p}), a_j^*(\mathbf{p}')\} = \delta_{ij} \delta_{\mathbf{p}\mathbf{p}'}, \\ [b(\mathbf{k}), b(\mathbf{k}')] = [b(\mathbf{k}), a_i(\mathbf{p})] = \{a_i(\mathbf{p}), a_j(\mathbf{p}')\} = 0, \end{cases}$$

g_{0i} are the unrenormalized coupling constants for the transitions $N_i \rightleftharpoons N_{i+1} + \pi$.

For the sake of simplicity we hereafter make the assumption that all g_{0i} are equal to a given g_0 , and all M_i equal to a given M (*).

The energy cut off operator $f(\Omega)$ is a given function of the operator $\Omega = \sum_{\mathbf{k}} \omega(\mathbf{k}) b^*(\mathbf{k}) b(\mathbf{k})$ total energy of the mesons, such that

$$f(\Omega) |\Omega'\rangle = f(\Omega') |\Omega'\rangle$$

for every state $|\Omega'\rangle$ satisfying $\Omega |\Omega'\rangle = \Omega' |\Omega'\rangle$.

(*) No essential change in the results would derive from the supposition that all M_i can be different, but fulfill the conditions $M_i < M_{i+1} + \mu$. Otherwise we should be faced with a problem of instable particles. This case, in the context of the Lee model, has been examined by GLASER and KÄLLÉN ().

(4) O. GLASER and G. KÄLLÉN: *Nucl. Phys.*, **2**, 706 (1957).

As it can be easily verified, the operators

$$(3) \quad \begin{cases} Q_1 = \sum_i i \sum_{p_i} a_i^*(\mathbf{p}_i) a_i(\mathbf{p}_i) - \sum_k b^*(\mathbf{k}) b(\mathbf{k}) \\ Q_2 = \sum_i \sum_{p_i} a_i^*(\mathbf{p}_i) a_i(\mathbf{p}_i), \end{cases}$$

commute with the total Hamiltonian. They are therefore constants of motion.

We shall consider as physical states the states with $Q_1 = 0$. For example, if we have a physical state with one nucleon and j mesons, the nucleon must be denoted by N_j . So the Hilbert space of our physical states, $\mathcal{H}_0$, will be the subspace of the total Hilbert space $\mathcal{H}$, spanned by the eigenstates of H_0 , belonging to the eigenvalue 0 of the operator Q_1 . These eigenstates will be denoted in an occupation number representation by symbols such as ⁽⁵⁾

$$(4) \quad |\{n_p^{(0)}\}_\alpha, \{n_p^{(1)}\}_\beta, \{n_k\}_i\rangle,$$

which expresses that α N_0 -nucleons, β N_i -nucleons, i mesons with given momenta are present in the state. They form a complete orthonormal system in $\mathcal{H}$.

Following the renormalization custom, we introduce:

- a) Mass renormalization terms of the form $-\delta M_i \sum_{p_i} a_i^*(\mathbf{p}_i) a_i(\mathbf{p}_i)$
- b) Renormalized field operators

$$(5) \quad a'_i(\mathbf{p}_i) = a_i(\mathbf{p}_i)/C_i$$

- c) Renormalized coupling constants g_i^r related to g_0 by

$$(6) \quad g_i^r = C_i C_{i+1} Z_i g_0$$

The eigenstates of the total Hamiltonian $\bar{H}$ (including mass renormalization terms) with $Q_1 = 0$ describe physical « clothed » states. They can be expressed as linear combinations of states (4). For example the physical « clothed » single nucleon state $|N_0\rangle$ can be developed in the following way ⁽⁵⁾:

$$(7) \quad |N_0\rangle = D_0 \{ |N_0\rangle + \sum_{i=1}^{\infty} \varphi_0^{(i)} [\{n_k\}_i] |N_i \{n_k\}_i\rangle \}$$

— — —

⁽⁵⁾ We use notations similar to those used in the paper by TH. W. RUIJGROK and L. VAN HOFVE: *Physica*, **22**, 880 (1956).

This particular state has to satisfy the equation

$$(8) \quad \bar{H} |N_0\rangle = M |N_0\rangle$$

That is, the state $|N_0\rangle$ has to be eigenstate of $\bar{H}$, with eigenvalue M , the « experimental » mass

All the other eigenstates of $\bar{H}$, with $Q_1 = 0$, and eigenvalues ranging from $M + \mu$ to ∞ , with proper boundary conditions will describe scattering states

In accord with our assumptions a state $|N_i\rangle$, with $i \neq 0$, satisfying the equation

$$\bar{H} |N_i\rangle = M_i |N_i\rangle,$$

has no physical meaning because it belongs to a subspace $\mathcal{H}_i$ of the total Hilbert space $\mathcal{H}$, with eigenvalue of $Q_1 \neq 0$. But a state of the form $b^*(\mathbf{k}_1) b^*(\mathbf{k}_i) |N_i\rangle$, having $Q_1 = 0$ should have a physical meaning: it should represent a state compound of a clothed nucleon plus i free mesons (obviously it is not an eigenstate of $\bar{H}$), something which we might call an asymptotic physical state ⁽⁶⁾

As it will appear from the following, where we shall use only special forms of the cut-off function, it is useful to redefine such asymptotic states as states $|N_i \{n'_k\}_i\rangle_{as}$ obtained by applying i creation operators of mesons to a state which is solution of the equation (*)

$$(9) \quad \bar{H}[\{n'_k\}_i] |N_i\rangle = M |N_i\rangle,$$

where $\bar{H}[\{n'_k\}_i]$ is the total Hamiltonian with the cut off operator $f(\Omega + \Omega'(\{n'_k\}_i))$ instead of $f(\Omega)$, $\Omega'(\{n'_k\}_i)$ being the total energy of the free mesons in the state $|N_i \{n'_k\}_i\rangle_{as}$

The equations (8) and (9) are sufficient to determine the quantities δM_i ; and the coefficients of the development of $|N_0\rangle$ and of $|N_i\rangle$

$$(10) \quad |N_i\rangle = D_i[\{n'_k\}_i] \left\{ |N_i\rangle + \sum_j \varphi_i^{(j)}[\{n'_k\}_i; \{n_k\}_j] |N_{i+j} \{n_k\}_j\rangle \right\}$$

We notice again that $|N_i\rangle$ in itself has no physical meaning; only the state $|N_i \{n'_k\}_i\rangle_{as}$ has the just described physical meaning of an asymptotic state

⁽⁶⁾ We use the word « asymptotic » in a sense similar to the one it has in the papers by L. VAN HOVE: *Physica*, **21**, 901 (1955); **22**, 343 (1956) or H. EKSTEIN: *Nuovo Cimento*, **4**, 1017 (1956)

(*) More care should be taken in defining asymptotical states with more than one nucleon, but we don't need them for our purposes

In order to determine the renormalization quantities C_i , D_i , and Z_i we impose the following conditions:

$$(11) \quad \langle 0 | a'_i | \mathbf{N}_i \rangle = 1 \quad (\text{field operators renormalization})$$

$$(12) \quad \langle \mathbf{N}_i | \mathbf{N}_i \rangle = 1 \quad (\text{normalization})$$

$$(13) \quad g_i^* f[\Omega'(\{n'_k\}_i) + \omega] = \\ = g_0 \langle \mathbf{N}_{i+1} | b(\mathbf{k}) \sum_j \sum_{\mathbf{p}_j = \mathbf{p}_{j+1} + \mathbf{k}} f[\Omega'(\{n'_k\}_i) + \Omega] a_j(\mathbf{p}_j) a_{j+1}^*(\mathbf{p}_{j+1}) b^*(\mathbf{k}) | \mathbf{N}_i \rangle \\ (\text{charge renormalization})$$

Conditions (11) gives immediately $D_i = C_i$

From the definition of $|\mathbf{N}_i\rangle$ it is evident that the C_i and the $\varphi_i^{(j)}$, for $i \neq 0$, are functions of the energy of the mesons present in the complete asymptotical state. As a consequence, if g_0 is a constant, the g_i^* , $i \neq 0$ will also be functions of the energy of the mesons.

The equations (8), (9) can be solved exactly. Omitting the lengthy but trivial calculations, we quote the results:

$$(14) \quad \varphi_i^{(j)}[\{n'_k\}_i; \{n_k\}_j] = \prod_{h=1}^j \frac{g_0}{(2v)^{\frac{1}{2}}} (n_k!)^{-\frac{1}{2}} \{ \omega^{-\frac{1}{2}}(k) f[\Omega'(\{n'_k\}_i) + \Omega'(\{n_k\}_h)] \},$$

where the symbol $\prod^s$ means symmetrized product. For example:

$$\varphi_1^{(2)}[\omega_0; \omega_1 \omega_2] = \begin{cases} \frac{g_0^2}{2v} \frac{1}{2\omega_1^{\frac{3}{2}} \omega_2^{\frac{3}{2}}} f(\omega_0 + \omega_1 + \omega_2) [f(\omega_0 + \omega_1) + f(\omega_0 + \omega_2)], & \text{for } \omega_1 \neq \omega_2, \\ \frac{g_0}{2v} \frac{1}{2^{\frac{1}{2}} \omega_1^{\frac{3}{2}}} f(\omega_0 + 2\omega_1) f(\omega_0 + \omega_1), & \text{for } \omega_1 = \omega_2, \end{cases}$$

$$(15) \quad \delta M_i[\{n'_k\}_i] = - \frac{g_0^2}{2v} \sum_k f^2[\Omega'(\{n'_k\}_i) + \omega] / \omega^2$$

From (12)

$$(16) \quad C_i^{-2}[\{n'_k\}_i] = 1 + \frac{g_0^2}{2v} \sum_k f^2[\Omega'(\{n'_k\}_i) + \omega] / \omega^3 + \frac{1}{2} \frac{g_0^4}{(2v)^2} \\ \sum_k \sum_{k'} \frac{1}{4} \frac{1}{\omega^3 \omega'^3} f^2[\Omega'(\{n'_k\}_i) + \omega + \omega'] \{ f[\Omega'(\{n'_k\}_i) + \omega] + f[\Omega'(\{n'_k\}_i) + \omega'] \}^2 +$$

and from (13)

$$(17) \quad Z_i[\{n'_k\}_i; \omega] = 1 + \frac{g_0^2}{2v} \sum_{k'} \frac{1}{\omega'^3} f^2[\Omega'(\{n'_k\}_i) + \omega + \omega'] \\ f[\Omega'(\{n'_k\}_i) + \omega'] f[\Omega'(\{n'_k\}_i) + \omega] +$$

3 - One-meson case

As a first simple example we consider the case in which the cut-off function has the form $f(x) = \theta(2\mu - x)$

The function $\theta(y)$ is the step function equal to 1 for $y > 0$ and to 0 for $y < 0$ (*)

We are now in the case considered by LEE, but with our particular cut-off function. One obtains immediately (using $\theta^2 = \theta$)

$$(18) \quad \varphi_0^{(1)}(\omega) = \frac{g_0}{(2v)^{\frac{1}{2}}} \theta(2\mu - \omega)/\omega^{\frac{3}{2}},$$

$$(19) \quad \varphi_i^{(j)} = 0 \quad \text{for } i + j \geq 2$$

$$(20) \quad \delta M_0 = -\frac{g_0^2}{2v} \sum_k \theta(2\mu - \omega)/\omega^2,$$

$$(21) \quad C_0^{-2} = 1 + \frac{g_0^2}{2v} \sum_k \theta(2\mu - \omega)/\omega^3,$$

$$(22) \quad C_i = 1 \quad \text{for } i \geq 1,$$

$$(23) \quad Z_i = 1 \quad \text{for all } i$$

The expectation value of the operator $\sum_k b^*(\mathbf{k})b(\mathbf{k})$, number of mesons in the state $|N_0\rangle$, must obviously be less than 1. We can write the inequality

$$(24) \quad 0 \leq \langle N_0 | \sum_k b^*(\mathbf{k})b(\mathbf{k}) | N_0 \rangle = C_0^2 \frac{g_0^2}{2v} \sum_k \theta(2\mu - \omega)/\omega^3 \leq 1$$

From (21) and (24) follow the well known results that the value of C_0 must lie between 0 and 1, and that there is an upper limit for $g_0^2 = C_0 C_1 Z_0 g_0 = C_0 g_0$,

(*) To avoid the troubles pointed out by PAULI and KÄLLÉN⁽²⁾ (p. 11, footnote) we can use a function $\theta'(y)$ of the form

$$\theta'(y) = \begin{cases} 1, & \text{for } y > 0, \\ \delta g(y), & \text{for } y < 0, \end{cases}$$

with $0 < \delta \leq 1$ and $g(y)$ a continuous function tending to 0 faster than $1/y$ as $y \rightarrow \infty$. So all the relations involving the function $\theta(y)$ which appear in the following, are to be considered approximate relations obtained dropping all terms involving $\delta g(y)$, or powers of it.

given by

$$(25) \quad g_0^{\text{crit}} = \left\{ \frac{1}{2v} \sum_k \theta(2\mu - \omega)/\omega^3 \right\}^{-\frac{1}{2}}$$

for $g_0^r > g_0^{\text{crit}}$ one obtains the inconsistencies pointed out by PAULI and KÄLLÉN

4 - Two-meson case

We are now going to consider the case in which

$$f(x) = \theta(3\mu - x)$$

The general results (14)–(17) assume the form:

$$(26) \quad \left\{ \begin{array}{l} \varphi_0^{(1)}(\omega) = \frac{g_0}{(2v)^{\frac{1}{2}}} \theta(3\mu - \omega)/\omega^{\frac{3}{2}}, \\ \varphi_1^{(1)}(\omega_0; \omega) = \frac{g_0}{(2v)^{\frac{1}{2}}} \theta(3\mu - \omega_0 - \omega)/\omega^{\frac{3}{2}}, \\ \varphi_0^{(2)}(\omega_1 \omega_2) = \begin{cases} \frac{g_0^2}{2v} \frac{1}{2\omega_1^{\frac{3}{2}} \omega_2^{\frac{3}{2}}} \theta(3\mu - \omega_1 - \omega_2) [\theta(3\mu - \omega_1) + \theta(3\mu - \omega_2)], & \text{for } \omega_1 \neq \omega_2, \\ \frac{g_0^2}{2v} \frac{1}{2^{\frac{3}{2}} \omega_1^3} \theta(3\mu - 2\omega_1) \theta(3\mu - \omega_1), & \text{for } \omega_1 = \omega_2, \end{cases} \\ \varphi_i^{(j)} = 0, & \text{for } i + j \geq 3 \end{array} \right.$$

$$(27) \quad \left\{ \begin{array}{l} \delta M_0 = -\frac{g_0^2}{2v} \sum_k \theta(3\mu - \omega)/\omega^2, \\ \delta M_1(\omega_0) = -\frac{g_0^2}{2v} \sum_k \theta(3\mu - \omega_0 - \omega)/\omega^2, \\ \delta M_i = 0, & \text{for } i \geq 2, \end{array} \right.$$

$$(28) \quad \left\{ \begin{array}{l} C_0^{-2} = 1 + \frac{g_0^2}{2v} \sum_k \theta(3\mu - \omega)/\omega^3 + \\ \quad + \frac{1}{2} \frac{g_0^4}{(2v)^2} \sum_k \sum_k \frac{1}{4} \frac{1}{\omega^3 \omega'^3} \theta(3\mu - \omega - \omega') [\theta(3\mu - \omega) + \theta(3\mu - \omega')]^2, \\ C_1^{-2}(\omega_0) = 1 + \frac{g_0^2}{2v} \sum_k \theta(3\mu - \omega_0 - \omega)/\omega^3, \\ C_i = 1, & \text{for } i \geq 2, \end{array} \right.$$

$$(29) \quad \left\{ \begin{array}{l} Z_0(\omega) = 1 + \frac{g_0^2}{2v} \sum_k \frac{1}{\omega'^3} \theta^{-1}(3\mu - \omega) \theta(3\mu - \omega') \theta(3\mu - \omega - \omega') = \\ \\ = 1 + \frac{g_0^2}{2v} \sum_k \theta(3\mu - \omega - \omega') / \omega'^3 = c_1^{-2}(\omega) \quad (*) , \\ \\ Z_i = 1 , \quad \text{for } i \geq 1 . \end{array} \right.$$

The impositions on the expectation value of the number of mesons give for the state $|N_1\rangle$

$$(30) \quad 0 \leq \langle N_1 | \sum_k b^*(\mathbf{k}) b(\mathbf{k}) | N_1 \rangle = C_1^2(\omega_0) \frac{g_0^2}{2v} \sum_k \theta(3\mu - \omega_0 - \omega) / \omega^3 \leq 1$$

We have now

$$g_1^r(\omega_0) = C_1(\omega_0) g_0 ,$$

from (30) we obtain a limitation to the values of

$$(31) \quad g_1^r(\omega_0) \leq \left\{ \frac{1}{2v} \sum_k \theta(3\mu - \omega_0 - \omega) / \omega^3 \right\}^{-\frac{1}{2}} = g_{1\text{crit}}^r(\omega_0)$$

for the state $|N_0\rangle$ we have the condition

$$(32) \quad 0 \leq \langle N_0 | \sum_k b^*(\mathbf{k}) b(\mathbf{k}) | N_0 \rangle = C_0^2 \left\{ \frac{g_0^2}{2v} \sum_k \theta(3\mu - \omega) / \omega^3 + \right. \\ \left. + \frac{g_0^4}{(2v)^2} \sum_k \sum_k \frac{1}{4} \frac{1}{\omega^3 \omega'^3} \theta(3\mu - \omega - \omega') [\theta(3\mu - \omega) + \theta(3\mu - \omega')]^2 \right\} \leq \\ \leq 2 - C_0^2 \frac{g_0^2}{2v} \sum_k (3\mu - \omega) / \omega^3$$

It is identical to the condition $0 \leq C_0^2 \leq 1$. When it is satisfied, pathological states are excluded.

5 - Concluding remarks

Further, it is possible to consider cases in which an increasing number of mesons are taken into account, assuming

$$f(x) = \theta(n\mu - x) ,$$

(*) This equality $Z_i = C_{i+1}^{-2}$ presents itself also in the model of Ruijgrok and Van Hove (5)

when $n \rightarrow \infty$, $f(x) \rightarrow 1$, i.e. the operator $f(\mathcal{Q})$ becomes the identity operator. The upper limit of the allowed energies of the mesons is shifted to infinity.

The results (14)–(17) assume the form

$$(33) \quad \left\{ \begin{aligned} \varphi_1^{(j)}[\{n'_k\}_i; \{n_k\}_j] &= \prod_1^i \frac{g_0}{(2v)^{\frac{1}{2}}} (n_k!)^{-\frac{1}{2}} \omega^{-\frac{3}{2}}(k), \\ \delta M_i[\{n'_k\}_i] &= -\frac{g_0^2}{2v} \sum_k \frac{1}{\omega^2}, \\ C_i^{-2}[\{n'_k\}_i] &= \exp \left[\frac{g_0^2}{2v} \sum_k \frac{1}{\omega^3} \right], \\ Z_i[\{n'_k\}_i; \omega] &= \exp \frac{g_0^2}{2v} \sum_k \frac{1}{\omega^3} \end{aligned} \right.$$

All these quantities become independent of $\{n'_k\}_i$ and i and coincide with the quantities $\varphi^{(j)}[\{n_k\}_j]$, δM , C , and Z obtained from the neutral scalar meson theory with point sources.

This supports our initial assertion that our theory can be considered as an approximation of the neutral scalar theory obtained, taking into account only states in which the mesons have an energy less than a fixed value (this is due to the choice of the step function for the $f(x)$ and the definition (9)).

We hope that the method outlined in this note may be of some use when applied to more realistic cases of meson theories, because of its possibility to give a sharp definition, in terms of energy, of the «ignorance zone», that is of the energy region in which other phenomena not taken into account in the schematization of the meson theory as a closed one, become important.

* * *

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RIASSUNTO

Si esaminano alcuni aspetti di una possibile generalizzazione del modello di Lee. Si mostra come essa possa essere considerata come un'approssimazione della teoria mesonica scalare neutra con sorgenti fisse ottenute con l'introduzione di un taglio sull'energia totale dei mesoni.