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CP Violation in Higher Dimensional Theories

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Abstract

It is shown how the CP violation in our 4-dimensional world is intimately related with the complex structure of the compactified space in general higher dimensional theories. We also point out that having fermions with definite higher dimensional chirality, as in the $D = 10$ Type I superstring theory, is another essential ingredient for realizing CP violation. In a “non-chiral” case, like in Type IIa theory, CP will not be broken explicitly even for the Calabi-Yau manifold determined by complex parameter(s). In a typical model of Type I, a phenomenological requirement turns out to prevent the theory from acquiring complex Yukawa couplings.

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There still seems to be no conclusive argument concerning the origin of CP violation, in spite of the recent impressive result at LEP supporting the three generation model by Kobayashi and Maskawa (K-M) [1]. In four-dimensional ($D=4$) theories the mechanisms of CP violation so far proposed are either due to explicit breaking by complex Yukawa couplings à la K-M model [1], or due to spontaneous breaking by the complex vacuum expectation values (VEV's) of Higgs fields [2]. We may have some new mechanism of the CP violation once the space-time is extended to higher dimensional ones, as in the (super-) string theories or Kaluza-Klein type theories. In fact the possibility of CP violation in our four-dimensional world due to the specific sort of compactification of the extra space in superstring theories, or in $D=10$ super Yang-Mills theories, has been discussed [3].

It should, however, be noticed that it is non-trivial to achieve the CP violation starting from the super Yang-Mills theory, whose Lagrangian has no complex parameter. On the other hand, if realized, it will serve as a new type of CP breaking mechanism. As long as getting CP violation is a non-trivial task, the condition for breaking CP may be utilized in order to restrict viable theories and/or the possible manner of extra-space compactification.

In this paper it is pointed out that the straightforward (simple-minded) generalizations of the 4-dimensional (4-dim.) CP operator to the higher dimensional cases does not yield desirable operators, which reduce to the original 4-dim. one after the dimensional reduction. Therefore the modification of the operators is necessary, and we will see how such modified operators are related to the so-called complex structure of the compactified spaces. This relation has been discussed for $D=10$ case in Ref.[3] by using a somewhat mathematical terminology. Here we will try to understand it in a more physics-oriented way. We will further take, as a concrete example, a model based on Calabi-Yau (C-Y) compactification [4,5], where the Yukawa coupling has been explicitly written down starting from first principle. The derived Yukawa coupling will be used to analyze if the possible complex phase(s) in Yukawa coupling can "survive" as physically meaningful CP violating phase(s). Other interesting issues to be discussed are whether the mechanism of CP violation is operative in other dimensionalities of extra space, and also whether it has something to do with the chirality (in the higher dimensional sense) of the adopted fermion.

An interesting thing to be noted is the similarity of the mechanism under consideration with the spontaneous breaking of CP [2] usually discussed in $D=4$. For example, the $D=10$ super Yang-Mills theory has only real gauge coupling and it is the compactification, fixing the vacuum state of the theory, which may break CP. (We, nevertheless, have to be aware of the fact that in odd-dimensional space-time CP can be broken explicitly, even if the parameters are real [6].) The CP symmetry, therefore, is expected to restore at very high energies, $E \gg M_c$, with M_c being the compactification scale ($\sim M_P$), or in other words the decoupling of heavy particles is essential to get the breaking.

To interpret the possible complex phase appearing after the compactification as the consequence of some sort of Higgs VEV's, following this similarity, is suggestive. Let us recall that four dimensional Yukawa couplings are given as the result of overlap integral of zero mode wave functions on the compactified space. Symbolically (in D=10 case),

$$g_{ijk} = \int d^6y \sqrt{g} \psi_i^0(y)^\dagger A_j^0(y) \psi_k^0(y), \quad (1)$$

where ψ_i^0 and A_j^0 are zero modes coming from D=10 fermion and gauge fields, respectively. Equation (1) should be the Yukawa coupling of the corresponding 4-dim. fermion and scalar fields, $\psi_i(x)$, etc.. The zero mode wave functions in (1) might be regarded as a sort of Higgs VEV's, since they are scalars in the 4-dim. sense. Once we bravely identify $\psi_i^0(y)$ as v_i (VEV), etc., the possible complex phase of g_{ijk} comes from the phase of $v_i^* v_j v_k$. One may, however, argue that the Yukawa coupling can be made all real by the "re-phasing" of the each 4-dim. field, $\psi_i(x) \rightarrow e^{i\phi_i} \psi_i(x)$ etc., chosen such that ϕ_i cancel the phase of v_i etc.. This is really the case when the compactification is, e.g., on a torus or a sphere, as we will see briefly below. The analogy discussed above, however, is not in its strict sense, as the zero modes are not just constants but the Yukawa couplings are obtained by the "averaging" over the compactified space. The argument anyway demonstrates the importance of checking the freedom of re-phasing, and also the non-triviality of getting CP violation.

First we have to answer a fundamental question, i.e., whether the "ordinarily" defined CP operator in D-dim. world reduces to the 4-dim. CP, whose breaking is of our real physical interest, after the compactification. As a typical example, we first consider D=10 type I (or N=1) super Yang-Mills theory, the field theoretical limit of the heterotic string.

The essential point is that only half of the full spinor, i.e., 16- component Majorana-Weyl spinor is used in the Type I theory. Since the spinor is Majorana, the charge-conjugation operator, ordinarily defined, naturally reduces to the 4-dim. one. The problem is in the parity operator. One may, as usual, define the operator on a fermion by $\psi \rightarrow \gamma_0^{(10)} \psi$ ($\gamma_M^{(D)}$ denote D-dim. gamma matrices $M=0 \sim D-1$). This operator, however, transforms ψ to a fermion with opposite chirality, simply because $\{\gamma_{FIVE}^{(10)}, \gamma_0^{(10)}\} = 0$, with $\gamma_{FIVE}^{(D)}$ being the chiral operator for even D dimension. In order to explain how the parity operator should be modified, it will be useful to write down D=10 gamma matrices $\gamma_M^{(10)}$ in the form of direct product of 4-dim. spinor space and so-called internal space parts (we use here a basis adopted in Ref.[7]),

$$\begin{aligned} \gamma_\mu \otimes \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} & \quad \text{for } M = 0 \sim 3 \\ iI \otimes \begin{pmatrix} 0 & -\alpha_i \\ \alpha_i & 0 \end{pmatrix} & \quad \text{for } M = 4 \sim 6 \\ \gamma_5 \otimes \begin{pmatrix} 0 & \beta_i \\ \beta_i & 0 \end{pmatrix} & \quad \text{for } M = 7 \sim 9, \end{aligned} \quad (2)$$

where γ_μ, γ_5 are ordinary 4-dim. matrices, I is a 4×4 unit matrix and α_i, β_j are 4×4 matrices satisfying $\{\alpha_i, \alpha_j\} = -2\delta_{ij}$, $[\alpha_i, \alpha_j] = -2\epsilon_{ijk}\alpha_k$, $[\alpha_i, \beta_j] = 0$, etc.. This is the "chiral-basis", i.e.,

$$\gamma_{FIVE}^{(10)} = I \otimes \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad (3)$$

and the 16-component spinor used for the Type I theory is, say, just the upper half of the full spinor. The charge-conjugation matrix $C^{(10)}$ satisfying, $C^{(10)\dagger} \gamma_M^{(10)} C^{(10)} = -(\gamma_M^{(10)})^T$, is given by (C being the 4-dim. charge-conjugation matrix $C = i\gamma_0\gamma_2$),

$$C^{(10)} = C \otimes \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \quad (4)$$

Then the charge-conjugation operator leads to $C^{(10)} : \psi \rightarrow \psi^C = C^{(10)} \bar{\psi}^T = (C\gamma_0 \otimes I_8) \psi^*$, where I_8 is a 8×8 unit matrix. The presence of I_8 clearly shows that the operator just reduces to the 4-dim. charge-conjugation. As for the parity, however, the straightforward extension of the 4-dim. case just does not work, as was explained above. We thus have to modify it into an operator defined by,

$$P^{(10)} : \psi \rightarrow (\gamma_0 \otimes I_8) \psi \equiv \Gamma_0 \psi, \quad (5)$$

which obviously reduces to the 4-dim. one, we are familiar with.

A remarkable outcome of the 10-dim. CP operator, $C^{(10)} P^{(10)}$, such modified is its intimate relation with the complex structure of the extra 6-dim. space. To see this connection we have to establish the transformation property of a vector, $X_M \equiv \bar{\psi} \gamma_M^{(10)} \psi$. The charge-conjugation operator causes the transformation, $C^{(10)} : X_M \rightarrow -X_M$, as usual, while the modified parity operator, Eq.(5), yields the following transformation for the extra space coordinates,

$$P^{(10)} : X_M \rightarrow \begin{cases} X_M & \text{for } M = 4 \sim 6 \\ -X_M & \text{for } M = 7 \sim 9 \end{cases}, \quad (6)$$

though the 4-dim. coordinates transform in the usual way. This rather funny transformation property can be easily shown by considering $[\Gamma_0, \gamma_M^{(10)}]$. By suitably choosing the transformation property of A_M , we can make the Lagrangian invariant under the $C^{(10)} P^{(10)}$. Once three complex coordinates Z_a are introduced as $Z_a = X_{6+a} + iX_{3+a}$ ($a = 1, 2, 3$), the transformation just corresponds to the complex conjugation of Z_a ,

$$C^{(10)} P^{(10)} : Z_a \rightarrow Z_a^*. \quad (7)$$

Now the intimate relation of the operator with the complex structure of the manifold is obvious. (Let us note that, by suitable $SO(6)$ rotation, we can always move to a basis where complex homogeneous coordinates are made just by the pairing shown above.)

To get CP violation, however, just having complex structure is not enough and the way of compactification has to be carefully chosen. To illustrate how the mechanism works, we take a model with 4-generations of quarks and leptons [4,5], whose compactified C-Y manifold is determined on CP^4 by a “defining equation” for 5 homogeneous complex coordinates,

$$\sum_{a=1}^5 Z_a^5 - C(Z_1 Z_2 \cdots Z_5) = 0. \quad (8)$$

If the parameter C is real the defining equation is invariant under (7) and, therefore, the Lagrangian of the resulting 4-dim. theory should preserve CP. The CP violation thus necessitates a complex parameter C [3]. The explicit construction of the Yukawa coupling [4,5] actually confirms this anticipation, as we will see below. The situation should be compared with the case of spheres. For example, a sphere S^{2n-1} can be regarded as a manifold with defining equation, $\sum_{a=1}^n Z_a Z_a^* - R^2 = 0$, where R , being the radius, is a real parameter and there should not be CP violation in the resulting Yukawa coupling. In general, however, we may have some complex phases even for spheres. For simplicity, let us take the S^1 case. The zero modes on the circle are just Fourier series, $\exp\{i[(my/R) + \phi_m]\}$ (y : coordinate of S^1 , m : integer). The Yukawa couplings are readily calculated in a similar way with Eq.(1) and are given by $g_{ijk} = (2\pi R) \delta_{-i+j+k} \exp\{i(-\phi_i + \phi_j + \phi_k)\}$, for zero modes characterized by integers i, j and k . As was advertised earlier, in this case the apparently complex Yukawa couplings can be made all real by re-phasing of the corresponding 4-dim. fields: $\psi_j(x) \rightarrow \exp(-i\phi_j)\psi_j(x)$, etc... Thus, as we expected, physically meaningful CP violating phase does not appear. This rather trivial example shows the importance of re-phasings of the 4-dim. fields. A similar thing happens for general S^{2n-1} spheres.

One may wonder what is the essential ingredient in the mechanism of CP violation discussed above. Though the violation clearly necessitates the absence of symmetry under the $Z_a \rightarrow Z_a^*$, it turns out that the higher-dimensional chirality plays a crucial role. In fact, if a full 32-component fermion with both chiralities ($16 + \overline{16}$) is used to construct a theory, as in the Type IIa ($N=2$) theory, CP is shown not to be broken even for the C-Y manifold with the complex parameter C . In this case, the straightforward extension of the ordinary parity transformation, $\psi \rightarrow \gamma_0^{(10)}\psi$, exchanging the two parts, $16 \leftrightarrow \overline{16}$, makes sense. Combining with the charge-conjugation due to (4) we get a CP transformation,

$$\psi \rightarrow (C \otimes \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix})\psi^*. \quad (9)$$

Strictly speaking this transformation does not reduce to the ordinary 4-dim. CP transformation, since the matrix corresponding to the internal space is not I_8 . We should, however, recall the fact that the necessary and sufficient condition of the CP invariance in the ordinary 4-dim. theory is the requirement that the theory is invariant under $\psi_i \rightarrow U_{ij} C \psi_j^*$, allowing the freedom of unitary

transformation described by U [8]. In our present case U can be chosen as

$$U = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \quad (10)$$

Accordingly a vector X_M transforms just as usual, and we notice that the defining equation (8) is always invariant under such transformation, $Z_a \rightarrow -Z_a$. We have to conclude that in this case the complex parameter C is not enough to break CP. Unfortunately any realistic example of Type IIa theory to explicitly demonstrate such our expectation has not been found.

Another interesting question to be raised is whether the mechanism of CP violation discussed above works in other space-time dimensions. We will consider only even space-time dimensions. (Let us note the trivial fact that in odd-dimensions “chiral fermions”, whose existence is essential for our mechanism, cannot be defined). In general even-dimensional space-time, even the charge conjugation operator may not automatically reduce to the 4-dim. one. This can be easily understood, by noting that Majorana fermions exist only for $D = 2, 4 \pmod{8}$ for even D . We, therefore, have to impose a criterion to choose suitable D-dim. operators, so that they reduce to the ordinary 4-dim. one. The following conditions are required for the charge-conjugation : (i) Majorana condition, $(\psi^C)^C = \psi$ (ii) chirality-preserving condition, $\gamma_{FIVE}^{(D)} \psi^C = (\gamma_{FIVE}^{(D)} \psi)^C$, which are written with respect to the charge-conjugation matrix $C^{(D)}$,

$$C^{(D)} \gamma_0^{(D)} (C^{(D)} \gamma_0^{(D)})^* = I \quad (11)$$

$$C^{(D)} \gamma_0^{(D)} (\gamma_{FIVE}^{(D)})^* - \gamma_{FIVE}^{(D)} C^{(D)} \gamma_0^{(D)} = 0, \quad (12)$$

respectively, assuming $\gamma_0^{(D)}$ is symmetric. The condition (ii) has been imposed so that the operator is meaningful for the “chiral-fermion”, since our exercise in $D = 10$ case shows the importance of having the chiral fermions. As for the parity operator, we just demand that its action on the internal spinor index is the same as in the charge-conjugation, satisfying the above two conditions, and that the 4-dim. spinor index is transformed by ordinary γ_0 . These requirements clearly have been fulfilled in the $D = 10$ case.

In order to find out such CP operators, it will be useful to see what kind of structure the higher dimensional gamma matrices do have in general. The gamma matrices for arbitrary even D-dimension, $\gamma_M^{(D)}$, can be written in the form of decomposition into 4-dim. and the internal spaces,

$$\gamma_M^{(D)} = \begin{cases} \gamma_\mu \otimes \gamma_{FIVE}^{(D-4)} & \text{for } M = 0 \sim 3 \\ iI \otimes \gamma_a^{(D-4)} & \text{for } M = 4 \sim D-1, \end{cases} \quad (13)$$

where $\gamma_a^{(D-4)}$ ($a = 1 \sim D-4$) are $D-4$ dimensional hermitian gamma matrices,

which can be iteratively constructed [9];

$$\gamma_a^{(D+2)} = \begin{cases} \gamma_a^{(D)} \otimes \tau_3 & a = 1 \sim D \\ I_D \otimes \tau_1 & \text{for } a = D+1 \\ I_D \otimes \tau_2 & a = D+2, \end{cases} \quad (14)$$

with Pauli matrices $\tau_{1\sim 3}$ and $\gamma_1^{(2)} = \tau_1, \gamma_2^{(2)} = \tau_2$. Accordingly $\gamma_{FIVE}^{(D)} = \gamma_5 \otimes \gamma_{FIVE}^{(D-4)}$. Let us note that in this basis $\gamma_{FIVE}^{(D-4)}$, the internal index part of the chiral operator, is diagonal, $\tau_3 \otimes \tau_3 \otimes \dots$, which means a D-dim. chiral fermion just decomposes into a set of 4-dim. Weyl spinors without any complex coefficients in front. This reality of the coefficients guarantees that Eq.(11) is really the Majorana condition. The charge-conjugation matrix (not necessarily what we want), satisfying $\hat{C}^{(D)\dagger} \gamma_M^{(D)} \hat{C}^{(D)} = -(\gamma_M^{(D)})^T$ can be written as, $\hat{C}^{(D)} = \gamma_5^{D/2} C \otimes \hat{C}^{(D-4)}$. The following relations are useful,

$$\gamma_{FIVE}^{(D)} \hat{C}^{(D)} = (-1)^{D/2} \hat{C}^{(D)} (\gamma_{FIVE}^{(D)})^* \quad (15)$$

$$\hat{C}^{(D)*} = (-1)^{D(D+2)/8} \hat{C}^{(D)\dagger}. \quad (16)$$

We are ready to check the conditions (11) and (12) for arbitrary even dimensions. First (16) tells us that the Majorana condition (11) can be satisfied when $D(D+2)/8$ is odd, i.e., $D = 2, 4 \pmod{8}$, nothing but the dimensions where Majorana spinors exist. Next from (15) it is easy to see that the chirality-preserving condition is satisfied when $D = 2 \pmod{4}$. Obviously $\hat{C}^{(D)}$ should be, in general, in order to meet these two conditions. For definiteness we consider below only the $D = 6$ case, though the similar argument holds in other even dimensions ($D = 8, 10$ cases have been explicitly checked).

The improved charge-conjugation matrix $C^{(6)}$ takes the form,

$$C^{(6)} = \hat{C}^{(6)} \cdot (I \otimes \gamma_{FIVE}^{(2)}) = \gamma_5 C \otimes \hat{C}^{(2)} \gamma_{FIVE}^{(2)}. \quad (17)$$

We have put $\gamma_{FIVE}^{(2)}$ in the internal space part, since it does not affect the successful relation (12), while changing the l.h.s. of Eq.(11), through $\hat{C}^{(2)} \gamma_{FIVE}^{(2)} \hat{C}^{(2)\dagger} = -(\gamma_{FIVE}^{(2)})^T$. Now both conditions have been satisfied. The resulting transformation of a vector, $X_M = \bar{\psi} \gamma_M^{(6)} \psi$, is $C^{(6)} : X_M \rightarrow X_M$ for the extra-space coordinates, $M = 4, 5$, while the 4-dim. part transforms as usual. Correspondingly the modified parity operator for $D = 6$ should be defined by replacing $C \gamma_0$ part in $C^{(6)} \gamma_0^{(6)}$ by γ_0 . Namely, $P^{(6)} = \gamma_5 \gamma_0 \otimes \hat{C}^{(2)}$, which yields, $P^{(6)} : X_M \rightarrow -\bar{\psi} (\gamma_M^{(6)})^T \psi$, for $M = 4, 5$. Combining these two improved operators we finally get

$$C^{(6)} P^{(6)} : X_M = \bar{\psi} \gamma_M^{(6)} \psi \rightarrow -\bar{\psi} (\gamma_M^{(6)})^T \psi, \quad (18)$$

for the extra-space coordinates. Our recursive construction shown in (14) clearly says that $\gamma_M^{(6)}$ for $M = 4, 5$ are half symmetric and half anti-symmetric. Thus just as in the $D = 10$ case, a complex coordinate defined by $Z = X_5 + iX_4$

transforms as $Z \rightarrow Z^*$. The essential thing which underlies this transformation property is the simple fact that $\hat{C}^{(D-4)}$ causes the transpose of the gamma-matrices in the internal space. On the other hand Eq.(14) shows that for arbitrary even dimensions $\gamma_a^{(D-4)}$ in Eq.(13) have an equal number of symmetric and antisymmetric matrices. We thus conclude that the close relationship between CP operator and the complex structure of the compactified space is a general feature of the even dimensional theories.

We will now discuss a 4-generation model with C-Y compactification [3,4,5] as a typical example of the theories with complex Yukawa coupling due to the complex structure of the compactified space. A similar argument has already been made in Ref.[5] on the CP violation in this model. The manifold is determined by the defining equation Eq.(8), supplemented by freely acting Z_5 symmetries, i.e., S: $Z_a \rightarrow Z_{a+1}$, T: $Z_a \rightarrow \alpha^a Z_a$ ($\alpha^5 = 1$). In spite of the lack of the metric of C-Y manifold, the Yukawa coupling can be fully computed by using Eq.(8) and the Z_5 symmetries. For example, the Yukawa coupling matrix responsible for $Q = 2/3$ quark masses in the basis of weak eigenstates u_i ($i = 1 \sim 4$) is given as [4,5]

$$\begin{pmatrix} s_2 h^{(3)} & r_2 h^{(2)} & s_1 h^{(1)} & q h^{(0)} \\ r_2 h^{(2)} & r_1 h^{(1)} & p h^{(0)} & s_2 h^{(4)} \\ s_1 h^{(1)} & p h^{(0)} & r_2 h^{(4)} & r_1 h^{(3)} \\ q h^{(0)} & s_2 h^{(4)} & r_1 h^{(3)} & s_1 h^{(2)} \end{pmatrix} \quad (19)$$

where $h^{(i)}$ ($i = 0 \sim 4$) denote Higgs fields, and the parameters $p, q, r_{1,2}$ and $s_{1,2}$ are all (except for p) complex when the constant C , appearing in Eq.(8), is complex. For instance, $r_1 = \alpha^{-1} + \alpha + (\alpha^{-2} + \alpha^2)C + C^3$. The presence of complex phases, however, does not immediately imply CP violation. Let us examine whether the Yukawa couplings can be made all real by means of re-phasing of the quark and the Higgs fields, $u_i \rightarrow e^{i\alpha_i} u_i$, $h^{(i)} \rightarrow e^{i\beta_i} h^{(i)}$, respectively. Though we first have 5 coupled linear equations for 4 variables α_i , as the result of the requirement to make Yukawa couplings of 5 Higgs fields real, we find that only three equations are independent each other. Thus the Yukawa couplings are made real only when the phases of the parameters satisfy a special relation, $\phi_p - \phi_q - (\phi_{r_1} + \phi_{r_2}) + (\phi_{s_1} + \phi_{s_2}) = 0$, where ϕ_p is the phase of parameter p , etc.. It turns out that this relation holds for the specific cases, $C = \alpha^n$ ($n = 1 \sim 4, C = 1$ should be excluded to avoid the singularity due to a fixed point). It is interesting to note that when $C = \alpha^n$ the “re-phasing” of the complex coordinates, $Z_a \rightarrow \alpha^{n_a} Z_a$ ($\sum_{a=1}^5 n_a = -n$) will remove the phase of the parameter C in the defining equation. (Actually, this Z_5 transformation is not compatible with S, T and cannot be qualified as the re-phasing of the zero-modes.) Strictly speaking, we further have to check whether the re-phasing invariant condition for CP violation [10,8], involving Yukawa couplings of both $Q = 2/3$ and $Q = -1/3$, is really satisfied. It is, however, very reasonable to expect that for a general choice of VEV’s of the Higgs fields the CP phase will survive.

Unfortunately, the Yukawa coupling discussed above causes a serious phenomenological problem. Namely the coupling of $Q = 2/3$ quarks with many (five) Higgs fields results in flavour-changing neutral current processes due to the Higgs exchange at the tree level. It is well known that the Yukawa coupling has to be restricted so that only one Higgs can couple to quarks with a definite charge in order to avoid the problem [11]. Such a restriction of the Yukawa coupling, on the other hand, will lead to another problem; now the Yukawa coupling can be trivially made real by re-phasing and therefore CP violation à la K-M is not possible.

One may wonder how CP can be broken in the theories with other types of compactification. In orbifold compactification, at least as far as the “untwisted sector” is concerned, a similar thing with the torus compactification will be expected to happen. As was discussed in the S^1 case possible complex phases can be always rotated away by re-phasing of 4-dim. fields. We may also construct Yukawa couplings [12] relying on the Gepner’s approach to realize string propagation on the C-Y manifold in terms of (super-)conformal theories [13]. Such constructed Yukawa couplings [12] again show no complex phase. In fact the models correspond to C-Y type manifolds, but with no complex parameter in the defining equation(s). Whether the extension of the model will yield some complex phases or not is an open question.

As summary, we have shown (especially in $D = 10$ for the details) how the CP operator, devized to reduce into 4-dim. one after the compactification, is closely related to the complex structure of the compactified manifold. Having fermions with definite higher-dimensional chirality like in $D = 10$ Type-I theory was essential for the CP breaking mechanism. It was also claimed that the same mechanism works in general even dimensional space-time, as well (though it has been explicitly confirmed only in $D = 6, 8$ and 10). A $D = 10$ model with C-Y compactification was used to examine the mechanism in some detail, and we have seen that in the phenomenologically acceptable case all Yukawa couplings can be made real. Some examples with other types of compactification also show that to get CP violation is a very non-trivial task. We thus have to conclude that to accommodate CP violation in a realistic string model seems to be highly challenging, even though it is not impossible. In other words, the condition of realizing CP violation will serve as a very useful criterion to construct a viable model. Of course there still remains another possibility to break CP through the complex VEV’s of Higgs fields, but at present we can say little about it since our knowledge on the scalar potential in supersymmetric theory is still poor (e.g., concerning non-renormalizable self-coupling of Higgs). Finally the most deep question will be that how a manifold leading to CP violation can be chosen as the result of the dynamics of the compactification starting from “real theory” like $D = 10$ super Yang-Mills theory.

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