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SUPERSYMMETRY BREAKING AND DYNAMICAL DETERMINATION OF SUPERSTRING PARAMETERS*

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Abstract

The characteristics of the effective potentials coming from phenomenologically promising compactified superstring theories are examined, paying special attention to the supersymmetry breaking issue. We briefly review the status and some of the recent work on the subject and present a mechanism for generating the large gauge hierarchy by gaugino condensation effect in the case that the hidden sector possesses more than one condensate. Explicit examples based on orbifold compactification in which this is realized are also given. Minimization of the effective potential not only determines the gauge hierarchy but also fixes other important parameters of the theory, in particular the gauge coupling constant at the unification point and the expectation values of the moduli which give the size and shape of the compactified space. These get reasonable values which may, in turn, lead to a determination of the family mass hierarchy.

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1 BRIEF REVIEW

Superstring theories [1], in particular the $E_8 \times E_8$ heterotic string [2], are the most promising candidates to unify all known interactions. However the connection between theory and observation has not been established yet, although significant progress has been made in the last few years [3–23]. The main problem standing in the way of experimental tests is perhaps the large number of classical vacuum states. The reason is that at present there are no dynamical criteria to prefer a single vacuum out of the infinite number of possibilities. (We shall see however that a dynamical selection is already possible if we restrict ourselves to a more reduced set of vacua.) Under these circumstances the best one can do is to study the phenomenological characteristics of the vacua in order to select the viable ones. In this sense one should require

- Four dimensional space-time
- $N=1$ supersymmetry at the Planck scale
- Gauge group $SU(3) \times SU(2) \times U(1)_Y$
- Three families of particles with the correct representations of the Standard Model
- Correct fermion masses and mixing angles,

etc.

Much work has recently been devoted to this task, and in fact several models have been constructed satisfying *almost* all the experimental requirements. This has been achieved using different methods: Calabi-Yau spaces [10–12], orbifolds [13–15], four dimensional strings [20,21], etc.

In spite of these achievements there remains the question why a specific ground state should be dynamically preferred. In fact we do not have knowledge about the connection between different schemes (e.g. different orbifolds with or without Wilson lines, Calabi-Yau spaces, etc.) within the string vacuum landscape. However not everything is lost. For a given scheme there is still an infinite number of continuously connected supersymmetric vacua, parametrized by the vacuum expectation values (vev's) of certain scalar fields, the *moduli*, which in general

admit a geometrical interpretation (e.g. the size of the compactified space). They include also the dilaton field, whose vev is related to the gauge coupling constant at the grand unification point. Therefore, by minimizing the effective potential *we can decide which vacuum is dynamically chosen*. However this potential is flat in the absence of supersymmetry breaking. Thus we conclude that supersymmetry breaking and dynamical selection of vacua are two intimately related issues. In this talk we will discuss how supersymmetry breaking effects (of non-perturbative origin) are in fact efficient to split the vacuum degeneracy, leading to a determination of the moduli vev's, and, consequently, of the gauge coupling constant and the size and shape of the compactified space. This in turn determines the size of the supersymmetry breaking mass hierarchy and fixes the Yukawa couplings of the theory and the associated fermion masses.

We know that in any realistic model supersymmetry should be broken at a hierarchically small scale relative to the Planck scale, otherwise supersymmetry would not be useful to solve the gauge hierarchy problem. More precisely the supersymmetry breaking mass of the scalars should be $m_\phi \lesssim 1$ TeV. On the other hand experimental results tell us that $m_\phi \gtrsim 100$ GeV. This must come about as a result of minimizing the effective potential of the theory. The most promising source of supersymmetry breaking, capable of generating a large mass hierarchy, is perhaps gaugino condensation [24–26]. The reason for this is that the scale of gaugino condensation corresponds to the scale at which the gauge coupling becomes large and this is governed by the corresponding renormalization group running. Since the running is only logarithmically dependent on the scale, the gaugino condensation scale is suppressed relative to the initial scale by an exponentially small factor $\sim e^{-\frac{1}{\beta_0 \alpha^2}}$ providing a promising source for the mass hierarchy (here β_0 is the one-loop coefficient of the beta function of the hidden sector gauge group, $\beta_0 = b_0/16\pi^2$). However in string theories it is not obvious this mechanism will work for the value of the gauge coupling at the starting scale for evolution is itself a dynamical variable to be determined by the vev of a scalar field in the effective theory after compactification. To be precise $1/g_{\text{eff}}^2 = S_r$ where S_r is the real part of the dilaton S . Thus it is necessary to study the effective potential which determines this vev before using the renormalization group equations to fix the gaugino condensate scale.

First derivations of effective Lagrangians [27-29] lead to no-scale models [30]. It was shown that these possessed a trivial (supersymmetric) minimum of the effective potential unless non-perturbative contributions (e.g. gaugino condensation effects) were included [25,26,31]. Unfortunately the form of the resultant supersymmetry breakdown was not satisfactory. For example, in the Witten's truncation model [27], the gaugino condensate $\langle \lambda\lambda \rangle = \mu^3 e^{-(3S/2\alpha)} (with \mu of order the compactification scale) contributes a positive definite term to the effective potential. Consequently at the minimum of this potential $S_r = \frac{1}{g^2} \rightarrow \infty, V = 0$ so gaugino condensation does not take place and there is no supersymmetry breaking at all. This problem was avoided by cancelling the positive gaugino condensate contribution against the contribution from the vev of the pseudoscalar degree of freedom associated with the four-dimensional antisymmetric tensor $\langle \epsilon^{ijk} H_{ijk} \rangle \equiv c$. Then the superpotential W takes the form $W = \mu^3 e^{-(3S/2\alpha)} + c$. Unfortunately it was shown in ref.[32] that this vev was quantized in units of the Planck scale and hence this cancellation mechanism requires the gaugino condensate and the corresponding supersymmetry breaking to occur at the Planck scale. This is not the only problem. At tree level the masses of the scalar fields and gauginos are still vanishing $m_\phi, m_\lambda = 0$ and the compactification radius R (related to the vev of a modulus field) becomes undetermined. One-loop radiative corrections do not improve the situation [33].$

Possible ways out for these problems have recently been proposed. Krasnikov [34] was the first who proposed to use several gaugino condensates. Nevertheless he found it impossible to generate a sufficiently small gravitino mass for a hidden sector resulting from E_8 . More recently the multiple gaugino condensation scenario has been considered in references [35,36]. In ref.[35] Dixon et al. also included in their analysis the one-loop contribution to g from the spectrum of charged, massive string excitations. For some cases these make g to acquire a certain dependence on the moduli defining the compactified space [37] in agreement with what was claimed in [38] from duality arguments. In ref.[36] the multiple condensates were considered in combination with the matter contribution to the superpotential, specially the world-sheet instanton corrections to Yukawa couplings (a previous suggestion in this direction was given in ref.[39]). This more detailed study of the effective potential is now possible since in the last two

years the knowledge of the low-energy effective Lagrangians has been significantly improved. This is the work that we mainly expound in what follows.

We will see that supersymmetry breaking cannot be achieved without gaugino condensation. It may be obtained with just one condensate, but in this case it cannot be realistic. With several gaugino condensates a suitable supersymmetry breakdown can take place, although additional ingredients are required for this, namely a stage of symmetry breaking after compactification and a moduli dependence of the superpotential via non-perturbative effects. (Even though these ingredients are usually present in physically relevant compactifications, they may be relaxed in some cases.) In this way, by minimization of the scalar potential (i.e. *dynamically*), a hierarchical supersymmetry breaking ($m_\phi < 1 \text{ TeV}$) can be obtained. As it was mentioned above this minimization also leads to a dynamical determination of the gauge coupling constant (which becomes small) and the radii of the compactified space (which acquire reasonable values, suitable for correct fermion mass generation). The rest of this talk is organized as follows. In sec.2 we write the form of the Kähler potential and the subsequent effective potential used throughout the paper and obtain some general results. In particular it is shown that without gaugino condensation supersymmetry breaking is not possible and with just one condensate it cannot be realistic. We also obtain very useful and general bounds on the gauge coupling constant that must be respected in any viable model. In sec.3 we argue that there are three essential (or at least very convenient) ingredients that should be present for a realistic supersymmetry breakdown. Namely multiple gaugino condensation, a stage of symmetry breaking after compactification and non-perturbative effects on the world-sheet. In fact these ingredients are very likely to be present, and this is illustrated with explicit examples. The relationship between the size and shape of the compactified space (obtained by minimization of the potential) and the hierarchy of fermion masses is explained at the end of this section. Finally, in sec.4 we present our conclusions.

2 GENERAL RESULTS

2.1 THE EFFECTIVE POTENTIAL

Our starting point is the effective field theory resulting from a compactified field theory. If one is to solve the gauge hierarchy problem this theory must have (at least) one supersymmetry left intact down to "low energies" of the order of 1 TeV. Compactifications of the superstring leaving such a supersymmetry intact have been extensively studied. The resulting low-energy locally-supersymmetric theory is parameterised by the superpotential, W , holomorphic in the chiral superfields, the Kähler potential, K , and the gauge kinetic function, f ($f=S$ is the most general form permitted at string tree level [27,31,40]). These functions are determined, in principle, in a given compactification scheme although in practice they are known completely only for some orbifold [5] compactification schemes. W may depend on the $(1,1)$ and $(1,2)$ untwisted moduli (N_i) and on the charged matter fields which, for an orbifold, include untwisted fields (U), twisted fields (T) and twisted moduli (C). The form of W has been completely determined for the trilinear sector (in the matter fields) of $(2,0)$ and $(2,2)$ orbifolds [41] and the structure of higher order terms is pretty well known too [41,42]. Finally the structure of the Kähler potential K (to the lowest order in the matter fields) has recently been determined by Dixon et al. and others [43,44] for $(2,2)$ vacua.

For the sake of definiteness we will work with the Z_3 orbifold [5] based on the torus R^6/Λ where Λ is the root lattice of $SU(3)^3$, but the results are easily extended to other Z_N orbifolds. In this case there are nine $(1,1)$ moduli and no $(1,2)$ moduli. For the considerations of this section it is not essential to take into account the whole set of moduli, and we consider just the three moduli N_1, N_2, N_3 whose real parts are associated with the size of the three $SU(3)$ sublattices and the corresponding imaginary parts are axion-like fields coming from the antisymmetric tensor components in the extra dimensions (from now on, we take the index i to be, $i=1,3,5$)

$$N_i = \frac{\sqrt{3}}{2} (R_i^2 - i B_i) \quad (1)$$

The radii of the compactified space, R_i , are in string units. For the rest of the $(1,1)$ moduli we

assume vanishing expectation values, i.e. we work with the cartesian orbifold. Then the Kähler potential takes the form [44]¹

$$K = -\log(S + \bar{S}) - \log \prod_i (N_i + \bar{N}_i) + \sum_i \frac{U_i \bar{U}_i}{N_i + \bar{N}_i} \delta_{ij} + \sum_j \frac{C_j \bar{C}_j}{\prod_i (N_i + \bar{N}_i)} \delta_{je} + \sum_j \frac{T_j \bar{T}_j}{\prod_i (N_i + \bar{N}_i)^{2/3}} \delta_{je} + \dots \quad (2)$$

where the dots stand for higher order terms in the matter fields and we are taking $k = \sqrt{8\pi G_N} = 1$. Eq.(2) is derived subject to the conditions

$$\begin{aligned} \sum_j |T_j|^2 &<< \prod_i (2N_i^*)^{2/3} \\ \sum_j |C_j|^2 &<< \prod_i (2N_i^*) \\ |U_i|^2 &<< 2 N_i^* \end{aligned} \quad (3)$$

where we henceforth use the convention that r means real part of the field. The form of eq.(2) is also supported by an expansion of the Kähler potential conjectured in ref.[45]. The tree level scalar potential, V_0 , given by

$$V_0 = e^G (G_i (G^{-1})^i_m G^m - 3) \quad (4)$$

where $G = K + \log |W|^2$, takes the form²

$$\begin{aligned} V_0 = e^K \{ & |W - 2S_r \frac{\partial W}{\partial S}|^2 + \sum_i (2N_i^*)^2 \left| \frac{\partial W}{\partial N_i} - \frac{W}{2N_i^*} \right|^2 + \\ & \sum_i 2N_i^* \left| \frac{\partial W}{\partial U_i} + \frac{\bar{U}_i W}{2N_i^*} \right|^2 + \sum_j \prod_i (2N_i^*) \left| \frac{\partial W}{\partial C_j} + \frac{\bar{C}_j W}{\prod_i (2N_i^*)} \right|^2 + \\ & \sum_j \prod_i (2N_i^*)^{2/3} \left| \frac{\partial W}{\partial T_j} + \frac{T_j W}{\prod_i (2N_i^*)^{2/3}} \right|^2 - 3|W|^2 \} \end{aligned} \quad (5)$$

¹It is worth noticing here that the Kähler potentials derived from Calabi-Yau compactifications have a similar structure [27,28].

²The D -terms, which are positive definite, are taken to be zero. This can always be arranged with a careful assignment of vev's to matter fields [14,15].

It is easy to see here that if all the charged matter fields of the theory have vanishing expectation values, the potential will be positive in agreement with the general result of ref.[46], where a more general class of Kähler potentials was considered.

Supersymmetry breaking occurs if the superpotential acquires a non-zero vev. In this case the gravitino mass, $m_{3/2}$, is given by

$$m_{3/2}^2 = e^K |W|^2 \quad (6)$$

and all the scalar fields acquire a supersymmetry breaking mass of order $\sim m_{3/2}$ coming from the corresponding F terms (see eq.(5)) for a non-vanishing value of W . It is straightforward to check that the fermionic partners remain massless. In order to write the properly normalized scalar masses, the non-canonical kinetic terms for the chiral fields must be taken into account. These are derived from (2) in the standard way and using them we finally find scalar masses equal to the gravitino mass, given by (6).

One-loop radiative corrections can also be included in the analysis. They do not modify sensitively all the results presented in this paper if supersymmetry is broken at a reasonable scale [36].

2.2 THE TRILINEAR SUPERPOTENTIAL

In order to determine the minimum of the potential, eq.(5), we need some information about the form of W . However a general result can be obtained if only the part of the superpotential trilinear in charged matter fields, say $W_T = \sum f(N) A A A$, where $f(N)$ is some general function of the moduli, is considered. The reason for considering this restricted class of superpotential is that the Kähler potential given in eq.(2) is derived only for small values of the matter fields vevs and in this limit higher order terms will be suppressed. Keeping only the trilinear terms we have

$$2 \operatorname{Re} \left(\sum_g \frac{\partial W_T}{\partial A_g} A_g \bar{W}_T \right) = 6 |W_T|^2 \quad (7)$$

where A_g means any charged matter field of the theory. It is clear that the potential of eq.(5) with $W = W_T$, is positive definite.

This result does not depend on the specific dependence of W on the various fields of the theory: it is just a consequence of the form of K and the fact that only trilinear couplings on the matter fields are considered. Therefore we conclude that the minimum of the effective potential is trivial (i.e. $W = 0$) with unbroken supersymmetry, and this holds for other Z_N compactifications. Actually this is similar to what happens in the no-scale effective Lagrangians of ref. [30]. Radiative corrections do not modify this result [36].

Since this situation is not a realistic one we should explore possible ways out. One possibility is to take into account the presence of higher order terms in W . Despite our arguments above to the contrary it could be possible for these terms to play an important rôle provided they involve fields which have flat directions along which the trilinear terms do not contribute to non-zero F -terms. We have not investigated this possibility in any generality and cannot rule out the possibility they lead to non-positive potentials, but we know of no examples of this type.

Another (more standard) possibility is to consider gaugino condensation in the hidden sector and we turn now to an investigation of this issue.

2.3 BOUNDS ON THE GAUGE COUPLING CONSTANT

The effect of a gaugino condensation in the effective Lagrangian is to modify the superpotential [26,47-49] $W \rightarrow W + \langle S \bar{\lambda} \lambda \rangle$. Let us denote by $f(S)$ the functional dependence of $\langle S \bar{\lambda} \lambda \rangle$ on S . In this subsection we wish to derive some general results which will allow us to bound the gauge coupling in a large class of models. In order to study the impact of this modification on the conclusions of the previous subsection we again consider only the part of the superpotential, W_T , trilinear in matter fields. Thus

$$W = f(S) + W_T \quad (8)$$

We are interested in the possibility that a non-trivial minimum with $V < 0$ occurs in the presence of the gaugino condensate. Following from eq.(8) the corresponding tree level potential is readily obtained from eq.(5). Analogous to eq.(7) we find the F -terms associated with the charged matter fields can be written as

2.4 PROBLEMS FOR SUPERSYMMETRY BREAKING WITH A GAUGINO CONDENSATE

Here we briefly discuss the simplest case of a *single* hidden sector gauge group triggering a gaugino condensate. This turns out to be an unviable possibility. The functional dependence of $\langle S \bar{\lambda} \lambda \rangle$ on S has been found to be dictated by certain classical symmetries[48,26,49].

$$\langle S \bar{\lambda} \lambda \rangle = f(S) = \mu^3 e^{-(3S/2\beta_0)} \quad (14)$$

where β_0 is the one-loop coefficient of the beta function of the hidden sector gauge group and μ is expected to be of order the compactification scale M_c , which in the Z_3 orbifold is given by (the right-hand side refers to background field values)

$$M_c^2 = \frac{\sqrt{3}}{8} \frac{1}{S_r (\prod_i N_i)^{1/3}} \quad (15)$$

To obtain this equation we used the fact that the volume of the compactified space is $V_\Lambda = (\sqrt{3}/2)^3 (\prod_i R_i^2)$. Other approaches studied in the literature are discussed in ref.[36]. They lead to a parametrization very similar to that of eq.(14)³. Now, it is easy to see that with the unique condensate of eq.(14), bound (12) reads

$$S_r < \frac{7}{18} \beta_0 \quad (16)$$

This in turn implies that the gauge coupling, $g_{\text{gauge}}^2 = 1/S_r$, is large and the theory is strongly coupled at the compactification scale. Furthermore it should be mentioned here that a very large value of the coupling constant jeopardizes the way in which the gaugino condensate has been parametrised. In fact, to obtain eq.(14) a perturbative expansion of the gauge beta function must be performed. This indicates that one should be cautious in extracting conclusions for small values of S_r . Again this result holds for other Z_N orbifold compactifications. Such a result is in conflict with the hope for perturbative unification and a large mass hierarchy. Explicit examples, in the framework of the Z_3 orbifold, of how these results are realized in practice are given in ref.[36], to which the interested reader is referred.

³The case of several gaugino condensates using the approach of ref.[47] was studied very recently in ref.[50].

$$e^K \{ 6 \operatorname{Re}(W_T \bar{W}) + \text{positive terms} \} \quad (9)$$

But $6 \operatorname{Re}(W_T \bar{W}) = 3(|W_T|^2 + |W|^2 - |f(S)|^2)$, so a negative value for V_0 (notice that from eq.(5) the only negative term to be added to the previous expression is $-3|W|^2$) can only be reached if

$$|W_T| < |f(S)| \quad (10)$$

This bound tells us that the scale of supersymmetry breaking will be essentially given by $|f(S)|$. Now it is very simple to see that the sum of the F -terms associated with S and with the charged matter fields can be written as

$$e^K \{ |W_T - 2S_r f'(S)|^2 + 2|W_T|^2 + 4|W|^2 - 3|f(S)|^2 - 4S_r \operatorname{Re}(f'(S) \bar{f}(S)) + \text{positive terms} \} \quad (11)$$

where $f' = \frac{\partial f}{\partial S}$. Taking into account that, for a given value of $f(S)$, the positive contribution to V_0 , $2|W_T|^2 + |W|^2$, has a minimum at $|W_T| = \frac{1}{3}|f(S)|$ a negative value for V_0 (eq.(5)) implies

$$-S_r \cos \theta < \frac{7}{12} \left| \frac{f(S)}{f'(S)} \right| \quad (12)$$

where θ is the angle between f and f' in the complex plane. Eq.(12) is very cumbersome to manage in practice because of the explicit dependence on the angle. A more useful bound can be obtained as follows: It is clear from eqs.(8) and (10) that $|W| < 2|f(S)|$; then if the effective potential as given by (5) is to be negative (requiring $|W - 2S_r f'(S)|^2 < 3|W|^2$) the condition

$$S_r < (1 + \sqrt{3}) \left| \frac{f(S)}{f'(S)} \right| \quad (13)$$

must be satisfied. It is important to remark that conditions (10), (12) and (13) are completely general for trilinear superpotentials, i.e. they hold for any form of the gaugino condensation. We will see below that the associated bounds on the gauge coupling constant for specific parametrizations of $f(S)$ are very restrictive (recall that in the heterotic string vacua $S_r = 1/g_{\text{gauge}}^2$).

Previous conclusions seem to exclude the case of a unique gaugino condensate. In the next section we discuss how it is possible to get realistic and stable solutions in the region of small coupling constant, thus offering an interesting way-out to this situation.

3 A NEW PERSPECTIVE FOR SUPERSYMMETRY BREAKING

3.1 THREE ESSENTIAL INGREDIENTS FOR REALISTIC SUPERSYMMETRY BREAKING

Now we argue that the three essential ingredients that should be present to expect a realistic supersymmetry breakdown are the following

- 1) At least two gaugino condensates in the hidden sector
- 2) A stage of spontaneous symmetry breaking after compactification
- 3) Moduli dependent superpotential by world-sheet non-perturbative effects

Let us first comment on the meaning and origin of these conditions. Then it will become clear why are they necessary. The first point refers to the possibility that the hidden sector possesses two (or more) separate sectors and hence two (or more) different gauge coupling constants. Apparently this is the only way to modify the gaugino condensation contribution to the superpotential $f(S)$, thus evading the previous bound (16) for the case of a unique sector. In the early Calabi-Yau vacua considered for phenomenology [3,9,10] this possibility did not occur since the $SU(3)_H$ group of the compactified space and the discrete symmetries by which it was divided were completely embedded in the observable sector of the $E_8 \times E_8$ heterotic string. Thus, the hidden sector gauge group was always the simple group E_8 . Subsequent developments have shown, even in the framework of (2,2) compactifications, that things may be quite different and the hidden sector is usually broken on compactification. (Notice for instance that the shift and Wilson lines can act non-trivially in both E_8 weight lattices simultaneously.) For a hierarchically small gaugino condensate this is essential for the E_8 beta function is so large it would trigger gaugino condensation at the Planck scale. Consequently, there can be an enormous variety of different hidden sector gauge groups (see for instance ref.[51]) and most of them consist in fact of several factors (i.e. they are the product of different simple Lie groups). The matter con-

tent is also very diverse. Therefore the scenario with several gaugino condensates is not odd or artificial, on the contrary it is the typical case.

In the next subsection it will become apparent that the mere existence of several gaugino condensates is not enough to obtain a sufficiently small gauge coupling. It is also required the presence of a stage of symmetry breakdown after compactification (i.e. the second ingredient in the previous list) which slightly modifies the form of $f(S)$ in the correct way. Several sources for such a breaking have been discussed in the literature: intermediate scale breaking via flat directions [9,10], Fayet-Iliopoulos breaking [52,14,15] ($F-I$), etc. For example, if an "anomalous" $U(1)$ factor is present in the unbroken gauge group after compactification (the generic case), then a $F-I$ D term is generated in string loop perturbation theory [52,53] and a subsequent process of symmetry breaking unavoidable takes place. Actually, this mechanism has been used to reduce the rank of the unbroken group and to construct $SU(3) \times SU(2) \times U(1)_Y$ orbifold vacua [14,15]. Furthermore it has been argued in ref.[19] that such a symmetry breaking process after compactification is essential in the context of orbifold models in order to have fermion mixing angles. The scale of these breakings is rather variable: it depends on the type of breaking and on the specific model considered. It is in general a large scale, close to the compactification scale (e.g. for the $F-I$ mechanism it is of order $\sim M_s^4/M_c^3$, where M_s and M_c are the string mass and the compactification mass respectively). We will call this scale M_I . Let us briefly discuss the modifications on the usual parametrization of the condensate due to the presence of such a gauge symmetry breaking. As it was previously stated, the form of the gaugino condensate, eq.(14), can be derived from symmetry properties of the theory, but it can also be understood by dimensional arguments to correspond to the scale at which coupling becomes strong [26]. This comes from the one-loop renormalization group equations (RGEs) for the coupling constant

$$\frac{1}{g^2(Q^2)} - \frac{1}{g^2(\mu^2)} = \beta_0 \log\left(\frac{Q^2}{\mu^2}\right) \quad (17)$$

The coupling becomes strong at

$$Q^2 = \mu^2 e^{-(1/\beta_0 g^2(\mu^2))} \quad (18)$$

Taking the gaugino condensate to be set by this scale, with $\mu \sim M_c$ and using $S = 1/g^2(M_c^2)$, we obtain

$$\langle \bar{\lambda}\lambda \rangle = Q^3 = M_c^3 e^{-(3S/2\beta_0)} \quad (19)$$

which essentially coincides with (14). Clearly, if there is a stage of symmetry breaking at a scale M_I between the compactification and the gaugino condensation scales that changes the beta function, eq.(19) should be corrected. Calling β'_0 and β'_0 the coefficients of the beta function between M_c and M_I , and between M_I and Q respectively; the corresponding RGEs read

$$\frac{1}{g^2(M_I^2)} - \frac{1}{g^2(M_c^2)} = \beta'_0 \log\left(\frac{M_I^2}{M_c^2}\right) \quad \frac{1}{g^2(Q^2)} - \frac{1}{g^2(M_I^2)} = \beta_0 \log\left(\frac{Q^2}{M_I^2}\right) \quad (20)$$

Thus, the coupling constant becomes strong at

$$Q^2 = M_c^2 \left(\frac{M_I^2}{M_c^2}\right)^{(1-\beta'_0/\beta_0)} e^{-(S/\beta'_0)} \quad (21)$$

We finally obtain the gaugino condensate as

$$\langle \bar{\lambda}\lambda \rangle = Q^3 = \mu'^3 e^{-(3S/2\beta'_0)} \quad \mu' = M_c \left(\frac{M_I}{M_c}\right)^{(1-\beta'_0/\beta_0)} \quad (22)$$

Finally, the third ingredient (N -dependent superpotential) is required to avoid a positive contribution to the effective potential from the N F -terms, which would destroy the possible existence of negative non-trivial minima. (This can be seen from eq.(5)). Fortunately world-sheet non-perturbative effects yield the desired N -dependence of the superpotential W . For instance it has been shown [41] that for the orthogonal Z_3 orbifold the coupling between three twisted fields has the form

$$L_{yuk} = g h \sqrt{V_\Lambda} \prod_{i=1,3,5} \left(\sum_{n_i, n_{i+1} \in \mathbb{Z}} e^{-\pi N_i (n_i^2 + n_{i+1}^2 + n_i n_{i+1} + 2n_i n_{i+1} + n_i)} \right) (T_1' T_2' T_3')$$

$$h = 3^{3/4} \left(\frac{\Gamma^2(2/3)}{\Gamma(1/3)} \right)^3 \quad (23)$$

where $\epsilon_i = 0, 1/3$ if the fields are attached in each i-sublattice to the same or different fixed points respectively and the primes denote fields with canonical kinetic energy terms. For $R_i > 1$ the dominant term of (23), behaves as

$$L_{yuk} = c g h \sqrt{V_\Lambda} e^{-\pi \sum_i \epsilon_i N_i} (T_1' T_2' T_3') \quad (24)$$

where $c = 1, 3, 9, 27$ for $3 \sum_i \epsilon_i = 0, 1, 2, 3$ respectively. Note that in the Z_3 orbifold the fields T_j' carry gauge quantum numbers which prevent terms of the form T_j^3 . Thus eq.(24) gives the dominant terms involving these fields which may lead to non-zero F -terms. The corresponding form of the superpotential

$$W = c \frac{h}{2} e^{-\pi \sum_i \epsilon_i N_i} (T_1 T_2 T_3) \quad (25)$$

can be extracted by using the Kähler potential (2) and the fact that $\prod_i (N_i + \tilde{N}_i) = 8V_\Lambda$ for the Z_3 orbifold.

This seems to be the only source for a N -dependence of the superpotential for the Z_3 orbifold, which has proved to be the most interesting one from the phenomenological point of view so far. For certain compactifications (e.g. other, less attractive, orbifolds) an additional dependence appears through the gauge coupling constant provided gaugino condensation is present. This can be shown by a RGEs calculation including massive modes [37] or by duality arguments [38]. The phenomenological consequences of this modification have been discussed in the first two papers of ref.[38]

3.2 EXPLICIT EXAMPLES WITH SEVERAL GAUGINO CONDENSATES

In order to bring the claims of the previous subsection into light we will illustrate the implications of several hidden sectors by means of an explicit example. Consider a vacuum in which the hidden sector has two separate sectors with β_0 and β'_0 the respective one-loop beta functions. The contribution of gaugino condensation to the superpotential is

$$f(S) = \mu^3 e^{-(3S/2\beta_0)} + \mu^3 e^{-(3S/2\beta_0')} e^{i\delta} \quad (26)$$

where we have allowed for a symmetry breaking modifying the second condensate according to (22) and we have also allowed for a relative phase, δ , between the condensates as the gauge interactions which give rise to gaugino condensation do not fix the phase of the condensate (this phase however turns out to be irrelevant). Note that it is not essential to have only two condensates or to allow just one of them to be affected by spontaneous symmetry breaking: we make this choice for simplicity. The superpotential has the form $W = f(S) + W_T$, where, as in the previous section, we take the trilinear part of the superpotential in the matter fields. The tree level effective potential is again given by (5). We will now show that it is possible to make $\frac{\partial f}{\partial S} = 0$ (but the S F -term different from zero) for a particular (and non-infinite) value of S , say S^0 , while the other F -terms are kept zero. In this case the scalar potential is negative, demonstrating the presence of non-trivial vacuum solutions.⁴ We find $\frac{\partial f}{\partial S} = 0$ for

$$\frac{3S_0^0}{2\beta_0} = \frac{1}{1 - \frac{\beta_0'}{\beta_0}} \log \left(\frac{\beta_0'}{\beta_0} \frac{\mu^3}{\mu^3} \right) \quad (27)$$

$$\text{Im } S^0 = \frac{(2m+1)\pi - \delta}{2 \left(\frac{1}{\beta_0} - \frac{1}{\beta_0'} \right)} ; m \in \mathbb{Z} \quad (28)$$

Notice that $\text{Im } S$ is fixed so that the condensates get opposite signs giving a cancellation between them. (It is obvious that with a unique condensate, only for the trivial case $S_r \rightarrow \infty$, can $\frac{\partial f}{\partial S}$ vanish.) The rôle played by the relative phase, δ , is irrelevant for what follows, as will become clear shortly. As we will presently show, S^0 corresponds, very closely, to the position of a true minimum.

In order to examine the other F -terms of the effective potential an explicit form for W_T must be taken. A simple choice is

⁴It is very interesting, from the phenomenological point of view, to notice that all the gauginos become massive for this solution due to the fact that the F -term associated with S is non vanishing. They all acquire a supersymmetric breaking contribution to their masses equal to the gravitino mass. This is also the case for the scalar fields as we saw in subsect. 2.1.

$$W = f(S) + \frac{27}{2} h e^{-\frac{2}{3}(N_1+N_2+N_3)} T_1 T_2 T_3 \quad (29)$$

As was discussed in the previous subsection this coupling is always present in the Z_3 orbifold vacua and the meaning of the various fields and constants involved in (29) was also given there. Other terms of the superpotential can consistently be ignored by assigning zero expectation value to any matter field not involved in (29). (This does not cause any problem with their F -terms, since the orbifold symmetries ensure that eq.(29) contains the only possible term involving more than one T_i , $i = 1, 2, 3$ [5,41]) It is now straightforward to write from (5) the corresponding effective potential and derive the conditions to get vanishing F -terms for the moduli and matter fields. These are

$$|T_f|^2 = \frac{6}{\pi} N^r ; f = 1, 2, 3 \quad (30)$$

$$\frac{e^{-3S_0^0/2\beta_0}}{S_0^{3/2}} \left(\frac{\beta_0'}{\beta_0} - 1 \right) = \frac{3^{3/4} 864}{\pi^{3/2}} h (N^r)^3 (1 + 2\pi N^r/3) e^{-\pi N^r} \quad (31)$$

where $N_i = N$; $i = 1, 3, 5$. It is worth noticing that this solution satisfies the requirement (3). Eq.(31) implies

$$\pi N^r \sim \frac{3S_0^0}{2\beta_0} \quad (32)$$

and the corresponding gravitino mass is

$$m_{3/2} = \frac{\pi^2 \sqrt{3}}{6144} \frac{e^{9/2\pi N^r}}{(N^r)^4 (1 + 2\pi N^r/3)^2} \left(1 - \frac{\beta_0'}{\beta_0} \right) \frac{e^{-3S_0^0/2\beta_0}}{S_0^4} \quad (33)$$

In deriving eqs. (31) and (33) we have chosen $\mu = M_c$, but it is clear from eqs. (31), (32) and (33) that the scale of the gravitino mass is insensitive to a possible constant of proportionality between μ and M_c , because of the rapidly changing exponential factors. It is clear from (33) that a large hierarchy (i.e. a reasonable gravitino mass) must be driven by a fairly large S_r ($\frac{3S_0^0}{2\beta_0} \sim 20$) and hence a small coupling constant. Furthermore, due to (32), this will necessarily lead to reasonable values for the moduli ($N_i^r \sim 10$). The actual value of S_r is determined by

eq.(27), i.e. by the characteristics of the hidden sector. It is easy to see that if no symmetry breaking (e.g. a $F-I$ breaking) is present, the desired large hierarchy cannot be achieved. For $\mu' = \mu$ then, from (27), the largest value for $\frac{3S_r}{2\beta_0}$ is obtained if β_0 and β'_0 differ by the addition of one fermion, i.e. $\beta'_0 - \beta_0 = 1/2$. But in this case, the value of the log is too small and as a consequence the maximum of $\frac{3S_r}{2\beta_0}$ is by far too small to generate a reasonable hierarchy. This means that in order to get a realistic large hierarchy a stage of symmetry breaking after compactification should take place.

Now it is easy to verify that there are reasonable values for β_0 , β'_0 and $(\mu/\mu')^3$ that lead to the correct hierarchy. In order to have a more quantitative flavor, we take the following representative values for the various parameters that appear in eqs. (22) and (27)

$$\begin{aligned} 16\pi^2 \beta_0 &= 12 - 2\frac{1}{2} \\ 16\pi^2 \beta'_0 &= 12 \\ 16\pi^2 \tilde{\beta}'_0 &= 12 - 2\frac{1}{2} \\ \left(\frac{\mu}{\mu'}\right)^3 &= 6 \end{aligned} \quad (34)$$

These correspond to a hidden gauge sector with gauge group $SU(4) \times SU(4) \times U(1)^2$ (which can occur in the Z_3 orbifold [51]) where each sector has a 4 and a 4 pair of matter representations and one pair acquires a mass at an intermediate scale with $\frac{\mu}{M_I} = 1300$, a perfectly reasonable value. Now, from eqs.(27), (30), (31) and (33) we can derive the corresponding values of S_r , N_I , T_I , $m_{3/2}$

$$\begin{aligned} S_r &= 1.047 \\ N_I &= 13.27 \\ T_I &= 5.03 \\ m_{3/2} &= 1.47 \times 10^{-16} \end{aligned} \quad (35)$$

which are acceptable for a realistic phenomenology (notice that $m_{3/2} = 294 \text{ GeV}$). In this particular example, the condensation scales are very close to the intermediate scale but this is

not a general feature. We can always obtain widely separated scales by having $\beta'_0 > \beta_0 > \tilde{\beta}'_0$. The new feature, namely the latter inequality, can be achieved here by having more matter in the primed sector before intermediate scale breaking⁵. It is not even necessary to have matter in the gaugino condensate sector. For example the hidden sector in the Z_3 orbifold may be $SU(3) \times SU(6) \times U(1)$ [51] with $16\pi^2 \beta_0 = 9$ and $16\pi^2 \beta'_0 = 18$. Provided $(\frac{\mu}{\mu'})^3 = 2 \times 10^5$ a large hierarchy develops with $m_{3/2} = 135 \text{ GeV}$. We obtain

$$\begin{aligned} S_r &= 0.98 \\ N_I &= 13.53 \\ T_I &= 5.1 \\ m_{3/2} &= 6.74 \times 10^{-17} \end{aligned} \quad (36)$$

This occurs if the $SU(6)$ hidden sector contains twelve 6 + $\bar{6}$ pairs of matter representations which acquire mass at an intermediate scale given by $\frac{\mu}{M_I} = 450$. The hidden sectors below this intermediate scale are pure gauge simplifying the analysis of gaugino condensation. These examples serve to emphasise the point that there are many possible models in which a large hierarchy results.

We have verified, by a numerical exploration, that the previous values, obtained by imposing $\frac{\partial f}{\partial S} = 0$, are extremely close to the actual values of the minimum. (The numerical minimization was performed taking into account the one-loop radiative contributions to the effective potential but, in fact, these are very small for this case. The fact that now the gauginos are massive do not modify this result.) Actually the effective potential can only take negative values within a narrow band around S_r^0 and in a second disconnected region of very small S_r (and hence large coupling constant). As it was mentioned in subsect. 2.4 this region is not trustworthy and, in addition, does not spoil the stability of the explicit solution obtained above.

Several comments are in order here. First, the numerical values of β_0 , β'_0 and $(\mu/\mu')^3$ depend on the specific string construction considered. In particular, as explained in subsect.3.1 $(\frac{\mu}{\mu'})^3 = \left(\frac{M_I}{\mu}\right)^{3(1-\tilde{\beta}'_0/\beta'_0)}$, with $\tilde{\beta}'_0$ and β'_0 the one loop beta functions before and after the breaking

⁵We thank L. Dixon for pointing out this possibility

respectively. It should be noticed here that for the case of a $F-I$ breaking the value of $(\frac{\mu}{\mu'})^3$ does not have any dependence on S . Second, there are many combinations of β'_0/β_0 and $(\mu/\mu')^3$ that lead to a good phenomenology too, i.e. to similar values to those of eqs. (35) and (36) without the necessity to fine-tune any parameter. Finally one may wonder whether these results are modified when other forms for W_T are considered. The answer is that they will remain basically unchanged. Examples of this have been given in ref.[36]. For a renormalizable superpotential this similarity is not an accident but follows using the general bounds obtained in subsec 2.3. We have represented in fig.1 the bound eq.(13) for one of the present examples ($f(S)$ as given by eq.(26) with the numerical values of eq.(34)). The value of $\text{Im } S$ is chosen in fig.1 so that the upper bound, $(1 + \sqrt{3})\frac{f(S)}{f(S_*)}$, is a maximum. Clearly there are two separate regions in which the bound is fulfilled. This implies two different, but stable minima (or sets of minima) kept apart by a high energy barrier[54]. Obviously, the region of large S_* , i.e.

$$1.041 < S_* < 1.051 \quad (37)$$

is the phenomenologically interesting one. This represents a remarkably narrow band. Notice that (37) corresponds to the neighbourhood of the point where $f'(S)$ vanishes and hence the upper bound goes to infinity (it is easy to check that for the range (37) the maximum upper bound is obtained with $\text{Im } S$ in such a way that the two condensates get opposite signs). Therefore the solution previously obtained, eqs.(27) and (28), belongs to the allowed bound given by (37). The other region of non-trivial solutions falls completely in the strong coupling zone (recall that $S_* = 1/g_{\text{eff}}^2$) so, as explained above, we cannot trust the results very much. It is remarkable that any non-trivial solution should lead to eq.(37), *independently* of the form of W_T (in particular this holds for other Z_N compactifications). Moreover, due to the constraint (10) the scale of the supersymmetry breakdown will be essentially fixed by (37). This explains why the previous result was not a mere accident. On the other hand, when the whole trilinear superpotential W_T is considered, we can expect slightly different values for N_1, N_3, N_6 , which is very convenient to get a realistic fermion mass hierarchy [19]. We have concentrated here on the possibility there are just two gaugino condensates in the hidden sector. However the hidden sector can involve more

than two gauge sectors capable of generating gaugino condensates and it is reasonable to ask if this will lead to a different pattern of hierarchy generation. The answer is yes for, with three or more gaugino condensates each with an arbitrary phase, it is possible, at tree level, to have $\frac{\partial f}{\partial S} = 0$ for arbitrary $\text{Re } S^0$ and $\text{Im } S^0$ (in the two condensate case only $\text{Im } S^0$ was undetermined at tree level, cf. eq.(28)). As a result the value of $\text{Re } S^0$, and hence the hierarchy will be fixed by radiative corrections (without the need for intermediate scale breaking). This will give a hierarchy of the form given in ref.[39], the size of the hierarchy being largely determined by the number of scalar fields left light after compactification.

3.3 FERMION MASSES AND MIXING ANGLES

As we have just seen, besides the scale of supersymmetry breaking, the minimization of the effective potential fixes other important parameters of the theory, in particular the gauge coupling constant at the unification point and the expectation values of the moduli which give the size and shape of the compactified space. At the minimum there is a cancellation between contribution of the exponentially suppressed gaugino condensates to the potential and the exponentially suppressed contribution coming from the vev of the superpotential (see eq.(31)). The cancellation fixes the moduli at reasonable values. In particular they are suitable to implement the mechanism for the generation of a family mass hierarchy explained in ref.[19]. Let us briefly summarize how this mechanism works.

The observed structure of fermion masses is a problem for which the standard model cannot provide any solution. It is then remarkable that orbifold spaces have a beautiful mechanism to generate a mass hierarchy at the renormalizable level. Namely the Yukawa couplings get suppression factors that depend on the relative position of the fixed points to which the relevant fields are attached [41] (Calabi-Yau spaces do not have so rich structure at least at the renormalizable level).

This geometrical origin for the hierarchy can work only if the physical matter fields belong to twisted sectors. The case of matter lying entirely in the untwisted sector has been discussed in ref.[19], where it was found that this possibility can hardly be realistic. Yukawa couplings between twisted fields are known for the Z_3 and Z_7 orbifolds [41,55]. For the Z_3 case the

complete expression is

$$LY_{\text{eff}} \sim \sum_i e^{-\beta|f_i|^2} (T_1 T_2 T_3); \quad \vec{v} \in (1 - \theta^2)(f_3 - f_2 + \Lambda) \quad (38)$$

where f_i are the positions of the fixed points associated with T_i , $\beta = \pi/2\sqrt{3}$ and Λ is the torus lattice. It is possible to check that eq.(38) coincides with eq.(23) for an orthogonal Z_3 orbifold. In general the Z_3 orbifold can get deformations compatible with the point group. These degrees of freedom correspond to the hermitician part of the nine (1,1) moduli surviving compactification. A possible set of these nine independent parameters are the radii of the three $SU(3)$ sublattices (say R_1, R_3, R_6) and the six angles ($\theta_{ij}, \theta_{i,j+1}; i, j = 1, 3, 5; i < j$) between the lattice vectors ($\mathbf{e}_k; k = 1, \dots, 6$). Taking for simplicity the case $R_i > 1$, it is possible to check that the coupling (38) is dominated by the term $e^{-3\beta d^2}$ where d is the shortest distance between fixed points, whose explicit dependence on the previous nine parameters can be found in ref.[19].

The point is that with a careful assignment of the physical fields to fixed points one can reproduce the desired pattern of fermion masses for particular values of the deformation parameters. This has been explicitly shown in ref.[19]. It was found that for this mechanism to work the values of the radii should be $R_i^2 \sim O(10)$, which is precisely what has been obtained in the previous subsection by minimization of the scalar potential, i.e. *dynamically*. These are good news for the scenario proposed.

What about quark mixing angles? It is remarkable that at the renormalizable level the Z_3 mass matrices are always diagonal [19]. If this were the whole story we should conclude that quark mixing angles are not possible within the Z_3 orbifold framework. Fortunately things are quite different when the gauge group is broken after compactification which precisely coincides with the second ingredient for realistic supersymmetry breaking (see subsection 3.1). This breaking can be provided by the above-mentioned F-I mechanism. Then off-diagonal mass matrix elements can appear for instance through higher order operators (although this is not the only possibility). As these operators are usually suppressed, this leads to a generic ansatz for the form that a viable quark mass matrix should exhibit in orbifold models

$$\mathcal{M} = \begin{pmatrix} \epsilon & a & b \\ \bar{a} & A & c \\ \bar{b} & \bar{c} & B \end{pmatrix}$$

where $\epsilon, a, \bar{a}, b, \bar{b}, c, \bar{c} \ll A \ll B$ in magnitude. The properties of this ansatz have been studied in ref.[19]. It turns out to be perfectly compatible with the experimental KM matrix, leading even to predictions for some of the most poorly known parameters. For example the weak CP violation angle should be, according to this ansatz, close to the maximal value, i.e. $\delta \sim \pm 90^\circ$.

4 SUMMARY AND CONCLUSIONS

We have reviewed the status of supersymmetry breaking in superstring theories and presented some recent work on the subject, which is showing a revived interest. This is in part due to the recent improvement of the knowledge about superstring effective Lagrangians, which allows for a reliable analysis of the effective potentials coming from physically relevant superstring theories. This is intimately related to supersymmetry breaking, since this must necessarily come as a result of the minimization of the effective potential. Some of our main results are the following

- i) In the absence of gaugino condensation no supersymmetry breaking is possible.
- ii) Supersymmetry breaking cannot be realistic with just one gaugino condensate.
- iii) In order to get a realistic supersymmetry breaking three essential ingredients should be present. Namely multiple gaugino condensation, a stage of symmetry breaking after compactification and a (non-perturbative) moduli dependence of the superpotential.

These results hold for a wide class of Z_N orbifold compactifications and are very likely to be maintained for other schemes. As it is explained in the text, the three ingredients of (iii) are usually present in string constructions. We have illustrated how the mechanism works with explicit examples in the Z_3 orbifold framework. For other, less attractive, schemes the point (iii) could be relaxed in part as we have seen in subsec.3.1. We have also presented very useful and general bounds on the gauge coupling constant that must be respected for any viable model. Besides the scale of supersymmetry breaking, the minimization of the effective potential fixes other important parameters of the theory, in particular the gauge coupling constant at the unification point and the expectation values of the moduli which give the size and shape

of the compactified space. At the minimum there is a cancellation between contribution of the exponentially suppressed gaugino condensates to the potential and the exponentially suppressed contribution coming from the vev of the superpotential. The cancellation fixes the moduli at reasonable values. In particular they are suitable to implement the mechanism for the generation of a family mass hierarchy explained in ref.[19], which is also reviewed in the text. All these results are, in our opinion, very encouraging. It is very suggestive that the calculation relates several a priori unconnected facts, namely an acceptable supersymmetry breaking scale, reasonable small coupling constants at the unification point, a large compactification mass ($M_c \lesssim M_P$) and the observed fermion mass hierarchy.

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Fig. 1 - Graphic representation of bound eq.(13)