CERN-TH 5890/90

GEF-Th-17/1990

UPRF-90-290

QCD radiative corrections to Z boson pair production in hadronic collisions

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Abstract

We present a calculation of the corrections of order α_s to Z pair production in hadronic collisions. We give analytic formulae for the differential parton cross sections. For the total parton cross sections we provide quite accurate fits to the numerically integrated results, which can be easily used for phenomenological applications. We also present a few phenomenological results for the total cross section at energies of interest. We find that the corrections are positive in all the cases that we have considered, and are of the order of 20% to 30% of the total.

CERN-TH 5890/90

October 1990

1. Introduction

Electroweak vector boson pair production is an issue of primary importance for physics at future high-energy hadron colliders. In fact, cross sections for the processes

$$pp \rightarrow WW, WZ, ZZ + X \quad (1.1)$$

arising in lowest order from the subprocesses

$$q\bar{q} \rightarrow WW, WZ, ZZ \quad (1.2)$$

become quite sizeable at LHC ($\sqrt{S} = 16$ TeV) and SSC ($\sqrt{S} = 40$ TeV) energies. Thus, the detailed study of the reactions (1.1) can provide by itself a test of the predictions of the standard model, and expose possible deviations from them. On the other hand, the standard vector boson pair production is a major background for new physics signals.

One of the main motivations for the construction of high-energy hadron colliders is the search for the Higgs boson or, more generally, the clarification of the electroweak symmetry breaking mechanism [1]. Higgs bosons are produced via gluon fusion[2], for low to moderate Higgs masses, and via vector boson fusion for high masses[3,4]. In the latter case, one has two accompanying jets in the final state. The two jets, however, are produced rather forward in rapidity and can easily escape detection. If the Higgs boson is heavier than twice the mass of the W or Z , its dominant decay mode is into pairs WW and ZZ [5]. In this case, Higgs production at hadron colliders would be detected via a resonant production of vector boson pairs. Therefore, the background to the Higgs signal comes from other sources of vector boson pairs. The $t\bar{t}$ production process, mediated by strong interactions, followed by the standard decay $t \rightarrow Wb$, becomes a very important source of WW pairs, and creates a severe background to the $H \rightarrow WW$ signal. The ZZ channel, in contrast, offers a particularly clear signal. In fact, in this case the main background comes from the direct production of ZZ pairs through $q\bar{q}$ annihilation, which proceeds via the electroweak interaction and thus has a moderate cross section. Moreover, for this channel one can completely reconstruct the leptonic final state from each Z decay.

Even if a Higgs boson with a mass less than about 1 TeV is not found, one

expects the electroweak sector of the standard model to become strongly interacting in the TeV energy range [6]. Then one should observe an anomalous signal from the scattering of W and Z vector bosons. In this framework as well, the ZZ pair final state seems to be one of the most interesting cases to study as far as the signal-to-background ratio is concerned [7].

As a consequence, it is particularly important to know the details of the continuum ZZ pair production at LHC and SSC. The cross section for the process

$$q\bar{q} \rightarrow ZZ \quad (1.3)$$

was calculated in the Born approximation in ref. [8]. In ref. [10], vector boson pair production accompanied by a large transverse momentum jet was studied in leading order QCD. For the ZZ case this process arises at the parton level through the channels

$$q\bar{q} \rightarrow ZZg \quad (1.4)$$

$$gg \rightarrow ZZq \quad (1.5)$$

which can give an important background for Higgs production at large p_T [9]. Also ZZ pair production accompanied by two large transverse momentum jets

$$pp \rightarrow ZZJJ + X, \quad (1.6)$$

arising in second order QCD, was studied in refs. [11,12]. In ref. [12] the purely electroweak contribution to the process (1.6), including Higgs boson exchange, was analysed as well. All these calculations are based on tree-level amplitudes and do not include virtual corrections.

A further relevant contribution to ZZ pair production comes from the process $gg \rightarrow ZZ$, proceeding through a quark loop [13]. Although important, this channel is not dominant over the $q\bar{q}$ annihilation process (1.3) at the LHC and SSC energies. However, one should stress that the gluon fusion $gg \rightarrow ZZ$ is only part of the order α_s^2 correction to the ZZ pair production in pp collisions. All the other subprocesses contributing to the QCD correction to this order should be included in order to have an accurate knowledge of the relevance of the process $gg \rightarrow ZZ$.

At this stage it seems necessary to get a more systematic determination of the complete QCD corrections at fixed order (including both virtual and real gluon

corrections) to vector boson pair production at hadron colliders. This is important both because it can further test perturbative QCD, and because it can better determine the backgrounds for new physics. Some work has already been done in this field. The complete order α_S corrections to $\gamma\gamma$ and $W\gamma$ pair production have been calculated in refs. [14,15] respectively.

In this paper we present the complete calculation of the order α_S QCD corrections to the total cross section for ZZ pair production in hadronic collisions. A general description of our computation is given in section 2, where the most important technical issues are illustrated. Our results for the partonic cross-sections are given in section 3. Finally, in section 4 we discuss phenomenological issues. Similar analyses for WZ and WW production will be given in forthcoming publications.

2. Description of the calculation

The diagrams involved in the process $q\bar{q} \rightarrow ZZ + X$ up to the order α_S are shown in figs. 1, 2, 3 and those for the process $qg \rightarrow ZZ + X$ are shown in fig. 4. We have not included self-energy corrections on the external fermion lines, since they vanish in dimensional regularization. The cross section to order α_S is given by the square of the Born term, plus the interference of the Born with the virtual graphs, plus the square of the sum of the real graphs

$$d\sigma_{ij} = d\sigma_{ij}^{(b)} + d\sigma_{ij}^{(v)} + d\sigma_{ij}^{(r)}, \quad (2.1)$$

where i, j stand for the different incoming parton types. We will never indicate any integral sign in our formulae, implying that in the sum of differential cross sections with different final states one must perform all the integrals satisfying given final state cuts in each term of the sum.

In order to control the ultraviolet, collinear and infrared divergences that appear in intermediate steps of the calculation, one must choose a regularization scheme. This choice is complicated by the presence of γ_5 in the fermion traces, which prevents a straightforward use of dimensional regularization. We can show, however, that in the case at hand one can avoid the γ_5 problem completely, and safely use dimensional regularization.

From figs. 2, 3 and 4, we can see that the QCD couplings do not involve any non-abelian vertex, and furthermore there are no divergent fermionic loops. In this case, an appropriate regularization exists which preserves the conservation of both the axial and vector currents. One can in fact replace the denominator of the gluon propagator by

$$\frac{1}{p^2} \rightarrow \frac{1}{p^2 - \lambda^2} - \frac{1}{p^2 - \Lambda^2} \quad (2.2)$$

with Λ large and λ small. In this way Λ regulates the ultraviolet divergences and λ the collinear and soft ones. Ultraviolet divergences cancel between wave function renormalization factors, vertex corrections and self-energy corrections. Therefore Λ disappears from the result. Logarithms of λ do instead remain in the inclusive cross section, and they collect themselves into collinear factors convoluted with the Born cross section. The separation of these collinear factors would yield a cross section which can be related to the $\overline{MS}$ cross section through a well defined transformation of scheme.

Let us define the vertex of the vector boson to be $-i\gamma^\mu(g_V - g_A\gamma_5)$. It is easy to see that in the regularization scheme proposed above, terms containing an even power of g_A can be collected as $g_V^4 + g_A^4 + 6g_V^2g_A^2$ times a common factor. The contribution of terms with an odd power of g_A is finite. In fact, all divergences must be factorizable in terms of the Born cross section, and there are no terms with an odd number of g_A in the Born term. In the case of the production of two identical vector bosons this contribution is zero, because it must be proportional to the antisymmetric tensor contracted with a set of independent momenta in the process, while it should also be symmetric with respect to the momenta of the two vector bosons. It is therefore sufficient to calculate our process setting $g_A = 0$, and then replacing g_V^4 with $g_V^4 + g_A^4 + 6g_V^2g_A^2$ at the end of the calculation. In this case, since we only have vector couplings around, we can use directly dimensional regularization, which has the advantage of yielding a result in the $\overline{MS}$ scheme.

In the following subsections we describe in some details the steps of our calculation, and the mechanism of factorization of the collinear singularities. We found it more convenient to calculate a completely exclusive formula for the process. Exclusive formulae contain regularized distributions, which are remnants of the cancellation of infrared and collinear singularities. In the form in which we present them, however, it is easy to implement a Monte Carlo integration program that allows us

to calculate numerically any desired inclusive quantity.

2.1. Kinematics

The process $q\bar{q} \rightarrow ZZ$ is described in terms of the invariants

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 - k_1)^2 = (p_2 - k_2)^2 \\ u &= (p_1 - k_2)^2 = (p_2 - k_1)^2 \end{aligned} \quad (2.3)$$

where p_1, p_2 are the momenta of the incoming quark and antiquark, and k_1, k_2 are the momenta of the Z 's. We have $p_1^2 = p_2^2 = 0$ and $k_1^2 = k_2^2 = m^2$, where m is the mass of the Z , and $s + t + u = 2m^2$.

The two-body phase space in $d = 4 - 2\epsilon$ dimensions is given by

$$d\Phi_2 = \frac{2^{2\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{4\pi}{s} \right) \frac{1}{16\pi} \beta^{1-2\epsilon} \sin^{-2\epsilon} \theta_1 d\cos\theta_1, \quad (2.4)$$

where $\beta = \sqrt{1 - \rho}$, $\rho = 4m^2/s$ and θ_1 is the angle between $\vec{p}_1$ and $\vec{k}_1$ in the $q\bar{q}$ center of mass system. Therefore

$$t = m^2 - \frac{s}{2}(1 - \beta \cos\theta_1). \quad (2.5)$$

The three-body processes $q\bar{q} \rightarrow ZZg$ and $qg \rightarrow ZZq$ are characterized by five independent scalar quantities:

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t_k &= (p_1 - k)^2 \\ u_k &= (p_2 - k)^2 \\ q_1 &= (p_1 - k_1)^2 \\ q_2 &= (p_2 - k_2)^2 \end{aligned} \quad (2.6)$$

where p_1, p_2 are the momenta of the incoming quark and antiquark (gluon for $qg \rightarrow ZZq$), k_1, k_2 are the momenta of the Z bosons and k is the momentum of the

outgoing gluon (quark). The remaining five invariants are related to these according to the equations

$$\begin{aligned} s_2 &= (k_1 + k_2)^2 = s + t_k + u_k \\ \hat{q}_1 &= (p_1 - k_2)^2 = 2m^2 - s - t_k - q_1 \\ \hat{q}_2 &= (p_2 - k_1)^2 = 2m^2 - s - u_k - q_2 \\ w_1 &= (k + k_1)^2 = m^2 - q_1 + q_2 - t_k \\ w_2 &= (k + k_2)^2 = m^2 + q_1 - q_2 - u_k. \end{aligned} \quad (2.7)$$

It will be convenient to introduce variables x and y , where $x = s_2/s$ and y is the cosine of the angle between $\vec{p}_1$ and $\vec{k}$ in the $q\bar{q}$ center of mass system. We have

$$\rho \leq x \leq 1, \quad -1 \leq y \leq 1 \quad (2.8)$$

and

$$\begin{aligned} t_k &= -\frac{s}{2}(1-x)(1-y) \\ u_k &= -\frac{s}{2}(1-x)(1+y). \end{aligned} \quad (2.9)$$

In the center of mass frame of the ZZ system, our four-momenta are given by

$$\begin{aligned} p_1 &= p_1^0(1, 0, 0, 1) \\ p_2 &= p_2^0(1, 0, \sin\psi, \cos\psi) \\ k &= k^0(1, 0, \sin\psi', \cos\psi') \\ k_1 &= \frac{\sqrt{s_2}}{2}(1, \beta_z \sin\theta_2 \sin\theta_1, \beta_z \cos\theta_2 \sin\theta_1, \beta_z \cos\theta_1) \\ k_2 &= \frac{\sqrt{s_2}}{2}(1, -\beta_z \sin\theta_2 \sin\theta_1, -\beta_z \cos\theta_2 \sin\theta_1, -\beta_z \cos\theta_1) \end{aligned} \quad (2.10)$$

where

$$p_1^0 = \frac{s + t_k}{2\sqrt{s_2}}, \quad p_2^0 = \frac{s + u_k}{2\sqrt{s_2}}, \quad k^0 = -\frac{t_k + u_k}{2\sqrt{s_2}}$$

$$\begin{aligned}
\cos \psi &= 1 - \frac{s}{2p_1^0 p_2^0}, \quad \sin \psi > 0 \\
\cos \psi' &= 1 + \frac{t_k}{2p_1^0 p_2^0}, \quad \sin \psi' > 0 \\
\beta_z &= \sqrt{1 - \frac{4m^2}{s_z}}.
\end{aligned} \tag{2.11}$$

The two remaining independent invariants q_1, q_2 are given by

$$\begin{aligned}
q_1 &= m^2 - \frac{s + t_k}{2}(1 - \beta_z \cos \theta_1) \\
q_2 &= m^2 - \frac{s + u_k}{2}(1 + \beta_z \cos \theta_2 \sin \theta_1 \sin \psi + \beta_z \cos \theta_1 \cos \psi).
\end{aligned} \tag{2.12}$$

Now all invariants are expressed in terms of x, y, θ_1, θ_2 and s through eqs. (2.7), (2.9), (2.11) and (2.12). We can now express also the three-body phase space in terms of the variables x, y, θ_1, θ_2 as

$$d\Phi_3 = HN d\Phi_2^{\frac{s^{1-\epsilon}}{2\pi}} (1-x)^{1-2\epsilon} (1-y^2)^{-\epsilon} dy \sin^{-2\epsilon} \theta_2 d\theta_2, \tag{2.13}$$

where

$$H = \frac{\Gamma(1-\epsilon)}{\Gamma(1+\epsilon)\Gamma(1-2\epsilon)} = 1 - \frac{\pi^2}{3}\epsilon^2 + \mathcal{O}(\epsilon^3) \tag{2.14}$$

$$N = \frac{(4\pi)^\epsilon}{(4\pi)^2} \Gamma(1+\epsilon) \tag{2.15}$$

and

$$d\Phi_2^{(s)} = \frac{2^{2\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{4\pi}{3z}\right)^\epsilon \frac{1}{16\pi} \beta_z^{1-2\epsilon} \sin^{-2\epsilon} \theta_1 d\cos \theta_1 dx. \tag{2.16}$$

Both θ_1 and θ_2 range between 0 and π .

Finally, we need the cross section for the process $q\bar{q} \rightarrow ZZ$ at the zeroth order in α_s , in $4-2\epsilon$ dimensions:

$$d\sigma_{q\bar{q}}^{(b)}(s, t) = \mathcal{M}_{q\bar{q}}^{(b)}(s, t) d\Phi_2 \tag{2.17}$$

where $d\Phi_2$ is given by eq.(2.4),

$$\mathcal{M}_{q\bar{q}}^{(b)}(s, t) = \frac{1}{2s} \frac{1}{4N_C^2} \sum_{\text{spin, colors}} |\mathcal{A}^{(b)}|^2 \tag{2.18}$$

and $|\mathcal{A}^{(b)}|^2$ is the invariant squared amplitude for the Born process:

$$\begin{aligned}
\sum_{\text{spin, colors}} |\mathcal{A}^{(b)}|^2 &= 4N_C (g_V^4 + g_A^4 + 6g_A^2 g_V^2) \\
&\quad \left\{ \left[\frac{t}{u} + \frac{u}{t} + \frac{4m^2 s}{tu} - m^4 \left(\frac{1}{t^2} + \frac{1}{u^2} \right) \right] (1-2\epsilon) \right. \\
&\quad \left. + \frac{2\epsilon}{tu} [m^2(2s+m^2) - tu] - \frac{\epsilon^2(s-2m^2)^2(m^4-tu)}{t^2 u^2} \right\},
\end{aligned} \tag{2.19}$$

where $N_C = 3$ is the number of colours.

2.2. Virtual contribution

The interference of the virtual graphs, represented in fig. 2, with the Born graphs, gives the virtual contribution to the production cross section. We have

$$d\sigma_{q\bar{q}}^{(v)} = \mathcal{M}_{q\bar{q}}^{(v)} d\Phi_2 \tag{2.20}$$

$$\mathcal{M}_{q\bar{q}}^{(v)}(s, t) = \frac{1}{2s} \frac{1}{4N_C^2} \sum_{\text{spin, colors}} [\mathcal{A}^{(b)}(\mathcal{A}^{(v)})^* + \mathcal{A}^{(v)}(\mathcal{A}^{(b)})^*] \tag{2.21}$$

where $\mathcal{A}^{(v)}$ is the virtual amplitude and $d\Phi_2$ is defined in eq. (2.4).

We performed the calculation of the virtual graphs in the following way. We carried out all the traces, and the contraction of indices. The scalar products appearing in the numerator of the resulting expression where then decomposed in terms of linear combinations of denominators. All these algebraic steps where performed by computer. We had two different programs performing the same task, one written in Macsyma and the other in Reduce, in order to avoid programming errors. At the end of this procedure one is left with a small number of virtual integrals of bosonic type. We evaluated these integrals by hand. The integrals are listed in Appendix A.

The complete result for the virtual contribution is given in Appendix B. We notice that the divergent part is proportional to the Born term in $4-2\epsilon$ dimensions.

We have

$$\mathcal{M}_{q\bar{q}}^{(v)} = -4\pi\alpha_s^{(b)} C_F N_C^{-\epsilon} \left(\frac{4}{\epsilon^2} + \frac{6}{\epsilon} \right) \mathcal{M}_{q\bar{q}}^{(b)} + \mathcal{M}_{q\bar{q}}^{(v, \text{reg})} \tag{2.22}$$

where $\mathcal{M}_{q\bar{q}}^{(u,r\bar{r})}$ is regular for $\epsilon = 0$. N is defined in eq. (2.15), $\mathcal{M}_{q\bar{q}}^{(\theta)}$ is given in eq. (2.18), $C_F = 4/3$ is the Casimir of the fundamental representation of the colour group and $\alpha_S^{(\theta)} = \alpha_S \mu^{2\epsilon}$ is the dimensionful coupling constant in d dimensions. After renormalization and subtraction of collinear divergences, some residual dependence upon the scale μ remains, and μ plays the role of the subtraction point.

2.3. Real graphs

The real graphs giving corrections of order α_S to the process $q\bar{q} \rightarrow ZZ + X$ are represented in Fig. 3. Their contribution to our cross section (in d dimension) is given by

$$d\sigma_{q\bar{q}}^{(r)} = \mathcal{M}_{q\bar{q}}^{(r)}(s, t_k, u_k, q_1, q_2) d\Phi_3 \quad (2.23)$$

$$\mathcal{M}_{q\bar{q}}^{(r)}(s, t_k, u_k, q_1, q_2) = \frac{1}{2s} \frac{1}{4N_C^2} \sum_{\text{spin, colour}} |\mathcal{A}^{(r)}|^2 \quad (2.24)$$

where $d\sigma_{q\bar{q}}^{(r)}$ stands for the 3-body cross section. $\mathcal{A}^{(r)}$ is the invariant amplitude, and as usual we sum over the final and average over the initial color and spin. The invariant cross section $\mathcal{M}_{q\bar{q}}^{(r)}$ has singularities in $t_k = 0$ and $u_k = 0$. Although double poles can appear in individual diagrams, they cancel in the total, and only single poles are left. Therefore the function

$$f_{q\bar{q}}(x, y, \theta_1, \theta_2) = 4t_k u_k \mathcal{M}_{q\bar{q}}^{(r)}(s, t_k, u_k, q_1, q_2) \quad (2.25)$$

is regular for $y = \pm 1$ and $x = 1$ (the dependence of $f_{q\bar{q}}$ upon m^2/s is not explicitly shown). Using equations (2.9) we get

$$\mathcal{M}_{q\bar{q}}^{(r)}(s, t_k, u_k, q_1, q_2) = \frac{f_{q\bar{q}}(x, y, \theta_1, \theta_2)}{s^2(1-x)^2(1-y^2)}. \quad (2.26)$$

The three-body contribution to our cross section, including the phase space, is then given by

$$d\sigma_{q\bar{q}}^{(r)} = HN d\Phi_2^{(z)} \frac{s^{-1-\epsilon}}{2\pi} dy \sin^{-2\epsilon} \theta_2 d\theta_2 (1-x)^{-1-2\epsilon} (1-y^2)^{-1-\epsilon} f_{q\bar{q}}(x, y, \theta_1, \theta_2) \quad (2.27)$$

We can now use the following expansions valid for small ϵ

$$(1-x)^{-1-2\epsilon} = -\frac{\beta^{-4\epsilon}}{2\epsilon} \delta(1-x) + \left(\frac{1}{1-x}\right)_\rho - 2\epsilon \left(\frac{\log(1-x)}{1-x}\right)_\rho + \mathcal{O}(\epsilon^2) \quad (2.28)$$

$$(1-y^2)^{-1-\epsilon} = -[\delta(1+y) + \delta(1-y)] \frac{2^{-1-2\epsilon}}{\epsilon} + \frac{1}{2} \left[\left(\frac{1}{1-y} \right)_+ + \left(\frac{1}{1+y} \right)_+ \right] + \mathcal{O}(\epsilon) \quad (2.29)$$

where the distributions in round brackets are defined according to the prescriptions

$$\int_\rho^1 h(x) \left(\frac{1}{1-x} \right)_\rho dx = \int_\rho^1 \frac{h(x) - h(1)}{1-x} dx \quad (2.30)$$

$$\int_\rho^1 h(x) \left(\frac{\log(1-x)}{1-x} \right)_\rho dx = \int_\rho^1 [h(x) - h(1)] \frac{\log(1-x)}{1-x} dx \quad (2.31)$$

$$\int_{-1}^1 h(y) \left(\frac{1}{1 \pm y} \right)_\pm dy = \int_{-1}^1 \frac{h(y) - h(\mp 1)}{1 \pm y} dy, \quad (2.32)$$

for any sufficiently regular test function $h(x)$. Now we can expand the term $(1-x)^{-1-2\epsilon}$ in eq. (2.27). We obtain

$$d\sigma_{q\bar{q}}^{(r)} = d\sigma_{q\bar{q}}^{(s)} + HN d\Phi_2^{(z)} \frac{s^{-1-\epsilon}}{2\pi} dy \sin^{-2\epsilon} \theta_2 d\theta_2 \left[\left(\frac{1}{1-x} \right)_\rho - 2\epsilon \left(\frac{\log(1-x)}{1-x} \right)_\rho \right] (1-y^2)^{-1-\epsilon} f_{q\bar{q}}(x, y, \theta_1, \theta_2) \quad (2.33)$$

where

$$d\sigma_{q\bar{q}}^{(s)} = HN d\Phi_2^{(z)} \frac{s^{-1-\epsilon}}{2\pi} dy \sin^{-2\epsilon} \theta_2 d\theta_2 \left[-\frac{\beta^{-4\epsilon}}{2\epsilon} \delta(1-x) \right] (1-y^2)^{-1-\epsilon} f_{q\bar{q}}(x, y, \theta_1, \theta_2). \quad (2.34)$$

To obtain $d\sigma_{q\bar{q}}^{(s)}$, we observe that for $x = 1$ all invariants, and therefore also $f_{q\bar{q}}$, become independent of y and θ_2 . Defining

$$f_{q\bar{q}}^{(s)}(\theta_1) = f_{q\bar{q}}(1, y, \theta_1, \theta_2) \quad (2.35)$$

and then integrating in x , y and θ_2 in eq. (2.34) we obtain

$$d\sigma_{q\bar{q}}^{(s)} = HN d\Phi_2 \frac{1}{4\epsilon^2} s^{-1-\epsilon} \beta^{-4\epsilon} f_{q\bar{q}}^{(s)}(\theta_1). \quad (2.36)$$

We now expand $(1-y^2)^{-1-\epsilon}$ in the second term of eq. (2.33), observing that we only need the expansion up to order ϵ^0 . We get

$$d\sigma_{q\bar{q}}^{(r)} = d\sigma_{q\bar{q}}^{(s)} + d\sigma_{q\bar{q}}^{(c+)} + d\sigma_{q\bar{q}}^{(c-)} + d\sigma_{q\bar{q}}^{(f)} \quad (2.37)$$

where

$$d\sigma_{q\bar{q}}^{(c\pm)} = N d\Phi_2^{(s)} \frac{s^{-1-\epsilon}}{2\pi} dy \sin^{-2\epsilon} \theta_2 d\theta_2 \left[\left(\frac{1}{1-x} \right)_\rho - 2\epsilon \left(\frac{\log(1-x)}{1-x} \right)_\rho \right] \left[-\frac{2^{-1-2\epsilon}}{\epsilon} \delta(1 \pm y) \right] f_{q\bar{q}}^{(c\pm)}(x, y, \theta_1, \theta_2) \quad (2.38)$$

and

$$d\sigma_{q\bar{q}}^{(f)} = N \frac{s^{-1}}{64\pi^2} \beta_x d \cos \theta_1 d\theta_2 dy dz \left(\frac{1}{1-x} \right)_\rho \left[\left(\frac{1}{1-y} \right)_+ + \left(\frac{1}{1+y} \right)_+ \right] f_{q\bar{q}}^{(f)}(x, y, \theta_1, \theta_2). \quad (2.39)$$

In order to evaluate $d\sigma_{q\bar{q}}^{(c\pm)}$ we observe that the invariants, and therefore also $f_{q\bar{q}}$, become independent of θ_2 when $y = \pm 1$. We define then

$$f_{q\bar{q}}^{(c\pm)}(x, \theta_1) = f_{q\bar{q}}(x, \pm 1, \theta_1, \theta_2) \quad (2.40)$$

Performing the y and θ_2 integrals in eq. (2.38) we obtain

$$d\sigma_{q\bar{q}}^{(c\pm)} = -N \frac{s^{-1-\epsilon}}{4\epsilon} d\Phi_2^{(s)} \left[\left(\frac{1}{1-x} \right)_\rho - 2\epsilon \left(\frac{\log(1-x)}{1-x} \right)_\rho \right] f_{q\bar{q}}^{(c\pm)}(x, \theta_1). \quad (2.41)$$

We will actually calculate directly $d\sigma_{q\bar{q}}^{(s)}$ and $d\sigma_{q\bar{q}}^{(c\pm)}$ starting from $d\sigma_{q\bar{q}}^{(b)}$ in d dimensions. In fact, the t_k and u_k singularities must factorize in terms of the Born cross section and the Altarelli-Parisi splitting function. We can decompose $\mathcal{M}_{q\bar{q}}^{(r)}$ as

$$\mathcal{M}_{q\bar{q}}^{(r)}(s, t_k, u_k, q_1, q_2) = -\frac{8\pi C_F \alpha_S}{t_k} \left[\frac{1+x^2}{1-x} - \epsilon(1-x) \right] \mathcal{M}_{q\bar{q}}^{(b)}(x, s, q_2) \quad (2.42)$$

$$-\frac{8\pi C_F \alpha_S^{(b)}}{u_k} \left[\frac{1+x^2}{1-x} - \epsilon(1-x) \right] \mathcal{M}_{q\bar{q}}^{(b)}(x, s, q_1) + \mathcal{M}_{q\bar{q}}^{(c,r,s)}(s, t_k, u_k, q_1, q_2) \quad (2.42)$$

where $\alpha_S^{(b)} = \alpha_S \mu^{2\epsilon}$ is the dimensionful coupling constant in d dimensions (the suffix (b) stands for bare). The term in the square bracket is the quark-quark Altarelli-Parisi splitting function in d dimensions, and the Born cross section must also be given in d dimensions. The term $\mathcal{M}_{q\bar{q}}^{(c,r,s)}$ is regular in t_k and u_k . Using eq. (2.42) (and eqs. (2.25, 2.35, 2.40)) we can immediately derive an expression for $d\sigma_{q\bar{q}}^{(s)}$ and $d\sigma_{q\bar{q}}^{(c\pm)}$. In fact we obtain

$$f_{q\bar{q}}^{(c+)}(x, \theta_1) = 32\pi C_F \alpha_S^{(b)} s(1+x^2 - \epsilon(1-x)^2) \mathcal{M}_{q\bar{q}}^{(b)}(x, s, q_2) \quad (2.43)$$

$$f_{q\bar{q}}^{(c-)}(x, \theta_1) = 32\pi C_F \alpha_S^{(b)} s(1+x^2 - \epsilon(1-x)^2) \mathcal{M}_{q\bar{q}}^{(b)}(x, s, q_1) \quad (2.44)$$

$$f_{q\bar{q}}^{(s)}(\theta_1) = 64\pi C_F \alpha_S^{(b)} s \mathcal{M}_{q\bar{q}}^{(b)}(s, q_1) \quad (2.45)$$

and from eqs. (2.36), (2.41)

$$d\sigma_{q\bar{q}}^{(c+)} = -\frac{s^{-\epsilon}}{\epsilon} \frac{C_F \alpha_S^{(b)}}{2\pi} \left[\left(\frac{1}{1-x} \right)_\rho - 2\epsilon \left(\frac{\log(1-x)}{1-x} \right)_\rho \right] (1+x^2 - \epsilon(1-x)^2) \mathcal{M}_{q\bar{q}}^{(b)}(x, s, q_2) d\Phi_2^{(s)} \quad (2.46)$$

$$d\sigma_{q\bar{q}}^{(c-)} = -\frac{s^{-\epsilon}}{\epsilon} \frac{C_F \alpha_S^{(b)}}{2\pi} \left[\left(\frac{1}{1-x} \right)_\rho - 2\epsilon \left(\frac{\log(1-x)}{1-x} \right)_\rho \right] (1+x^2 - \epsilon(1-x)^2) \mathcal{M}_{q\bar{q}}^{(b)}(x, s, q_1) d\Phi_2^{(s)} \quad (2.47)$$

$$d\sigma_{q\bar{q}}^{(s)} = 16\pi C_F \alpha_S^{(b)} N d\Phi_2 \frac{1}{\epsilon^2} H s^{-\epsilon} \beta^{-4\epsilon} \mathcal{M}_{q\bar{q}}^{(b)}(s, q_1) \quad (2.48)$$

where

$$\frac{1}{\epsilon} = \frac{1}{\epsilon} - \gamma_E + \log(4\pi). \quad (2.49)$$

At this point one can see that the $1/\epsilon$ divergence in the collinear term assumes the form dictated by the factorization theorem. According to the factorization theorem[16] the partonic cross section can be written as

$$d\sigma_{ij}(p_1, p_2) = \sum_k d\hat{\sigma}_{ik}(x_1 p_1, x_2 p_2) \Gamma_k(x_1) \Gamma_j(x_2) dx_1 dx_2 \quad (2.50)$$

$$\Gamma_{ij}(x) = \delta_{ij}\delta(1-x) - \frac{1}{\epsilon} \frac{\alpha_S}{2\pi} P_{ij}(x) + \mathcal{O}(\alpha_S^2). \quad (2.51)$$

where $d\hat{\sigma}$ is free of singularities as ϵ goes to zero. The collinear factors Γ are usually reabsorbed into the hadronic structure functions, and the physical cross section is then expressed as

$$d\sigma_{H_1 H_2}(p_1, p_2) = \sum_H d\hat{\sigma}_H(x_1 p_1, x_2 p_2) f_{H_1}(x_1) f_{H_2}(x_2) dx_1 dx_2 \quad (2.52)$$

where $f_{H_{1(2)}}$ is the structure function of the parton k in the hadron $H_{1(2)}$. As a consequence of this procedure, both $d\hat{\sigma}$ and the structure functions become dependent upon the scale μ . Expanding equation (2.50) order by order in perturbation theory, we find for our case

$$\begin{aligned} d\hat{\sigma}_{q\bar{q}}(p_1, p_2) &= d\sigma_{q\bar{q}}(p_1, p_2) + \frac{1}{\epsilon} \frac{\alpha_S}{2\pi} d\sigma_{q\bar{q}}^{(b)}(x_1 p_1, p_2) P_{q\bar{q}}(x_1) dx_1 \\ &\quad + \frac{1}{\epsilon} \frac{\alpha_S}{2\pi} d\sigma_{q\bar{q}}^{(b)}(p_1, x_2 p_2) P_{q\bar{q}}(x_2) dx_2 \end{aligned} \quad (2.53)$$

or

$$\begin{aligned} d\hat{\sigma}_{q\bar{q}}(p_1, p_2) &= d\sigma_{q\bar{q}}(p_1, p_2) + \frac{1}{\epsilon} \frac{\alpha_S}{2\pi} P_{q\bar{q}}(x) \mathcal{M}_{q\bar{q}}^{(b)}(x s, q_2) d\Phi_2^{(x)} \\ &\quad + \frac{1}{\epsilon} \frac{\alpha_S}{2\pi} P_{q\bar{q}}(x) \mathcal{M}_{q\bar{q}}^{(b)}(x s, q_1) d\Phi_2^{(x)} \end{aligned} \quad (2.54)$$

where

$$\begin{aligned} P_{q\bar{q}}(x) &= C_F \left[\left(\frac{1}{1-x} \right)_+ (1+x^2) + \frac{3}{2} \delta(1-x) \right] \\ &= C_F \left[\left(\frac{1}{1-x} \right)_\rho (1+x^2) + \delta(1-x) \left(\frac{3}{2} + 4 \log \beta \right) \right]. \end{aligned} \quad (2.55)$$

2.4. Analytical results

The final expression for the short distance cross section, after subtraction of the collinear divergences (eq. 2.54)), becomes

$$d\hat{\sigma}_{q\bar{q}} = d\sigma_{q\bar{q}}^{(b)} + d\hat{\sigma}_{q\bar{q}}^{(c+)} + d\hat{\sigma}_{q\bar{q}}^{(c-)} + d\hat{\sigma}_{q\bar{q}}^{(v)} + d\sigma_{q\bar{q}}^{(f)} \quad (2.56)$$

$$d\hat{\sigma}_{q\bar{q}}^{(c+)} = \frac{C_F \alpha_S}{2\pi} \left\{ \left[\log \frac{s}{\mu^2} \left(\frac{1}{1-x} \right)_\rho + 2 \left(\frac{\log(1-x)}{1-x} \right)_\rho \right] (1+x^2) + (1-x) \right\} \mathcal{M}_{q\bar{q}}^{(b)}(x s, q_2) d\Phi_2^{(x)} \quad (2.57)$$

$$d\hat{\sigma}_{q\bar{q}}^{(c-)} = \frac{C_F \alpha_S}{2\pi} \left\{ \left[\log \frac{s}{\mu^2} \left(\frac{1}{1-x} \right)_\rho + 2 \left(\frac{\log(1-x)}{1-x} \right)_\rho \right] (1+x^2) + (1-x) \right\} \mathcal{M}_{q\bar{q}}^{(b)}(x s, q_1) d\Phi_2^{(x)} \quad (2.58)$$

$$d\hat{\sigma}_{q\bar{q}}^{(v)} = 4\pi C_F \alpha_S N \left(\frac{s}{\mu^2} \right)^{-\epsilon} \left[\frac{4}{\epsilon^2} H \beta^{-4\epsilon} + \frac{2}{\epsilon} (3 + 8 \log \beta) \left(\frac{s}{\mu^2} \right)^\epsilon \right] \mathcal{M}_{q\bar{q}}^{(b)}(s, t) d\Phi_2. \quad (2.59)$$

The remaining singularities in $d\hat{\sigma}_{q\bar{q}}^{(s)}$ cancel with the singularities in $d\sigma_{q\bar{q}}^{(v)}$ (eqs. (2.20), (2.22)), so that the full expression for $d\hat{\sigma}_{q\bar{q}}$ is finite. In fact the soft-virtual contribution is given by

$$\begin{aligned} d\hat{\sigma}_{q\bar{q}}^{(sv)} &= d\hat{\sigma}_{q\bar{q}}^{(s)} + d\sigma_{q\bar{q}}^{(v)} = \frac{C_F \alpha_S}{4\pi} \left[\log \frac{s}{\mu^2} (6 + 16 \log \beta) + 32 \log^2 \beta - \frac{4}{3} \right] \mathcal{M}_{q\bar{q}}^{(b)}(s, t) d\Phi_2 \\ &\quad + \mathcal{M}_{q\bar{q}}^{(v, \text{reg})}(s, t) d\Phi_2. \end{aligned} \quad (2.60)$$

For the $q\bar{q}$ process, depicted in Fig. 4, one follows an analogous procedure. The invariant cross section can be decomposed as

$$\begin{aligned} \mathcal{M}_{q\bar{q}}^{(v)}(s, t_k, u_k, q_1, q_2) &= -\frac{8\pi T_F \alpha_S}{u_k} [x^2 + (1-x)^2 - 2\epsilon x(1-x)] \mathcal{M}_{q\bar{q}}^{(b)}(x s, q_1) \\ &\quad + \mathcal{M}_{q\bar{q}}^{(v, \text{reg})}(s, t_k, u_k, q_1, q_2) \end{aligned} \quad (2.61)$$

and for the collinear term one finds

$$\begin{aligned} d\hat{\sigma}_{q\bar{q}}^{(c-)} &= -\frac{s^{-\epsilon} T_F \alpha_S}{\epsilon} \frac{1}{2\pi} [1 - 2\epsilon \log(1-x)] [x^2 + (1-x)^2 - 2\epsilon x(1-x)] \\ &\quad \mathcal{M}_{q\bar{q}}^{(b)}(x s, q_1) d\Phi_2^{(x)}. \end{aligned} \quad (2.62)$$

For the short distance cross section one gets then

$$d\hat{\sigma}_{q\bar{q}} = d\hat{\sigma}_{q\bar{q}}^{(c-)} + d\sigma_{q\bar{q}}^{(f)} \quad (2.63)$$

$$d\sigma_{q\bar{q}}^{(c-)} = \frac{T_F \alpha_S}{2\pi} \left[\log \frac{s}{\mu^2} + 2 \log(1-x) \right] (x^2 + (1-x)^2) + 2x(1-x) \left[\mathcal{M}_{q\bar{q}}^{(b)}(xs, q_1) d\Phi_2^{(x)} \right] \quad (2.64)$$

$$d\sigma_{q\bar{q}}^{(f)} = N \frac{s^{-1}}{32\pi^2} \beta_x \cos \theta_1 d\theta_2 dy dx \left(\frac{1}{1+y} \right) f_{q\bar{q}}(x, y, \theta_1, \theta_2) \quad (2.65)$$

where

$$\begin{aligned} f_{q\bar{q}}(x, y, \theta_1, \theta_2) &= -2s u_k \mathcal{M}_{q\bar{q}}^{(r)}(s, t_k, u_k, q_1, q_2) \\ &= s^2(1-x)(1+y) \mathcal{M}_{q\bar{q}}^{(r)}(s, t_k, u_k, q_1, q_2). \end{aligned} \quad (2.66)$$

The expressions for $\mathcal{M}_{q\bar{q}}^{(r)}$ and $\mathcal{M}_{q\bar{q}}^{(f)}$ can be easily obtained directly in four dimensions, using an algebraic manipulation program. We evaluated the real cross section in d dimensions, using two independent programs in Reduce and Macsyma. The collinear and soft terms were extracted algebraically, and we verified that they coincide with eqs. (2.57), (2.58), (2.59) and (2.64). The collinear and soft terms, together with $\mathcal{M}_{q\bar{q}}^{(r)}$ and $\mathcal{M}_{q\bar{q}}^{(f)}$ in four dimensions, and the $\mathcal{M}_{q\bar{q}}^{(u,r,q)}$ contribution given in Appendix B, are all what one needs to compute the full $\mathcal{O}(\alpha_S)$ cross section. For completeness, in Appendix C we give expressions for $\mathcal{M}_{q\bar{q}}^{(r)}$ and $\mathcal{M}_{q\bar{q}}^{(f)}$ in four dimensions. Therefore, our final analytic result is contained in the Appendices B and C, and in eqs. (2.56), (2.39), (2.57), (2.58), (2.60) for the $q\bar{q}$ process, and (2.63), (2.64), (2.65) for the qg process.

2.5. Redefinition of the structure functions

The results presented in the previous subsections are all given in the $\overline{\text{MS}}$ subtraction scheme. If we want to use our results in conjunction with structure function sets defined in a different scheme (like the DFLM^[22] sets) we must correct our short distance cross section in order to account for the redefinition. In general, a redefinition of the structure function will have the form

$$f'_i(x) = f_i(x) + \frac{\alpha_S}{2\pi} \sum_j \int_z^1 K_{ij} \left(\frac{x}{z} \right) f_j(z) \frac{dz}{z} + \mathcal{O}(\alpha_S^2) \quad (2.67)$$

Quantities of physical interest must be independent of the scheme one uses. Therefore, if we have a generic partonic cross section $\sigma_i(xP)$, associated with some hard

event caused by the parton type i carrying a fraction x of the momentum P of the incoming hadron, we must have:

$$\sum_i \int dx f_i(x) \sigma_i(xP) = \sum_i \int dx f'_i(x) \sigma'_i(xP) + \text{NNL} \quad (2.68)$$

where NNL stands for next-to-next-to-leading terms. From eq. (2.67) we get immediately:

$$\sigma'_i(p) = \sigma_i(p) - \frac{\alpha_S}{2\pi} \sum_j \int_0^1 \sigma_j^{(0)}(xp) K_{ji}(x) dx + \text{NNL}. \quad (2.69)$$

For a process initiated by two partons, the formula for the change of scheme becomes

$$\begin{aligned} \sigma'_{ik}(p_1, p_2) &= \sigma_{ik}(p_1, p_2) - \frac{\alpha_S}{2\pi} \sum_j \int_0^1 \sigma_{jk}^{(0)}(xp_1, p_2) K_{ji}(x) dx \\ &\quad - \frac{\alpha_S}{2\pi} \sum_l \int_0^1 \sigma_{il}^{(0)}(p_1, xp_2) K_{lk}(x) dx + \text{NNL}. \end{aligned} \quad (2.70)$$

The transformation functions $K_{ij}(x)$ may contain singularities in x . We will restrict ourselves to the following general form

$$\begin{aligned} K_{ij}(x) &= K_{ij}^{(d)} \delta(1-x) + K_{ij}^{(+)}(x) \left(\frac{1}{1-x} \right)^+ \\ &\quad + K_{ij}^{(l)}(x) \left(\frac{\log(1-x)}{1-x} \right)^+ + K_{ij}^{\text{reg}}(x) \end{aligned} \quad (2.71)$$

where $K_{ij}^{\text{reg}}(x)$ is an integrable function.

The change of scheme modifies both the collinear and the soft-virtual cross sections in the following way. Using the following expression for $K_{ij}(x)$

$$K_{ij}(x) = \bar{K}_{ij}^{(d)} \delta(1-x) + \bar{K}_{ij}(x) \quad (2.72)$$

$$\bar{K}_{ij}^{(d)} = K_{ij}^{(d)} + 2K_{ij}^{(+)}(1) \log \beta + 2K_{ij}^{(l)}(1) \log^2 \beta \quad (2.73)$$

$$\bar{K}_{ij}(x) = K_{ij}^{(+)}(x) \left(\frac{1}{1-x} \right)_p + K_{ij}^{(l)}(x) \left(\frac{\log(1-x)}{1-x} \right)_p + K_{ij}^{\text{reg}}(x) \quad (2.74)$$

we get

$$d\sigma_{q\bar{q}}^{(c+)} = d\sigma_{q\bar{q}}^{(c+)} - \frac{\alpha_S}{2\pi} \bar{K}_{q\bar{q}}(x) \mathcal{M}_{q\bar{q}}^{(b)}(xs, q_1) d\Phi_2^{(x)}$$

$$\begin{aligned}
d\sigma_{q\bar{q}}^{(c-)} &= d\sigma_{q\bar{q}}^{(c-)} - \frac{\alpha_S}{2\pi} \overline{K}_{q\bar{q}}^{(c-)}(x) \mathcal{M}_{q\bar{q}}^{(b)}(xs, q_2) d\Phi_2^{(c-)} \\
d\sigma_{q\bar{q}}^{(sv)} &= d\sigma_{q\bar{q}}^{(sv)} - \frac{\alpha_S}{2\pi} \left[\overline{K}_{q\bar{q}}^{(d)} + \overline{K}_{q\bar{q}}^{(d)} \right] \mathcal{M}_{q\bar{q}}^{(b)}(s, t) d\Phi_2 \\
d\sigma_{q\bar{q}}^{(c-)} &= d\sigma_{q\bar{q}}^{(c-)} - \frac{\alpha_S}{2\pi} \overline{K}_{q\bar{q}}^{(c-)}(x) \mathcal{M}_{q\bar{q}}^{(b)}(xs, q_2) d\Phi_2^{(c-)} \\
d\sigma_{q\bar{q}}^{(sv)} &= d\sigma_{q\bar{q}}^{(sv)} - \frac{\alpha_S}{2\pi} \overline{K}_{q\bar{q}}^{(d)} \mathcal{M}_{q\bar{q}}^{(b)}(s, t) d\Phi_2.
\end{aligned} \tag{2.75}$$

Observe that a two-body contribution may arise also in the quark-gluon short distance cross section.

We give here, for completeness, the transformation functions we need in order to go from the $\overline{\text{MS}}$ scheme to a scheme in which the deep inelastic structure function F_2 receives no corrections to order $\alpha_S^{[20,21]}$, which we will call from now on DIS scheme

$$\begin{aligned}
K_{q\bar{q}}^{(d)} &= -C_F \left(\frac{9}{2} + \frac{\pi^2}{3} \right) \\
K_{q\bar{q}}^{(+)}(x) &= C_F \left[-\frac{3}{2} - (1+x^2) \log(x) + (1-x)(3+2x) \right] \\
K_{q\bar{q}}^{(l)}(x) &= C_F(1+x^2) \\
K_{q\bar{q}}^{\text{reg}}(x) &= T_F \left[(x^2 + (1-x)^2) \log \frac{1-x}{x} + 8x(1-x) - 1 \right] \\
K_{q\bar{q}}^{\text{reg}} &= 0, \quad K_{q\bar{q}}^{(d)} = 0, \quad K_{q\bar{q}}^{(+)} = 0, \quad K_{q\bar{q}}^{(l)} = 0.
\end{aligned} \tag{2.76}$$

Observe that they do not generate any soft-virtual term in the quark-gluon cross section. The above transformation functions must be used in conjunction with the structure functions of ref. [22].

3. Total parton cross sections

The total cross sections for the processes $q\bar{q} \rightarrow ZZ + X$, $qg \rightarrow ZZ + X$ can be written in the following way:

$$\hat{\sigma}_{ij}(s, m^2, \mu^2) = \frac{g_V^4 + g_A^4 + 6g_V^2 g_A^2}{16\pi N_C m^2} h_{ij} \left(\rho, \frac{\mu^2}{m^2} \right), \tag{3.1}$$

where $\rho = 4m^2/s$ and μ is the renormalization scale. The indices i and j refer to the type of the colliding partons. As explained in subsection (2.5), the functions $h_{ij}(\rho, \mu^2/m^2)$ depend on the choice of the renormalization scheme; we will give our results both in the $\overline{\text{MS}}$ and in the DIS scheme defined in subsection (2.5).

The quantities $h_{ij}(\rho, \mu^2/m^2)$ have a perturbative expansion in α_S :

$$h_{ij} \left(\rho, \frac{\mu^2}{m^2} \right) = h_{ij}^{(0)}(\rho) + \frac{\alpha_S}{4\pi} \left[h_{ij}^{(1)}(\rho) + \hat{h}_{ij}^{(1)}(\rho) \log \frac{\mu^2}{m^2} \right] + \mathcal{O}(\alpha_S^2). \tag{3.2}$$

Integrating eq. (2.18) over the two-body phase space one easily obtains

$$h_{q\bar{q}}^{(0)}(\rho) = \rho \left[\frac{1+\rho^2/4}{1-\rho/2} \log \frac{1+\beta}{1-\beta} - \beta \right], \tag{3.3}$$

where $\beta = \sqrt{1-\rho}$, while $h_{q\bar{q}}^{(0)}(\rho) = 0$ since there is no $\mathcal{O}(\alpha_S^0)$ contribution to the qg process. The function $h_{q\bar{q}}^{(0)}(\rho)$ is plotted in fig. 5.

In eq. (3.2), the dependence on the renormalization scale μ has been shown explicitly. The coefficient functions $\hat{h}_{ij}^{(1)}$ can be expressed in term of the lowest order functions $h_{ij}^{(0)}$:

$$\hat{h}_{ij}^{(1)}(\rho) = -2 \sum_k \int_\rho^1 P_{ki}(x) h_{kj}^{(0)} \left(\frac{\rho}{x} \right) dx - 2 \sum_k \int_\rho^1 P_{kj}(x) h_{ki}^{(0)} \left(\frac{\rho}{x} \right) dx. \tag{3.4}$$

The above equation is obtained by differentiating both sides of eq. (2.52) with respect to $\log \mu^2$. The left-hand side of eq. (2.52) is μ -independent, and the derivative of the structure functions are given by the Altarelli-Parisi equation. In our case we get

$$\begin{aligned}
\hat{h}_{q\bar{q}}^{(1)}(\rho) &= -4 \int_\rho^1 P_{q\bar{q}}(x) h_{q\bar{q}}^{(0)} \left(\frac{\rho}{x} \right) dx \\
\hat{h}_{qg}^{(1)}(\rho) &= -2 \int_\rho^1 P_{qg}(x) h_{q\bar{q}}^{(0)} \left(\frac{\rho}{x} \right) dx
\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}
P_{q\bar{q}}(x) &= C_F \left[\frac{1+x^2}{(1-x)_\rho} + \left(\frac{3}{2} + 4 \log \beta \right) \delta(1-x) \right] \\
P_{qg}(x) &= T_F \left[x^2 + (1-x)^2 \right]
\end{aligned} \tag{3.6}$$

are the relevant Altarelli-Parisi kernels.

We do not have analytic expressions for $h_{ij}^{(1)}(\rho)$ and $\hat{h}_{ij}^{(1)}(\rho)$. We will provide for them a fit that agrees to better than 1% with the results obtained by numerical integration of the formulae of the previous Section.

For the $q\bar{q}$ process, the behaviour of $h_{q\bar{q}}^{(1)}(\rho)$ and $\hat{h}_{q\bar{q}}^{(1)}(\rho)$ near the threshold of ZZ production, $s \sim 4m^2$ or $\rho \sim 1$, is dominated by the contribution of the soft-virtual term, and therefore can be read off eq.(2.60). We find

$$\begin{aligned} h_{q\bar{q}}^{(1)\overline{\text{MS}}}(\rho) &= \left\{ C_F \left[(6 + 16 \log \beta) \log \frac{s}{m^2} + 32 \log^2 \beta - \frac{4}{3} \pi^2 \right] \right. \\ &+ a_0 + a_1 \rho - a_2 \log \beta - a_3 \rho \log \beta - a_4 \log \rho \\ &- \left. a_5 \rho \log \rho + a_6 \log^2 \rho + a_7 \rho \log^2 \rho \right\} h_{q\bar{q}}^{(0)}(\rho) \end{aligned} \quad (3.7)$$

$$\begin{aligned} h_{q\bar{q}}^{(1)\text{DIS}}(\rho) &= \left\{ C_F \left[(6 + 16 \log \beta) \log \frac{s}{m^2} + 16 \log^2 \beta + 12 \log \beta + 18 \right] \right. \\ &+ b_0 + b_1 \rho - b_2 \log \beta - b_3 \rho \log \beta - b_4 \log \rho \\ &- \left. b_5 \rho \log \rho + b_6 \log^2 \rho + b_7 \rho \log^2 \rho \right\} h_{q\bar{q}}^{(0)}(\rho) \end{aligned} \quad (3.8)$$

$$\begin{aligned} \hat{h}_{q\bar{q}}^{(1)}(\rho) &= \left\{ -C_F(6 + 16 \log \beta) \right. \\ &+ c_0 + c_1 \rho - c_2 \log \beta - c_3 \rho \log \beta - c_4 \log \rho \\ &- \left. c_5 \rho \log \rho + c_6 \log^2 \rho + c_7 \rho \log^2 \rho \right\} \hat{h}_{q\bar{q}}^{(0)}(\rho), \end{aligned} \quad (3.9)$$

with the coefficients a_i , b_i and c_i given in Table I. The functions $h_{q\bar{q}}^{(1)\overline{\text{MS}}}(\rho)$, $h_{q\bar{q}}^{(1)\text{DIS}}(\rho)$ and $\hat{h}_{q\bar{q}}^{(1)}(\rho)$ are plotted in fig. 6.

In the qg case our result is

$$\begin{aligned} h_{qg}^{(1)\overline{\text{MS}}}(\rho) &= \left[d_0 + d_1 \rho - d_2 \log \beta - d_3 \rho \log \beta - d_4 \log \rho \right. \\ &- \left. d_5 \rho \log \rho + d_6 \log^2 \rho \right] \rho \beta \log \rho \end{aligned} \quad (3.10)$$

$$\begin{aligned} h_{qg}^{(1)\text{DIS}}(\rho) &= \left[e_0 + e_1 \rho - e_2 \log \beta - e_3 \rho \log \beta - e_4 \log \rho \right. \\ &- \left. e_5 \rho \log \rho + e_6 \log^2 \rho \right] \rho \beta \log \rho \end{aligned} \quad (3.11)$$

j	a_j	b_j	c_j
0	30.2	21.075	-0.16875
1	-2.56	-5.5530	6.9135
2	-67.5	-29.430	-2.4114
3	93.0	42.245	2.3582
4	-6.96	-16.597	-1.2961
5	17.9	9.5026	-3.5857
6	4.82	4.2457	-0.092729
7	0	-12.941	5.2871

Table I: Coefficients for the fit of $h_{q\bar{q}}^{(1)\overline{\text{MS}}}(\rho)$, $h_{q\bar{q}}^{(1)\text{DIS}}(\rho)$ and $\hat{h}_{q\bar{q}}^{(1)}(\rho)$.

j	d_j	e_j	f_j
0	-2.431	-2.957	0.6104
1	6.096	-0.8835	2.366
2	-12.46	3.134	-2.778
3	22.93	2.552	2.665
4	-0.4829	-0.1864	0.2999
5	5.339	6.566	-0.4115
6	-0.6835	-0.5562	-0.01072

Table II: Coefficients for the fit of $h_{qg}^{(1)\overline{\text{MS}}}(\rho)$, $h_{qg}^{(1)\text{DIS}}(\rho)$ and $\hat{h}_{qg}^{(1)}(\rho)$.

$$\begin{aligned} \hat{h}_{qg}^{(1)}(\rho) &= \left[f_0 + f_1 \rho - f_2 \log \beta - f_3 \rho \log \beta - f_4 \log \rho \right. \\ &- \left. f_5 \rho \log \rho + f_6 \log^2 \rho \right] \rho \beta \log \rho, \end{aligned} \quad (3.12)$$

with d_i , e_i and f_i given in Table II. The functions $h_{qg}^{(1)\overline{\text{MS}}}(\rho)$, $h_{qg}^{(1)\text{DIS}}(\rho)$ and $\hat{h}_{qg}^{(1)}(\rho)$ are plotted in fig. 7.

4. Phenomenological applications

In this Section we present results for the total cross section for Z pair production that are relevant to phenomenological applications. In order to give a consistent result at the next-to-leading level, we must use sets of structure functions which are accurate to next-to-leading order. We have chosen as reference set the DFLM^[22] set with $\Lambda_4 = 260$ MeV. We use the two-loop expression for α_s .

The weak couplings of the Z boson to up and down quarks are given by

$$\begin{aligned} g_V^{up} &= \frac{g}{2 \cos \theta_W} \left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \right) \\ g_A^{up} &= \frac{g}{2 \cos \theta_W} \frac{1}{2} \\ g_V^{down} &= \frac{g}{2 \cos \theta_W} \left(-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \right) \\ g_A^{down} &= -\frac{g}{2 \cos \theta_W} \frac{1}{2}, \end{aligned} \quad (4.1)$$

where θ_W is the weak mixing angle. Furthermore we set $M_Z = 91.18$ GeV, $\sin^2 \theta_W = 0.228$, $\alpha_{em} = 1/128$ and $g^2 = 4\pi\alpha_{em}/\sin^2 \theta_W$.

We begin by considering the scale dependence of our final result. In Figs. 8 and 9 we plot the total cross section as a function of the scale μ for $\sqrt{S} = 2$ TeV and 16 TeV respectively. Both the leading and next-to-leading results are shown. We can see that in both cases the next-to-leading result is less sensitive to scale changes, as expected. At LHC energy we observe that the scale dependence of the next-to-leading result is surprisingly mild. Looking at the LHC energy only, one would be tempted to conclude that $\mathcal{O}(\alpha_s^2)$ effects are indeed very small. On the other hand, the much stronger scale dependence that we observe at 2 TeV indicates that some accidental cancellation may take place at 16 TeV. We have verified that this is the case. We observed that the structure functions in the relevant x region grow with μ at LHC energy, and decrease with μ at 2 TeV, the turning point being around 6 TeV. This is also apparent from the slope of the Born cross section in Figs. 8 and 9. It turns out that the growth of the structure functions around $\sqrt{S} = 16$ TeV compensates the decrease of α_s in the $\mathcal{O}(\alpha_s)$ contribution to the cross section. Since the remaining scale dependence cancels

		DFLM160	DFLM260	DFLM360	HMRSE	HMRSE
$p\bar{p}$	Born	0.0551 ^{+0.01} _{-0.007}	0.0475 ^{+0.01} _{-0.007}	0.0417 ^{+0.01} _{-0.007}	0.0882 ^{+0.01} _{-0.009}	0.0571 ^{+0.01} _{-0.008}
	Full	0.0693 ^{+0.003} _{-0.004}	0.0610 ^{+0.003} _{-0.004}	0.0542 ^{+0.003} _{-0.004}	0.0889 ^{+0.003} _{-0.005}	0.0766 ^{+0.003} _{-0.005}
$p\bar{p}$	Born	0.925 ^{+0.07} _{-0.06}	0.869 ^{+0.07} _{-0.06}	0.820 ^{+0.07} _{-0.06}	1.02 ^{+0.07} _{-0.04}	0.972 ^{+0.03} _{-0.07}
	Full	1.17 ^{+0.04} _{-0.04}	1.12 ^{+0.03} _{-0.04}	1.07 ^{+0.06} _{-0.03}	1.25 ^{+0.04} _{-0.04}	1.22 ^{+0.03} _{-0.04}
pp	Born	3.00 ^{+0.07} _{-0.1}	3.02 ^{+0.05} _{-0.1}	2.99 ^{+0.02} _{-0.07}	3.01 ^{+0.1} _{-0.2}	3.29 ^{+0.1} _{-0.1}
	Full	3.97 ^{+0.08} _{-0.08}	4.06 ^{+0.1} _{-0.1}	4.06 ^{+0.1} _{-0.1}	3.80 ^{+0.06} _{-0.03}	4.21 ^{+0.09} _{-0.05}
pp	Born	9.8 ^{+0.9} _{-1.1}	10.7 ⁺¹ ₋₁	11.4 ⁺¹ ₋₁	8.13 ^{+0.9} ₋₁	9.75 ⁺¹ ₋₁
	Full	13.7 ^{+0.04} _{-0.08}	15.2 ^{+0.01} _{-0.02}	16.3 ^{+0.1} _{-0.1}	11.0 ^{+0.1} _{-0.1}	13.2 ^{+0.1} _{-0.08}
pp	Born	24.2 ⁺⁴ ₋₄	29.2 ⁺⁴ ₋₄	34.0 ⁺⁵ ₋₅	17.7 ⁺⁴ ₋₃	23.5 ⁺⁴ ₋₄
	Full	35.8 ⁺¹ ₋₁	43.6 ⁺¹ ₋₁	50.8 ^{+0.7} ₋₁	25.6 ^{+0.9} ₋₁	33.9 ⁺¹ ₋₁
pp	Born	48.6 ⁺¹² ₋₁₁	64.6 ⁺¹⁴ ₋₁₄	82.2 ⁺¹⁷ ₋₁₈	36.8 ⁺¹⁰ ₋₉	53.2 ⁺¹³ ₋₁₃
	Full	77.4 ⁺³ ₋₅	103 ⁺³ ₋₅	131 ⁺³ ₋₆	56.3 ⁺³ ₋₅	81.3 ⁺³ ₋₆

Table III: Cross sections (in pb) for Z pair production at various colliders, and for different sets of structure functions. The scale μ is chosen equal to 90 GeV, and the errors quoted correspond to $\mu = 45$ and $\mu = 180$ GeV. Both the Born and the full (Born + $\mathcal{O}(\alpha_s)$) cross sections are shown.

because of the renormalization group equations, this results in an almost exact scale independence. On the other hand, at 2 TeV the scale dependence of α_s and of the structure functions go in the same direction, and strengthen each other, as is clear from Fig. 8. In the following we will quote cross sections together with their variation coming from taking $\mu = 45, 90, 180$ GeV, keeping however in mind that this cannot always be considered as a good estimate of the theoretical error due to $\mathcal{O}(\alpha_s^2)$ and higher order corrections.

In Fig. 10 we plot the cross section for Z pair production versus energy. We see that radiative corrections are positive and of the order of 20% to 30% in the energy range considered.

In Table III we give cross sections for Z pair production for energies of interest,

using the three sets of DFLM structure functions with $\Lambda_s = 160, 260$ and 360 MeV, and the two sets of structure functions from ref. [23]. We see that most of the uncertainty in the cross section comes from structure function dependence. The most extreme case is always given by the HMRSE set, which is based upon EMC data. If we exclude this set, we find that at LHC energy the cross section ranges between 13.1 and 16.4 picobarns, while at the SSC it goes from 32.9 to 51.5 picobarns.

Acknowledgements

We wish to thank G. Altarelli and C. M. Becchi for helpful discussions.

APPENDIX A: One loop integrals

In this Appendix we list the d -dimensional loop integrals that appeared in the calculation of the virtual graphs. The kinematic invariants are given by

$$s = (p_1 + p_2)^2, \quad t = (p_1 - k_1)^2, \quad k_1^2 = (p_1 + p_2 - k_1)^2 = m^2 \quad (\text{A.1})$$

Our integrals are performed in $d = 4 - 2\epsilon$ dimensions. In the following formulae, terms of order ϵ are always dropped. The integrals are given for $t < 0$, $s > 0$, $m^2 > 0$. For completeness, we include the imaginary parts which arise when continuing from the euclidean region to positive s and m^2 , although they do not contribute to the final result since they cancel between complex conjugate graphs. The box diagram is given by:

$$\begin{aligned} I_4 &= \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l+p_1)^2(l+p_1-k_1)^2(l-p_2)^2} \\ &= N i \frac{s^{-\epsilon}}{st} \left[\frac{1}{\epsilon^2} - \frac{2}{\epsilon} \log \frac{-t}{m^2} - 4 \log \frac{m^2-t}{m^2} \log \frac{-t}{m^2} + 2 \log^2 \frac{-t}{m^2} - 4 \text{Li}_2 \frac{t}{m^2} \right. \\ &\quad \left. - i\pi \left(\frac{1}{\epsilon} + 4 \log \frac{m^2-t}{m^2} - 2 \log \frac{-t}{m^2} \right) \right] \end{aligned} \quad (\text{A.2})$$

where the dilogarithm is defined as

$$\text{Li}_2(z) = - \int_0^z \frac{dz}{z} \log(1-z) \quad (\text{A.3})$$

and the normalization factor N is given by

$$N = \frac{1}{16\pi^2} (4\pi)^\epsilon \Gamma(1+\epsilon) \quad (\text{A.4})$$

Two of the three triangle graphs we encounter are singular when $\epsilon \rightarrow 0$. They are

$$\begin{aligned} I_{3s} &= \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l+p_1)^2(l-p_2)^2} = N i \frac{s^{-1-\epsilon}}{\epsilon^2} \left[1 + i\epsilon\pi - \frac{2}{3}\pi^2\epsilon^2 \right] \quad (\text{A.5}) \\ I_{3t} &= \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l+p_1)^2(l+p_1-k_1)^2} \\ &= N i \frac{s^{-\epsilon}}{m^2-t} \left[\frac{1}{\epsilon} \log \frac{-t}{m^2} + \frac{1}{2} \log^2 \frac{m^2}{s} - \frac{1}{2} \log^2 \frac{-t}{s} - \frac{\pi^2}{2} + i\pi \left(\frac{1}{\epsilon} - \log \frac{m^2}{s} \right) \right] \end{aligned} \quad (\text{A.6})$$

The third triangle graph is given by

$$I_{3m} = \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l+p_1)^2(l+p_1-k_1)^2(l-p_2)^2} = Ni \frac{1}{s\beta} \left[\frac{4\text{Li}_2}{1+\beta} \left(\frac{1-\beta}{1+\beta} \right) + \frac{\pi^2}{3} + \log^2 \frac{1-\beta}{1+\beta} \right] \quad (\text{A.7})$$

where $\beta = \sqrt{1-4m^2/s}$. Finally, the two-propagator integrals are

$$I_2 = \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2(l+q)^2} = Ni \frac{(-q^2)^{-\epsilon}}{\epsilon} (1+2\epsilon) \quad (\text{A.8})$$

and they arise with $q^2 = s, t$, and m^2 .

It is possible to perform a very valuable semi-numerical test of the virtual integrals. We observe that the integral

$$I_f = \int \frac{d^4 l}{(2\pi)^4} \frac{1+A((l+p_1)^2+(l-p_2)^2)+B(l+p_1-k_1)^2}{l^2(l+p_1)^2(l-p_2)^2(l+p_1-k_1)^2} \quad (\text{A.9})$$

$$A = \frac{m^2-t}{ts}, \quad B = -\frac{1}{t}$$

has no collinear or soft divergences. In fact, expanding the numerator one obtains

$$I_f = \int \frac{d^4 l}{(2\pi)^4} \frac{(2A+B)l^2 + 2l \cdot (A(p_1-p_2) + B(p_1-k_1))}{l^2(l+p_1)^2(l-p_2)^2(l+p_1-k_1)^2} \quad (\text{A.10})$$

The numerator has no terms of the order l^0 . Furthermore, the term of order l involves a projection of l in a direction orthogonal to p_1 and p_2 , since

$$p_1 \cdot (A(p_1-p_2) + B(p_1-k_1)) = 0$$

$$p_2 \cdot (A(p_1-p_2) + B(p_1-k_1)) = 0 \quad (\text{A.11})$$

and therefore damps the soft and collinear divergences. The terms of order l^2 are trivially finite. From eq. (A.9) one obtains immediately the relation

$$I_f = I_4 + 2AI_{3l} + BI_{3s} \quad (\text{A.12})$$

One can easily verify that the combination on the right hand side of eq. (A.12) is indeed finite. The integral I_f can be easily performed semi-numerically, and we checked eq. (A.12) numerically with five digit accuracy.

APPENDIX B: Virtual contribution to the cross section

$$\mathcal{M}_{q\bar{q}}^{(v)} = -4\pi\alpha_s C_F N \left(\frac{s}{\mu^2} \right)^{-\epsilon} \left(\frac{4}{\epsilon^2} + \frac{6}{\epsilon} \right) \mathcal{M}_{q\bar{q}}^{(v)} + \mathcal{M}_{q\bar{q}}^{(v, \text{res})} \quad (\text{B.1})$$

$$\begin{aligned} \mathcal{M}_{q\bar{q}}^{(v, \text{res})} = & (g_V^4 + g_A^4 + 6g_V^2 g_A^2) \frac{4\pi\alpha_s C_F N}{N_C} \frac{1}{2} \left\{ 2 \frac{(t+m^2)^2}{\beta^2 stu} + \frac{4s}{(t-m^2)u} \right. \\ & - \frac{16t^3 + (28s-68m^2)t^2 + (18s^2-36m^2s+88m^4)t + 18m^4s-36m^6}{t^2 su} \\ & + \left[\frac{12s}{(t-m^2)u} - \frac{4m^2s}{(t-m^2)^2 u} + 2 \frac{t+4s}{su} - \frac{s^2+m^4}{6stu} + 6m^4 \frac{2m^2-s}{t^2 su} \right] \log \frac{-t}{m^2} \\ & + \left[- \frac{5t^3 + (8s-18m^2)t^2 + (6s^2+25m^4)t + 6m^4s-12m^6}{t^2 su} \right. \\ & - \frac{12m^2(t+m^2)^2}{\beta^4 s^2 tu} + \frac{3t^2-26m^2t-25m^4}{\beta^2 stu} \left. \right] \log \frac{s}{m^2} \\ & + \left[\frac{-2t^2+8m^2t-2s^2-12m^4}{u} + \frac{4m^4}{4m^4} \frac{2m^2-s}{tu} \right] \\ & + \left[\frac{t^2-5m^2t+s^2+10m^4}{su} + 4m^4 \frac{-s^2+2m^2s-10m^4}{stu} + 8m^4 \frac{2m^2-s}{t^2 su} \right] \\ & + \left[\frac{(2s-3m^2)t^2 + (6m^4-8m^2s)t + 2s^3-4m^2s^2+12m^4s-3m^6}{stu} \right. \\ & + \frac{12m^4(t+m^2)^2}{\beta^4 s^2 tu} - \frac{m^2t^2-30m^4t-27m^6}{\beta^2 stu} \left. \right] \\ & + \left[\frac{t+4s-4m^2}{3su} + \frac{4(s-2m^2)^2}{3stu} \right] \pi^2 \left. \right\} + (t \leftrightarrow u) \quad (\text{B.2}) \end{aligned}$$

APPENDIX C: Real matrix elements

$$\begin{aligned}
\mathcal{M}_{q\bar{q}}^{(\nu)} = & (g_V^4 + g_A^4 + 6g_V^2 g_A^2) \frac{4\pi\alpha_s C_F}{N_C} \\
& \frac{1}{2} \left\{ - \frac{t_k u_k^2 + 2s u_k^2 - t_k^2 u_k - 2st_k u_k + m^2(t_k^2 - u_k^2 + 2s u_k - 2st_k - 2s^2)}{\hat{q}_1 \hat{q}_2 s u_k} \right. \\
& + \frac{t_k u_k^2 - m^2 u_k(s + 3t_k) + m^4(2u_k - s)}{2 \hat{q}_1 \hat{q}_2 s t_k u_k} \\
& + \frac{t_k u_k(u_k + s) - m^2(u_k^2 + 3t_k u_k + 3s u_k + st_k) + 2m^4(u_k + t_k + 2s)}{\hat{q}_1 \hat{q}_2 s t_k u_k} \\
& + \frac{t_k(u_k^2 + t_k u_k - s^2) + m^2(4s u_k - 3t_k u_k - t_k^2 + 4s^2)}{\hat{q}_1 \hat{q}_2 s t_k u_k} \\
& - \frac{t_k u_k + s u_k - st_k - s^2 + 2m^2(s - t_k)}{\hat{q}_1 \hat{q}_2 s u_k} + \frac{t_k u_k - s u_k - 2st_k - 2s^2}{\hat{q}_1 \hat{q}_2 s t_k u_k} \\
& + \frac{t_k u_k - t_k^2 - st_k - s^2 + m^2(2s - u_k)}{\hat{q}_1 s t_k u_k} - \frac{2m^2 u_k^2 - 2m^2 u_k + 2m^4}{\hat{q}_1^2 \hat{q}_2 s u_k} \\
& + \frac{2s u_k + t_k^2 + 3st_k + 2s^2}{\hat{q}_1 s t_k u_k} + \frac{q_1(u_k + s)(u_k + t_k)}{\hat{q}_1 \hat{q}_2 s t_k u_k} \\
& + \frac{t_k u_k + s u_k + 3st_k + 2s^2 - m^2(u_k + t_k + 2s)}{\hat{q}_1 \hat{q}_2 s t_k} \\
& + \frac{u_k - t_k}{2\hat{q}_1 \hat{q}_2 s} \left[\frac{q_1(u_k + s)}{q_2 t_k} - \frac{q_2(t_k + s)}{q_1 u_k} \right] \\
& + \frac{(t_k - 2m^2)(u_k - 2m^2)}{\hat{q}_1 \hat{q}_2 q_2} - \frac{q_1^2 + q_2^2}{\hat{q}_1 \hat{q}_2 t_k u_k} - \frac{2m^2(q_2 - 2m^2)}{q_1^2 s u_k} \Bigg\} \\
& + (t_k \leftrightarrow u_k, q_1 \leftrightarrow q_2, \hat{q}_1 \leftrightarrow \hat{q}_2) \\
& + (q_1 \leftrightarrow \hat{q}_1, q_2 \leftrightarrow \hat{q}_2) \\
& + (t_k \leftrightarrow u_k, q_1 \leftrightarrow \hat{q}_1, q_2 \leftrightarrow \hat{q}_1). \tag{C.1}
\end{aligned}$$

The cross section for the quark-gluon subprocess is easily obtained by crossing:

$$\mathcal{M}_{q\bar{q}}^{(\nu)} = - \frac{t_k}{s} \frac{\bar{I}F}{C_F} \bar{\mathcal{M}}_{q\bar{q}}^{(\nu)} \tag{C.2}$$

where $\bar{\mathcal{M}}_{q\bar{q}}^{(\nu)}$ is obtained from $\mathcal{M}_{q\bar{q}}^{(\nu)}$ by applying the following substitutions

$$s \leftrightarrow t_k, q_2 \leftrightarrow w_2, \hat{q}_2 \leftrightarrow w_1. \tag{C.3}$$

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Figure Captions

Fig. 1: Born diagrams for the hadronic production of a pair of Z bosons.

Fig. 2: Virtual diagrams.

Fig. 3: Real diagrams.

Fig. 4: Quark-gluon graphs.

Fig. 5: The function $h_{q\bar{q}}^{(0)}(\rho)$.

Fig. 6: The functions $h_{q\bar{q}}^{(1)\overline{MS}}(\rho)$, $h_{q\bar{q}}^{(1)DIS}(\rho)$ and $\hat{h}_{q\bar{q}}^{(1)}(\rho)$.

Fig. 7: The functions $h_{qg}^{(1)\overline{MS}}(\rho)$, $h_{qg}^{(1)DIS}(\rho)$ and $\hat{h}_{qg}^{(1)}(\rho)$.

- Fig. 8: Total cross section for Z pair production in pp collisions at $\sqrt{S} = 2$ TeV, as a function of the subtraction scale μ . The structure functions are the DFLM set with $\Lambda_4 = 260$ MeV.
- Fig. 9: Same as in Fig. 8, for $\sqrt{S} = 16$ TeV.
- Fig. 10: Total cross section for Z pair production in pp collisions as a function of the center of mass energy.

Fig. 1

+ crossed diagrams

Fig. 3

+ crossed diagrams

Fig. 4

+ crossed diagrams

Fig. 2

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10