SOLID AND LIQUID ČERENKOV COUNTERS

by

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Introduction

Although the application of Čerenkov radiation for the threshold counting of charged particles is fairly old, its use for the measurement of particle velocities and their separation is still a relatively recent development. The use of the directional properties of Čerenkov radiation and its plane polarization, together with better developed photomultiplier tubes offers a powerful tool in the analysis of very high energy beams.

The principal application of the Čerenkov radiation in this analysis is in the measurement of particle velocities. There are several sources of errors which limit the accuracy of this measurement. Some are inherent to the particle beam, others to the Čerenkov counter itself. Most of these errors cannot therefore be eliminated altogether, but with a careful design and choice of parameters they can be minimised so as to enable us to obtain the maximum information from a given arrangement. It is the purpose of this note to indicate some of the ways of minimising the errors and to explore the possibilities of some counter designs.

When a charged particle passes through a transparent medium whose index of refraction \( n \) is greater than one, Čerenkov radiation is emitted if the product of the particle velocity \( \beta \) and \( n \) is greater than unity. This light is emitted in a cone similar to a shock wave of a supersonic aircraft. The angle of light emission with respect to the particle direction depends on \( \beta \) and \( n \) and is given by

\[
\cos \theta = \frac{1}{n \beta}.
\]
The intensity of light emitted is small compared to that which can be given off in the process of scintillation and is measured in photons per centimetre. Thus an important requirement placed upon a radiator material is that it should not scintillate appreciably. The intensity of Čerenkov radiation emitted by a particle of charge z is

\[ I = \frac{2\pi z^2}{137c} \Delta \nu \sin^2 \Theta \]  

(2)

where \( I \) is the number of photons emitted per cm, \( \Theta \) is the Čerenkov angle, \( \Delta \nu \) is the frequency interval of the radiation considered and \( c \) is the velocity of light.

**Dispersion and Scattering**

The Čerenkov light is emitted at all frequencies for which the above expressions hold and can be used at all those frequencies for which the radiator is transparent. Since \( n \) varies with the frequency of light it follows inevitably that light of different colour is emitted at different Čerenkov angles.

For a small interval \( \Delta \Theta \) and a corresponding variation \( \Delta n \) we may assume that the spread in Čerenkov angle due to dispersion is

\[ \Delta \Theta_d = \frac{\Delta n}{n} \frac{1}{\tan \Theta} . \]  

(3)

Let us define dispersion of the radiator material as

\[ \frac{dn}{d\nu} = D . \]  

(4)

Assuming that this is linear for small intervals \( \Delta \nu \),

\[ \Delta \Theta_d = \frac{D\Delta \nu}{n} \frac{1}{\tan \Theta} . \]  

(5)
Using expression (2) we may then say

\[ \Delta \theta_d = \frac{137 \cdot I \cdot c}{2 \pi^2 n \sin^2 \theta \tan \theta}. \]  

(5)

If we differentiate equation (1) with respect to \( \beta \) and remembering that \( \tan \theta \) is a function of \( \beta \) and \( n \) alone, we obtain

\[ \frac{d\theta}{d\beta} = -\frac{1}{\beta(\sqrt{n^2 \beta^2 - 1})}. \]  

(7)

In Fig. 1, \( \theta \) is plotted as a function of \( \beta \) for different values of \( n \) corresponding to threshold values of \( \beta \) from 0.5 to 0.80. Values of \( \frac{d\theta}{dn} \) are plotted in Fig. 2 for various values of \( \beta \). Similarly, Fig. 3 represents equation (7) where \( \frac{d\theta}{db} \) is plotted as a function of \( n \).

We see from the above expressions and graphs that if we want to measure a given \( \beta \) accurately we must choose low indices of refraction close to the threshold. On the other hand, this is not the only way to measure accurately \( \beta \). The expression giving the relative error in \( \beta \)

\[ \frac{\Delta \beta}{\beta} = \Delta \theta \sqrt{n^2 \beta^2 - 1} \]  

(8)

can be kept low not only by reducing \( n \) but also by reducing \( \Delta \theta \). On the other hand, we are interested in measuring \( \theta \) accurately and if the error in this measurement is fixed rather by light collecting system, beam divergencies etc., than by dispersion and scattering in the radiator, then, since \( \Delta \theta \) is well within this "technical" error, there is no point in reducing it. For the moment, however, let us find a way to reduce \( \Delta \theta \).

For a given \( n \), \( D \) is a constant and it becomes apparent from Eqs. (5) and (6) that a good way to decrease \( \Delta \theta_d \) is by decreasing \( \Delta I \). This may be done by light filters. This reduces the effective intensity \( I \), and to make up for this we must use a longer radiator. The latter in turn increases the multiple scattering with a consequent spread in Čerenkov angle.
due to this second process. Fig. 4 shows the variation of $\Delta \nu$ with the thickness of a lucite radiator for a given number of photons (in this case 10) required at the photocathode.

Let the number of photo-electrons required at the photocathode be $a$. Let the efficiency of Čerenkov light producing these photo-electrons be $\eta$, and let the length of particle path in the radiator be $\chi$. It follows that

$$I = \frac{a}{\chi \eta}.$$  \hfill (9)

The particle in passing through the radiator suffers multiple scattering. The spread in Čerenkov angle due to this scattering is twice the r.m.s. projected angle of scattering. The total spread in Čerenkov angle due to scattering and dispersion is then (*)

$$\Delta \Theta (d, s) = \frac{137 D \cdot a \cdot c}{2 \pi z^2 n \chi \eta \sin^2 \Theta \tan \Theta} + \frac{\sqrt{2} E_s}{\beta^2 E} \sqrt{\frac{\chi}{\chi_0}}$$  \hfill (10)

where $E_s$ is the energy constant of scattering

$$E_s = \left(\frac{4\pi}{\alpha}\right)^{\frac{1}{2}} \frac{Z}{A} c^2 = 21 \text{ MeV}$$  \hfill (11)

and $\chi_0$ is the radiation length defined by (3)

$$\frac{1}{\chi_0} = 4\alpha \frac{N}{A} Z^2 r_e^2 \ln \left(183 Z^{-\frac{1}{3}}\right).$$  \hfill (12)

$E$ is the total energy of the particle, $E = \gamma E_0$ where $E_0 = m_0 c^2$. In Eqs. (11) and (12) $\alpha$ is the fine structure constant, $N$ is Avogadro's number, $A$ is the atomic mass number of the material, $Z$ is the charge number, and $r_e$ is the classical electron radius.

(*) It should be stressed that here two entirely different errors are added together. The first is fixed, the second is statistical.
Fig. 5 shows plots of $\Delta \theta (d,s)$ as a function of $\chi$ at various values of $\beta$ for protons in lucite.

Differentiation of $E_C$ (10) with respect to $\chi$ yields the optimum value of $\chi$ which, expressed in terms of $\beta$ and $n$, may be written as

$$m = 9.84 \left( \frac{x_0 E_0^2 \beta^2 a^2 c^2}{E_s n^2} \right)^{\frac{1}{3}} \left[ \frac{\beta^4}{n^2 (n^2 \beta^2 + \frac{3}{n^2 \beta^2} - \frac{1}{n^4 \beta^4} - 3)(1 - \beta^2)} \right]^{\frac{1}{3}} \text{. (13)}$$

If we plot $\chi_m$ as a function of $\beta$ we find that for both low and high values of $\beta$, $\chi_m$ increases rather rapidly. This is due to the fact that near threshold the small $\Theta$ is very strongly sensitive to variations in $n$ (see Figs. 1 and 2) and thus dispersion plays an important role and has to be reduced. At high velocities we can afford to cut down dispersion because scattering is smaller and the intensity, reduced by light filters, can now be compensated by longer radiators. For protons in lucite the above plot is shown in Fig. 6.

**Slowing Down**

Particles will slow down in passing through the radiator and this changes the Čerenkov angle.

One can easily show that if $E_K$ is the kinetic energy of the particle, then

$$\frac{d\theta}{\beta} = \frac{dE_K}{\gamma (\gamma + 1) E_K} \text{ (14)}$$

where

$$\frac{E_K}{E_0} = \gamma - 1 \text{.}$$
Combining (14) and (3)

\[
\frac{dE}{d\Theta} = \frac{\gamma}{\gamma(\gamma+1)E} \tan \Theta = \frac{d\gamma}{\gamma(\gamma+1)(\gamma-1) \tan \Theta}.
\]

(15)

Since the Čerenkov angle gets smaller as the particle slows down this may be compensated for by making tapered radiators if one or more total internal reflections are to be used\(^1\). On the other hand, if no total internal reflections are used then probably one way to compensate for the above effect is to make a radiator with a varying index of refraction in the direction of particle motion\(^2\).

If a tapered radiator is used then the rate of change of angle that a light ray makes with the axis of the cone due to multiple reflections is given by\(^1\)

\[
\frac{d\Theta}{dx} = -\frac{\alpha \tan \Theta}{t}
\]

(16)

where \(\alpha\) is the full angle of the cone and \(t\) is the width of the cone at the position considered. This expression is valid for small values of \(\alpha\) and it follows that

\[
\frac{d}{dx} = -\frac{\tan \Theta}{x}
\]

(17)

where \(x\) is the distance of the apex of the cone from the position considered. It is interesting to note that \(\frac{d\Theta}{dx}\) does not depend on the value of the angle \(\alpha\) as long as the product of this angle and the number of total internal reflections of the light ray remains constant.

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2) A remark to this effect was made by Dr. Lundby at the 1956 Geneve Conference.
In order to find the value of $x$ needed to compensate for the decrease in the Čerenkov angle due to slowing down of a particle, the above expression is combined with (15) to yield

$$x = - \frac{\gamma(\gamma+1)E_K \tan^2 \Theta}{\frac{dE_K}{dx}} = - \frac{\gamma(\gamma+1)(\gamma-1)E_0 \tan^2 \Theta}{\frac{dE_K}{dx}}$$

(13)

The negative sign is due to the fact that $\frac{d\Theta}{dx}$ is negative. Since $\frac{dE_K}{dx}$ is also negative, the final value of $x$ will come out positive. The above expression is true only as long as $\frac{dE_K}{dx}$ remains constant for the distance the particle travels through the radiator. If this condition is not sufficiently satisfied for a required thickness of the radiator, then the latter should be split up into several sections and each section computed separately. For low energies the resulting radiator would then be trumpet-shaped. However, this will rarely be required.

It should be noted that $\frac{dE_K}{dx}$ in the above expression is the familiar collision loss but expressed in MeV·cm$^{-1}$. For $\frac{dE_K}{dx}$ expressed in MeV/g·cm$^{-2}$ one can use the expression

$$- \frac{dE_K}{dx} = \frac{2\pi NZZr^2m^2c^2}{A \beta^2} \left[ \ln \frac{4m^2 c^4 \beta^4}{(1-\beta^2)^2 I^2(Z)} - 2\beta^2 \right]$$

(19)

where all the symbols have the usual meaning and $I(Z)$ is the ionization potential. For most purposes we can say that $I(Z) = I_H Z$ where $I_H$ can be taken to be 13.5 eV.$^3$

A curve giving $x$ as a function of $\gamma$ and $\beta$ for protons in lucite is shown in Figs. 3 and 9. We see that for protons with $\gamma = 1.9$ the decrease in Čerenkov angle in a 5 cm long radiator is

$$\Delta \Theta = 0.189^\circ.$$
For protons with velocities above, say, 0.5 c there seems little point in using tapered radiators.

**Photomultiplier Errors**

So far we have ignored the spread in pulse height due to the photomultiplier alone. This has been considered by Morton\(^4\) and is given by

\[
\frac{\langle \Delta p \rangle^2}{p^2} = \frac{1}{a} \cdot \frac{\sigma^{m+1}}{\sigma^m (\sigma - 1)} \approx \frac{1}{a} \cdot \frac{\sigma}{\sigma - 1}
\]

(20)

where \(\frac{\langle \Delta p \rangle^2}{p^2}\) is the fractional mean square deviation of the output pulse \(p\), \(\sigma\) is the average secondary emission ratio, \(m\) is the number of stages and \(a\), as before, is the number of photo-electrons. Since \(a\) is here in the denominator while in (10) it is in the numerator, some compromise depending on the photomultiplier is obviously necessary.

**Some Special Counters**

In the following several counters are considered for use with high energy proton beams.

The first counter considered is shown schematically in Fig. 10. The particles produce Čerenkov light in a radiator of a special shape. This light then strikes the conical surface and most of it is refracted at an angle \(i_2\) with the normal to the surface. The angle \(\beta\) (= \(\frac{\pi}{2} - i_2\)) is chosen so small that the emerging light continues as a narrow parallel beam. This light is then reflected by a mirror into a photomultiplier. Owing to the fact that the Čerenkov light is polarized with its electric vector in the plane of incidence it will be 100% transmitted at the Brewster angle and this transmission will fall off relatively slowly for greater angles \(i_2\). We first assume that the radiator is made of Methyl Methacrylate Polymer (hereafter referred to as M.M.P.) whose index of refraction for the D line is

\(^4\) See, for example, J.V. Jelley, Progress in Nuclear Physics, Vol. 3
\[ n_D = 1.4913 . \]

If we decide to fix \( i_2 \) at 30° then \( i_1 \) will be 41°. The amount of light totally internally reflected will be 23.7% and the amount transmitted then 76.3%. If, on the other hand, one decreases \( i_2 \) to 70° the transmission rises to more than 95%. For the present consideration we shall keep \( \beta \) to 10°.

Let us now estimate the suitability of this counter for counting protons at two velocities, \( \gamma = 2 \) and \( \gamma = 4 \).

\[ 1/ \quad \gamma = 2 \quad \beta = 0.866 \quad \cos \Theta = 0.7743 \quad \Theta = 0.6846 \quad r = 39°15' . \]

As can be seen in the figure the effective thickness of the radiator varies, being the smallest at the axis and increasing by \( \frac{r}{\tan \Theta} \) where \( r \) is the maximum useful radius of the radiator. This increase in thickness has the purpose of compensating for the loss of light emitted by particles passing at a distance from the axis and thus giving insufficient number of photons on one side of the radiator and also giving a number of photons with angular momenta around the axis which will cause them to be lost to the light collecting system. In order to make all particles scatter equally in passing through the counter (on account of other possible detectors placed behind the counter) a cone of M.M.P. is placed behind the radiator to make the total thickness of matter transversed substantially equal across the whole beam section. In computing the optimum thickness of the radiator we may decide tentatively that

\[ X_m = X_{\text{min}} + \frac{r}{2 \tan \Theta} . \quad (21) \]

In computing \( X_m \) by means of the expression (13) we should carefully decide the value of \( \eta \), for it has to denote now the total efficiency of the conversion of light into photo-electrons. This \( \eta \) is now
the product of efficiencies of the photocathode, the mirror, the refractive surfaces and the transparency of the radiator itself:

$$\eta_{\text{tot}} = \prod_{n} \eta_{n}$$

(22)

where \(n\) denotes all processes where losses of light occur. We shall assume that the transparency of M.M.P. is close to 100\%, which is particularly true in this case where we shall filter our light and can always choose the band for which the transmission of both M.M.P. and the photomultiplier window is at a maximum. Another assumption will be that the efficiency of the photocathode is 0.1 which is a factor of two lower than the best photocathodes made and is very nearly correct in the case of such tubes as RCA 7029 and RCA C7237. The efficiency of the mirrors will be assumed to be 92\% which is substantially correct for well-evaporated aluminium. Finally, 0.763 and 0.96 are the efficiencies of the refracting surfaces of the radiator and photomultiplier window\(^\text{*}\). Finally we get

$$\eta_{\text{tot}} = 0.0674.$$ 

From the above assumptions and taking \(a = 10\) we obtain

$$\chi_{m} = 3.8 \text{ cm}.$$ 

This corresponds to a frequency band of \(6.4 \times 10^{13} \text{ c/sec}\) and an amount of light of about 40 photons per cm used. The angular spread of Čerenkov light is then

$$\Delta \theta(d, s) = 1.072 \times 10^{-2} \text{ rad. or}$$
$$= 0.614^\circ.$$ 

From this

$$\Delta \beta = 7.58 \times 10^{-3} \text{ or}$$

$$\frac{\Delta \beta}{\beta} = 8.76 \times 10^{-3} = 0.876\%.$$ 

\(^\text{*}\) These may be improved by the technique of optical coating (blooming) but this is tedious.
The error introduced by the slowing down of protons is only

\[ \Delta \theta = 1.76 \times 10^{-3} \text{ rad} \]

i.e., nearly an order of magnitude lower than above and it need not be considered for higher energies.

2/ \( \gamma = 4 \), \( \beta = 0.968 \), \( \cos \theta = 0.6927 \), \( \theta = 0.8058\degree \) or 46°10'.

With the same assumptions as in the previous case we get

\[ \chi_m = 5.0 \text{ cm.} \]

This corresponds to about 50 photons per cm used and a frequency band \( \Delta \nu = 3.3 \times 10^{13} \text{ c/sec.} \) Also we obtain

\[ \Delta \theta_{(d,s)} = 4.84 \times 10^{-3} \text{ rad} \]
\[ = 0.277\degree. \]

From this it follows that

\[ \Delta \beta = 4.88 \times 10^{-3} \]
\[ \frac{\Delta \beta}{\beta} = 5.04 \times 10^{-3} \text{ or } 0.504\%. \]

So far we have been able to estimate fairly accurately the errors introduced by elements which are "inherent" to the counter. The resolution of the light collecting system must be adapted to these "inherent" errors. But in spite of this we are confronted with inefficiencies and spread in resolution, all the way down to the pulse output. We may well find, for example, that \( a = 10 \) is not acceptable for our particular type of photomultiplier tube. Its increase to, say, 20 photo-electrons would change \( \chi_m \) by a factor of \( 2^3 \), with a consequent change in all other parameters.
The angle $\Delta \Theta_{(d,s)}$ changes upon the emergency of light from the radiator to approximately $n \Delta \Theta_{(d,s)}$ and the light collection efficiency should be flat over this range. In case of 1) this is about $0.916^\circ$, and in case of 2) $0.413^\circ$. From Fig. 10 one can see that the required resolution at the light collecting system may in a way be adapted by the mean distance $d$ of the main mirror. The amount of light totally internally reflected at the surface of the radiator can be collected and brought to a rough focus at another photomultiplier. The resulting pulse may be pulse-height selected to correspond to about $24\%$ of the total light produced and used in conjunction with the first pulse to increase the resolution of the counter. In this case, however, one may have to change the parameters used above.

One of the most serious disadvantages of the counter first described is its relatively small useful cross-sectional area of the beam. Photons emitted with angular momenta get rapidly lost as these angular momenta increase. A modification of this counter intended to overcome some of these difficulties is shown schematically in Fig. 11. Here the radiator is extended radially in such a way that Čerenkov light suffers several total internal reflections before being finally refracted out of the radiator and used. Thus the particles can be accepted all over the flat surface of the radiator, the beam width can be considerably increased, but for very "thick" beams several photomultipliers may have to be used because of the angular momentum criterion. Several disadvantages of this counter are obvious, however. Firstly, one of the important properties of Čerenkov light, its polarization, used successfully to reduce refraction losses in the previous model, is now to a large extent lost due to multiple reflections. Thus the percentage of $23.7\%$ of light lost at refraction in the previous case would now rise to a little less than $40\%$. Thus our total efficiency $\eta$ would now fall to nearly $0.053$. Another apparent property of this counter would be its inability to handle Čerenkov angles for which

$$\sin \Theta < \frac{1}{n} \quad (23)$$
for in this case the Čerenkov light would escape out of the radiator at the flat surfaces. This difficulty could, however, be overcome either by optical coating of the flat surfaces or, more easily but less efficiently, by evaporating a reflecting metallic layer of them. Let us work out \( \chi_m \) for this counter assuming the use of reflecting layers. Let the efficiency of these be 0.92 each. With the above assumptions it turns out that

\[
\eta_{\text{tot}} = 0.0456.
\]

For protons with \( \gamma = 4 \) in lucite this gives

\[
\chi_m = 6.4 \text{ cm}
\]

and this corresponds to \( I = 3 \% \) photons/cm and a frequency band of \( 4.27 \times 10^{13} \text{ c/sec} \). We can write equation (10) as

\[
\Delta \theta(d,s) = \frac{D \Delta v}{n\nu n^2 \beta^2 - 1} + \frac{\sqrt{2} E_s}{\beta E} \sqrt{\frac{\chi}{\chi_0}}
\]  

from which it turns out that

\[
\Delta \theta(d,s) = 5.51 \times 10^{-3} \quad r = 0.315^\circ.
\]

This gives

\[
\Delta \beta = 5.54 \times 10^{-3} \quad \text{or}
\]

\[
\frac{\Delta \beta}{\beta} = 5.73 \times 10^{-3} \quad \text{or} \ 0.573\%.
\]

The increase in error therefore is about 15% for a drop in \( \eta_{\text{tot}} \) of 30%. This is what one would expect by looking at Eqs. (10) and (13). We can therefore conclude that although the second version of the counter gives a greater error in \( \beta \) its shortcomings may be, under certain circumstances, compensated by its greater freedom in the choice of the
beam size. In any case, however, the diameter of the radiator, the
size of the mirrors lenses and photomultiplier windows should be chosen
in such a way that the equation

\[ r_2 = n r_1 \frac{\sin \theta}{\sin \theta'} \]  \hspace{1cm} (25)

is not violated\(^1\). Here \( r_1 \) and \( r_2 \) denote the distance from the axis
of the photon in the radiator and outside of it. \( \theta' \) is the angle that
this photon makes with the axis at any given point and other symbols
have the usual meaning. Let us now estimate the above errors assuming
that the radiator material is a liquid of a low index of refraction as,
for example, FC-75. Of course, it is obvious that it will have to be
contained in a container with transparent windows where reflections and
refractions occur. These solid windows will be made of some higher-
index solid. In order to avoid introducing new dispersion due to the
refraction of light between this solid and FC-75, the light should be
made to strike the surface of the solid at right angles. If the ex-
tended radiator counter is to be used, its possible mode of construction
is suggested in Fig. 12.

So let us take \( \gamma = 4 \) for protons in FC-75

\[ \cos \Theta = 0.8140 \quad \Theta = 35^\circ 30' \quad \beta = 0.968 \quad n_D = 1.2769. \]

Though efforts are still being made to obtain the fullest possible data
on dispersion and transmission of FC-75\(^\text{*} \), for the moment we can as-
sume that the dispersion is

\[ D = 2.5 \times 10^{-17} \text{ c}^{-1} \text{ sec.} \]

Using the same requirements as for the previous case and as-
suming similar transparency for FC-75 as for lucite, we obtain

\[ \chi_n = 7.1 \text{ cm.} \]

\(^*\) The manufacturers of this compound have been approached with this
in view.
Other quantities of interest are now readily derived.
\[ \Delta \nu = 5.56 \times 10^{13} \text{ c/sec, } I = 31 \text{ ph/cm}. \]

\[ \Theta_{d,s} = 4.91 \text{ rad} = 0.281^\circ \]
\[ \Delta \beta = 3.39 \times 10^{-3} \]
\[ \frac{\Delta \beta}{\beta} = 3.50 \times 10^{-3} \text{ or } 0.35\%. \]

Thus we see the advantage of a smaller index of refraction, for although the higher density of FC-75 almost offsets its lower dispersion in the contribution to \( \Delta \Theta_{d,s} \), the error in \( \beta \) is considerably lower than in the case of \( \text{M.M.P.} \). At higher energies therefore the FC-75 would be preferable to M.M.P., in spite of much greater difficulties of fabrication of such a counter.

We shall next consider another kind of solid-liquid counter which, although not of a focusing type, can still be used to measure \( \beta \) and thus for particle separation. It is based on the total internal reflection at the front surface of the radiator. This principle has been used by several authors\(^5\) and the design considered here, shown in Fig. 13, has been suggested by von Dardel and Winter.

The radiator consists of a high refractive index \( n_1 \) material in contact with a low refractive index \( n_2 \) material, possibly a liquid, with light "guides" of a material the refractive index of which \( n_3 \) is close to \( n \), but slightly lower. The purpose of using a material with a low index of refraction is to be able to change the value of \( \beta \) by changing \( n_2 \). The indices \( n_1 \) and \( n_2 \) have to be chosen so that \( \beta \) to be measured satisfies the expression

\[ \beta^2 > \frac{1}{\left( \frac{n^2_1 - n^2_2}{n^2_2} \right)} \]  
(26)

for only in this case will the Čerenkov light be totally internally reflected. On the other hand, \( n_3 \) has to be chosen so that it just fails

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to satisfy the above criterion. This light will pass from \( n_1 \) to \( n_3 \) and will then be refracted at a small angle into the photomultiplier. 

Let us first suppose that we want to measure the velocity \( \beta = 0.866 \) for protons of \( \gamma = 2 \). For the moment we shall assume that \( n_2 = 1.276 \) (FC-75). We see from the condition (25) that

\[ n_1 > 1.722. \]

Let us assume \( n_1 = 1.722 \). This gives a Čerenkov angle \( \Theta = 48^\circ \). If we now want the light to be refracted from \( n_3 \) to \( n_2 \) at \( 80^\circ \) at a loss of about 50\%, then we should let this light be refracted, say twice, in order to extract 75\% of it travelling in the direction which is perpendicular to the boundary \( n_1 \rightarrow n_3 \).

In order to satisfy this condition we must have \( n_3 = 1.708 \). It is our intention that \( n_3 \) should be so close to \( n_1 \) that for a small change in the value of \( \beta \) the light is totally internally reflected in \( n_3 \) also and is absorbed at the end of the light guide. In the above case this happens at \( \beta = 0.877 \), i.e., \( \Delta \beta = 0.011 \) of \( \frac{\Delta \beta}{\beta} = 1.27\% \). As we shall see below, this "technical" error is below that introduced by the dispersion and scattering if a glass radiator is used. On the other hand, this can hardly be avoided since the highest refractive plastics do not have such high refractive indices. For example, polyvinyl carbazole has \( n_D = 1.683 \), polyvinyl naphtalene has \( n_D = 1.6818 \), lead methacrylate polymer has \( n_D = 1.645 \), methacryl-phenyl salicylate polymer has \( n_D = 1.6006 \), and 2-6-dichloro-styrene polymer has \( n_D = 1.6248 \). Even if plastics of the above type could be used, their high dispersion would probably offset their low density. Assuming that we have managed to find a glass with \( n = 1.722 \) and a very low dispersion (we could hardly hope to go below \( 6 \times 10^{-17} \) \( \text{cm}^{-1} \) sec at these indices) we still are confronted with its high density and a short radiation length.

Taking the values of \( \chi_0 = 3 \) cm, \( D = 6 \times 10^{-17} \) \( \text{cm}^{-1} \) sec, \( a = 20 \) and \( \eta = 0.04 \), we obtain

\[ \chi_m = 4.4 \text{ cm} \]
from which \( I = 114 \) photons/cm \( \Delta \nu = 1.35 \times 10^{14} \text{ c/sec} \) and \( \Delta \theta (d,s) = 16.96 \times 10^{-3} \text{ rad} = 0.973^\circ \). From this

\[
\Delta \beta = 16.31 \times 10^{-3}
\]

and

\[
\frac{\Delta \theta}{\beta} = 1.88%.
\]

If the radiator is divided into three parts as in Fig. 13, then the length of the light pipes must be at least 3 cm and this makes the minimum distance between the radiators 0.50 cm. Since the two spaces are occupied by a liquid, in this case FC-75, this adds about 1.7 g/cm\(^2\) of material in the way of the beam. This is small, however, compared with the over-all thickness of probably about 15 g/cm\(^2\). This, together with the fact that the glass will contain elements of high Z number (heavy flint with \( n = 1.66 \) contains nearly 50% of PbO), renders the slowing-down process in the radiator probably quite important even at these energies.

Let us now see how the counter would behave at higher proton energies. As in previous cases and for the purpose of comparison we shall now take \( \gamma = 4 \). To satisfy the criterion (25) we should either increase \( n_2 \) to 1.377 or lower \( n_1 \) to 1.6416. But since the whole point of using the arrangement is in the ability of changing the liquid \( n_2 \) without taking the counter apart, we shall assume the first possibility.

Thus for \( \gamma = 4 \), \( \beta = 0.968 \) for protons and for \( n = 1.722 \) the Čerenkov angle \( \Theta = 53^\circ 10' \). From this

\[
\chi_m = 3.1 \text{ cm}.
\]

Also \( I = 161 \) photon/cm \( \Delta \nu = 1.64 \times 10^{14} \text{ c/sec} \) and \( \Delta \theta (d,s) = 12.84 \times 10^{-3} \text{ rad} = 0.736^\circ \). Thus \( \Delta \beta = 16.59 \times 10^{-3} \) and \( \frac{\Delta \theta}{\beta} = 17.14 \times 10^{-3} \) or 1.714%.

We see that we have not gained anything by going from \( \gamma = 2 \) to \( \gamma = 4 \) for protons.
If we abandon the idea of using a liquid in the spaces between the radiators but use only air, we lose the flexibility of the counter velocity band, but gain in the accuracy of measurement. In this case criterion (25) becomes

$$\beta^2 > \frac{1}{n^2 - 1}$$

and for $\beta = 0.968$ this gives $n > 1.438$. For the radiator it would be an advantage to use a plastic. The nearest to the requirement is triethoxy-silicol methacrylate polymer with $n_D = 1.436$, but cheaper and presumably more readily obtainable would be isopropyl methacrylate polymer with $n_D = 1.4728$. However, let us suppose that we use the former. Making the same assumption as before, i.e. the refracting angle at the light guide to be $80^\circ$, we get $n_3 = 1.427$. Total refraction in the light guide occurs for $\beta = 0.982$. This gives $\Delta\beta = 14 \times 10^{-3}$ or $\frac{\Delta\beta}{\beta} = 1.44\%$. This is obviously going to be the limiting error because that introduced by dispersion and scattering will be of a similar order of magnitude as for M.M.P. described earlier, i.e. about 0.5%. From these results we may conclude that the counter described is not likely to yield accuracies better than those obtained by the above calculations at least for velocities considered here.

For indices of refraction below that of FC-75 we must resort to more complicated techniques as no solids of liquids exist at ordinary temperatures and pressures with indices lower than that of the above compound. One may prefer lower temperatures to higher pressures, and very cheap and easily accessible material is liquid air with $n_D = 1.21$. Assuming this approximate value one gets for protons with $\gamma = 2$ $\Theta = 17^\circ 25'$ and for $\gamma = 4$ $\Theta = 31^\circ 20'$. These small angles enable us to make use of the fact that $\Theta$ diminishes at refracting surfaces which are perpendicular to the particle motion if the Čerenkov light passes from a lower to a higher refractive medium. We can thus make a Marshall-type counter, which would be fairly free of aberrations since the angle of light with the axis
would, in case of $\Theta = 31^\circ 20'$, be only $24^\circ 50'$. The focusing of the ring image is then achieved in a way analogous to that of Marshall (Fig. 14). The dispersion is reduced by the lens material, and may be totally cancelled by other standard optical methods, or finally overcome by filtering as in the previous cases.

Conclusions

We may devise a multitude of other counters making use of one or more properties of Čerenkov radiation. As far as the visible part of this radiation is concerned, several things are obvious.

If we keep $\Theta$ small, then it follows from Eq. (7) that we can afford greater changes in $\Theta$ for a given error $\beta$. In fact, we can see that dispersion and scattering taken together yield

$$\frac{\Delta\beta}{\beta} = 6.66 \left( \frac{E^2 D e c n}{E^2 \chi_0 \beta^2 z^2 n} \right)^{\frac{1}{3}}.$$  (28)

This tells us a lot, but not all. It follows that we should choose low values of $n$ and light material. The slowing down of the particles in the radiator has not been taken into account here but this would not change qualitatively the above conclusion. On the other hand, low $n$ means a rise in the effect of diffraction. The error in $\Theta$ introduced by this is

$$\Delta\Theta = \frac{0.38 \lambda_m}{n \sin \Theta}.$$  (29)

In the present note this effect was not considered as it was small, but for very low values of $n$ it should be.

Finally it should be stressed that the above conclusions have been drawn under the assumption that the effect of dispersion can be reduced by filtering the light. This is not the only way, however,

because similar results can be achieved by cancelling dispersion by means of prisms, etc.\textsuperscript{6,4}. To adapt this method to our counters would be a fairly difficult problem, even in principle, because of the angular momenta of photons and consequently varying angles at which they cross the optical surfaces. But for counters with very low refractive indices and thus small angles \( \theta \) where perfect ring focusing of the Čerenkov "circle at infinity" is possible due to low optical aberrations, this is quite a feasible method.

Counters which depend on total internal reflections at surfaces orthogonal to the particle paths can obviously be made only with material with high refractive indices. Their high sensitivity to the change in \( \theta \) for high refracting angles and their almost complete freedom to handle large beam sizes, makes them suitable for some purposes in spite of the fact that on account of large angles \( \theta \) they are inherently inaccurate for the measurement of \( \beta \).

* * *

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\( \theta = 17^\circ 50' \quad \beta = 0.7 \)

\( \theta = 33^\circ 30' \quad \beta = 0.8 \)

\( \theta = 42^\circ 10' \quad \beta = 0.9 \)

\( \theta = 47^\circ 40' \quad \beta = 0.99 \)
Logar. Teilung 1
Division 1
Einheit 1
Unité 1
62,5 mm

$x$ cm.
reflecting surfaces

\[ n = 1.276 \]

\[ \theta = 41^\circ \]

\[ \phi = 35^\circ \]

\[ n = 1.50 \]