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To Jens Vigen <Jens.Vigen@cern.ch>
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Carmen Vasini

G DELL'ANTONIO and F DUIMIO
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On the Ruijgrok-Van Hove Model

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In a recent paper RUIJGROK and VAN HOVE ⁽¹⁾ propose a model of renormalizable field theory, which can be considered as a generalization of the Lee model

It consists of a field of spin zero neutral bosons (θ particles) coupled with fixed nucleons, which can be in n different states N_i , through interactions causing the transitions:

$$N_0 \rightleftharpoons N_1 + \theta$$

$$N_1 \rightleftharpoons N_2 + \theta$$

$$N_n \rightleftharpoons N_0 + \theta$$

The strength of each interaction is determined by the (unrenormalized) coupling constant $g_i^{(0)}$; $i = 0, 1, \dots, n$

It follows that an unrestricted number of θ particles can be emitted and absorbed by a nucleon and, in contrast to the Lee model, that a mass renormalization for every nucleon state and a non trivial charge renormalization are needed. The renormalization constants can be calculated explicitly.

A very interesting feature of this model is that the renormalized coupling constants g_i , which are in general different from each other for finite cut off, become equal in the limit of point source.

In this letter we examine, following the method which has been developed by PAULI and KÄLLÉN ⁽²⁾ in their analysis of the mathematical structure of the Lee

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⁽¹⁾ TH. W. RUIJGROK and L. VAN HOVE: *Physica* **22** 880 (1956)

⁽²⁾ G. KÄLLÉN and W. PAULI: *Dan. Mat. Fys. Medd.* **30** No. 7 (1955)

model ⁽³⁾, the dependence of the renormalized coupling constants on the cut off function and the limitations to be imposed on them to avoid the appearance of unphysical (« ghost ») states

The Hamiltonian of the system (for simplicity we consider the case $n = 2$) is given, with the same notations as in (1), by

$$(1) \quad \begin{cases} H = H_0 + H_I \\ H_0 = (M - \delta M_0) \sum_p \psi_0^*(p) \psi_0(p) + (M - \delta M_1) \sum_p \psi_1^*(p) \psi_1(p) + \sum_k \omega(k) a^*(k) a(k), \\ H_I = - (2v)^{-\frac{1}{2}} \sum_{p=p+k} f(k) \omega^{-\frac{1}{2}}(k) [g_0^{(0)} \psi_1^*(p) \psi_0(p') a^*(k) + g_1^{(0)} \psi_0^*(p) \psi_1(p') a^*(k) + \text{h.c.}] \end{cases}$$

For the renormalization constants one obtains:

$$(2) \quad \begin{cases} \delta M_0 = - (2v)^{-1} (g_0^{(0)})^2 \sum_k f^2(k) / \omega^2 \\ \delta M_1 = - (2v)^{-1} (g_1^{(0)})^2 \sum_k f^2(k) / \omega^2 \quad (\text{mass renorm}) \end{cases}$$

$$(3) \quad \begin{cases} Z_2^{-1}(0) = \cosh (g_0^{(0)} g_1^{(0)} L) + \frac{g_0^{(0)}}{g_1^{(0)}} \sinh (g_0^{(0)} g_1^{(0)} L) \\ Z_2^{-1}(1) = \cosh (g_0^{(0)} g_1^{(0)} L) + \frac{g_1^{(0)}}{g_0^{(0)}} \sinh (g_0^{(0)} g_1^{(0)} L) \quad (\text{nucleon amplitude renorm}) \\ L = (2v)^{-1} \sum_k f^2(k) / \omega^3, \end{cases}$$

$$(4) \quad \begin{cases} Z_1(0) = Z_2(1) \\ Z_1(1) = Z_2(0) \quad (\text{charge renorm}) \end{cases}$$

The connection between renormalized and unrenormalized coupling constants is given by

$$(5) \quad \begin{cases} g_0 = Z_2^{\frac{1}{2}}(0) Z_2^{-\frac{1}{2}}(1) g_0^{(0)}, \\ g_1 = Z_2^{\frac{1}{2}}(1) Z_2^{-\frac{1}{2}}(0) g_1^{(0)} \end{cases}$$

From (3) and (5) we can obtain $Z_2(0)$ and $Z_2(1)$ as functions of g_0 and g_1 :

$$(6) \quad \begin{cases} Z_2(0) = \cosh (g_0 g_1 L) - \frac{g_0}{g_1} \sinh (g_0 g_1 L), \\ Z_2(1) = \cosh (g_0 g_1 L) - \frac{g_1}{g_0} \sinh (g_0 g_1 L) \end{cases}$$

⁽³⁾ T. D. LEE: *Phys. Rev.*, **95**, 1329 (1954).

These expressions can assume negative values. As they cannot be simultaneously negative, when one of them is < 0 , the unrenormalized $g_0^{(0)}$ and $g_1^{(0)}$ have to be imaginary with the consequence that the Hamiltonian is non hermitian.

If we impose the conditions

$$(7) \quad \begin{cases} Z_2(0) \geq 0 \\ Z_2(1) \geq 0 \end{cases}$$

it is easy to see that g_0 and g_1 must satisfy the inequalities

$$(8) \quad \operatorname{tgh}(g_0 g_1 L) \leq \frac{g_0}{g_1} \leq \operatorname{cotgh}(g_0 g_1 L),$$

i.e. for any given g_0 (L is kept fixed), g_1 can assume only the values contained in an interval around g_0 , which is the more narrow, the more the product $g_0 L$ becomes large.

Where (8) is satisfied, it follows from (6) that

$$(9) \quad Z_2(0), Z_2(1) \leq \frac{1}{\cosh(g_0 g_1 L)} \leq 1$$

Therefore the general theorem ⁽⁴⁾, stating that the constants Z_2 must lie between 0 and 1, is valid here. When the conditions (8) are violated, the theory gives unphysical results, i.e. «ghost» states with negative probabilities appear.

This can be simply shown, as indicated by KÄLLÉN ⁽⁵⁾, evaluating the expectation value in the vacuum of the anticommutator of the renormalized operators $\psi'_0(p) = \psi_0(p) Z^{-\frac{1}{2}}(0)$ and $\psi'^*_0(p)$. We obtain

$$(10) \quad \langle 0 | \{ \psi'_z(p), \psi'^*_0(p) \} | 0 \rangle = \sum_z |\langle z | \psi'^*_0(p) | 0 \rangle|^2 = Z_2^{-1}(0),$$

where the sum is extended over all the complete set of eigenstates of H . Obviously, when $Z_2(0)$ is < 0 , the sum has to contain some terms with negative norm.

Another indication of the pathological behaviour of the theory when the conditions (8) do not hold, comes out from the evaluation of the expectation value in the physical N_0 state

$$(11) \quad |N_0\rangle = Z_2^{\frac{1}{2}}(0) \left\{ |N_0\{0\}_0\rangle + \sum_{n=0}^{\infty} \sum \alpha_0^{(2n+1)}(\{l_k\}_{2n+1}) |N_1\{l_k\}_{2n+1}\rangle + \right. \\ \left. + \sum_{n=1}^{\infty} \sum \alpha_0^{(2n)}(\{l_k\}_{2n}) |N_0\{l_k\}_{2n}\rangle \right\},$$

⁽⁴⁾ See for ex. H. UMEZAWA and S. KAMEFUCHI: *Progr. Theor. Phys.* **6**, 543 (1951).

⁽⁵⁾ G. KÄLLÉN: unpublished.

of the operators number of N_0 and N_1 (bare) particles:

$$(12) \quad \left\{ \begin{array}{l} \mathcal{N}_0 = \langle N_0 | \sum_p \psi_0^*(p) \psi_0(p) | N_0 \rangle = Z_2(0) \cosh (g_0^{(0)} g_1^{(0)} L) = \\ \quad = \cosh^2 (g_0 g_1 L) - \frac{g_0}{g_1} \sinh (g_0 g_1 L) \cosh (g_0 g_1 L) , \\ \mathcal{N}_1 = \langle N_0 | \sum_p \psi_1^*(p) \psi_1(p) | N_0 \rangle = Z_2(0) \frac{g_0^{(0)}}{g_1^{(0)}} \sinh (g_0^{(0)} g_1^{(0)} L) = \\ \quad = \frac{g_0}{g_1} \sinh (g_0 g_1 L) \cosh (g_0 g_1 L) - \sinh^2 (g_0 g_1 L) \end{array} \right.$$

The sum of $\mathcal{N}_0$ and $\mathcal{N}_1$ gives obviously 1 (law of conservation of nucleons) But, when (8) is violated, one of them turns out to be negative and the other >1

Finally, let us consider the case $L \rightarrow \infty$, i.e. the point source limit: g_0 and g_1 tend to the common value $^{(1)} (g_0^{(0)} g_1^{(0)})^{\frac{1}{2}}$

It is evident that the Lee model is a particular case of the Ruijgrok Van Hove model with $g_1^{(0)}$ (or $g_0^{(0)}$) equal to zero It follows that in the Lee model the renormalized coupling constant has to be equal to zero in the point source limit It is now apparent that this result is a consequence of the very special structure of the Lee model, and no conclusion can be drawn for a more general model

In fact, in the model discussed here, even in the point source limit it is possible to obtain physically meaningful results, which are essentially the results of a neutral scalar theory with fixed sources

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