Multiple meson production from p-p collisions at 2.75 GeV according to a statistical theory by F. Cerulus and R. Hagedorn
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Multiple meson production from p-p collisions at 2.75 GeV according to a statistical theory

by

F. Cerulus and R. Hagedorn

ABSTRACT

The multiple meson production in p-p collisions at 2.75 GeV is calculated using a Fermi-type statistical theory. Conservation of isospin, energy and momentum as well as the indistinguishability of like particles are taken into account. The phase-space integrals are evaluated by a Monte-Carlo method from their exact expression.

Final state N-N interactions are introduced by assuming the production of isobars in the collision. With this feature and the conventional interaction volume agreement with the experimentally measured multiplicities is good. The ratios of the charge-states for the different multiplicities are
Introduction.

Since exact calculations of phase-space densities have become available \(^1\), it seemed advisable to work out a case of meson-production at an energy where experimental results are available or could be awaited in the future.

Experiments at 2.75 GeV proton energy have been carried out, and the branching ratio between single, double and triple production has been measured \(^2\).

Comparison between the calculated values and the experimental points gives a surprisingly good agreement, as shown in Table III and Fig. 1, provided one uses a Fermi-type theory which takes into account the "interactions in the final state" between nucleons and pions. The experiment \(^2\) yields only two values with which to compare the theory, and it would be interesting to test more detailed predictions, e.g. spectra of pions or ratios between different charge states. It is one of the aims of this report to make such statements as can be tested experimentally.

We feel that a test at lower energies would be less conclusive, because the energy available to the end particles is smaller, and the number of available states decreases, such that it will no longer be admissible to neglect the variations of the matrix element with
In this report we shall try to obtain the maximum information which a Fermi-type of meson production theory can yield, because up to now its potentialities have not been fully developed. We shall include in the formulation of the theory the strict conservation of energy, momentum and isospin, and we take into account the indistinguishability of particles of the same kind. No approximations will be made, and the results should reflect the properties of the Fermi model within a few percent; this deviation is only due to the statistical fluctuations of Monte-Carlo results, which are used for computing the phase-space density.

An appreciable number of these calculations have been performed by S.Z. Belenki et al. 3) but their work is not readily available and we feel that it might be useful to repeat some of their numbers here, for easy reference. In this report, moreover, we calculate the charged meson spectra which had not been available so far. We shall do this rather in detail to show how the method works, because it may of course be used in many other cases.

The Fermi theory of multiple production.

The probability for a transition from a state $A$ to a state $B$ can always be written as
In a production process state $A$ will consist, e.g., of an incoming proton of kinetic energy $T$ and a target nucleon at rest. The state $B$ will contain two nucleons and a certain number of mesons. (We shall not consider here strange particle or antiparticle production).

In the absence of any reliable theory of strong interactions, one cannot compute $T_{AB}$. But some of its properties may be obtained by general argument.

If we choose for $A$ a pure state of two nucleons, viz. two incoming plane waves (in the C.M.S.), it may be completely described by the following set of quantum numbers

\begin{align*}
E & : \text{total energy} \\
P & : \text{total momentum} \\
I & : \text{total isospin} \\
I_x & : \text{third component of isospin} \\
B & : \text{baryon number} \\
U & : \text{displacement of multiplet centre}.
\end{align*}

We believe that all those are good quantum numbers for strong interactions, and consequently we know that $T_{AB}$ must vanish if $B$ has other values for those quantum numbers. This may be safely
Suppose then that we have as final state B 2 protons and 3 pions e.g. with a total energy E, a total momentum \( P = 0 \), (i.e. C.M.S.), etc. We imagine the whole system enclosed in a box of size \( V \), and we ask what is the probability of those five particles going over by the inverse reaction in state A, which consists of only two protons. If the particles are mutually apart by distances larger than the range of the strong interactions certainly no reaction will occur. For the transition \( B \rightarrow A \) to happen, a second particle has to come within the range of a first, then a third has to be simultaneously in the range of the first two, and a fourth, etc.

From purely geometrical reasons we expect therefore the probability for the transition \( B \rightarrow A \) to contain a factor \( \frac{\sqrt{n-1}}{\sqrt{V}} \) where \( n \) is the total number of particles and \( \sqrt{V} \) an "interaction volume" that is closely related to the range of the meson producing forces, and that might in principle depend on \( n \). By the reciprocity theorem this factor must be present also in \( |T_{AB}|^2 \).

The Fermi model assumes now that

a) all other factors are slowly varying with energy and multiplicity, and we may suppose them to be constant in first approximation,

b) the interaction volume \( \sqrt{V} \) is independent of multiplicity but is Lorentz contracted.
The $|T_{AB}|^2$ has still to be multiplied with the density of states

$$
\rho_B(E) = (2\pi)^{-3n} \frac{d}{dE} \int_{V} d\vec{x}_1 \cdots d\vec{x}_n \int_{V} d\vec{p}_1 \cdots d\vec{p}_n
$$

(3)

where the region of integration is confined to $V$ for the configuration space and is restricted by the available energy for the momentum space. If we calculate in the centre of mass system (C.M.S.) there exists a relation between the $\vec{x}_1, \ldots, \vec{x}_n$ viz. their C.M. is at the origin of coordinates. This reduces the number of integrations by three, and we get

$$
\rho_B(E) = (2\pi)^{-3n}(V)^{n-1} \int_{V} d\vec{p}_1 \cdots d\vec{p}_n \delta \left( \sum_{i=1}^{n} \vec{p}_i \right) \delta \left( \sum_{i=1}^{n} \sqrt{\vec{p}_i + m_i^2} - E \right)
$$

(4)

where $m_i$ is the mass of the $i^{th}$ particle, and because of the $\delta$-functions the integration is over the whole of momentum space. For each of the linear momentum states whose density has thus been calculated there correspond possibly different states of spin and isospin, which one has to count.

Up to now we assumed implicitly that all particles were distinguishable. Like particles are not, and this reduces the number of possible different states. We express those two effects by a function $f^{(n)}$ whose value depends on the number of particles, on
The relative probability for the production of $\sqrt{\nu}$ mesons
and $n$ nucleons in a collision (C.M.S. energy $E$) is therefore
given by

$$w_n(E) = \left( \frac{\sqrt{\nu}}{2n!^2} \right)^{n-1} f_{T,S}^{(n)} \int n, (E)$$

with

$$f_{T,S}^{(n)} = \int \prod_{i=1}^{n} \frac{d^4 \pi_i}{(2\pi)^4} \delta \left( \sum_{i=1}^{n} \pi_i^2 \right) \times \delta \left( \sum \sqrt{\nu \pi_i^2 + M^2} + \sum \sqrt{\pi_i^2 + \mu^2} - E \right)$$

$M$: nucleon mass
$
\mu$: meson mass.

We have stated the principles of the Fermi-theory rather in
the abstract. It is not difficult to see, however, which are the
physical reasons behind the assumptions a) and b). One pictures
the particle-producing strong interactions as of such a large
strength that equilibrium between all possible and states is reached
in the very short time the collision act lasts. The weight of a
certain configuration after the collision is then only given by the
density of states which produce such kind of configuration, i.e. by
The interaction volume corresponds not to mean distances of interaction, whatever that means, but to such distances that, as soon as the particles are further apart they no longer undergo interactions which could change their number. They can still scatter, or decay, but it is assumed that, once they have left the interaction volume, they can be considered as individual particles with a definite rest mass, energy, spin and isospin.

The requirement that $T_{AB}$ is slowly varying can be satisfied even if $T_{AB}$ would exhibit small and rapid fluctuations, provided that in the mean, i.e. by summing over all final states, these fluctuations cancel. It is conceivable that due to angular momentum conservation e.g. this is not the case. Such a state of affairs would show up in the angular distribution e.g. As the experimentally observed anisotropy is small $^{4^1,5^1}$ at low energies, we feel confident that it is justified to use the Fermi hypothesis.

Isobaric states.

For fast collisions the interaction time is of order $\frac{1}{\beta}$, i.e. $10^{-24}$ s. Any metastable state with a much larger lifetime has therefore - in this theory - to be considered as a particle $^1$.
This is a little uncomfortable at high energies, because one is not sure to know all metastable states of nucleons and pions with lifetimes of $10^{-23}$ or longer. However, at the low energy of 2.75 GeV only the lowest will contribute appreciably. This is the $S = 3/2$ $I = 3/2$ pion-nucleon state which shows up as a resonance at 180 MeV in pion-nucleon scattering.

Its rest mass *) is $1.31 M$ and its lifetime **) $10^{-23}$ s.

If one would apply this statistical theory of meson-production at higher energies one would have to review carefully the evidence for the higher excited state of the nucleon or for possible $\pi - \pi$ isobars.

In the present case we shall limit ourselves to three kinds of particles as products of the primary collision:

<table>
<thead>
<tr>
<th>Mass $/M$</th>
<th>Spin</th>
<th>Isospin</th>
</tr>
</thead>
<tbody>
<tr>
<td>nucleons</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>pions</td>
<td>0.147</td>
<td>0</td>
</tr>
<tr>
<td>isobars</td>
<td>1.31</td>
<td>$\frac{3}{2}$</td>
</tr>
</tbody>
</table>

Of course the isobar as such is not observed, and we shall have to calculate into what states of a nucleon and a pion it will decay, after the collision.
Spectra of the collision particles.

Suppose that one is not only interested in the number and kind of particles that are produced in a collision, but wants to know the energy of one of them (say the $\nu^\text{th}$) as well. It is then necessary to take such end states as are compatible with not only the given $\nu$ and $E$, but with a prescribed $|\vec{p}_\nu|$. The integral over final states has then to be carried out over $\vec{q}_1, \ldots, \vec{q}_n, \vec{p}_1, \ldots, \vec{p}_{n-1}$, i.e. we want

$$
\rho_{n, n^*, \nu-1}(E-\sqrt{\vec{p}_\nu^2 + \mu^2}, \vec{p}_\nu) = \int d\vec{p}_1 \ldots d\vec{p}_{n-1} n^{n^*+\nu} \delta\left(\sum_{i=1}^{n^*+\nu} \vec{p}_i\right) \delta\left(\sum_{i=1}^{n^*+\nu} \sqrt{\vec{p}_i^2 + m_i^2} - E\right) 
$$

(7)

The density of those end states where $|\vec{p}_\nu|$ lies between $P$ and $P+dP$ (in the C.M.S. of all particles) is consequently proportional to

$$
\rho_{n, n^*, \nu-1}(E-\sqrt{\vec{p}_\nu^2 + \mu^2}, \vec{p}) \propto n \vec{p}^2 d\vec{p} 
$$

(8)

Integration of (8) yields back $\rho_{n, n^*, \nu}(E)$ as it should be.
is the (unnormalized) probability to produce \( \psi \) mesons, \( n \) nucleons and \( n \) isobars with total energy (in the C.M.S.) \( E \) where the \( \psi \) meson (a quite definite one, the indistinguishability is accounted for in \( f(\psi) \)) has a momentum \( p_\psi \) such that \( P < p_\psi < P + dP \). In the same way one can obtain the spectrum of a nucleon or of an isobar. The spectrum, instead of being expressed as a function of the momentum \( P \) can just as well be given as a function of the energy \( E = \sqrt{P^2 + m^2} \).

The Monte Carlo method \(^1\) yields the functions \( \rho_{n,n,\psi}(E,E) \) automatically, so we will assume them to be known.

**Spectra of the decay pions from the isobars.**

The isobars - whose spectrum is known - decay into a nucleon and a pion, with a Q-value of 0.16 MeV. We have therefore to calculate the spectrum of the decay-pion in the C.M.S. of the collision, if we want to calculate the measured pion spectrum which is composed of pions from the collision and from decay pions.

We have to transform the motion of the pion from the isobar system to the C.M.S. This transformation depends on the velocity of
with \[ \frac{\varepsilon_0^*}{\varepsilon_1^*} = \left[ \frac{\eta \varepsilon}{\mu} + \sqrt{\left(\frac{\eta \varepsilon}{\mu}\right)^2 - \left(\varepsilon^2 + k^2\right)} \right] \frac{M^*}{\mu} \]

\[ W(\varepsilon)d\varepsilon = \text{probability to find the decay pion in the energy range } \varepsilon, \varepsilon+d\varepsilon \text{ (total energy of } J \text{ in the C.M.S.)} \]

\[ w(\varepsilon^*)d\varepsilon^* = \text{probability to find the isobar in the energy range } \varepsilon^*, \varepsilon^*+d\varepsilon^* \]

\[ k = \text{momentum of the decay pion in the isobar system} \]

\[ \eta = \text{energy} \]

The statistical factors.

The factor \( f_{T,S}^{(v)} \) in (5) is a product of three factors, each of which is due to a different principle.

In the present case, when we have two baryons (nucleons or isobars) and \( \nu \) pions in the end state, it can be written

\[ f_{T,S}^{(v)} = (2\lambda_1+1)^n (2\lambda_2+1)^{n^*} \frac{1}{n! n^*! \nu!} \gamma_{n, n^*, \nu} (T) \]
The second part takes the indistinguishability of like particles into account. Because of this factor it is allowed to consider in all arguments the particles as being labelled, and distinguishable. The error one makes this way is corrected by the factor \( \frac{1}{n!n^*!n^+!n^-!} \).

The factor \( p_{n,n^*,n^+} \) is proportional to the number of states in isospin space that are compatible with a given total isospin, total charge and the number and kind of particles one considers. Such factors have been computed by Yeivin and de-Shalit \(^6\) for nucleons and pions, and by Saraženkov and Barbašev \(^7\) for nucleons, pions and isobars.

**Distribution of charge states.**

The theory as sketched hitherto is able to predict multiplicities and spectra averaged over the three charge states of the pions.

If one wants to know for example the ratio between \( \pi^+ \) and \( \pi^- \) single production, or the complete spectrum of neutral pions, one needs more information about the distribution of charge states in the different end channels of the collision process.

We shall, as an example, treat the case \( p+p \rightarrow N+N^*+\pi^- \).
The total isospin is 1, the \( T_3 \)-component is 1. Consequently we have two possible Clebsch-Gordan series (We use the notation \( \frac{j_1 j_2 J}{m_1 m_2 M} \) for the Clebsch-Gordan coefficient and \( |j, m\rangle \) for the states)

\[
a) \quad |1,\frac{1}{2},\frac{1}{2}, \frac{1}{2}\rangle = C_{\frac{1}{2} \frac{1}{2} \frac{1}{2}} \left| 2, \frac{1}{2} \right\rangle N_N N_N \left| 1, \frac{1}{2} \right\rangle + C_{\frac{1}{2} \frac{1}{2} \frac{1}{2}} \left| 2, -\frac{1}{2} \right\rangle N_N N_N \left| 1, -\frac{1}{2} \right\rangle
\]

and

\[
b) \quad |1,1\rangle = C_{\frac{1}{2} \frac{1}{2} \frac{1}{2}} \left| 1, \frac{1}{2} \right\rangle N_N N_N \left| 1, 0 \right\rangle + C_{\frac{1}{2} \frac{1}{2} \frac{1}{2}} \left| 1, -\frac{1}{2} \right\rangle N_N N_N \left| 1, -1 \right\rangle
\]

The representations \( |2,2\rangle, |2,1\rangle, |2,0\rangle \) and \( |1,0\rangle \) of the \( N-N^* \) system in turn split up according to the Clebsch-Gordan series, e.g.

\[
|1,\frac{1}{2}\rangle = C_{\frac{1}{2} \frac{1}{2} \frac{1}{2}} \left| 1, \frac{1}{2} \right\rangle N_N N_N \left| 1, \frac{1}{2} \right\rangle + C_{\frac{1}{2} \frac{1}{2} \frac{1}{2}} \left| 1, -\frac{1}{2} \right\rangle N_N N_N \left| 1, -\frac{1}{2} \right\rangle
\]

The isobars will decay in a different way, according to their isospin state:

\[
|\frac{3}{2}, \frac{1}{2}\rangle = \sqrt{\frac{2}{3}} |\frac{1}{2}, \frac{1}{2}\rangle N_N \left| 1,0\right\rangle + \sqrt{\frac{1}{3}} |\frac{1}{2}, -\frac{1}{2}\rangle N_N \left| 1, 1\right\rangle
\]

\[
|\frac{3}{2}, \frac{3}{2}\rangle = \left| \frac{1}{2}, \frac{1}{2}\right\rangle N_N \left| 1, 1\right\rangle
\]

If one collects all coefficients, one finds the following relative
In the spirit of the statistical theory, case a) and case b) have equal probability, and the actual distribution is the mean, i.e.

<table>
<thead>
<tr>
<th></th>
<th>Case a)</th>
<th>Case b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pp + -</td>
<td>37/60</td>
<td>3/4</td>
</tr>
<tr>
<td>np + 0</td>
<td>16/60</td>
<td>1/12</td>
</tr>
<tr>
<td>pp 0 0</td>
<td>9/60</td>
<td>1/12</td>
</tr>
<tr>
<td>nn + +</td>
<td>1/60</td>
<td>1/12</td>
</tr>
</tbody>
</table>

We can treat the other cases in the same way, and for every number of $N, N^*, N$ calculate the relative probability of the charge states. The results appear in Table IV.

From these numbers one gets at once the relative probabilities for $\eta^+, \eta^0$ or $\eta^-$. In the example given

\[
\begin{align*}
\eta^+ & : \frac{1}{2} \left(0.350 + 0.483 + 2 \cdot 0.050\right) = 0.466 \\
\eta^- & : \frac{1}{2} \left(0.350\right) = 0.175 \\
\eta^0 & : \frac{1}{2} \left(0.483 + 2 \cdot 0.117\right) = 0.353
\end{align*}
\]
The column for $2N+3$ in Table IV cannot be calculated this way, because the isospin state of three pions is not uniquely defined by the total $T$ and $T_3$. One has to take the symmetry of the three particle wave-functions into account. The relevant numbers may be found in $^3$), but we do not want to describe the procedure in detail for this relatively unimportant case here. For higher energies and larger multiplicities it is on the contrary very important to consider the effect on the charge distribution of the symmetry of the wave function for $n$ pions.

The spectrum of the charged pions requires still one more step.

Take for example the 2 pion spectrum. This is built up via the channels $N+N+\pi$, $N+N+\bar{N}$, $2\pi$. Every channel yields a different form of spectrum; the second one is a superposition of the spectra of the directly produced pion and of the decay pion.

In each of these four spectra the charge distribution is different. Knowing the decomposition of the total isospin state, as was sketched before, it is easy to draw up tables of charge distributions for pions produced directly or for those produced via an isobar. Table V gives these probabilities for single, double and triple production.
The total spectrum.

The phase space calculations and the factors \( f_{T,S}^{(v)} \) have given, in the present case, spectra of the directly produced pions (denoted by \( \mathcal{W}_{n_1 n_2}^{(n)}(\xi) \)) and of the isobars. From the isobar spectra one calculates the spectra of the decay pions, which we denote \( \mathcal{W}_{n_1 n_2}^{(n)}(\xi) \).

If we want for example the total spectrum of the neutral pions, we have to multiply every \( \mathcal{W} \) with the probability to find a \( \pi^0 \) in the channel considered and add up the results for every point \( \xi \) of the spectrum.

The numbers in Table V give the charge distribution in every channel. The \( \mathcal{W}(2) \) give the probability to find a "labelled" meson between the total energy \( \xi \) and \( \xi + d\xi \), in the channel considered. To have the probability to find any meson of this energy one must multiply \( \mathcal{W} \) with the number of mesons in the channel: e.g.

\[
\begin{align*}
\mathcal{W}_{1,1}^{(n)}(\xi) \text{ is multiplied with } & 0.3166 \\
\mathcal{W}_{2,0}^{(n)}(\xi) \text{ } & 0.3250 \times 2 \\
\mathcal{W}_{0,0}^{(n)}(\xi) \text{ } & 0.4666 \times 2
\end{align*}
\]
The total spectrum of the positive, neutral or negative pions is then
\[ \mathcal{W}_{i,\alpha} \frac{\varepsilon}{d\varepsilon} = \sum_{\lambda} \frac{f^{(\lambda)}}{f} \mathcal{W}_{i,\alpha}^{(n)} + \sum_{\lambda} \frac{f^{(\lambda)}}{f} \mathcal{W}_{i,\alpha}^{(n\pi)} \]
\[ + \sum_{\lambda} \frac{f^{(\lambda)}}{f} \mathcal{W}_{i,\alpha}^{(n\eta)} \]

The values of the factors \( f, g, h, l, m \) for \( \pi^+, \eta^0 \) or \( \eta^- \) are given in Table V.

The Monte Carlo method gives of course the spectrum in the form of a histogram. In the present case the blocks of the histogram have a width of 0.02 \( \text{MeV}^2 \), and the whole spectrum is made up of about 40 blocks. The ordinates of the histogram are scattered around the true curve, and it is necessary to use some smoothing procedure to balance the errors of the single blocks. In the present case we used a least square Fourier analysis (based on 24 equidistant points in the histogram), and tried to produce a reasonable smooth curve that fitted the histograms by trying the curves given by the first terms.
Results, tables and curves.

Parameters of the problem:

Reactions considered

\[ p + p \rightarrow \begin{cases} \text{N} + \nu + \nu^* \\ \text{N} + \nu + \nu^* + \text{N} \\ 2\text{F}^* + \nu + \nu \end{cases} \]

Incident kinetic energy \( 2.75 \text{ GeV} \)

C.M.S. energy \( 3.14 \text{ MeV}^2 \)

Masses:

- Nucleon \( M = 1.000 \)
- Isobar \( M^* = 1.310 \)
- Pion \( \mu = 0.1471 \)

Momentum of decay pion from isobar at rest: \( k = 0.2402 \text{ MeV} \)

Energy: \( y = 0.2816 \text{ MeV}^2 \)

Interaction volume \( \Omega = \frac{2m}{E} \Omega_0 = 3.32 \times \left( \frac{E}{\text{MeV}} \right)^3 \)

with \( \Omega_0 = \frac{4\pi}{3} \lambda^2 \frac{3}{2} \pi \times (2\pi)^{-3} \)

The curves shown are interpolated by a least squares method from the Monte Carlo results. They may, in their detailed behaviour, still show some error due to statistical fluctuations.

Figs. 3, 4, 5 and 6 are unnormalized spectra, and the ordinates are proportional — with the same factor — to the intensities of
<table>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>W</th>
</tr>
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<tbody>
<tr>
<td>N</td>
<td>$n$</td>
<td>$N_\nu$</td>
<td>$P_{n_\nu n_\nu}$ (1)</td>
<td>$\nu$</td>
<td>$\psi_{n_\nu n_\nu} (3.14)$</td>
<td>$\sum_{n=n_\nu+\nu}^{n_\nu+n_\nu+\nu-1} W$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>11.92</td>
<td>3.32</td>
<td>79.15</td>
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<td>2</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>16.08</td>
<td>11.02</td>
<td>708</td>
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<td>2</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>2.04</td>
<td>36.57</td>
<td>298</td>
<td></td>
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<td>2</td>
<td>3</td>
<td>0.333...</td>
<td>9</td>
<td>3</td>
<td>$5.61 \times 10^{-2}$</td>
<td>121.4</td>
<td>20.4</td>
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<tr>
<td>2</td>
<td>4</td>
<td>0.0833...</td>
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<td>$3.76 \times 10^{-4}$</td>
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<td>0</td>
<td>8</td>
<td>1</td>
<td>10.38</td>
<td>3.32</td>
<td>275.7</td>
</tr>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>2</td>
<td>5.93</td>
<td>11.02</td>
<td>1045</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>20</td>
<td>$2.95 \times 10^{-1}$</td>
<td>36.57</td>
<td>215</td>
</tr>
<tr>
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<td>Mean kinetic energy of isobars $\bar{T}_{N^*}$</td>
<td>Mean kinetic energy of pions $\bar{T}$</td>
<td>Probable error of Monte Carlo calculation in %</td>
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</table>

**TABLE II**

Mean kinetic energies in the C.M.S. of nucleons, isobars and pions from a $p-p$ collision at 2.75 GeV incident energy,
### TABLE III

Calculated probabilities for multiple pion production in p-p collisions at 2.75 GeV without and with isobars. Experimental data 2).
<table>
<thead>
<tr>
<th>Baryons after Collision</th>
<th>NN</th>
<th>NN*</th>
<th>N<em>N</em></th>
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<tr>
<td><strong>Observed End states</strong></td>
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<td>1 meson</td>
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<td>pp 0</td>
<td>0.25</td>
<td>0.166</td>
<td>0</td>
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<td>pn +</td>
<td>0.75</td>
<td>0.833</td>
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<td>2 mesons</td>
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<td>0.350</td>
<td>0.200</td>
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<tr>
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<td>0.100</td>
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<td>0.1777</td>
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<td>0.5777</td>
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<td>pn + +</td>
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<td>0.0444</td>
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<td>3 mesons</td>
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<tr>
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<td>0.2800</td>
<td>0.2444</td>
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<td>np + + +</td>
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</table>

**TABLE IV**

Distribution of charged end states for different intermediate states.
### Table 1: Charge of Directly Produced Λ

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<tr>
<th>Channel</th>
<th>NN* <em>f</em> (1)</th>
<th>NN2* <em>f</em> (2)</th>
<th>NN3* <em>f</em> (3)</th>
<th>NN* <em>g</em> (1)</th>
<th>NN* <em>g</em> (2)</th>
<th>NN* <em>h</em> (1)</th>
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<tr>
<td>-</td>
<td>0</td>
<td>0.1500</td>
<td>0.2000</td>
<td>0.3000</td>
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<td>0.2000</td>
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<tr>
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### Table 2: Charge of Isobaric

<table>
<thead>
<tr>
<th>Channel</th>
<th>NN* <em>l</em></th>
<th>NN* <em>l</em> (1)</th>
<th>NN* <em>l</em> (2)</th>
<th>NN* <em>m</em> (0)</th>
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<tbody>
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<td>-</td>
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<td>0.1600</td>
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<td>0.5266</td>
<td>0.4333</td>
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REFERENCES

1) F. Cerulus and R. Hagedorn; to appear in Nuovo Cimento Suppl.

2) M.H. Block, CERN Symposium 1956.


Total spectrum of pions from p-p collisions at 2.75 Gev. (normalised to unit area) in the C.M.S. Histogram and interpolated curve.
\[ \varepsilon = \frac{\text{kinetic energy}}{M_0^2} \]

The graph shows the function \( W_0(\varepsilon) \) versus \( \varepsilon \).
Fig. 5 Total spectrum of negative pions.

\[ \varepsilon = \frac{\text{kinetic energy}}{MC^2} \]