Modulator CERN Synchro-Cyclotron

by

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GENEVE
Note

The present report contains the work for which - as member of the CERN Synchro-Cyclotron Division - the author was mainly responsible. It covers the period July 1952 - August 1957. The work was discussed at several meetings of the Division during this period.

The members and the consultants of the Division, who were participating in the design, the construction and the testing of the machine are mentioned below:

C.J. Bakker (General Director),
W. Gentner (Division Director),
F. Blythe, F. Bonaudi, G. Boyadjian, E. Braunersreuther,
D.H. Broadbent, J.M. Cassels, P. Debraine, E. Hedin,
C. Fredriksson, M. Georgijević, M. van Gulik, F. Kriven,
M. Lažanski, M.J. Moore, M. Morpurgo, W. Paul,
T.G. Pickavance, K.H. Schmitter, H.W.B. Skinner,
Summary

This report deals with the design of the tuning fork modulator for the 600 MeV Synchro-Cyclotron, which accelerator has come into operation for the first time in August 1957. The intricacy of the modulator in the accelerator as a whole, required the development of the principle of scaling. Basic design considerations have been put forward. These considerations have initiated an experimental program, which in its turn verified and corrected the starting point. The mechanics of the tuning fork and of the driving system have been dealt with in extenso. Suitable mechanical and electrical analogues have been derived, which in many respects clarify the nature of the problem. The control and initial operation of the system have been described and possible improvements are mentioned. The report as such is drawn up from author's unpublished divisional reports, engineering notes and communications with contractors.

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1. Introduction

1.1 One of the leading concepts for the design of the 600 MeV Synchro-Cyclotron was the scaling up of a known Synchro-Cyclotron of lower energy. We compute this scale factor and show how the factor reflects on the high frequency requirements. The starting relation is:

\[ \omega = \frac{eB}{m} = \frac{v}{r} \]

in which \( \omega \) is the angular frequency of the high frequency field, \( v \) is the velocity, \( r \) the radius, \( e \) the charge, \( m \) the relativistic mass of the particle and \( B \) the magnetic flux density at radius \( r \). The second identity of (1) \( eBr = mv = \) gives with \( \frac{p^2 c^2}{c^2} = E^2 - E_0^2 \) in which \( E \) is the total energy and \( E_0 \) is the rest energy:

\[ E^2 = E_0^2 = (ecBr)^2 \]
Now multiplication with a factor $g$ of all parts of a magnetic circuit including the number of ampere turns, leaves unaltered the field configuration, whatever the amount of magnetic saturation may be. From this we conclude that the critical radius at which the $n$-value of the machine

$$n = -\frac{1}{B} \frac{dB}{dr} = 0.2$$

also increases with the factor $g$. Consequently the scale factor between two similar machines with total energy $E_1$ and $E_2$ respectively is given by:

$$g^2 = \frac{E_1^2 - E_0^2}{E_2^2 - E_0^2}$$

The first identity of (1) $\omega = 2\pi f = eB/m$ gives with $E = mc^2$

$$f = \frac{eBo^2}{2\pi E}$$

From this we conclude that the upper frequency limit for both machines is inversely proportional with the total energy.

Now the electrode system which accelerates the protons is part of a transmission line, the total length of which is closely related to the upper frequency limit.

$$L \approx \mu \lambda_{\min}$$

This relation becomes plausible if one agrees that the system should work in the lowest possible mode. The standing wave associated with the upper frequency limit has necessarily a voltage node at a radius about equal to the pole radius, hence the lowest mode for the frequency could be a half wave, open line section, and the frequency variation could be obtained by shunting a variable capacitance at the open end.

Apparently the construction of bigger machines becomes increasingly difficult. Not only the frequency interval to be covered by the modulator increases, but the high frequency system as a whole tends to get squeezed between the magnet poles and coils.
Mostly by trial and error improvement of a preliminary design is obtained by shifting the variable capacitance along the line and changing the characteristic impedance of some line sections to suit special requirements with respect to the voltage distribution, the capacitance variation and loading, and the oscillator tube behaviour. It appeared always essential to keep the total length as short as possible and to minimize the total stored electro-magnetic energy. As a result of the above considerations a suitable place for the variable capacitance could be found in the region where the stray field of the magnet goes through zero. The problem has also been approached by considering Foster's reactance theorem in connection with some properties of the energy-function (Morpurgo), leading to essentially the same results.

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1.2 A variable capacitance is not the only way to modulate the frequency. Serious attempts have been made to use ferrite loaded tuning elements. The results were not encouraging. The heat dissipation in the ferrite slabs were estimated to be about half a megawatt high frequency, and 100 kW low frequency for a machine of this size. However for smaller machines ferrites are very well possible (Kriienen).

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1.3 For the variable capacitance two possible designs were considered. One is the conventional rotating capacitor, the other a tuning fork type of capacitor.

In designing a rotating capacitor one has to face, in addition to a formidable engineering on machining and balancing, such problems as rotating vacuum seals, high frequency bypassing of the bearings, heavy duty stand off insulators and the suppression of parasitic modes along the rotor. A tuning fork design also requires formidable engineering on machining and balancing but the other requirements are less stringent. We therefore concentrated on the latter design, although it is now clear that a rotating capacitor for a high energy Synchro-Cyclotron can be made to work (Mints).
2. Design considerations

2.1 Electrical system

Many investigations were needed before the design of the modulator could be frozen and the manufacturing could be started. Experiments to that end were undertaken at the Philips Physical Laboratory, Eindhoven (Netherlands) by Mr. B. Bollée, who under the supervision of Mr. G. de Vries carried out High Frequency tests on a 1/10 scale model working in the U.H.F. range of 150 - 300 MHz with the Philips tubes EC 56 and EC 55. The results were in fair agreement with preliminary calculations. Thus the various line sections and the capacitance range were determined. In the figure below are given the most important high frequency parameters:

![Diagram](https://via.placeholder.com/150)

- Dee = 1350 pF
- $Y_{eq} = 0.17$ mho
- Tuning fork data
- Stem $Y_{eq} = 0.17$ mho, $Y_{eq} = 0.125$ mho
- Stub $Y_{eq} = 0.17$ mho, $Y_{eq} = 0.125$ mho
- Cmin 230 pF, 20 kV, 29.2 MHz
- Cmax 2300 pF, 10 kV, 16.8 MHz

**fig. 1** Transmission line analogue.

Although in principle the electrical system was herewith determined some further experiments were done at the manufacturing department of Philips by Mr. S. Kortleven, under the supervision of Mr. L. van Mechelen. Work was done on a model in vacuum, operating at true frequencies but scaled up in the impedances. A slice of a true size tuning fork was used to modulate. Also a life size model was constructed in wood. It worked in air with the true oscillator tube at slightly reduced voltage and was modulated by a flap capacitor, considering the electrical respect equivalent to a tuning fork.
Both series of experiments indicated that throughout the desired frequency range the conditions for stable oscillations could be met, but the adjustments were very critical. Considerable improvement could be obtained if the conventional anode and cathode coupling of the oscillator tube was abandoned. Instead the grid driving power is derived from the anode directly through phasing elements, the transmission line acting now as an electrical flywheel (Schmitter). Figure 2 shows a radial section through the system.

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2.2 Tuning fork capacitor

The tuning fork capacitor resembles in cross section a normal tuning fork with two prongs and a handle which is transformed into a baseplate. The variable capacitance is formed between the stator and the tips of the tuning fork, as is shown in figure 3. In order to obtain sufficient capacitance the tuning fork has to be rather wide. The above requirements were met with an overlap of 100 mm. and a width of 2 meters. The minimum gap is 1.5 mm. and the maximum gap is 25 mm.

The stator is tapered so as to have the minimum gap constant. Therefore, in the position of minimum capacitance the gap is not constant. This effect added to the stray capacitance accounts for the fact that notwithstanding the gap ratio of 1/16 the minimum capacitance is about one tenth of the maximum capacitance. The width of 2 meters of the tuning fork is a convenient size to connect up to the Dee structure and leaves enough space for accommodating the two vacuum diffusion pumps. The machining of the tuning fork out of a solid block, although difficult was considered to be better than to clamp two flat blades on a common beam. (See fig. 17). Since the stresses in the material are proportional to the frequency of free vibration, a safe figure for the latter was found to be 55 Hz. The high frequency voltage requirements associated with this modulator frequency could be met also. In section 3 the mechanics of the tuning fork is discussed.

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2.3 Tuning fork drive

The tuning fork is driven in free vibration by means of an electro-mechanical transducer attached to the baseplate. This underlies a reversal of the principle of a tuning fork: when in motion and resting on a soundboard, it will transmit energy to the latter and so produce sound. Thus if energy is fed in the reverse direction and in the proper frequency the prongs will be set in vibration. For a discussion of the mechanics see section 4. The proper frequency is maintained by means of a feedback loop which in the final execution took shape as an optical device.

As a result of the low losses in the tuning fork proper, very little power is required to obtain the full amplitude. This makes the system very sensitive to additional losses, among which the eddy currents caused by the stray field of the magnet appeared to be of importance. The tuning fork is located symmetrical with respect to the median plane of the magnet, the motion is therefore nearly parallel with the magnetic flux. Indeed no losses were measured on a quarter scale model placed in 800 Gauss but the conditions were not quite reproducible and special measures had to be taken in the actual machine. See under 6. The electrostatic force of attraction between tuning fork tip and stator, due to the high frequency voltage between them, contributes in the case of pulsed operation some driving power to the tuning fork. Since the voltage is only on during the time that the gap decreases the force has the same direction as the velocity and work is done on the moving system (of the order of 2 Watt).

The heating up of various parts of the electrode system and the vacuum tank may cause some deformation and reflect in a twist or a translation of the tuning fork with respect to the stator. Also the switching on of the magnetic field or the vacuum pumps may cause similar non reproducible effects. From the above it will be clear that elaborate measures have to be taken to ensure the proper amplitude and position of the tuning fork. This will be discussed in section 5.

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2.4 Tuning fork cooling

The high frequency currents in the tuning fork blades cause heat dissipation. To this may be added the deformation heat developed with the vibration. The high frequency current flows on the outside of the blades. Since the path is short compared with the wavelength the (surface) current density can be assumed constant, and likewise the specific heat dissipation, except at the tip where also current flows on the inside.

The current in an open transmission line is proportional to the Dee voltage, the line admittance and the sine of the electrical length. If one averages these quantities, a representative current is found to be 1600 A, and thus a surface current density \( J = 400 \, \text{A/m} \). The corresponding heat dissipation is found with:

\[
(7) \quad S = \sqrt{\frac{\mu}{c}} \omega K^2
\]

This turns out to be 160 \( \text{W/m}^2 \) for aluminium, if the representative frequency is chosen to be 20 MHz and \( \sigma = 2 \cdot 10^7 \, \text{mho/m} \). This figure has still to be multiplied with the duty cycle \( = 1/2 \) since the h.f. current only flows when the high frequency decreases. If this heat should be reradiated the temperature would be probably too high. Therefore watercooling is provided for at the base plate of the tuning fork. The temperature is now highest at the tip and lowest at the base plate. Neglecting the effect of reradiation the equation for the temperature gradient becomes:

\[
(8) \quad \int_{0}^{x} S_{x} \, dx + \tau h_{x} \frac{dT}{dx} = 0
\]

in which \( x \) is the distance from the tip,

\( \tau \) the coefficient for heat conduction \( \approx 140 \, \text{Watt/m}^0 \text{C} \),

and \( h_{x} \) is the thickness of the blade.

In first approximation the temperature gradient is constant because the blade is tapered linearly from tip to base; substitute only \( S_{x} \, dx = S \, x \) and \( h_{x} = ax \). This is very fortunate, because under these conditions the temperature dilatation would be stress free and the much feared buckling of the blade would not occur. The
actual situation reveals a total temperature drop of 13°C and both
tip and base about 2 degrees lower than constant temperature gradient
would prescribe (Bollée). Figure 19 shows measurement of temperatu-
re dilatation.

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2.5 Experimental work and manufacturing

Considering the desired life time of the tuning fork
in the order of 10^9 vibrations and the high stresses in the alumi-
rium, selection of a metal with a high fatigue limit is indicated.
In addition the metal should be homogeneous in large slabs of
2000 x 550 x 130 mm and should not require special treatment after
machining. The emphasis is here with put on rather simple alloys
among which type 57 S was selected. The supplier was the Netherlands
Aluminium Company (N.A.M.), Utrecht, who delivered blocks of the
above size ready for machining. Supersonic tests were carried out
on each block, and x-ray test on samples. The machining was entrus-
ted to the Rotterdam Drydock Company (R.D.M.), Rotterdam (Netherlands),
who used the method of successive approximation to obtain the desi-
red profile. Special care was taken on the finish of the interior
curved part, since maximum stresses occur in that region. Thickness
deviations measured on lines parallel to the long side could be kept
below 0,1 mm. Straightness of the blades was within 0,2 mm. Tests
on two quarter scale models in vacuum were undertaken. They provided
data on the life time, drive power, constancy of frequency and ampi-
tude and undesired modes of vibration. Since tests on full size tu-
ning forks could only be performed on the completely assembled
Synchro-Cyclotron, preliminary tests were done in a hydrogen atmos-
phere. The damping compared with air atmosphere is an order less
and next best to vacuum. Data were obtained on undesired modes,
temperature dilatation and the suspension of the tuning fork. The
final tests in which the vibrating tuning fork carries the full
high frequency current, could only be done in situ and is discussed
in section 6.

* * *
3. Mechanics of the tuning fork

3.1 Introduction

The calculation of the motion of the tuning fork and the stresses in the material has to be done in steps. It is plausible to consider as a first approximation the vibration of flat rectangular blades, which are clamped rigidly along one side and tapered towards the tip. This is done in para. 2, 3 and 4. We then proceed in para. 5 to consider the motion of two such blades which at the thicker part are curved along a quarter of a circle and so joined together. Finally in para. 6 the higher modes of vibration are discussed.

Already Kirchhoff considered the vibration of sharp wedges and sharp cones. Compared with the vibrations of cylindrical bars of rectangular or circular cross-section respectively he found considerably lower stresses in the tapered bars, provided the material, the mode of vibration, the frequency and the displacement of the free end is the same.

This is of some importance for our Cern-machine if we adopt the tuning fork capacitance for varying the frequency. Unfortunately we cannot have the blades tapered to zero thickness for constructional and electrical reasons. We therefore have to analyse the problem anew and to look what can be gained with a tapering to finite thickness. We shall see that the general solution is the same as in Kirchhoff's case.

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3.2 Equation of motion

Let \( \frac{E}{\rho} \) be Young's modulus, \( \rho \) the density, \( S \) the area of the cross-section and \( I \) the moment of inertia of the cross-section about an axis through its centre of gravity perpendicular to the plane of bending. \( I \) and \( S \) being a function of the independent variable \( u \) which is thought along the axis of the bar. If \( t \) is the time variable then the displacement \( x \) is considered to be a function of \( u \) and \( t \) only.
\[ x = x(u, t) \]

The slope at a point of the neutral zone is given by:

\[ g = \frac{\partial x}{\partial u} \]

The bending moment we find from:

\[ M = -Q \cdot I \cdot \frac{\partial^2 x}{\partial u^2} \]

The shearing force as:

\[ F = \frac{\partial M}{\partial u} = -Q \cdot \frac{\partial}{\partial u} (I \cdot \frac{\partial^2 x}{\partial u^2}) \]

The resultant force acting on a length \( du \) of the bar is given by:

\[ \frac{\partial F}{\partial u} \cdot du = -Q \cdot \frac{\partial^2}{\partial u^2} (I \cdot \frac{\partial^2 x}{\partial u^2}) \cdot du \]

which must be equal to the mass in question \((\rho \cdot S \cdot du)\) multiplied with the acceleration \((\frac{\partial^2 x}{\partial t^2})\).

So the equation of motion is given by:

\[ (9) \quad \frac{\partial^2 x}{\partial t^2} = -\frac{Q}{\rho \cdot S} \cdot \frac{\partial^2}{\partial u^2} (I \cdot \frac{\partial^2 x}{\partial u^2}) \]

Splitting off the harmonic time variable with \( x = y \cdot \exp(-i \cdot \omega t) \) we get:

\[ \omega^2 \cdot y = \frac{Q}{\rho \cdot S} \cdot \frac{\partial^2}{\partial u^2} (I \cdot \frac{\partial^2 y}{\partial u^2}) \]

Confining ourselves to a linear taper as mentioned:

\[ S = S_o \cdot u \quad \text{and} \quad I = I_o \cdot u^3 \]

we then have to solve:

\[ \omega^2 \cdot y = \frac{Q \cdot I_o}{\rho \cdot S_o \cdot u} \cdot \frac{\partial^2}{\partial u^2} (u^3 \cdot \frac{\partial^2 y}{\partial u^2}) \]
Changing the independent variable $u$ into $v$ with:

$$\omega \cdot u \cdot (S_0 \cdot \rho)^{\frac{1}{3}} = v \cdot (I_0 \cdot q)^{\frac{1}{3}}$$

we get:

$$v \cdot \frac{d^2 y}{dv^2} \left( v^3 \cdot \frac{d^2 y}{dv^2} \right)$$

Now a general solution of both equations:

$$v \cdot \frac{d^2 y}{dv^2} + 2 \frac{dy}{dv} = \pm y$$

is also a solution of the former equation. The general solution of this equation is therefore a linear combination of these two. They are respectively:

$$y = \frac{1}{2} \cdot v^{-\frac{1}{2}} \cdot Z_1 \left( 2 \cdot v^{\frac{1}{2}} \right)$$

$$y = \frac{1}{2} \cdot v^{-\frac{1}{2}} \cdot Z_1 \left( 2 \cdot i \cdot v^{\frac{1}{2}} \right)$$

in which $Z_1$ stands for a linear combination of Bessel functions of the first order. For the real argument we choose:

$$Z_1 \left( 2 \cdot v^{\frac{1}{2}} \right) = A \cdot J_1 \left( 2 \cdot v^{\frac{1}{2}} \right) + B \cdot N_1 \left( 2 \cdot v^{\frac{1}{2}} \right)$$

and for the imaginary argument:

$$Z_1 \left( 2 \cdot i \cdot v^{\frac{1}{2}} \right) = - i \cdot C \cdot J_1 \left( 2 \cdot i \cdot v^{\frac{1}{2}} \right) - D \cdot H_1^{(1)} \left( 2 \cdot i \cdot v^{\frac{1}{2}} \right)$$

Upon changing the independent variable $v$ into $w$ by $2 \cdot v^{\frac{1}{2}} = w$ our displacement is given by:

$$(10 \ a) \ y = \left( 1/w \right) \cdot \left[ Z_1(w) + Z_1(i \cdot w) \right]$$

or written in full:

$$y = \left( 1/w \right) \cdot \left[ A \cdot J_1(w) + B \cdot N_1(w) - i \cdot C \cdot J_1(i \cdot w) - D \cdot H_1^{(1)}(i \cdot w) \right]$$

For obtaining $g$, $M$ and $F$ we note that:
\[
\frac{d}{du} = \omega \cdot \left( \frac{S_o \cdot \rho}{I_o \cdot Q} \right)^{\frac{1}{2}} \quad \frac{d}{dv} = \omega \cdot \left( \frac{S_o \cdot \rho}{S_o \cdot Q} \right)^{\frac{1}{2}} \cdot \frac{2}{w} \cdot \frac{d}{dw}
\]

\[
S = \frac{S_o}{\omega} \cdot \left( \frac{I_o \cdot Q}{\rho} \right)^{-\frac{1}{2}} \quad v = \frac{S_o}{\omega} \cdot \left( \frac{I_o \cdot Q}{S_o \cdot \rho} \right)^{-\frac{1}{2}} \cdot \frac{w^2}{4}
\]

\[
I = \frac{I_o}{\omega^3} \cdot \left( \frac{I_o \cdot Q}{S_o \cdot \rho} \right)^{-\frac{1}{2}} \cdot \frac{w^6}{64}
\]

\[
\frac{d}{dw} \left[ w^{p_k} Z_p(a \cdot w) \right] = a \cdot w^{p_k-1}(a \cdot w)
\]

\[
\frac{d}{dw} \left[ w^{p_k} Z_p(a \cdot w) \right] = -a \cdot w^{-p_k+1}(a \cdot w)
\]

With the time factor \(\exp(-i \cdot \omega \cdot t)\) omitted we find successively:

\[
g = - \left( \frac{g_o}{w^2} \right) \cdot \left[ Z_2(w) + i \cdot Z_2(i \cdot w) \right]
\]

\[
M = - M_o \cdot w^3 \cdot \left[ Z_3(w) - Z_3(i \cdot w) \right]
\]

\[
F = - F_o \cdot w^2 \cdot \left[ Z_2(w) - i \cdot Z_2(i \cdot w) \right]
\]

in which:

\[
g_o = 2 \cdot \omega \left( \frac{S_o \cdot \rho}{I_o \cdot Q} \right)^{\frac{1}{2}}
\]

\[
M_o = \frac{Q \cdot I_o}{16 \cdot \omega} \left( \frac{I_o \cdot Q}{S_o \cdot \rho} \right)^{\frac{1}{2}}
\]

\[
F_o = Q \cdot I_o / 8
\]

Three other quantities are of interest, namely the bending stress \(\sigma\), the shearing stress \(\tau\), the total stored energy \(W\). From \(M = 2 \cdot I \cdot \sigma / h\) in which \(h\), the thickness of the bar, expressed in \(I\) and \(S\) is given by:

\[
h = 2 \cdot (3 \cdot 1/3)^{\frac{1}{2}}
\]

we find:

\[
\sigma = - (3 \cdot Q \cdot \rho)^{\frac{1}{2}} \cdot (\omega/w) \cdot \left[ Z_3(w) - Z_3(i \cdot w) \right]
\]

and from \(F = S \cdot \tau\) we find:
\[ \tau = -\frac{1}{2} \cdot \omega \cdot (p \cdot q)^{\frac{1}{2}} \cdot \left( \frac{I_0}{g_0} \right)^{\frac{1}{2}} \cdot \left[ Z_2(w) - i \cdot Z_2(i \cdot w) \right] \]

Assuming \( y \) to be the amplitude of the displacement, the amplitude of the velocity is given by \( \omega \cdot y \). The elementary mass having this velocity is given by \( dM = p \cdot S \cdot du \) so the total stored energy is given by:

\[ w = \frac{1}{2} \cdot \int_{u''}^{u'} \rho \cdot S \cdot \omega^2 \cdot y^2 \cdot du \]

written with \( w \) as independent variable, we get:

\[ W = \frac{I_0 \cdot Q}{16} \cdot \int_{w''}^{w'} w \cdot \left[ Z_1(w) + Z_1(i \cdot w) \right]^2 \cdot dw = \]

\[ = \frac{I_0 \cdot Q}{16} \cdot \left[ \left. \frac{w^2}{2} \cdot \left[ Z_1(w) \cdot Z_1(w) - Z_0(w) \cdot Z_2(w) + Z_1(i \cdot w) \cdot Z_1(i \cdot w) - Z_0(i \cdot w) \cdot Z_2(i \cdot w) \right] \right|_{w''}^{w'} + i \cdot w \cdot Z_1(w) \cdot Z_0(i \cdot w) - w \cdot Z_0(w) \cdot Z_1(i \cdot w) \right] \]

\[ * * * \]

3.3 Boundary conditions

If the bar is clamped at \( u = u' \), equivalent to \( w = w' \), then we have \( y = y' = 0 \) so \( Z_1(w') + Z_1(i \cdot w') = 0 \)

we also have \( g = g' = 0 \) or \( Z_2(w') + i \cdot Z_2(i \cdot w') = 0 \)

This condition however reduces to:

\[ Z_0(w') + i \cdot Z_0(i \cdot w') = 0 \]

as \( Z_2(w') + i \cdot Z_2(i \cdot w') = \frac{2}{w} \left[ Z_1(w') + Z_1(i \cdot w') \right] - \left[ Z_0(w') + i \cdot Z_0(i \cdot w') \right] \)

At the free end \( u = u'' \) or alternatively \( w = w'' \)

we have \( M = M'' = 0 \) or \( Z_3(w'') = Z_3(i \cdot w'') = 0 \)

also \( F = F'' = 0 \) or \( Z_2(w'') - i \cdot Z_2(i \cdot w'') = 0 \)

written in full:

\[ \]
\[ A \cdot J_c(w') + B \cdot N_0(w') + C \cdot J_0(i \cdot w') - i \cdot D \cdot H_0(1)(i \cdot w') = 0 \]
\[ A \cdot J_1(w') + B \cdot N_1(w'') - i \cdot C \cdot J_1(i \cdot w') - D \cdot H_2(1)(i \cdot w') = 0 \]
\[ A \cdot J_2(w'') + B \cdot N_2(w'') = C \cdot J_2(i \cdot w'') + i \cdot D \cdot H_2(1)(i \cdot w'') = 0 \]
\[ A \cdot J_3(w'') + B \cdot N_3(w'') + i \cdot C \cdot J_3(i \cdot w'') + D \cdot H_3(1)(i \cdot w'') = 0 \]

These conditions admit a solution if the determinant formed by the Bessel functions is zero, giving at the same time a functional relation between \( w' \) and \( w'' \).

\[
\Delta(w', w'') = \begin{vmatrix}
J_0(w') & N_0(w') & J_0(i \cdot w') & -i \cdot H_0(1)(i \cdot w') \\
J_1(w') & N_1(w') & -i \cdot J_1(i \cdot w') & -H_1(1)(i \cdot w') \\
J_2(w'') & N_2(w'') & -J_2(i \cdot w'') & i \cdot H_2(1)(i \cdot w'') \\
J_3(w'') & N_3(w'') & i \cdot J_3(i \cdot w'') & H_3(1)(i \cdot w'') \\
\end{vmatrix} = 0
\]

In figure 4 is plotted the relation between \( w' \) and \( w'' \) for the fundamental mode of vibration. As it is important to obtain accurate values of \( w' \) and \( w'' \) we consider also the determinant \( \Delta(w' + \delta w', w'' + \delta w'') \) in which \( \delta w' \) and \( \delta w'' \) are arbitrary small increments of \( w' \) and \( w'' \) respectively. Omitting this operation, which is straightforward, we get:

\[
\Delta(w' + \delta w', w'' + \delta w'') = (1 - \frac{\delta w'}{w'} - \frac{\delta w''}{w''}) \cdot \Delta(w', w'') +
\]

\[
\begin{vmatrix}
0 & 0 & -i \cdot J_1(i \cdot w') & -H_1(1)(i \cdot w') \\
J_1(w') & N_1(w') & 0 & 0 \\
J_2(w'') & N_2(w'') & -J_2(i \cdot w'') & i \cdot H_2(1)(i \cdot w'') \\
J_3(w'') & N_3(w'') & i \cdot J_3(i \cdot w'') & H_3(1)(i \cdot w'') \\
\end{vmatrix}
\]
\[ \begin{bmatrix}
J_0(w') & N_0(w') & J_0(i\cdot w') & -iH_0(1)(i\cdot w') \\
J_1(w') & N_1(w') & -iJ_0(i\cdot w') & H_1(1)(i\cdot w') \\
0 & 0 & iJ_3(i\cdot w') & H_3(1)(i\cdot w') \\
J_3(w'') & N_3(w'') & 0 & 0
\end{bmatrix} + 2\cdot \delta w'' \]

\[ + \ldots \ldots \ldots \ldots \] higher powers of \( \delta w' \) and \( \delta w'' \).

With a given pair of \( w' \) and \( w'' \) we find the coefficients \( A, B, C \) and \( D \) as the minors of the elements of a certain row of \( \Delta(w', w'') \), that is to say in proportion.

The bending stress at the clamped end reduces to a simple form:

\[ \sigma' = -(3\cdot Q\cdot p)^{3/2} \cdot (\omega/w'') \cdot \left[ Z_3(w') - Z_3(i\cdot w') \right] = \]

\[ = (3\cdot Q\cdot p)^{3/2} \cdot (2\omega/w') \cdot Z_1(w') \]

as \( Z_3(w') - Z_3(i\cdot w') = (4/w') \cdot \left[ Z_2(w') + i\cdot Z_2(i\cdot w') \right] + \]

\[ - \left[ Z_1(w') - Z_1(i\cdot w') \right] \]

in which the first term is zero on behalf of \( g' = 0 \) and the second term reduces to \( -2\cdot Z_1(w') \) on behalf of \( y' = 0 \).

The displacement of the free end can also be written in a simpler form:

\[ y'' = (1/w'') \cdot \left[ Z_1(w'') + Z_1(i\cdot w'') \right] = -(2/w'') \cdot Z_3(w'') \]

as \( Z_1(w'') + Z_1(i\cdot w'') = (4/w'') \cdot \left[ Z_2(w'') - i\cdot Z_2(i\cdot w'') \right] + \]

\[ - \left[ Z_3(w'') + Z_3(i\cdot w'') \right] \]

in which the first term is zero on behalf of \( F'' = 0 \) and the second term reduces to \( -2\cdot Z_3(w'') \) on behalf of \( M'' = 0 \).
Now $\sigma'$ expressed in terms of $\omega$ and $y''$ yields:

$$\sigma' = \sigma_0 \cdot \omega \cdot y'' = -(3 \cdot Q \cdot \rho)^{\frac{1}{2}} \cdot \frac{w''}{w'} \cdot \frac{Z_1(w')}{Z_2(w')} \cdot \omega \cdot y''$$

in which $\sigma_0$ can be regarded as a mode-shape factor. The slope at the free end expressed in terms of $\omega$ and $y''$ results in:

$$g'' = \left(\frac{S_0}{I_0} \cdot \frac{\rho}{Q}\right)^{\frac{1}{2}} \cdot \frac{2}{w'} \cdot \frac{Z_2(w'')}{Z_3(w'')} \cdot \omega \cdot y''$$

The total stored energy can be reduced to:

$$W = \frac{I_0 \cdot Q}{16} \cdot \left[\left\{\frac{w'}{Z_1(w')}\right\}^{2} - \left\{\frac{w''}{Z_3(w'')}\right\}^{2}\right]$$

or expressed in the displacement $y''$ squared:

$$W = \frac{I_0 \cdot Q}{64} \cdot \frac{1}{w'} \cdot \frac{Z_1(w')}{Z_3(w'')} \cdot \left[\frac{w''}{w'} \cdot \frac{Z_1(w')}{Z_3(w'')} \left(\frac{w''}{w'} \cdot \frac{Z_1(w')}{Z_3(w'')}\right)^{2} - \frac{w''}{w'} \cdot \frac{Z_1(w')}{Z_3(w'')}^{4}\right] \cdot y''^{2}$$

* * *

3.4 Example

A suitable pair is formed by $w'' = 3,00$ and $w' = 5,65$.

From this we find: $A = 1,472$; $B = 2,280$; $C = 0,0144$; $D = 9,77$
also: $Z_1(w') = -0,6516$; $Z_2(w'') = 0,3495$; $Z_3(w'') = -0,773$

consequently:

(12 a) $\sigma' = -0,447 \cdot (3 \cdot Q \cdot \rho)^{\frac{1}{2}} \cdot \omega \cdot y''$

$$g'' = -0,301 \cdot \left(\frac{S_0}{I_0} \cdot \frac{\rho}{Q}\right)^{\frac{1}{2}} \cdot \omega \cdot y''$$

$$W = 1,94 \cdot I_0 \cdot Q \cdot y''^{2}$$

With the practical values: $(Q/\rho)^{\frac{1}{2}} = 5000 \text{ m/sec}$; $\omega = 314$;
$y'' = 0,0125$ m; $g'' = -0,04$ we find $(I_0/S_0)^{\frac{1}{2}} = h_o/12^{\frac{1}{2}}$ in which $h_o$ is
the thickness of the bar at unity distance from the origin. In our case $h_o = 0,0205$ (dimensionless)

with $u = \left(\frac{I_0 \cdot Q}{S_0 \cdot \rho}\right)^{\frac{1}{2}} \cdot \frac{w^{2}}{4 \cdot \omega} = 0,0236 \cdot w^{2}$
we find the coordinate of the free end \( u'' = 0.212 \) m; and of the clamped end \( u' = 0.755 \) m.

With \( (\frac{u}{p})^{\frac{3}{2}} = 4.06 \cdot 10^{7} \text{ kg/m}^2 \cdot \text{ sec (Steel)} \) we find:
\[
\sigma' = 123 \cdot 10^6 \text{ N/m}^2 \quad (= 12.5 \text{ kg/mm}^2),
\]

with \( (\frac{u}{p})^{\frac{3}{2}} = 1.4 \cdot 10^7 \text{ kg/m}^2 \cdot \text{ sec (Aluminium)} \) we find:
\[
\sigma' = 42.5 \cdot 10^6 \text{ N/m}^2 \quad (= 4.3 \text{ kg/mm}^2).
\]

Per meter width of the bar the total stored energy is found to be:

\[
W = 46 \text{ J (for steel) and } W = 16 \text{ J (for aluminium)}.
\]

In figure 5 is shown the dimensions of the bar, provided:

\[
(\frac{u}{p})^{\frac{3}{2}} = 5000 \text{ m/sec (steel or aluminium)}
\]
\[
\omega = 314
\]
\[
g'' = 0.04 \text{ at } y'' = 0.0125 \text{ m}.
\]

Without going into detail we found for a sharp wedge, \( w'' = 0, w' = 4.611 \):

\[
(12 \ b) \quad \sigma' = -0.222 \left(3 \cdot \frac{u}{p} \right)^{\frac{3}{2}} \cdot \omega \cdot y''
\]

and without taper, i.e. beam of constant section:

\[
(12 \ c) \quad \sigma' = -(3 \cdot \frac{u}{p})^{\frac{3}{2}} \cdot \omega \cdot y''
\]

The chosen taper seems to be a good compromise between a reduction of the bending stress and the possibility of making the bar.

One can ask for still better (non linear) tapers, solutions of which can be found if \( S = S_0 \cdot u^n \) and \( I = I_0 \cdot u^n \), \( m \) and \( n \) being integers, see also Nicholson. However, the machining of curved surfaces seems to be a tedious task.

* * *
3.5 Actual case

In the preceding paragraphs fairly exact solutions could be obtained, for which a simple indication is the measured resonance frequency. In the actual case of a tuning fork no such exact solutions could be found. However, of practical interest is to answer the question whether the stresses in the tuning fork at given frequency and amplitude are much higher than in corresponding straight bars. The experimental evidence is now that the resonance frequency of a tapered straight bar does not change much if the bar were bend to form one half of the tuning fork. Furthermore stroboscopic measurements reveal practically identical amplitude distribution. We conclude then than in both cases the kinetic energy is the same and likewise the potential energy which is here deformation energy.

We easily calculate that the bulk of deformation energy is found in the thick part of the beam. If we now assume that the neutral zone of the curved bar remains in the half way position, then we could be confident that the stress distribution in both cases are identical. However, some studies on the motion of rings reveal that the position of the neutral zone is not fixed and as a result we have to include a safety margin of some 20 %. In the course of the experimental programme we were able to apply strain gauges with sufficient accuracy. The measured maximum stress was found on the inside at the bend and amounts to 6.5 kg/mm² for the actual tuning fork at full amplitude.

Even in the ground mode the motion is not purely sinusoidal. The one-dimensional treatment for slender straight bars is accurate enough and holds also for the tuning fork insofar the stresses and the transversal motion are concerned. But with the help of a stroboscope one sees that the tuning fork moves also perpendicular to the base plate. The pattern of that motion is very much dependent on the suspension and the driving force. Since in actual fact the driving force does work against this longitudinal motion and so transmits energy to the tuning fork, it is of interest to find a suitable expression. For this we consider the free vibration of the freely suspended tuning fork. The centre of gravity
remains then at rest and we obtain as working quantity the displacement of the base plate (see fig. 7).

The rotation of one section with respect to another spaced ds is given by (see also section 3.2):

\[ \frac{d\alpha}{ds} = \frac{M}{\psi I} \]

and happens to be practically constant over the quarter circle AB and thus A'B stays part of a circle. In our case \( \alpha = 6 \cdot 10^3 \) radians at full amplitude. The projection of OD of AA' is easily found to be:

\[ OD = (1 - \frac{2}{n}) r\alpha \approx 0.14 \text{ mm.} \]

For symmetry reasons we neglect the effect of straight parts of the tuning fork. At the most they contribute to the second harmonic of the longitudinal motion. If now the curved part of the tuning fork, the base plate and the dead weight attached to the base plate had no mass, the amplitude of the base plate with respect to the centre of gravity would be the above calculated 0.14 mm. Upon bringing the above masses also into consideration the amplitude is somewhat less \( \approx 0.1 \text{ mm.} \)

* * *

3.6 Higher modes of vibration

In the course of the experiments many higher modes of vibration were discovered with the help of the stroboscope or by means of a sand pattern. In analogy with the discussion of 3.5 we may identify one class of these modes with the vibration of thin rectangular plates clamped along one edge. In this class opposite points on the prongs of the tuning fork always move in opposite direction. We may designate these modes by the number of node lines and require two indices, but the index associated with solutions of the one-dimensional case 3.3 may be discarded as the frequencies involved are many times the ground frequency (as a matter of fact the pureness of tone of a tuning fork is based on this). A second class exists which has about the same node lines but lacks the symmetry, opposite points on the prongs move always the same way.
There are two reasons why one should be concerned with these modes. If the frequency of one of these is very nearly equal to the ground mode frequency there is a risk that the self-consistent oscillating system locks into wrong vibration. Secondly the harmonics of the driving force may coincide with one of the higher modes of vibration, the effect of it being a superposition of this mode on the ground mode.

We can only deal qualitatively with the problem. Suppose we plot the frequency vs the number of lateral nodes and connect the discrete points for each class separately. Then we may consider the ground mode as the starting point for the symmetrical class. The frequency of the next point (one node) must be necessarily higher because there is an additional deformation energy due to the lateral bending and so on. The lateral deformation energy decreases with increasing width of the tuning fork, which sets a limit to the width in view of the possibility of interference. The a-symmetrical class starts obviously with one node, which frequency may be lower than the ground mode. This is not surprising because a considerable amount of the deformation energy results from a twist, which is an entirely different mechanism. So any of the modes belonging to the a-symmetrical vibration may be dangerously close to the ground mode. With respect to harmonics of the driving force the second provides a dangerous zone. In figure 3 are shown the two classes experimentally measured by Philips-Eindhoven.

In the course of the experiments such a dangerous zone was successfully shifted by increasing the lateral stiffness. To this end a L-shaped aluminium rib was screwed on the tuning fork (see fig. 20). The mechanical requirements are fortunately not high; as the rib is there to prevent the occurence of lateral modes, shearing forces on the screws do not exist.

At first sight it seems accidental that an harmonic of the driving force lies within the natural bandwidth of an unwanted mode. The explanation must be sought in the so called crossing of modes (Thornton). The frequency of free vibration depends on the temperature through dilatation, density variation and change of the
modulus of elasticity. There is also evidence, at least for aluminium, that the frequency is depending on the amplitude. The effects are not the same for every mode of vibrations and consequently their relative position on the frequency scale shifts. If we subject the tuning fork to all conceivable regimes of temperature and excitation, the shift is measured to be many natural bandwidths of the system.

* *

4. Driving System

4.1 Principle

Various principles have been considered, among which we may mention magnetostriction, working on a higher harmonic of the base plate motion; magnetized rotating disks pushing and pulling the base plate, and moving coil arrangements, enabling to apply purely sinusoidal forces. The forces on the base plate are fairly high, supposing they are in phase with the velocity:

\[ P = \frac{1}{2} \omega |x| F \]

in which \( P \) is the net power to be supplied to the tuning fork, say 16 Watt, \(|x|\) is the amplitude of the base plate, 0.1 mm., \( \omega \) is angular frequency of about 350. Consequently the amplitude of the force amounts to 1000 Newton. In virtue of this an electro-mechanical transducer had to be designed in which the magnetic forces could be reduced by a factor of 10. It was also advisable to suspend tuning fork plus driving system as a whole, so as to prevent forces being transmitted to the framework. The transducer is schematically arranged as follows (see fig. 9).

A core plus coil are fixed to the base plate. The yoke plus the attached mass can move relatively to the base plate through a set of springs. The coil carries a current of suitable periodicity and shape. The current may be harmonic with half the frequency of vibration, or DC biased or pulsed, each method works and has advantages; essential is that the magnetic force sets the yoke plus attached mass into vibration relatively to the base plate. Mass and spring are tuned approximately to the tuning fork frequency.
(16) \[ M'' S'' + M S' - \omega_o^2 M M'' = 0 \]

Apparently the amplitude of the yoke is 10 times the base plate amplitude (in order to reduce the magnetic force a factor of 10). This determines the minimum stiffness of the spring: \( S'' = 1000 \text{ Newton/mm} \) and consequently the minimum mass is found to be \( M'' = 10 \text{ kg} \) \( (\omega^2 = 10^5) \).

The above arrangement with the forces everywhere in phase with the velocity is however far too critical for practical use and we have to choose considerably higher mass and stiffness (approx. 2 times).

* * *

4.2 Tuning fork analogue

It is useful to have an expression for the driving point admittance of the tuning fork. The driving point here is obviously the base plate. Although the tuning fork is a continuous system, it can be represented in the neighbourhood of the resonance frequency at the ground mode by two masses connected through a spring (see fig. 10).

The idea is this: the tuning fork originates from a flat plate swinging in two nodes. Upon concentrating the mass in the centre and at the ends we get a workable analogue, but the displacements may be incorrect. Increase of the mass in the centre at the expense of the mass at the ends brings probably the proper displacements and finally to replace the transverse vibrations of the two springs by a longitudinal vibration of one spring is a matter of convenience. Indeed with the available parameters \( M' \), \( S' \) and \( M \) we may equate the total energy \( W \), the amplitude \( |x| \) at the driving point (M) and the frequency of free vibration \( \omega_o \). Losses are brought into the picture as friction \( B \) acting on the masses. If \( \mathbf{x} \) is the displacement of \( M \), \( \mathbf{x}' \) the displacement of \( M' \) and \( f_t \) is the driving force, the equations of motion are given by:

(17) \[ M \ddot{x} + B \dot{x} + S'(x - x') = f_t \]

\[ M' \ddot{x}' + B' \dot{x}' + S'(x' - x) = 0 \]

From this one finds easily that in the case of a harmonic force, by substituting off the time factor \( \exp(-i\omega t) \), the driving point
admittance is given by:

\[(18) \quad Y_t = \frac{\omega^2 M' - S' + i\omega B'}{B' (\omega^2 M - S') + B (\omega^2 M' - S') + i\omega (M'S' + MS' - \omega^2 MM') + BB'} \]

where \(Y_t\) is the complex ratio of velocity and driving force

\[(19) \quad \dot{X} = Y_t F \]

obviously the power fed into the system is given by:

\[(20) \quad P = \frac{1}{2} |F|^2 \text{Re}(Y_t) \]

We may associate \(B'\) with the deformation loss of the material and eventually eddy current losses as they occur in the tuning fork due to the stray field of the magnet. \(B\) may represent the suspension losses which in first approximation can be neglected.

\[(21) \quad \text{Re}(Y_t) = \frac{B'S'^2}{B'^2 (\omega^2 M - S')^2 + \omega^2 (M'S' + MS' - \omega^2 MM')^2} \]

We are now in the position to express \(M, M', S'\) and \(B'\) in terms of measurable quantities. The resonance condition is determined by the maxima in \(\text{Re}(Y)\). With sufficient accuracy, anticipating \(S' \ll \omega_o^2 M\), we find:

\[(22) \quad M'S' + MS' - \omega_o^2 MM' = 0 \]

Half the power absorption occurs at:

\[(23) \quad \mp \omega (M'S' + MS' - \omega^2 MM') = B' (\omega^2 M - S') \]

This equation is connected with the quality factor \(Q\) by introducing the detuning \(\Delta\omega\). Approximately one obtains:

\[(24) \quad Q = \frac{\omega_o}{2\Delta\omega} \approx \omega_o M'/B' \approx S'/\omega_o B' \]

At off-resonance one finds:

\[(25) \quad 1/Y_t \approx -i\omega M \]
In this case we expect the tuning fork to behave as its true mass so that $M$ of the analogue equals the true mass of the tuning fork (this includes of course dead weight as well).

The power absorption at resonance may be expressed in the amplitude $|x|$ of the base plate. One finds using (22) and (24):

$$P_{res} = \frac{\omega^2 |x|^2 B'}{M' \frac{M''}{M}^2 + \frac{1}{Q^2}}$$

By measuring $P_{res}$ or deducting this value from

$$Q = \frac{\omega W}{P}$$

in which $W$ is the total stored energy, known from 3.4, the equations (22), (24) and (26) may be solved for $M'$, $S'$ and $B'$.

* * *

4.3

Having thus established the characteristics of the tuning fork proper, we may now consider its behaviour in combination with the spring loaded mass of the iron yoke ($S''$, $M''$) and eventually refine the approximate expression (16)

![Diagram](image)

*fig. 11*

We assume equal and opposite forces $f$ between $M$ and $M''$, they are of course magnetic forces. The resulting force $f_\tau$ acting on the tuning fork will then be:

$$f_\tau = f + S''(x'' - x) = -M'' \ddot{x}'' - B'' \dot{x}''$$
We may associate $B''$ with deformation loss in the spring $S''$ and the hysteresis and eddy current losses in the magnetic circuit.

The complex equivalent of (28) works out as:

\[(29) \quad \mathbf{\dot{x}} - \mathbf{\ddot{x}} = \mathbf{Y F} \]

in which the driving point admittance for the magnetic force is given by:

\[(30) \quad \mathbf{Y} = \frac{1}{\frac{S''}{-i\omega} + \frac{1}{Y_t + \frac{1}{B'' - i\omega M''}}} \]

$\mathbf{Y}$ is presented here in a form to emphasize the analogy with electrical circuits. We may do the same for $Y_t$ in (18) and find the circuit presented in figure 12 ($Y_t$ is the upper branch).

The resonance condition is found upon maximizing $\text{Re}(\mathbf{Y})$. One finds in due course that the figures quoted in section 4.1 are minimum values. One may try to realize an electrical network according to (30). However, the damping is, even with the damping due to eddy currents, higher than say a thousand. Equivalent qualities are found in quartz crystals. Indeed if one draws up the dual network, the branch representing the tuning fork proper is in this case exactly the electrical analogue of a quartz crystal, and numerical equivalence can be obtained easily.

* * *

4.4

Assuming the flux density $B$ homogeneous in all parts of the magnetic circuit shown in figure 9, the force is given by:

\[(31) \quad F = \frac{1}{2} B^2 \frac{a}{\mu_0} \]

in which $a$ is the area crossed by the magnetic flux. $B$ is related to the current $I$ and the voltage $V$ in a somewhat complicated manner since the path of integration for $\int H \, dl$ is not constant. Assuming no saturation in the iron we find readily:
(32) \[ nI = B(h - 2(x - x''))/B_0 \]

\[ h = \frac{1}{\sqrt{\frac{B}{R}}} + 2d \]

\[ V = IR + \frac{1}{2}na dB/dt \]

in which \( n \) is the number of turns of the coil and \( h \) is a constant, and \( R \) is the resistance of the coil. Apparently \( f, B, I \) and \( x - x'' \) cannot be simple harmonic functions, even if one of them is so, but they will be in any case periodic functions and can be expressed adequately as a Fourier series, for instance the force:

(33) \[ f = \frac{1}{2\pi} \sum_{-\infty}^{+\infty} F_n e^{-i\omega t} \]

with the restriction that \( F_{-n} \) is the conjugate of \( F_n \) and \( F_0 \) is real. The equations of motion (17) and (28) are linear, according to them we may add solutions associated with each force component and obtain:

(34) \[ \dot{x} - x'' = \frac{1}{2} \sum_{-\infty}^{+\infty} Y_n F_n e^{-i\omega t} \]

in which \( Y_n \) is the driving point admittance associated with the frequency \( n\omega \) and \( Y_{-n} \) is the complex conjugate of \( Y_n \). As may be verified \( Y_0 = 0 \).

The velocity (34) may be integrated to provide the displacement in (32). Only a given series in \( B \) allows a straightforward computation for \( I \) and \( V \).

With a few components in \( B \) the general trend in the system becomes clear and optimum design parameters may be found. The introduction
of the driving point admittance on the basis of current and voltage is less suitable in this case. However, a moving coil system in a constant magnetic field lends itself to such a treatment very well.

* * *

5. Control

5.1 Suspension

The tuning fork suspension is entirely built inside the stub and is therefore in a region free from high frequency (see fig.14). The high frequency current passes from the outside of the tuning fork through thin phosphor bronze strips S to the stub liner. The system is hung from two steel straps attached at the bottom corners of the base plate and kept up by one steel strap attached to an outrigger in the centre of the base plate. The effective length of the steel straps is about 10 cm. The section is 20 x 0.2 mm. Corner pieces enable to adjust each strap independently. This adjustment is motor driven (M).

* * *

5.2 Feelers

In view of the minimum gap of 1.5 mm., section 2.1, the relative position of stator and tuning fork tip has to be kept with an accuracy of 0.1 mm. This relative position is measured mechanically by means of teflon feelers shown schematically in figure 15. If they are touched by the moving tuning fork, a built-in microswitch opens, giving a signal to the control system. The rest position of the feeler is equivalent to minimum gap of 1.4 mm. From time to time the feeler is brought up in two successive steps of 0.1 mm. each, to indicate minimum gaps of 1.5 and 1.6 mm. respectively. The signals received from the microswitches are translated in the control system to give the required commands to the servomotors M and the amplitude control motor. Assuming the tuning fork motion at all times to perform reproducible, full position indication can be achieved with four feelers, requiring the translation of 3^4 possible positions. The problem becomes much easier if for instance the information on the inclination is omitted. One requires in this case three feelers all
at the same distance from the stator, as shown in figure 15. The information on the inclination is secured by two limit switches which act independently as safety switches.

The principles of signalling, control and faultfinding are identical with the standards adopted elsewhere in the machine (Debraine et al.)

* * *

5.3 Feedback Loop

Another part of the control system is the signal derived from the tuning fork motion and fed into the amplifier which supplies energy for the driving system. Several methods have been tried. One of them consisted of a barium titanate crystal glued on the inside of the tuning fork. The voltage conditions were found to be quite good but the gluing was delicate, high frequency filtering is required and also separation from the bias voltage. Another method consisted of a capacitive sonde mounted in the stator, the capacitance variations providing a frequency modulated signal. Also this system and variants thereof had some merits except for flashovers. The system now in use is an optical device. Light is projected on the edge of the tuning fork, the reflected light passes through a collimator onto a phototransistor. The system works quite well, is easily accessible as it is outside the vacuum tank. Light passes through a perspex window.

Whatever pick-up system is used, a phasing element has to be introduced to fulfill the conditions for self-oscillation. A flywheel circuit allows the system to miss a beat and a filter helps to suppress spurious modes of vibration. In the block diagram (16) also, frequency halving is required in conjunction with the magnetic circuit which operates in unbiased condition.

* * *

5.4

From the tuning fork motion are derived various timing pulses required to operate the machine. One of them serves to
gate the high frequency oscillator. Others may be used to time physical apparatus. The primary pulse is obtained from the preamplifier stage of the feedback loop which proved to perform highly reproducible.

* * *

6. Initial operation

The relative position of stator and tuning fork suffers a random shift of about 0.5 mm., as a result of repeated switching on and off of magnet field and vacuum pumps. This is quite favourable in view of the much larger deflections occurring elsewhere. The starting of the machine requires adjustment before the high frequency generator can be turned on. This adjustment is affected automatically and displayed on the position indicators. Once the machine is set for continuous operation the drift due to heating up appears to be very small, and the system can work for hours before the automatic position control has to adjust. The amplitude control is more often in action. The reason for this is under investigation. The ultimate goal in this matter is of course to reduce the automatic control system to a safety device.

As mentioned in section 2.3, the stray field of the magnet caused more damping than originally anticipated. Measurements revealed several cm. shift of the median plane with respect to mid plane of the stator. This was corrected locally by an auxiliary coil of 10 turns carrying 400 amperes. The eddy current losses in the tuning fork tips reduced by this procedure from 20 Watt to 6 Watt. Nevertheless the existing springs S" of the driving system, (see section 4.3) are loaded to peak value. They will be replaced in due course by heavier springs. The effective total power delivered by the amplifier is about 30 Watt but voltage and current are not at all in phase \( \cos \varnothing \approx 0.1 \). Possible improvements are in study.

The cooling circuit requires constancy of in-and outlet temperature; the setting is not very critical. The cooling pipe of square section is clamped on the base plate of the tuning fork. Flexible junctions of a corrugated type connect up to the cooling conduit on the stub. The vibrations transmitted to the latter had to be tuned
away. This was done by adding some 60 kg mass to the aluminium frame of the stub. Otherwise no noticeable penetration of vibration has been found on the vacuum tank.

We may calculate the influence of this by considering the analogue of cooling pipe and phosphor bronze strips $S$, fig. 14. It is like a spring $S'''$ attached between base plate and an infinite mass (surrounding). Obviously one has to make $S'''$ sufficiently small compared with $S''$. The extra damping (cooling pipe junction) is incorporated in the function $B$, fig. 11.

The voltage conditions between tuning fork and stator seem to be favourable. No excessive flashover or pitting of the aluminium occur. The stator dc bias voltage is -2200 volt; the tuning fork dc bias voltage is -800 volt. The h.f. peak voltage at maximum gap is estimated to be 20 kV; the h.f. peak voltage at minimum gap is about 10 kV.

Figure 21 shows a picture of the mounted tuning fork taken through the porthole on the left of the stubtank. Figure 18 shows on the right coils plus part of the magnetic circuit screwed on the base plate of the tuning fork; and on the left joke plus brass slabs to form $M''$ and the two springs to form $S''$. 

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Fig. 3

- Stator
- Prong
- Length of path AB: 540 mm
- Taper of path AB linear from 4.5 to 16 mm
- Frequency: 55 periods/sec
- Bending stress: < 95 kg/mm²
- Material: Aluminium 575
- Width: 2000 mm

Fig. 4

- \( w'' = 3.00; w' = 5.65 \) (point A of figure 4)
- \( \frac{Q}{g} \frac{1}{2} = 5000 \text{ m sec}; \bar{g}'' = 0.0125 \text{ m}; \bar{g}'' = 0.04 \)
- \( \omega = 314 \)

Fig. 5

- \( h_0 = 20.5 \text{ mm} \)
Fig 7.

1/4-scale-model, 33 cm. (experimental, air)
1/4-scale-model, 50 cm. (experimental, air)

 Frequencies of the symmetric- and asymmetric modes of the first kind as a function of the number of nodes at the edges.
Fig. 9
Transducer

Fig. 10
Fig 12: Electrical Analogue

Fig 14: Suspension.
Fig. 15

3 Position Indicators

2 Limit Switches

Stator

Metal casing

Teflon feeler

Fig. 16. Block diagram control + feedback system.

Timing sequence

Control relays

Position indicators

Grid block pulser

Amplifier

Flywheel

Phasing

Servomotor

Amplitude motor

Driving system

Tuning fork limit switches

Servomotor

Lamp X

Phototransistor

Fig. 20
Fig. 17 Tuning fork made from a solid block of aluminium. Dimensions 2000 $\times$ 550 $\times$ 130 mm.

Fig. 18 Electromechanical transducer.
Fig. 19 Measurement of thermal expansion.
(by courtesy of Messrs Philips - Eindhoven).

Fig. 21 The tuning fork seen through the stub tank window.