STRAange PARTICLE PHYSICS

Lectures given at CERN in 1958 and 1959

Theory

by

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Selected experimental topics

by

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# STRANGE PARTICLE PHYSICS

## TABLE OF CONTENTS

- Introductory note.
- Table of fundamental particles.

### I. INTRODUCTION.

1. Classification of particles. J.M. Jauch
2. Isospin and strangeness. J.M. Jauch
3. Cross-sections and lifetimes. J.M. Jauch
4. Strong and weak interactions. J.M. Jauch
5. Fundamentals on heavy mesons and hyperons. C. Peyrou

### II. STRONG INTERACTIONS.

1. Isospin wave functions. Y. Yamaguchi
2. Experimental tests of charge independence (especially for strange particles). Y. Yamaguchi
3. Global symmetry. Y. Yamaguchi
4. Experimental test of restricted symmetry. Y. Yamaguchi
5. Experimental tests of global symmetry, using antihyperons. B. d'Espagnat

### III. WEAK INTERACTIONS.

1. Asymmetry in non-leptonic decays. B. d'Espagnat
2. The $\left| \Delta I \right| = \frac{1}{2}$ rule. B. d'Espagnat
3. Experimental tests of $\left| \Delta I \right| = \frac{1}{2}$ rule. B. d'Espagnat
   (Branching ratios and asymmetries).
4. PC invariance (elementary notions on). B. d'Espagnat
   ($K^0$ and $K^0$ decays, etc.).
5. Leptonic decays – Proposed $\Delta Q \Delta S = 1$ rule. B. d'Espagnat
STRANGE PARTICLE PHYSICS

TABLE OF CONTENTS

IV. THEORETICAL.
(1) Some basic concepts of quantum mechanics. J.M. Jauch
(2) Field theory. J.M. Jauch
(3) Observables in field theory. J.M. Jauch
(4) The transformations P, T and C. J.M. Jauch
(5) The mass and lifetime of antiparticles. J.M. Jauch

V. STRUCTURE OF WEAK INTERACTIONS IN SPACE-TIME.
(1) Introduction. Y. Yamaguchi
(2) $\pi^0, K^0, (\mu = \mu \text{ or } e)$. Y. Yamaguchi
(3) Beta radioactivity. B. d'Espagnat
(4) Muon decay. B. d'Espagnat
(5) Primary weak interactions. Y. Yamaguchi
(6) $K^0$-decays. Y. Yamaguchi
(7) Leptonic decays of hyperons. Y. Yamaguchi
(8) Non-leptonic decays of hyperons. B. d'Espagnat

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Introductory Note

A series of lectures on strange particle physics was given in 1958 and 1959 to a group of CERN physicists planning experiments with the Proton Synchrotron. The aim of these lectures was to familiarize the audience with those aspects of the theory of elementary particles which seem of most immediate use in strange particle physics. The attempts at a systematization of the most recent knowledge in this field were therefore presented with some emphasis on those aspects of current theories which, due to a lack of experimental knowledge, are still controversial.

As a result of the special aim of these lectures, it was felt desirable to avoid lengthy calculations and to replace them wherever possible by simple physical pictures describing the general trend of the phenomena under study. Formal developments were sometimes replaced by mere plausibility arguments or, where they proved absolutely necessary for the introduction of a new and useful concept, reduced to a minimum.

For didactic reasons, the lectures were given in an order of increasing complexity. This order has been kept in the present report with the effect that Part I summarizes those subjects which can be understood without knowledge of Dirac matrices or of the Hamiltonian formalism in field theory.

Some of the notes collected in these volumes were taken by helpful listeners (namely Mrs. M. Fidecaro, and Messrs. F.J.H. Parley, P. Hillman, D.E.C. Michiélis, W. Middelkoop, J.C. Sens, T. Yamagata, E. Zavattini and A. Zichichi), to whom we should like to express our heartfelt appreciation for undertaking this thankless task. The rest was written by the authors themselves, usually on the spur of the moment, as papers to be distributed just before the lectures. Under these conditions unfortunate variations in presentation could not be avoided; for instance, in some of these notes the original papers were quoted extensively whereas in others not even the authors were mentioned. Recasting this rather heterogeneous mass into a uniform mould would have been a time-consuming process which, for that reason, was not attempted. Indeed the lecture notes appear here as they were originally produced for the benefit of the participants in the course.
<table>
<thead>
<tr>
<th>Particle</th>
<th>Symbol</th>
<th>Spin</th>
<th>Mass (Me)</th>
<th>Mass (keV)</th>
<th>Mean Life (sec)</th>
<th>Decay modes</th>
<th>Branching ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon</td>
<td>γ</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leptons</td>
<td>ν̅e</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>(2.22 ± 0.02) × 10^-6</td>
<td>e^- + ν̅ + ν</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e^+</td>
<td>1/2</td>
<td>1</td>
<td>0.510 976±7 × 10^-6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>μ^-</td>
<td>1/2</td>
<td>206.86 ± 0.11</td>
<td>105.70 ± 0.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Photons</td>
<td>μ^+</td>
<td>0</td>
<td>264.37 ± 0.6</td>
<td>135.09 ± 0.09</td>
<td>4 × 10^-16</td>
<td>μ^+ + ν̅ + ν̅</td>
<td>98.8</td>
</tr>
<tr>
<td></td>
<td>μ^-</td>
<td>0</td>
<td>273.27 ± 0.11</td>
<td>139.63 ± 0.06</td>
<td>(2.56 ± 0.05) × 10^-8</td>
<td>μ^+ + ν̅ + ν̅</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>π^+</td>
<td>0</td>
<td>264.37 ± 0.6</td>
<td>135.09 ± 0.09</td>
<td>4 × 10^-16</td>
<td>π^+ + ν̅ + ν̅</td>
<td>(0.004) *</td>
</tr>
<tr>
<td></td>
<td>K^+</td>
<td>0</td>
<td>494.0 ± 0.2</td>
<td>(1.222 ± 0.013) × 10^-8</td>
<td></td>
<td>π^+ + π^- + π^0</td>
<td>(1 × 10^-6) *</td>
</tr>
<tr>
<td></td>
<td>K^-</td>
<td>0</td>
<td>965.6 ± 1.2</td>
<td>493.4 ± 0.6</td>
<td>(1.25 ± 0.11) × 10^-8</td>
<td>μ^- + ν̅ + ν̅</td>
<td>6.120</td>
</tr>
<tr>
<td></td>
<td>K^0</td>
<td>0</td>
<td>497.1 ± 0.8</td>
<td>(8 ± 3) × 10^-10</td>
<td></td>
<td>μ^- + ν̅ + ν̅</td>
<td>27 ± 2</td>
</tr>
<tr>
<td></td>
<td>K^-</td>
<td>0</td>
<td>965.6 ± 1.2</td>
<td>493.4 ± 0.6</td>
<td>(1.25 ± 0.11) × 10^-8</td>
<td>μ^- + ν̅ + ν̅</td>
<td>1.9 ± 0.4</td>
</tr>
<tr>
<td></td>
<td>K^0</td>
<td>0</td>
<td>497.1 ± 0.8</td>
<td>(8 ± 3) × 10^-10</td>
<td></td>
<td>μ^- + ν̅ + ν̅</td>
<td>59 ± 2</td>
</tr>
<tr>
<td></td>
<td>K^-</td>
<td>0</td>
<td>965.6 ± 1.2</td>
<td>493.4 ± 0.6</td>
<td>(1.25 ± 0.11) × 10^-8</td>
<td>μ^- + ν̅ + ν̅</td>
<td>3.3 ± 1</td>
</tr>
<tr>
<td>Nucleons</td>
<td>p</td>
<td>1/2</td>
<td>1836.118 ± 0.02</td>
<td>938.213 ± 0.01</td>
<td></td>
<td>e^- + p + ν̅</td>
<td>63 ± 33</td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>1/2</td>
<td>1836.645 ± 0.02</td>
<td>939.50620 ± 0.01</td>
<td></td>
<td>e^- + p + ν̅</td>
<td>37 ± 3</td>
</tr>
<tr>
<td></td>
<td>Λ^0</td>
<td>1/2</td>
<td>2182.5 ± 0.3</td>
<td>1115.2 ± 0.14</td>
<td>(2.60 ± 0.15) × 10^-10</td>
<td>p + ν̅ + ν̅</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Σ^+</td>
<td>1/2</td>
<td>2527.7 ± 0.5</td>
<td>1189.4 ± 0.25</td>
<td>(0.83 ± 0.05) × 10^-10</td>
<td>p + ν̅ + ν̅</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Σ^-</td>
<td>1/2</td>
<td>2341.6 ± 1</td>
<td>1196.5 ± 0.5</td>
<td>(1.72 ± 0.17) × 10^-10</td>
<td>p + ν̅ + ν̅</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Σ^0</td>
<td>1/2</td>
<td>2329.9 ± 2</td>
<td>1190.5 ± 1</td>
<td>&lt;10^-11</td>
<td>p + ν̅ + ν̅</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Λ^-</td>
<td>1/2</td>
<td>2383 ± 5.5</td>
<td>1319.7 ± 2.6</td>
<td>(1.8 ± 10^-10) Λ^0 + γ</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Σ^0</td>
<td>1/2</td>
<td>2579 ± 16</td>
<td>1311 ± 28</td>
<td>(10^-10) Λ^0 + π</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*) Theoretical expectation.
STRANGE PARTICLE PHYSICS

I. INTRODUCTION.

(1) Classification of particles. J.M. Jauch
I. INTRODUCTION.

(1) Classification of particles.
(2) Isospin and strangeness.
(3) Cross-sections and lifetimes.
(4) Strong and weak interactions.

(1) Classification of Particles.

Reference is made to the table of fundamental particles, distributed at the beginning of the lecture. The following general review is also useful:


We divide the various particles into certain groups which are listed below. The motivation for these divisions are certain general properties which the particles in each group have in common and which are listed under each group.
(a) Photons. Rest mass \( m = 0 \), spin 1, and gauge invariance.

Photons interact through a universal constant \( e = \sqrt{\frac{4 \mu}{137}} \) with charged particles. It is a remarkable feature that this interaction seems to be the only interaction which photons have with anything else. One may speak of the principle of "minimal interaction", formulated for instance tentatively in the following manner: "Every effect involving photons in interaction involves only the fundamental electromagnetic interaction". For instance the so-called anomalous magnetic moment of particles is not a new interaction, but is derivable from the fundamental electromagnetic interaction with great accuracy and complete agreement with experiment.

Important Question: What are basic and what are derived interactions. So far this question is only solved for the photons. For other fundamental particle interactions the answer is not known.

(b) Leptons. Relatively light, spin \( \frac{1}{2} \), weakly interacting.

The most important property of the leptons is that they seem to satisfy a conservation law. Leptons, being particles of spin \( \frac{1}{2} \) occur always in pairs of particles and antiparticles. The lepton conservation is true, if we associate with the antiparticles the opposite lepton number as to the particles. Thus for instance the following is a possible assignment of lepton numbers \( \ell \) to the six known leptons:

<table>
<thead>
<tr>
<th>lepton</th>
<th>( \nu )</th>
<th>( \bar{\nu} )</th>
<th>( e^- )</th>
<th>( e^+ )</th>
<th>( \mu^- )</th>
<th>( \mu^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ell )</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>
The lepton conservation law, if true, would be stated as the selection rule

$$\Delta \ell = 0$$  \hspace{1cm} (1.1)$$

It has observable consequences, for instance in the two reactions

$$\bar{\nu}_\mu^+ \rightarrow \mu^+ + \nu$$
$$\bar{\nu}_\mu^- \rightarrow \mu^- + \bar{\nu}$$  \hspace{1cm} (1.2)$$

the two neutrinos are different. Similarly

$$\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$$  \hspace{1cm} (1.3)$$

leads to a pair of different neutrinos.

(c) **Mesons.** Intermediate mass, spin 0 (Bosons).

There are two subgroups: pions and K-mesons. The pions occur in a charge triplet with slightly different masses (see table).

$$\mu_{\pi^+} \approx \mu_{\pi^-} > \mu_{\pi^0}$$

The K-mesons occur also with charges $^+\!\!$ and 0. But there are important differences from the pions. Within the experimental accuracy the masses of the K-mesons are all the same. Furthermore there exist two neutral K-mesons which can be experimentally differentiated by different decay schemes with different lifetimes.

$$K^0 \rightarrow \pi^+ + \pi^- \stackrel{\tau \sim 10^{-10}}{\longrightarrow} \pi^0 + \pi^0$$  \hspace{1cm} (1.4)$$
(d) Nucleons, together with the antinucleons comprise four particles \( n, p, \bar{n}, \bar{p} \). Strong interactions with the pion system! Spin \( \frac{1}{2} \).

(e) Hyperons. \( \Sigma^+, \Sigma^0, \Lambda^0, \) and \( \Xi^- \). Masses \( > \) nucleon masses. There are theoretical reasons for believing in the existence of a neutral \( \Xi^0 \). Furthermore all of them having spin \( \frac{3}{2} \) (or half-integer at least) are expected to have antiparticles. None of these have been discovered yet. *

The two last groups are conveniently placed into a larger group called the "baryons", because there seems to be a law, called the conservation of baryon numbers, which is similar to the law of lepton conservation.

*) Note added in August 1959: Both the anti-\( \Lambda \) and the \( \Xi^0 \) have been observed in the meantime.
I. INTRODUCTION

(2) Isospin and strangeness. J.M. Jauch
(2) Isospin and strangeness.

(a) I-spin-space. We refer to the basic fact that nuclear forces are charge independent. Thus if we could switch off the electric charge and all other weak interactions, the neutron and the proton would be distinguishable.

This is similar to the spin in a magnetic field. As long as the magnetic field is not present the two spin states will be degenerate and they cannot be distinguished. This analogy is the basis of the I-spin formalism.

Two nucleons at rest are represented in this formalism by a column vector consisting of two components. For instance

\[
p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

When we have more than one particle we use the method of product spaces. For instance 2 particles are described in a four dimensional space. A system of orthogonal vectors in such a space is \( pp, pn, np, nn \). Explicitly we may write

\[
pp = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad pn = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad np = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad nn = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
\]

This can be generalized in an obvious manner to a system of \( N \) particles. The dimension of the space is then \( 2^N \).
(b) I-Spin Operators. In analogy to the ordinary spin we introduce the I-spin operators, for instance

\[ \tau_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \] (2.3)

and

\[ \tau_+ = \frac{1}{2}(\tau_1 + i \tau_2) \]

with the properties

\[ \tau_3 p = p \quad \tau_3 n = n \] (2.4)

\[ \tau_+ n = p \quad \tau_- p = n \]

These operators satisfy the commutation rules

\[ \tau_1 \tau_2 = -\tau_2 \tau_1 = i \tau_3, \text{ etc.} \] (2.5)

In the case of many particles we introduce an I-spin operator for each particle

\[ T = \sum_{n=1}^{N} \tau_3^{(n)} \quad \underline{T} = (\tau_1, \tau_2, \tau_3) \] (2.6)

and

\[ I = \frac{1}{2} \underline{T} \] (2.7)

so that

\[ I^2 = I(I+1) \] (2.8)

for the substates with total I-spin equal to I.
Warning:

While the analogy of the I-spin formalism to the ordinary spin has always been stressed, it is perhaps not quite so well known, that the analogy is not complete. (see for instance L.L. Foldy, Phys.Rev. 93, 1395 (1954) on this point). The reason is that there exist general symmetry transformations which multiply every state vector in anyone of the subspaces of constant charge with a different phase factor. As a consequence any operator which does not commute with $T_2$ or $T_3$ cannot be an observable. In particular the I-spin operators $T_1$ and $T_2$ cannot be observables. This is the main reason why the physical significance of the I-spin formalism is much less transparent than that of the ordinary spin.

(c) Relation of I-Spin to Charge. In the case of one single nucleon the charge has the eigenvalues 0 and +1. (in units of $e$). We can therefore represent it by an operator

$$Q = I^3 \cdot \frac{1}{2}$$

(2.9)

and we find

$$Qp = p$$

$$Qn = 0.n$$

(2.10)

In the general case of $N$ nucleon we find

$$Q = \sum_{i=1}^{N} Q^{(i)} = I^3 + \frac{N}{2}$$

(2.11)
If we decompose the states of the I-spin system in the usual manner in multiplets we expect the eigenvalues of \( Q \) to occur in charge multiplets with values

\[
q = -I + \frac{N}{2}, -I + \frac{N}{2} + 1, \ldots, +I + \frac{N}{2}
\]  

(2.12)

All these multiplets have their centre of charge at the value \( \frac{N}{2} \). Thus we conclude the important relation, valid for any system of nucleons:

\[
\frac{\text{Baryon number}}{2} = \text{Centre of charge}
\]

(2.13)

(d) Extension of I-Spin to Pions. Pions occur in a triplet of charges \( \pi^+, \pi^0 \). We attribute to them a total I-spin = 1 and we can define the I-spin operators in complete analogy to angular momentum states

\[
I = \mathcal{P} = (P_1, P_2, P_3)
\]

(2.14)

\[
P_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad P_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0-1 & 0 \\ 1 & 0-1 \\ 0 & 1 & 0 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

(2.15)

\[
\begin{align*}
\pi^+ & = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, & \pi^0 & = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, & \pi^- & = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\end{align*}
\]

(2.16)
The following useful relations are immediately obtained from these definitions:

\[ \int_1 \overline{\eta}^+ = \frac{1}{\sqrt{2}} \overline{\eta}^0, \int_1 \overline{\eta}^0 = \frac{1}{\sqrt{2}} (\overline{\eta}^+ + \overline{\eta}^-), \int_1 \overline{\eta}^- = \frac{1}{\sqrt{2}} \overline{\eta}^0, \text{ etc.} \]

Notice:
The relation of the charge operator to the I-spin is now

\[ Q = I_3 \] \hspace{1cm} (2.17)

and differs from (2.9) by the term \( \frac{1}{2} \). But the Eq. (2.13) is still correct. The centre of charge is zero and so is the baryon number!

(e) Strangeness. For pions and nucleons we can write quite generally

\[ Q = I_3 + \frac{1}{2} N \] \hspace{1cm} (2.11)

where \( N \) is the number of nucleons.

Question: Can this relation be generalized, so that it is valid for other charge multiplets of strange particles?
Let us see how it is with the \( \Sigma \) -particles, since there exist \( \Sigma^+ \) and \( \Sigma^0 \) one would naturally assign to it the I-spin 1, and since the centre of charge is zero we have in this case

\[
Q = I_3
\]  

(2.18)

On the other hand, the \( \Sigma \)-meson is a baryon and therefore one would have expected \( N=1 \). Thus relation (2.11) is violated. But we can still write

\[
Q = I_3 + \frac{1}{2} U
\]  

(2.19)

where \( U \) is for the time-being an unknown integer. It is a strange fact that it differs from the baryon number \( N \) and so we write

\[
U = N + S
\]  

(2.20)

and call \( S \) the strangeness number. Instead of (2.13) we have now

\[
\frac{\text{Baryon number} + \text{Strangeness}}{2} = \text{Centre of charge}
\]  

(2.21)

Note:

The assignment of strangeness is not unique. It depends of course on the assignment of I-spin. When the charges occur in multiplets (as for instance the \( \Sigma^- \) and \( \Sigma^0 \)) we can try an enlightened guess.
There is, however, a much better guess possible if we combine the assignment with a selection rule. This selection rule was the starting point of the scheme and was invented first to explain the absence (or weakness) of certain reactions which otherwise would have to be observed.

These selection rules are

\[
| \Delta S | = 0 \quad \text{strong} \\
| \Delta S | = 1 \quad \text{weak} \\
| \Delta S | = 2 \quad \text{negligible}
\]

(2.22)

We shall study below some of the consequences of these selection rules. But before doing it, let us complete the assignment of the various quantum numbers with the following table:

<table>
<thead>
<tr>
<th>S</th>
<th>N</th>
<th>I</th>
<th>U=S+N</th>
<th>Q</th>
<th>Particle</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>not known</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>±1,0</td>
<td>pion</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1/2</td>
<td>1</td>
<td>0,1</td>
<td>nucleons</td>
<td>1/2</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>1/2</td>
<td>-1</td>
<td>-1,0</td>
<td>K^-, \bar{K}^0</td>
<td>0 ?</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>±1,0</td>
<td>\Sigma^-, \Sigma^0</td>
<td>1/2 ?</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
<td>1/2</td>
<td>-1</td>
<td>-1,0</td>
<td>\Xi^-, \Xi^0(?)</td>
<td>1/2 ?</td>
</tr>
</tbody>
</table>

**TABLE**

Assignment of I-spin and strangeness to the fundamental particles
The table must be completed for the antiparticles by adding the rule: strangeness of antiparticles = minus strangeness of particles.

This rule fits well into the already known rule that the pion is its own antiparticle \( \pi^- \rightarrow \pi^+ \). On the other hand, it leads to the conclusion that there should exist two neutral K-mesons, viz.

\[
K^0 \text{ with } S = +1, \ N = 0, \ I = \frac{1}{2}, \text{ and }
\]

\[
\bar{K}^0 \text{ with } S = -1, \ N = 0, \ I = \frac{1}{2}.
\]

This is one of the predictions of the scheme which has been strikingly confirmed by experiments.

\( \text{(f) Consequences of Selection Rules.} \)

\( \text{(A) Associated production and slow decay.} \)

Consider for example the reaction

\[ \pi^- + p \rightarrow \Lambda^0 + K^0 \]

\[ S = 0 \quad 0 \quad 0 \quad -1 \quad +1 \quad \Delta S = 0 \]

Thus this process is fast, while the decay

\[ \Lambda^0 \rightarrow p + \pi^- \]

\[ S = -1 \quad 0 \quad 0 \quad \Delta S = 1 \]

must be slow. And this is just what is observed!
(β) Exclusion of unobserved processes.

The reaction

\[ n + n \rightarrow \Lambda^0 + \Lambda^0 \quad (|\Delta S| = 2) \]

is excluded but

\[ n + n \rightarrow \Lambda^0 + n + K^0 \quad (|\Delta S| = 0) \]

should occur. Similarly

\[ \pi^- + p \rightarrow \Sigma^+ + K^- \quad (|\Delta S| = 2) \]

is not observed but

\[ \pi^- + p \rightarrow \Sigma^- + K^+ \quad (|\Delta S| = 0) \]

occurs easily.

(γ) Prediction of new particles.

(i) \( \Sigma^0 \) was predicted as part of a triplet and was later discovered;

(ii) A \( \eta^0' \) with \( S = N = I = 0 \) (first line in table) would fit very well into the scheme. It is not yet discovered.

(iii) \( \Xi^{-0} \) would be part of a I-spin doublet. It is not yet discovered.*

(δ) The \( K^0 \)-doublet.

The occurrence of two neutral K-mesons with different decay scheme and lifetimes confirms this prediction. The analysis of this process will be discussed later.

*) See note on p.4.
I. INTRODUCTION.

(3) Cross-sections and lifetimes. J.M. Jauch
(3) Cross-Sections and Lifetimes.

(a) Motivation. In Quantum Mechanics we describe the state of a system by a Schrödinger wave function \( \psi(t) \) which satisfies a Schrödinger wave equation

\[
  i \frac{\partial \psi(t)}{\partial t} = H \psi(t)
\]  

(3.1)

Here \( H \) denotes the total Hamiltonian of the system. In principle if we knew the Hamiltonian and if we knew how to solve the Eq. (3.1) for certain initial conditions we could derive all observable results from it. Neither of these conditions is met and this is why the theory of fundamental particles is fragmentary in character.

In spite of this fragmentary character it is possible to derive certain results which are very useful in the study of fundamental processes and which lead to important clues. These results are obtained if we express the observable quantities in terms of the scattering operator and the clues to which they lead are the classification of the interactions into strong and weak interactions and various selection rules and branching ratios.

In order to express these relations we proceed now to express the observable quantities in terms of the scattering matrix.

(b) Definition of the Scattering Operator. Let us consider an example first, for instance the reaction

\[
  \begin{align*}
    \text{(before)} & \quad \overline{\pi} + N \quad \rightarrow \quad \Lambda + K \\
    \text{(after)} & \quad t \rightarrow -\infty \quad t \rightarrow +\infty
  \end{align*}
\]
This process would be described by a wave function $\Psi(t)$ which in the remote past ($t \to -\infty$) would behave like two free particles $\pi$ and $N$, and in the distant future ($t \to +\infty$) it would behave like two free particles again, namely a $\Lambda$ and a $K$.

Let us denote by $\Psi_{-}(t)$ the free motion of a $\pi$ and a $N$ and by $\Psi_{+}(t)$ the free motion of a $\Lambda$ and a $K$. We also define $\Psi_{-} = \Psi_{-}(0)$, $\Psi_{+} = \Psi_{+}(0)$. The connection between the two state vectors $\Psi_{+}$ and $\Psi_{-}$ is then a linear transformation

$$\Psi_{+} = S \Psi_{-}$$

(3.2)

One can prove that, whatever the total Hamiltonian for the whole system, provided the above-mentioned limiting property is true, the operator $S$ must be a unitary operator, that is it satisfies

$$S^{*}S = SS^{*} = I$$

(3.3)

The operator $S$ is called the scattering operator. The scattering matrix is the system of matrix elements

$$(\beta | S | \alpha) = (\beta, S \alpha)$$

(3.4)

The important property of the scattering matrix is the fact that the general transition probability for the transition from an "initial state" $\alpha$ into a final state $\beta$ is determined by the square of the matrix element

$$P_{\beta \alpha} = |(\beta | S | \alpha)|^2$$

(3.5)
A little care is needed in relating this expression to observable quantities since we are dealing in general with transitions from one state in a continuum of states to another state again in a continuum. We shall give the relations of the observable quantities to the matrix elements of $S$ in the following subsection:

(c) Lifetimes and Cross-Sections. We write for the general $S$-matrix

$$S = I + R$$

and express the general matrix element of $R$ in a representation in which the momenta of the incident and final states are diagonal. Let

$P_1, P_2, \ldots \ldots$ the momentum four vectors of the initial state "i"

$P'_1, P'_2, \ldots \ldots$ " final " "f"

and

$$P_i = p_1 + p_2 + \ldots \ldots$$
$$P_f = p'_1 + p'_2 + \ldots \ldots$$

The general matrix element of $R$ has then the form

$$(f\mid R \mid i) = \frac{1}{i} (P_f - P_i) (f\mid M \mid i)$$
In (3.) the occurrence of the (four-dimensional) $J^+$-function expresses the conservation of momentum and energy. The matrix element $(f | M | i)$ is then in general free from $J^+$-functions. It will be called the reduced matrix element.

Let $n$ be the number of initial particles. There are two cases to consider:

$n = 1$ Decay of an unstable particle. The experimental quantity is the transition rate $\Gamma = \frac{1}{\tau}$. $\tau$ is the lifetime. One can derive the following relation

$$
\Gamma = \frac{1}{2 \pi} S_f \overline{S_i} \int (P_f - P_i) |(f | M | i)|^2 \tag{GR_1}
$$

$n = 2$ Collision of two particles. The experimentally observed quantity is the cross-section

$$
\sigma = (2 \pi)^2 \frac{1}{I} S_f \overline{S_i} \int (P_f - P_i) |(f | M | i)|^2 \tag{GR_2}
$$

The symbols $S_f \overline{S_i}$ occurring in these formulae are the summation over the final and possible averaging over the initial states. These summations are to be interpreted as integrations if the continuous momentum variables are involved and summation over the discrete spin variables if any. The quantity $I$ is the "total incident current". In the units used here it is simply the sum of the "dimensionless" velocities of the two incident particles. Only two cases are of practical importance, viz.

*) c.f. J.M. Jauch and F. Rohrlich, Theory of Photons and Electrons, Section 8-6, P.163.
(i) Centre of mass system:

\[ I = q \left( \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} \right) \quad \varepsilon_{1,2} = \sqrt{m_{1,2}^2 + q^2} \]

\( q = \) total momentum of particle 1 or 2

(ii) Laboratory system:

\[ I = \frac{p}{\xi} \quad \xi = \sqrt{m^2 + p^2} \]

\( p = \) momentum of incident particle.

The logical situation which shows the connection between Hamiltonian, Scattering operator and observable quantities can be summarized by the diagram

\[ \text{GR}_{1,2} \]

unknown region  \hspace{1cm} observable quantities

Note: \( \text{GR}_{1,2} \) denotes "golden rule" number 1 and 2 , a terminology adapted from Fermi.
I. INTRODUCTION.

(4) Strong and weak interactions. J.M. Jauch
(4) Strong and Weak Interactions.

(a) Types of Interactions. We shall first distinguish different types of interactions according to the different types of particles which undergo transitions.

There is an important distinction between Bosons and Fermions. Bosons can be absorbed and emitted singly, Fermions are always emitted and absorbed in pairs, or a Fermion is absorbed and another one emitted in its place. We shall use the following notation:

- \( \phi \) represents a Boson absorption or emission,
- \( \overline{\psi} \psi \) represents a Fermion absorption, emission or exchange process.

A typical term showing such a process together with its Feynmann diagram would be

\[ g \phi \overline{\psi} \psi \]

The most general term could be written symbolically as

\[ g \phi^B \overline{\psi}^F \psi^F \]

- \( B \) = number of Bosons
- \( 2F \) = number of Fermions

Note: This is a symbolic notation and it may represent different Bosons and Fermions.
(b) Dimensions of the Coupling Constant. Our aim is to obtain ultimately the strength of the various interactions in terms of certain dimensionless numbers. We must first realize that the coupling constant for a general type of process is in general not dimensionless. Let us determine the dimension of \( g \). To this end we need to know the dimensions of \( \phi \) and \( \psi \). Note that in our system of units the only free dimension is a length \( L \), the other two dimensions are already expressed in terms of \( \hbar \) and \( c \). Thus

\[
\left[ \text{energy} \right] = L^{-1}, \quad \left[ \text{mass} \right] = L^{-1}, \text{etc.}
\]

Since

\[
\left[ m^2 \phi^2 \right] = L^{-4}
\]

we must have

\[
\left[ \phi \right] = L^{-1}
\]

and since

\[
\left[ m \bar{\psi} \psi \right] = L^{-4}
\]

we have

\[
\left[ \psi \right] = L^{-3/2}
\]

Therefore

\[
\left[ \bar{g} \right], \quad L^{-B-3F} = L^{-4} \quad \text{or}
\]

\[
\left[ \bar{g} \right] = L^{-4} + B + 3F \quad \text{(4.1)}
\]

The following table gives a summary of the dimensions of the coupling constants for the various elementary processes:
<table>
<thead>
<tr>
<th>Symbol</th>
<th>B</th>
<th>F</th>
<th>$-4 + B + 3F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \overline{\Psi} \Psi$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\phi^4$</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\phi^3$</td>
<td>3</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\phi^5$</td>
<td>5</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>$\phi^2 \overline{\Psi} \Psi$</td>
<td>2</td>
<td>1</td>
<td>+1</td>
</tr>
<tr>
<td>$\phi^3 \overline{\Psi} \Psi$</td>
<td>3</td>
<td>1</td>
<td>+2</td>
</tr>
<tr>
<td>$(\overline{\Psi} \Psi)^2$</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\phi (\overline{\Psi} \Psi)^2$</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$\phi^2 (\overline{\Psi} \Psi)^2$</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

**TABLE**

Dimensions of the "coupling constant" for various processes

**Note:** There exists exactly two kinds of interactions with dimensionless coupling constants. It is interesting that the fundamental electromagnetic interaction is among them.
(c) Interactions of type $\phi \not\rightarrow \gamma$. We shall now look more
in detail at some specific interactions. We begin with the
interaction $\phi \not\rightarrow \gamma$. The following are systems which interact
with this type:

(i) The pion-nucleon system, $(\pi^+, N, N)$ and the
$(K,Y,N)$ systems. These are strong interactions.

(ii) The electromagnetic interaction. Here we write
$g = e$ and $\frac{G^2}{\hbar c} = \frac{e^2}{137}$.

(iii) Weak interactions: $(\mu, \nu, K), (\pi, N, \mu), (\pi, \mu, \nu), (\bar{\nu}, e, \nu)$ etc.

The transition probability for a $\phi \not\rightarrow \gamma$ interaction is
obtained from GR: We simplify immediately by carrying out the
integration over one of the sets of 3-momentum vectors occurring
in the final state sum and also integrating over the angles of the
remaining 3-momentum vectors. The result is

$$\int_{0}^{\infty} \frac{1}{2\pi} \frac{4\pi}{\hbar c} \int \frac{p^2 dp}{\sqrt{p^2 + M_0^2}} \left| \langle \phi | M | i \rangle \right|^2$$

$$= \frac{1}{2\pi} \frac{4\pi}{\hbar c} \int \frac{p^2 dp}{\sqrt{p^2 + M_0^2}} \left| \langle \phi | M | i \rangle \right|^2$$

(4.2)

where $M_0 = \text{energy of initial particle at rest} = \text{rest mass and}$
$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = \text{energy of decay products, and}$
$\mathcal{E}_1 = \sqrt{p^2 + M_1^2}$,
$\mathcal{E}_2 = \sqrt{p^2 + M_2^2}$.

The integration over the only remaining $\mathcal{E}$-function
proceeds according to the general formula

$$\int_{0}^{\infty} f(p) \frac{\mathcal{E}}{\sqrt{\mathcal{E}^2 - M_0^2}} = \left( \frac{f(p)}{\frac{d\mathcal{E}}{dp}} \right) \mathcal{E} = M_0$$
This gives after some simplification (using \( \frac{d\epsilon}{dp} = \frac{p}{\epsilon_1} + \frac{p}{\epsilon_2} \), etc.)

\[
\Gamma = 2 \left( \frac{p \epsilon_1 \epsilon_2}{M_0^2} \right) \left| \langle f \mid M \mid i \rangle \right|^2
\]

(4.3)

This is as much as one can obtain without knowing anything about the value of the matrix element. We get a little further by extracting from the matrix element those factors which must always occur for dimensional reasons and leave the rest in the value of a dimensionless effective coupling constant and spin factors of order 1.

\[
\langle f \mid M \mid i \rangle \equiv M = \frac{\hbar}{\sqrt{2\pi}} \frac{1}{\sqrt{2\omega}} \quad (\ast)
\]

(4.4)

The factor \( \frac{1}{\sqrt{2\pi}} \) is the result of the following consideration. Each plane wave gives a normalizing factor \( (2\pi)^{-3/2} \). The over-all \( \delta \) -function contains a factor \( (2\pi)^4 \). Hence

\[
(2\pi)^{-3/2}, (2\pi)^4 = \frac{1}{\sqrt{2\pi}}
\]

The factor \( \frac{1}{\sqrt{2\omega}} \) is the matrix element for the emission or absorption of a Boson of energy \( \omega \). (See for this the lecture notes by Weisskopf).

The term \( (\ast) \) contains the spin factors of the Fermions. They are the various matrix elements connecting the different spin states. In this part of the lecture we shall only be concerned with the course features of the transitions. Hence we shall completely ignore these terms and simply set them equal to 1.
We now distinguish two cases.

(i) Decay of a Boson into two Fermions

Examples
\[ \bar{\eta} \rightarrow \mu + \nu \]
\[ \bar{\eta} \rightarrow e + \nu \]
\[ K \rightarrow \mu + \nu \text{, etc.} \]

(ii) Decay of a Fermion into a Boson and a Fermion.

Examples
\[ \Lambda^0 \rightarrow p + \pi^- \]
\[ \Lambda^0 \rightarrow n + \pi^0 \]
\[ \Sigma^+ \rightarrow p + \pi^0 \text{, etc.} \]

**ad**(i) In this case we have

\[
\Gamma \approx \frac{\varepsilon^2}{4\pi} M_0 \left( \frac{2p \varepsilon_1 \varepsilon_2}{M_0^3} \right) \quad \varepsilon = M_0 \quad (4.5)
\]

**ad**(ii)

\[
\Gamma \approx \frac{\varepsilon^2}{4\pi} M_0 \left( \frac{2p \varepsilon}{M_0^2} \right) \quad \varepsilon = M_0 \quad (4.6)
\]

In the last formula \( \varepsilon \) = energy of the Fermion in the decay products.
The two formulae may be written in the form

\[ R \simeq \frac{\pi^2}{4\pi M_0} F(\mu_1, \mu_2) \]  

(4.7)

where \( F(\mu_1, \mu_2) \) is a dimensionless function of the two variables \( \mu_1 = \frac{M_1}{M_0} \) and \( \mu_2 = \frac{M_2}{M_0} \). In the two cases under discussion this function is easily obtained as

\[
F(\mu_1, \mu_2) = \begin{cases} 
\frac{1}{2}(1 - (\mu_1^2 - \mu_2^2)^2 \left[ 1 - 2(\mu_1^2 + \mu_2^2) + (\mu_1^2 - \mu_2^2)^2 \right]^{1/2} & \text{Case (i)} \\
\frac{1}{2}(1 + \mu_1^2 - \mu_2^2) \left[ 1 - 2(\mu_1^2 + \mu_2^2) + (\mu_1^2 - \mu_2^2)^2 \right]^{1/2} & \text{Case (ii)}
\end{cases}
\]

(4.8)

In the second case \( M_1 \) is the mass of the Boson and \( M_2 \) of the Fermion. The factors \( F \) are numbers of order 1.

Numerical values:

In order to obtain orders of magnitude of numerical values quickly it is convenient to introduce a characteristic time constant which in ordinary units is given by \( (m = \text{electron mass}) \)

\[
\frac{t_o}{c} = \frac{\hbar}{mc^2} \simeq 1.29 \times 10^{-21} \text{ sec.}
\]

(4.9)
We obtain then for the decay time from (4.7)

\[ \tau = \tau_0 \left( \frac{g^2}{4\pi} \right) -1 \left( \frac{m}{M_0} \right) \frac{1}{p} \]  
(4.10)

From this and the fact that the characteristic decay times which are known for these processes, one sees immediately that the interactions which produce these decays must be transmitted with a coupling constant of order $10^{-14}$.

In the following table we give a few characteristic cases of decays of this type together with their numerical values.

<table>
<thead>
<tr>
<th>Process</th>
<th>$\frac{M_0}{m}$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$F(\mu_1 / \mu_2)$</th>
<th>$\frac{g^2}{4\pi}$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda^0 \to n + \pi^-$</td>
<td>2583</td>
<td>0.106</td>
<td>0.845</td>
<td>0.181</td>
<td>$1.8 \times 10^{-10}$</td>
<td>$1.530 \times 10^{-14}$</td>
</tr>
<tr>
<td>$\eta \to n + \pi^-$</td>
<td>2342</td>
<td>0.117</td>
<td>0.785</td>
<td>0.253</td>
<td>$1.67 \times 10^{-10}$</td>
<td>$1.31 \times 10^{-14}$</td>
</tr>
<tr>
<td>$\rho^+ \to p + \eta^0$</td>
<td>2328</td>
<td>0.114</td>
<td>0.787</td>
<td>0.257</td>
<td>$0.83 \times 10^{-10}$</td>
<td>$2.60 \times 10^{-14}$</td>
</tr>
<tr>
<td>$\pi^- \to p + \pi^-$</td>
<td>2182</td>
<td>0.125</td>
<td>0.840</td>
<td>0.146</td>
<td>$2.60 \times 10^{-10}$</td>
<td>$1.56 \times 10^{-14}$</td>
</tr>
<tr>
<td>$\pi^+ \to \mu^+ + \nu$</td>
<td>966</td>
<td>0.214</td>
<td>0</td>
<td>0.248</td>
<td>$1.224 \times 10^{-8}$</td>
<td>$4.4 \times 10^{-16}$</td>
</tr>
<tr>
<td>$\pi^- \to \mu^- + \nu$</td>
<td>273</td>
<td>0.759</td>
<td>0</td>
<td>0.071</td>
<td>$2.56 \times 10^{-8}$</td>
<td>$2.60 \times 10^{-15}$</td>
</tr>
</tbody>
</table>

**Table**

Examples of decays through weak $\phi \overline{\tau} \overline{\nu}$ -interactions
(d) Interactions of type \((F\psi)(\overline{F}\psi)\). We consider first the decay of a Fermion into three other Fermions.

We use \(GR_1\) with

\[
S_\Gamma = \int d^3p_1 \quad d^3p_2 \quad d^3p_3
\]

The integration over \(p_3\) can be carried out and the space-part of the \(\delta\) -function can be omitted. The three final momentum vectors are then related by

\[
P_1 + P_2 + P_3 = 0
\]

which expresses the conservation of momentum for a decaying particle initially at rest.

\(GR_1\) now gives after angle integrations

\[
\Gamma = \frac{1}{2\pi} \quad (4\pi)^2 \quad \int \int p_1^2 dp_1 \quad p_2^2 dp_2 \quad (\varepsilon - M_0) \quad \varepsilon M^2
\]

The matrix element \(M\) has the form

\[
M = \frac{\tau}{(2\pi)^2} \quad (\ast) \quad \text{spinor wave amplitudes} \quad \sim 1.
\]

The numerical factor \(\frac{\tau}{(2\pi)^2}\) was obtained by the following reasoning:

number of plane wave amplitudes = 4, each contributing a factor \((2\pi)^{-3/2}\). Overall \(\delta\) -function giving a factor \((2\pi)^4\). Result:

\[
(2\pi)^{-6} \quad (2\pi)^4 = (2\pi)^{-2}.
\]
The result is now

\[ R \approx \frac{f^2}{2 \pi^3} \int \int p_1^2 dp_1 \; p_2^2 dp_2 \; \mathcal{J} (\mathcal{E} - M_0) \]  \hspace{1cm} (4.13)

with

\[ \mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 \]

\[ \mathcal{E} = \sqrt{p^2 + M_r^2} \quad (r = 1, 2, 3) \]

We consider now several special cases:

(i) **Decay of the neutron**

\[ n \Rightarrow p + e^- + \bar{\nu} \]

Let

- \( M_0 \) = neutron mass
- \( M \) = proton mass
- \( \Delta = M_0 - M \) = mass difference
- \( M_1 = m \) = electron mass
- \( M_2 = 0 \) = neutrino mass.

Furthermore

- \( p_1 = p \) = electron momentum
- \( p_2 = p' \) = neutrino momentum

Since \( \Delta \ll M \) the proton momentum can be neglected so that

\[ \mathcal{E} = M + \sqrt{p^2 + m^2} + p' \]  \hspace{1cm} (4.14)
The integrals over $p'$ can be carried out giving

$$
\int p'^2 \, dp' \int (E-M_0) = (\Delta - \sqrt{p'^2 + m^2})^2
$$

Thus we can write finally

$$
\Gamma = \int d\Gamma
$$

with

$$
d\Gamma \approx \frac{f^2}{2 h^3} p^2 (\Delta - \sqrt{p^2 + m^2})^2 \, dp \tag{4.16}
$$

Differential decay rate of free neutrons

The total decay rate is then obtained by integration

$$
\Gamma = \frac{f^2}{2 h^3} \int_0^{\sqrt{\Delta^2 - m^2}} p^2 (\Delta - \sqrt{p^2 + m^2})^2 \, dp \tag{4.17}
$$

If we neglect $m$ we have instead of (4.17) the integral

$$
\int_0^{1} p^2 (\Delta - p)^2 \, dp = \Delta^5 \int_0^{1} x^2(1-x^2) \, dx = \frac{\Delta^5}{30}
$$

$$
= \frac{1}{30}
$$
Actually $m \neq 0$ and a more detailed calculation gives

$$\sqrt{\Delta^2 - m^2} \int_0^{\ldots} dp \simeq 0.47 \frac{5}{30}$$

blocking factor depends on $\frac{m}{\Delta}$

The final result is thus

$$\Gamma \sim 0.47 \frac{f^3}{60 \pi^3} \Delta^5 = \frac{0.47}{60 \pi^3} \left( \frac{f m_{\pi}^2}{m_{\pi}} \right)^2 \cdot \Delta \cdot \left( \frac{\Delta}{m_{\pi}} \right)^4$$

(4.18)

When the known numerical values of $\frac{1}{f} \sim 702$ sec and $\Delta = 2.53$ m, $m_{\pi} = 273$ m are substituted, one finds

$$\left( \frac{f m_{\pi}^2}{m_{\pi}} \right)^2 \sim 10^{-14}$$

(4.19)

(ii) **Decay of the muon**

$$\mu \rightarrow e + \gamma + \bar{\nu}$$

We denote by

- $p$ = momentum of electron
- $P_1$ = " " neutrino
- $P_2$ = " " antineutrino
We put \( m = \text{mass of electron} \sim 0 \). Hereby neglecting terms of order \( \left( \frac{m}{m^2} \right)^2 \sim \frac{1}{200} \). The differential decay rate into the electron momentum is then with GR,

\[
d = \frac{1}{2\pi} \frac{4\pi p^2 dp}{4\pi} \int d^3 p_1 \int d^3 p_2 \delta (p - p_1 - p_2) \delta (M_e - p - p_1 - p_2) \frac{r^2}{(2\pi)^4}
\]

The integration \( d^3 p_2 \) can be carried out, giving

\[
d\Gamma = \frac{1}{2\pi} \frac{4\pi p^2 dp}{4\pi} \frac{r^2}{(2\pi)^4} F(p), \quad (4.20)
\]

where

\[
F(p) = \int d^3 p_1 \delta (M_e - p - p_1 - |p + p_1|) \quad (4.21)
\]

This integral can be evaluated with the following method:

Let

\[
\Psi(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 
\end{cases}
\]

then

\[
\frac{d\Psi}{dx}(x) = \frac{d\Omega}{dx}
\]

So that

\[
\frac{dG(p)}{dM_e} = F(p)
\]

with

\[
G(p) = \int d^3 p_1 \Psi (M_e - p - p_1 - |p + p_1|)
\]
This integral has a geometrical interpretation which facilitates its evaluation. The domain in the $p_1$-space for which

$$M_0 - p - p_1 - |p + p_1| > 0$$

represents an ellipsoid of revolution with semi-major axes $a$ and $b$

$$a = \frac{1}{2}(M_0 - p)$$
$$b = \sqrt{\frac{M_0^2}{4} - \frac{M_0 p}{2}}$$

The integral $G(p)$ is therefore the volume of this ellipsoid and is given by

$$G(p) = \frac{4}{3} \mu \frac{ab^2}{6} = \frac{\mu}{6} \left( M_0^3 - 3M_0^2p + 2M_0p^2 \right)$$

Therefore

$$F(p) = \frac{\mu}{6} \left( 3M_0^2 - 6M_0p + 2p^2 \right)$$
$$= \frac{\mu}{6} M_0^2 (3 - 6x + 2x^2) \quad (x = \frac{p}{M_0})$$

We find therefore for

$$d\Gamma \sim \frac{\mu^2}{48 \pi^3} \frac{M_0^5}{x^2} (3 - 6x + 2x^2) \, dx \quad (4.22)$$

$$\left( x = \frac{p}{M} \right)$$
For the total probability of decay we integrate \( d \Gamma \) from \( x=0 \) to the maximum momentum value \( (x = \frac{1}{3}) \). Since

\[
\int_{0}^{\frac{1}{3}} x^2 (3 - 6x + 2x^2) \, dx = \frac{7}{160}
\]

this becomes

\[
\Gamma \sim \frac{7}{160} f^2 M_0^5
\]  \hspace{1cm} (4.23)

If we substitute numbers we obtain for the dimensionless coupling constant

\[
(f m_n^2/H)^2 \sim 10^{-14}
\]  \hspace{1cm} (4.24)

(iii) Muon capture in nuclei

We consider the capture of a bound Muon by a nucleon inside a nucleus

\[
\mu + N \rightarrow N' + \nu
\]

Since the initial state is a bound state the pair \( \mu + N \) is treated like one particle. Hence the appropriate formula is still GR1.
We write for the capture probability of one nucleon

\[ \Gamma_o = \frac{1}{2\pi} \frac{S_f}{S_i} \int (P_f - P_i) \left| \frac{M}{\hbar} \right|^2 \]  

Now

\[ P_i = (M, 0, 0, 0) \]

\[ M_o = M^* + M_N - \text{Binding energy} \]

\[ p \sim M^* = \text{momentum of neutrino} \]

The only new feature is in the matrix element, since one plane wave is now replaced by a bound state wave function we must omit a factor \( \frac{1}{(2\pi)^{3/2}} \) and replace it by \( \frac{1}{\sqrt{\pi a^3}} \), the normalization factor of a bound state. Thus

\[ \left| \frac{M}{\hbar} \right|^2 \sim \frac{\hbar^2}{2\pi} \frac{1}{\pi a^3} \]

Consequently

\[ \Gamma_o \sim \frac{1}{2\pi} 4\pi \frac{\hbar^2}{2\pi} \frac{1}{\pi a^3} \sim \frac{\hbar^2}{\pi} \left( \frac{1}{137} \right)^3 \frac{M^5}{\hbar} \quad (4.25) \]

For the capture probability in nuclei one has approximately

\[ \Gamma = z^4 \Gamma_o \quad (4.26) \]

Putting numerical values, one finds again

\[ \left( \frac{\hbar}{m} \right)^2 \sim 10^{-13} \quad (4.27) \]

Conclusion: The three processes discussed in the preceding sections although occurring with widely different time constant have roughly the same interaction strength. It is expressed by a dimensionless
(e) Interaction of type $\phi^3$. A typical case would be $K^+ \rightarrow \pi^0 + \pi^-$. We use $G_{1}$ to obtain the decay rate in the form

$$
\Gamma = \frac{1}{2\pi} \int k_1^2 dk_1 \delta(M_0 - \omega_1 - \omega_2) |M|^2.
$$

$$\omega_1 = \sqrt{k_1^2 + M_1^2}, \quad \omega_2 = \sqrt{k_2^2 + M_2^2}.
$$

For the special case of a decay into two pions we have

$$M_1 = M_2 = m_{\pi}, \quad \omega_1 = \omega_2 = \omega.
$$

$$k_1 = k_2 = k.
$$

$$M_0 \simeq 2 \sqrt{k^2 + M_1^2} = 2 \omega.
$$

or

$$k^2 = \frac{M_0^2}{4} - m_{\pi}^2.
$$

The matrix element is

$$M \simeq \frac{f}{\sqrt{2\pi}} \frac{1}{\sqrt{2M_0}} \frac{1}{\sqrt{2\omega_1}} \frac{1}{\sqrt{2\omega_2}}.
$$

$$\Gamma = \frac{4\pi}{2\pi} \frac{\omega_1 \omega_2}{\omega_1 + \omega_2} \frac{f^2}{16\pi} \frac{1}{M_0 \omega^2}.
$$

$$\Gamma = \frac{f^2}{8\pi} \frac{p}{M_0^2}.
$$

(4.28)
From the known decay rate of this process we find

\[
\left( \frac{\tau}{m_{\mu}} \right)^2 \simeq 5 \times 10^{-14} \tag{4.29}
\]

(f) Interaction of the type $\phi^2 \psi \bar{\psi}$. A typical interaction of this kind is the production of hyperons by collision of pions and nucleons (associated production)

$$\mu + N \rightarrow Y + K$$

The initial and final mass values are denoted by

$$M_0, M_1; \ M_2, M_3.$$  

We calculate the cross-section with $G_{R_2}$ in the C.M.-system. The current operator in the C.M.-system would be

$$I = q \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon} \right) \quad \omega_1 = \sqrt{M_1^2 + q^2} \quad \epsilon = \sqrt{M_0^2 + q^2}$$

where $q$ is the momentum of one of the colliding particles. For the total cross-section we obtain then

$$\sigma' \sim (2 \pi)^2 \frac{4 \pi}{\mu} \int_0^\infty \frac{p^2 dp}{\epsilon^2} \left( \epsilon_0 + \omega_1 - \epsilon - \omega_2 \right) \left| M \right|^2 \frac{1}{q \left( \frac{1}{\omega} + \frac{1}{\epsilon_1} \right)}$$

$$M \sim \frac{\epsilon}{(2 \pi)^2} \frac{1}{\sqrt{\mu}} \frac{1}{\sqrt{\epsilon_1}}$$
Collecting terms we find

$$\sigma \sim \frac{n}{4} \varepsilon^2 \left( \frac{p}{q} \right) \left( \frac{E_0 E}{E^2} \right)$$

where \( E = E_0 + \omega_1 \) is the total energy of the collision.

For moderately high energies the factor \( \frac{p}{q} \frac{E_0 E}{E^2} \) is of order one and we have then

$$\sigma \sim \frac{n}{4} \varepsilon^2 = \frac{n}{4} \left( g \frac{m_\pi}{m_\pi} \right)^2 \frac{1}{(m_\pi)^2}$$

Defining a standard cross-section as

$$\sigma_o = \frac{1}{(m_\pi)^2} \sim 2 \times 10^{-26} \text{ cm}^2,$$

we find

$$\sigma \sim \sigma_o \frac{n}{4} \left( g \frac{m_\pi}{m_\pi} \right)^2$$

The observed cross-sections for this process are of order \( 10^{-26} \text{cm}^2 \). Hence

$$\left( g \frac{m_\pi}{m_\pi} \right)^2 \sim 1$$

We conclude that the interaction for this process is strong.
I. INTRODUCTION.

(5) Fundamentals on heavy mesons and hyperons. C. Peyrou

We will give the essential facts about the newer particles listed in the table and already classified by Prof. Jauch (Introduction page 1).

We will first describe briefly how the different particles were identified and then show that

1) all the K-mesons are different decay modes of the same particle,
2) the K's are produced either in pairs or associated with a hyperon,
3) how the Gell-Mann scheme is justified.

Phenomenological description of these particles.

The $K^+_\pi^+ (\Xi^+) \rightarrow \pi^+ + \pi^+ + \pi^-$ was first identified in emulsion (Bristol), and it is, indeed, the event best to be observed with this technique, because of the low energy of the pions which can easily be recognized by the ionization and the subsequent decay or absorption at the end of their short range.

(1) This talk was a complement to the theoretical lectures. It just restates some very elementary and basic facts of the discovery of strange particles. The shortness of this paper is responsible for gross oversimplifications.

(2) Notes collected by N. Fidecaro.
A second mode of the $K_{\pi^3}^+$ is the so-called $\Upsilon'$:

$$K_{\pi^3}^+ \rightarrow \pi^0 + \pi^0 + \pi^+$$ (also identified in emulsion) which was postulated from the existence of many K-mesons decaying in a $\pi^+$, whose spectrum falls on the spectrum of the pions from the $\Upsilon$'s. The $\pi^0$'s were not identified.

The $K_{\pi^2}^+$ was first identified in emulsions (Bristol) as a K particle giving a monoenergetic $\pi$. The $\pi$ being of relatively high energy this identification demanded very good measurements. The $\pi^0$ was inferred from momentum and energy balance. It has been directly identified in a multpile chamber (M.I.T.), through the associated gammas.

The $K_{\mu^3}^+$ ($= K$) $\rightarrow \mu^+ + \pi^0 + \nu$ was observed in nuclear emulsion (Bristol) as a K emitting $\mu$'s of different energies. The $\pi^0$ was only later identified through a Dalitz pair.

The $K_{\mu^2}^+ \rightarrow \mu^+ + \nu$ was first observed in a double cloud chamber (Ecole Polytechnique). (The high momentum of the $\mu$ from $K_{\mu^2}$ makes difficult the mass measurement in nuclear emulsion). K particles were observed stopping in the lower chamber (multiplate) after having traversed the upper one, imbedded in a magnetic field. The combined knowledge of the momentum and of the residual range allowed a rather good mass measurement of the incident particle. The existence of a two body decay was shown by a group of secondaries with unidic range. This range was so large that the secondary could not be a pion. The absence of soft radiation and the dynamics left only the $\nu$ as a second decay product.

The $K_{e^3}^+ \rightarrow e^+ + \pi^0 + \nu$ was also first observed in nuclear emulsion (Bristol). The identification of the electron was done by measuring the variation of energy along the tracks. Large losses of energy (due to Bremsstrahlung) are characteristics of an electron. From the tail in the spectrum of the electrons which is a little too long, one could argue the existence of a $K_{e^2}^+ \rightarrow e^+ + \nu$ but such a spectrum is still very poor.

The negative K-mesons interact strongly with the matter and when brought to rest are usually captured by a nucleus giving rise to stars. Therefore their decays can only be observed in flight.
The Neutral kaons were identified in cloud chambers. Nuclear emulsions being made of dense material they are more difficult to scan for the decay of a neutral particle among the large background of stars.

The $K^0_{\pi 2} (= \theta^0) \rightarrow \pi^+ + \pi^-$ was recognized (Thompson) from the existence of a characteristic group of V events (they were V-shaped) having all the same Q-value. The vicinity of the Q values $Q = 214$ MeV excludes the existence of a third neutral-body.

Concerning the $K^0_{\pi 2}$ number 2, one should refer to the Lederman lectures.

The $\Lambda^0 \rightarrow P + \pi$ was again found as a V event in cloud chamber. The proton was identified by momentum and ionisation measurement. The unicity of the Q value $= 37$ MeV and the coplanarity with the origin of the particle exclude three body decay.

The $\Sigma^+$ going in $\pi^0 + P$, was first observed in nuclear emulsion (Milano) the mass was measured as a hyperprotonic one. The nature of the neutral particle was guessed from the dynamics. The $\Sigma^+$ decays also in $\pi^+ + N$.

The $\Sigma^- \rightarrow \pi^- + N$ was first observed in the Brookhaven diffusion chamber exposed to the 1.5 GeV pions. The existence of such a particle, was confirmed by the observation in nuclear emulsion of a hyperon, decaying in flight into a pion which stopped in the emulsion and interacted with a nucleus. It is obvious that the $\Sigma^-$ cannot decay in $\pi^0 + \bar{P}$ because it is a particle and not an antiparticle (conservation of baryonic number).
The $\Sigma^0 \rightarrow \Lambda^0 + \gamma$ was predicted by the Gell-Mann scheme and thereafter found with good evidence in a propane bubble chamber (Columbia) (A reaction $\pi^- + P \rightarrow \phi^0 + \Lambda^0$ did not fit the dynamic and the $\gamma$ materialized in a pair).

The $\Xi^- \rightarrow \pi^- + \Lambda^0$ (which is called also cascade decay) was first observed in a cloud chamber by the Manchester group and confirmed by several other cloud chambers groups.

The $\Xi^0$ has been predicted but, at present, it has not yet been found (it has been found recently in the Berkeley hydrogen bubble chamber).

It is worthwhile to note that all the "zoology" of the strange particles which was the object of a long controversy during 4 years, has been settled, in 3 months or so, with the G-strack experiment, when big stacks of nuclear emulsion were flown at high altitude, and a very accurate study of the secondaries was made possible.

Just at the same time, started the systematic work with accelerators which definitely proved that all the kaons are different decay modes of the same particle; they have indeed the same mass, lifetime, and very probably the same spin and parity.

The equality of the masses has been established in two different ways:

a) from measurements of the $Q$-values, that is, of the energy released in K-decays.
b) from the magnetic rigidity of individual particles and their individual range. The apparatus used in Berkeley is sketched in the figure; all the kaons were found to have the same range as the \( \pi^- \)'s which entered the emulsion with the same momentum. The \( \pi^- \)-mass being well known from the Q-values, the mass of the positive kaons was measured in this way with an accuracy of \( 2 \, \text{m}_e \).

![Diagram of apparatus]

The measurement of the mass of negative kaons was more complicated because the momentum has to be deduced from the residual range of positive kaons channelled and focused in the same way, by reversing all the magnetic fields (The identity of conditions was checked by range measurements of \( \pi^+ \) and \( \pi^- \)). The ratio of the mass of the \( K^- \) to that of \( K^+ \) is: \( 0.998 \pm 0.013 \) as one would expect, the \( K^- \) being the antiparticle of the \( K^+ \).
For the mean life, we have absolute and relative measurements, made with counters and with emulsions. The absolute measurements are precise to 5 o/o. In such an experiment the K\(^+\) were brought to rest in a scintillator surrounded by a system of counters which could distinguish between the secondaries from K\(\pi^2\), K\(\mu^2\), K\(\pi^3\) on the basis of their range and their interaction (for the gammas related with the K\(\pi^2\)).

Higher precision can be attained in relative measurements, determining the branching ratio for the different decay modes in emulsions put at different distances from the target. In such a way, the equality of the life times has been established for values between 3.10\(^{-8}\) and 10\(^{-12}\) sec.

The kaons show also an identical behaviour in interacting with nuclei, as has been shown by the M.I.T. emulsion group, who determined the branching ratio of the various modes after the K particles had been scattered at large angles in a piece of copper.

In such a situation one would have expected that all the kaons have the same spin and parity, but there was the so-called K\(\pi^2\) - K\(\pi^3\) (\(\theta - \tau\)) riddle.

In the K\(\pi^2\) decay, the final system has even (odd) parity for even (odd) values of the angular momentum. This, in turn, is equal to the spin of the \(\kappa\)-meson. So, if the parity is conserved, the K-meson should have :

either parity + and spin 0, 2, 4 ... \hspace{1cm} (1)
or parity - and spin 1, 3, 5 ... \hspace{1cm} (2)
For $K^+_{n3}$, the two equal pions in the final system must be described by a symmetric wave function, therefore $p_1 = 0, 2 \ldots$. If the spin of the $K$-meson is zero, the third pion should have an even value for $p$, and therefore the total parity of the system is odd (the pions are pseudoscalar particles) which is in contradiction with (1). If the spin is not zero, one can always find a final situation which is identical to that of the $K^+_{n2}$ from the point of view of the spin and parity, provided that one allows the pions to be emitted in a state of high angular momentum ($\geq 2$) which is incompatible with the pion spectrum. This spectrum indicates strongly that the spin is zero.

The problem was solved by the discovery that parity is not conserved in weak interactions, which means that the total parity of the final system can be different in two different decay modes of the same particle.

The other basic fact is the associated production scheme. The reason for assuming this was that the new particles have a large production cross-section and at the same time a long mean life, which seemed a contradiction, because strong interacting particle such as pions were at the same time among the producing particles and the decay products. The associated production theory assumes that it is possible to produce many new particles provided two of them are produced at the same time.
This follows from the section rules for a new quantum number, the "strangeness". See Prof. Jauch lectures, page 12. Cosmic ray experiments did show some cases of associated production. But the bias of observation in particular the difficulty of detecting \( K^+ \) decays made it difficult to prove that associated production was the only way of producing strange particles.

On the contrary, clear evidence was obtained by shooting a negative pion beam in a hydrogen diffusion chamber.

Up to the present only cases of associated production of a \( K^0 \) with a \( \Lambda^0 \) and of \( K^+ \) with a \( \Sigma^- \), have been directly observed, but the theory predicts also the production of pairs of \( K^- \) and \( K^+ \).

Other facts which are explained in the framework of the same scheme are:

a) the positive excess. Because the hyperons made out of the nucleons of our matter do not have positive strangeness, it comes out that the number of \( K^+ \) is larger than the number of \( K^- \). This was shown either indirectly in emulsions (few stars from \( K^- \) mesons) or, directly, by the double cloud chamber (10 over 1). The magnitude of the positive excess depends obviously on the relative probability of having associated production of a hyperon with a \( K^- \) meson, and of a \( K^+ \) with a \( K^- \); therefore it is larger in the Berkeley experiments (the energy is close to the threshold for production of \( K^- \) pairs) than in cosmic rays.
b) the existence of the $\Sigma^0$.

c) the $K^+$ cannot be absorbed from the nuclear matter, because it cannot give either a hyperon (strangeness conservation) or a antihyperon (conservation of the barionic number) etc. It can only be rejected. The $K^-$ is strongly absorbed giving a star in which a $\Sigma$ or a $\Lambda$ is produced, and therefore the energy release is low. This emission of a hyperon has been observed in photographic emulsion and in a hydrogen bubble chamber.
II. STRONG INTERACTIONS.

(1) Isospin wave functions. 

Y. Yamaguchi
(1) Isospin wave functions.

(a) Isospin wave functions for particles and antiparticles.
In the following we shall denote the isospin wave functions by the particle symbols (see Table I).

The isospin wave functions for nucleons and pions have already been discussed in page 5 and 8.

The isospin wave functions for $\bar{\Sigma}$'s, $K$'s and $\bar{K}$'s can be written in the same way as those for nucleons.

The $\Sigma$'s are represented in the same way as the pions.

The $\Lambda^0$ is described by a single wave function which is invariant in the isospin space.

The isospin wave functions for antiparticles are related to those for particles:

\[
\begin{align*}
\bar{p} & \sim -n \\
\bar{n} & \sim p \\
K^0 & \sim -K^- \\
K^+ & \sim \bar{K}^0
\end{align*}
\] (1.1) (1.2)

($\sim$ means the same transformation property in the isospin space)
This can be justified by observing that \( \frac{\bar{\eta} n + pp}{\sqrt{2}} \) is a scalar in the isospin-space. On the other side we know that such a scalar can be written \( \frac{\bar{\eta} n - pp}{\sqrt{2}} \) from which it follows (1.1).
(b) Isospin wave functions for two particles systems.

The addition of two or more isospins follows the same rules as in the case of ordinary angular momenta. The eigenfunction $| I \ I_3 \rangle$ of $I = I^{(1)} + I^{(2)}$, with $I_3$ as third component of $I$, can be written as a product of the eigenfunctions $\psi(I^{(1)}_3 I^{(2)}_3)$, $\psi(I^{(2)}_3 I^{(2)}_3)$ of $I^{(1)}$ and $I^{(2)}$:

$$| I \ I_3 \rangle = \sum_{\ell_3} \left( I^{(1)}_3 I^{(2)}_3 \right) \psi(I^{(1)}_3 I^{(1)}_3) \psi(I^{(2)}_3 I^{(2)}_3) \left| I^{(1)}_3 \right\rangle \left| I^{(2)}_3 \right\rangle$$

$(I^{(1)}_3 I^{(2)}_3 \left| I^{(1)}_3 I^{(2)}_3 \right) \psi(I^{(1)}_3 I^{(1)}_3) \psi(I^{(2)}_3 I^{(2)}_3)$ are the so-called Clebsch-Gordon coefficients which are tabulated in: Condon and Shortley, The Theory of Atomic Spectra, pages 76-77. Some results are shown in Table II.

We can also derive the above expressions in the following way, for instance, for a K-N system. The isospin for such a system can have the eigenvalues:

$$I^K + I^N = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
to which correspond for $I_3 = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix}$.

Only the system $K^+p$ has $I=1$, $I_3=1$. Therefore

$$\left| 1, +1 \right\rangle = (K^+p) \quad (1.3)$$
Reminding that the operator \( I_+ = \tau^K_+ + \tau^N_+ \) (see page 6) has the properties

\[
I_+ | I I_3 \rangle = \text{const} \left| I I_3^{+1} \right\rangle \quad \text{for} \quad |I I_3^{+1}| \leq I
\]

\[
= 0 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{for} \quad |I I_3^{+1}| \geq I
\]

we get

\[
I_- | 1, +1 \rangle = \text{const} \left( (K^0 p_+^+)(K^+ n_0) \right) = | 1, 0 \rangle \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (1.4)
\]

\[
I_- | 1, 0 \rangle = \text{const} (K^0 n_0) = | 1, -1 \rangle \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (1.5)
\]

The normalization requires that

\[
\text{const} = \frac{1}{\sqrt{2}} \quad \quad \text{in} \quad (1.4)
\]

\[
= 1 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{in} \quad (1.5)
\]

Because \( | 0, 0 \rangle \) has to be orthogonal to \( | 1, 0 \rangle \), one gets

\[
| 0, 0 \rangle = \frac{(K^0 p_+^+)(K^+ n_0)}{\sqrt{2}}
\]

The isospin eigenfunctions for the systems: \( N-N, \quad \Xi^+ - N, \quad \Xi^- - N \), are the same as those for the \( K-N \) system.

For the \( \Xi^- - N \) system we can observe that, as far as the isospin is concerned

\[
\Xi^+ \quad \text{behaves like} \quad (K^+ p')
\]

\[
\Xi^0 \quad " \quad " \quad \frac{(K^0 p_+^+)(K^+ n_0')}{\sqrt{2}} \quad I=1 \quad I_3=0
\]

\[
\Xi^- \quad " \quad " \quad (K^0 n')
\]
The total isospin for $\Sigma^- N$ is $I = I_1^{1+\frac{3}{2}}$ with $I_3 = I_3^{\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}}$

By operating on $(\Sigma^+ p) = \left| \frac{3}{2}, \frac{3}{2} \right>$ with $I_- = \tau^K_-, \tau^N_-, \tau^N_-$, we get

$$I_- \left| \frac{3}{2}, \frac{3}{2} \right> \rightarrow \text{const } (K^0 p + K^+ n')_p + (K^+ p')_n$$

$$\rightarrow \text{const } \left[ \sqrt{2}(\Sigma^0 p) + (\Sigma^+ n) \right] = \left| \frac{3}{2}, \frac{1}{2} \right>$$

where the const $= \frac{1}{\sqrt{3}}$ is obtained by normalization.

All the other states can be formed in the same way.
II. **Strong interactions.**

(1) Isospin wave functions (by Y. Yamaguchi)

---

**TABLES I, II**
TABLE I

Charge wave functions (elementary particles)

$I = 0$

<table>
<thead>
<tr>
<th>$I_3$</th>
<th>0</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Λ</td>
</tr>
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</table>

$I = \frac{1}{2}$

<table>
<thead>
<tr>
<th>$I_3$</th>
<th>$N$</th>
<th>$\bar{N}$</th>
<th>$K$</th>
<th>$\bar{K}$</th>
<th>$\Xi$</th>
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</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$p$</td>
<td>$\bar{n}$</td>
<td>$K^+$</td>
<td>$\bar{K}^0$</td>
<td>$\Xi^0$</td>
</tr>
<tr>
<td>$-\frac{1}{2}$</td>
<td>$n$</td>
<td>$\bar{p}$</td>
<td>$K^0$</td>
<td>$-K^+ = -\bar{K}^0$</td>
<td>$\Xi^-$</td>
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$I = 1$

<table>
<thead>
<tr>
<th>$I_3$</th>
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<th>$\Sigma$</th>
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<tr>
<td>1</td>
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<tr>
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<td>$\Sigma^0$</td>
</tr>
<tr>
<td>-1</td>
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<td>$\Sigma^-$</td>
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TABLE II

Charge wave functions
(Two body system)

\[ N+N \]

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<th>I₃</th>
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<tr>
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<td>( p₁p₂ )</td>
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<tr>
<td>0</td>
<td>( \frac{p₁n₂ + n₁p₂}{\sqrt{2}} )</td>
<td>( \frac{p₁n₂ - n₁p₂}{\sqrt{2}} )</td>
</tr>
<tr>
<td>-1</td>
<td>( n₁n₂ )</td>
<td></td>
</tr>
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</table>

\[ \pi+N \]

<table>
<thead>
<tr>
<th>I₃</th>
<th>I</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/2</td>
<td>( p \pi^+ )</td>
<td></td>
</tr>
<tr>
<td>1/2</td>
<td>( \frac{n \pi^+ + \sqrt{2}p \pi^0}{\sqrt{3}} )</td>
<td>( \frac{\sqrt{2}n \pi^+ - p \pi^0}{\sqrt{3}} )</td>
</tr>
<tr>
<td>-1/2</td>
<td>( \frac{\sqrt{2}n \pi^0 + p \pi^-}{\sqrt{3}} )</td>
<td>( \frac{n \pi^0 - \sqrt{2}p \pi^-}{\sqrt{3}} )</td>
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### N + Λ

<table>
<thead>
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### N + Σ

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<td>$p\Sigma^+$</td>
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<td>$1/2$</td>
<td>$\frac{n\Sigma^+ + \sqrt{2}p\Sigma^0}{\sqrt{3}}$</td>
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<tr>
<td>$-1/2$</td>
<td>$\frac{\sqrt{2}n\Sigma^0 + p\Sigma^-}{\sqrt{3}}$</td>
<td>$\frac{n\Sigma^0 - \sqrt{2}p\Sigma^-}{\sqrt{3}}$</td>
<td></td>
</tr>
<tr>
<td>$-3/2$</td>
<td></td>
<td>$n\Sigma^-$</td>
<td></td>
</tr>
</tbody>
</table>
$$K+N$$

<table>
<thead>
<tr>
<th>$I_3$</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$K^+ p$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$\frac{K^+ n + K^0 p}{\sqrt{2}}$, $\frac{K^+ n - K^0 p}{\sqrt{2}}$</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>$K^0 n$</td>
<td></td>
</tr>
</tbody>
</table>

$$\pi + \pi$$

<table>
<thead>
<tr>
<th>$I_3$</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\pi_1^+ \pi_2^+$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$\frac{\pi_1^0 \pi_2^+ + \pi_1^+ \pi_2^0}{\sqrt{2}}$, $\frac{-\pi_1^0 \pi_2^+ + \pi_1^+ \pi_2^0}{\sqrt{2}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$\frac{\pi_1^- \pi_2^{+2} + \pi_1^0 \pi_2^0 + \pi_1^+ \pi_2^-}{\sqrt{6}}$, $\frac{-\pi_1^- \pi_2^{+2} + \pi_1^0 \pi_2^0 - \pi_1^+ \pi_2^-}{\sqrt{6}}$, $\frac{\pi_1^- \pi_2^- + \pi_1^0 \pi_2^0 + \pi_1^+ \pi_2^-}{\sqrt{2}}$</td>
<td></td>
<td>$\sqrt{3}$</td>
</tr>
<tr>
<td>-1</td>
<td>$\frac{\pi_1^- \pi_2^0 + \pi_1^0 \pi_2^-}{\sqrt{2}}$, $\frac{-\pi_1^- \pi_2^0 + \pi_1^0 \pi_2^-}{\sqrt{2}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>$\pi_1^- \pi_2^-$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>$3/2$</td>
<td>$1/2$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>$I_3$</td>
<td>$k^+ \Sigma^+$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3/2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1/2$</td>
<td>$\frac{k^0 \Sigma^+ + \sqrt{2} k^+ \Sigma^0}{\sqrt{3}}$</td>
<td>$\frac{\sqrt{3} k^0 \Sigma^+ + \sqrt{2} k^+ \Sigma^0}{\sqrt{3}}$</td>
<td>$k^+ \Lambda$</td>
</tr>
<tr>
<td>$-1/2$</td>
<td>$\frac{\sqrt{2} k^0 \Sigma^+ + k^+ \Sigma^-}{\sqrt{3}}$</td>
<td>$k^0 \Sigma^+ - \sqrt{2} k^+ \Sigma^-$</td>
<td>$k^0 \Lambda$</td>
</tr>
<tr>
<td>$-3/2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
II. STRONG INTERACTIONS.

(2) Experimental tests of charge independence
    (especially for strange particles).  
    Y. Yamaguchi
(2) Experimental tests of charge independence (esp. for strange particles).

(a) Introduction. The charge independence (C.I.) of the nuclear forces has been first postulated from the striking equality of the interactions p-p and p-n, in the $^1S$ state. Also the equality with the n-n interaction has been established, but in a less precise way.

Later on, the appearance of the isospin multiplets in the levels of the isobars gave a strong support to this idea. For instance, the following level scheme has been found:

\[ \begin{array}{c}
\text{B}^{12} \quad \text{C}^{12} \quad \text{N}^{12} \\
1^+ \quad \text{Isotriplet} \quad I=1
\end{array} \]

\[ \frac{\text{C}^{12}}{\text{C}^{12}} \quad 0^+ \quad \text{Isosinglet} \quad I=0 \quad (\text{ground state}) \]

In the pion physics, the direct tests (see later) are not very accurate, but the analysis based on C.I. have been extremely successful.

As it has been shown, also the strange particles can be grouped in isomultiplets. As a test, we can report the:

<table>
<thead>
<tr>
<th>Fractional mass difference $\Delta m/m$ (mass difference divided by the average mass)</th>
<th>Referred to</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.137 \times 10^{-2}$</td>
<td>n-p</td>
</tr>
<tr>
<td>$0.59 \times 10^{-2}$</td>
<td>$\Sigma^- - \Sigma^0$</td>
</tr>
<tr>
<td>$3.3 \times 10^{-2}$</td>
<td>$\Pi^+ - \Pi^0$</td>
</tr>
<tr>
<td>$0.3 \times 10^{-2}$</td>
<td>$K^+ - K^0$</td>
</tr>
</tbody>
</table>
The values of \( \Delta m/m \) are of the order of \( e^2/4\pi \approx 1/137 \); therefore it is possible that the differences among the terms of a multiplet, have only an electromagnetic origin. *)

All the facts mentioned above can be expressed by saying that when the electromagnetic interactions are neglected:

- a physical situation does not depend on a choice of coordinates in the isospin space;
- or the total isospin is conserved;
- or the interaction has to be a scalar in the isospin space.

**Remark:**

While charge independence requires that all physical processes should be invariant under any rotation in the isospin space, charge symmetry (C.S.) requires only the invariance under a rotation of 180° around the 1st (or 2nd) axis in the same space. That is, we can make the interchange \( p \leftrightarrow n, \pi^0 \leftrightarrow \pi^0, \pi^\pm \leftrightarrow \pi^\mp \), but not \( \pi^0 \leftrightarrow \pi^\mp \).

---

(b) Some consequences of C.I. Let us consider first the K-N scattering and call \( R \) the operator which relates the final state \( f \) with the initial one \( i \). C.I. states that if the initial (and the final) states differ only in the isospin quantum numbers:

\[
\frac{1}{\sqrt{2}} \left| R \right| \frac{K^0 p + K^+ n}{\sqrt{2}} = \left| R \right| \frac{K^0 p + K^+ n}{\sqrt{2}} = \left| R \right| \frac{K^0 n}{\sqrt{2}} = R_T
\]

\[
\frac{1}{\sqrt{2}} \left| R \right| \frac{K^0 p - K^+ n}{\sqrt{2}} = R_S
\]

*) In a naive way, if pions are considered as uniformly charged spheres of radius \( a \), the energy difference between \( \pi^\pm \) and \( \pi^0 \) should be \( \sim 3/5 e^2/a \), which gives a \( \sim 2 \times 10^{-14} \) cm
All other matrix elements being zero, e.g., \( \left\langle \frac{K^+ n - K^0 p}{\sqrt{2}} | R | \frac{K^0 n + K^+ p}{\sqrt{2}} \right\rangle = 0 \)

<table>
<thead>
<tr>
<th>That is, the processes allowed by charge conservation:</th>
<th>are characterised, under, C.I. hypothesis, by the matrix elements (complex numbers):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^+ p \rightarrow K^+ p )</td>
<td>( R_T )</td>
</tr>
<tr>
<td>( K^+ n \rightarrow K^+ n )</td>
<td>( \frac{(R_T + R_S)}{2} )</td>
</tr>
<tr>
<td>( K^+ n \rightarrow K^0 + p )</td>
<td>( \frac{(R_T - R_S)}{2} )</td>
</tr>
<tr>
<td>( K^0 + n \rightarrow K^0 + n )</td>
<td>( R_T )</td>
</tr>
<tr>
<td>( K^0 + p \rightarrow K^0 + p )</td>
<td>( \frac{(R_T + R_S)}{2} )</td>
</tr>
<tr>
<td>( K^0 + p \rightarrow K^+ + n )</td>
<td>( \frac{(R_T - R_S)}{2} )</td>
</tr>
</tbody>
</table>

In order to test C.I. one has to check the identities

\[ a = d \quad (2.1) \]
\[ b = e \quad (2.2) \]
\[ c = f \quad (2.3) \]
\[ b+c = a \quad (2.4) \]
\[ e+f = d \quad (2.5) \]

but (2.1), (2.2), (2.3) hold also under the hypothesis of C.S. alone. A test sensitive to C.I. is the check of (2.4) and/or (2.5).

Remark:

The \( K-N \) scattering can be analyzed in terms of \( 6 \times 2 - 1 = 11 \) parameters if only the isospin states are concerned - This number is reduced to 5 by C.S. and to 3 by C.I.
The relationships among the matrix elements magnitudes, which can be derived from a to f, are more useful. What we measure are, indeed, the differential cross-sections for the different processes: \( \frac{d\sigma}{d\Omega} \sim \left| \langle f \mid R \mid i \rangle \right|^2. \)

These relationships can be visualized by considering the matrix elements as vectors in the complex plane

\[ \vec{a} = R_T, \quad \vec{b} = \frac{R_T + R_S}{2}, \quad \vec{c} = \frac{R_T - R_S}{2} \]

If C.I. holds, these vectors form a triangle

therefore

\[
\begin{align*}
|a| + |b| & \geq |c| \\
|b| + |c| & \geq |a| \\
|c| + |a| & \geq |b|
\end{align*}
\]

which in the following will be written briefly

\( \triangle (a,b,c) \geq 0 \)

As an example let us consider the \( K^+ \) scattering on hydrogen and call

\[
\begin{align*}
\sigma_p &= \sigma(K^+ p \rightarrow K^+ p) \\
\varepsilon \sigma n &= \sigma(K^+ n \rightarrow K^+ n) \\
(1-\varepsilon) \sigma n &= \sigma(K^+ n \rightarrow K^0 p)
\end{align*}
\]
\( \sigma_p \sim 14\, \text{mbarns} \) from hydrogen target data

\( \sigma_n \sim 9\, \text{mbarns} \) from emulsion data (rather inaccurate)

\((1-\xi)\sigma_n\) is not known. From the triangle relationship:

\[
\frac{\sqrt{\xi} + \sqrt{1-\xi}}{\sqrt{2}} \geq \sqrt{\frac{\sigma_p}{\sigma_n}} \geq \frac{\sqrt{\xi} - \sqrt{1-\xi}}{\sqrt{2}}
\]

we get

\[
\frac{r + \sqrt{2-r^2}}{2} \geq \sqrt{\xi} \geq \frac{r - \sqrt{2-r^2}}{2}
\]

that is

\(0.96 \geq \xi \geq 0.3\)

and the charge exchange cross-section can be evaluated. On the other hand, if it is known, C.I. can be tested.
(o) Tests of Charge Independence. The $R$ matrices for several reactions are tabulated in table III. In the calculation of these $R$ matrices we have restricted ourselves to charge independence, i.e. to the influence of the charge wave-functions. For examples of charge independence applications other than those discussed in these notes, the reader is referred to D. Feldman, Phys.Rev. 102, 254 (1956), and T.D. Lee, Phys.Rev. 92, 337 (1955).

a) $\sigma \rightarrow N$ scattering ($N =$ nucleon, $n =$ neutron)

This subject is too well known to be treated in this lecture. See for example: Bethe - de Hoffmann, Mesons and Fields, Vol. 2.

b) $N+N \rightarrow N+N+\sigma$

Initial state : $I = 0, 1$
Final state of the two nucleons : $I_N = 0, 1$
Final state of the nucleons and pion : $I = 0, 1, 2$
The four possible cases with the corresponding $R$ matrices are listed in table III, page 2.

The case in which the two final nucleons appear as a deuteron ($I_N = 0$) has special interest. The two possible reactions for this case are

(i) $p + p \rightarrow d + \gamma^+$
$I = 1$  $I = 1$

(ii) $n + p \rightarrow d + \gamma^0$
$I = 0, 1$  $I = 1$

*) Notes collected by P. Hillman and W. Middelkoop.
Conservation of isospin forbids the reaction (ii) half the time \((I = 0)\), giving the ratio

\[
\frac{\text{Int (i)}}{\text{Int (ii)}} = \frac{\left|R_{10}(d)\right|^2}{\frac{1}{2}\left|R_{10}(d)\right|^2} = 2.
\]

From our table we can also construct the equality

\[
\sigma(pn \rightarrow \pi^0) = \frac{1}{3} \sigma(pp \rightarrow \pi^+) + \sigma(pn \rightarrow \pi^+) - \sigma(pp \rightarrow \pi^0)
\]

If the neutral meson production due to p-n collisions were larger than the right hand side, then there would be the following possibilities:

1. Violation of charge independence,
2. Existence of a second neutral pion \(\pi^0_o\) with \(I=0\)
3. Wrong approximations made in the derivation of \(\sigma(pn \rightarrow \pi^+)\) from \(\sigma(pd \rightarrow \pi^+)\)

If a \(\pi^0_o\) exists, one has to expect, in order not to disturb the excellent fits in pion physics, that its mass is much larger than the \(\pi^0\) mass. So one can try to distinguish the two by kinematics.

However, a cleaner proof of the existence of a \(\pi^0_o\) could be obtained from the following reactions:

\[
d + d \rightarrow \text{He}^4 + \pi^0
\]

\(I = 0\) \(0\) \(0\) \(1\) Forbidden by conservation of isospin

\[
d + d \rightarrow \text{He}^4 + \pi^o_o
\]

\(I = 0\) \(0\) \(0\) \(0\) Allowed

(see Y. Yamaguchi, Progress of Theoretical Physics 12, 662 (1958)).
A third test is given by the comparison of the two reactions

(i) \( p + d \rightarrow \text{He}^3 + \pi^0 \)

(ii) \( p + d \rightarrow \text{H}^3 + \pi^+ \)

We know that

\[
\begin{align*}
\text{H}^3 & \text{ has } I = \frac{1}{2} \text{ and } I_3 = -\frac{1}{2} \\
\text{He}^3 & \text{ " } I = \frac{1}{2} \text{ " } I_3 = +\frac{1}{2}
\end{align*}
\]

Hence we can treat \( \text{H}^3 \) as a proton and \( \text{He}^3 \) as a neutron. This, and the fact that the initial state has \( I=\frac{1}{2} \) leads, with the help of table II, page 2, for the charge wave functions of the pion-nucleon system, to the ratio

\[
\frac{\text{Int (i)}}{\text{Int (ii)}} = \frac{1}{2}
\]

Note that the coefficients on p.2 involve both the Clebsch-Gordon coefficients and the antisymmetrization of the identical particle wave functions. For this reason in the case of identical particles the corresponding equalities apply only to the total cross-sections, while in all other cases the results are equally valid for the differential cross-sections. See references cited in the table.

c) \( \pi^+ + N \rightarrow Y + K \) where \( Y \) is a \( \Sigma \) or a \( \Lambda \) particle.

So for the final system \( \Sigma + K \)  \( I = 3/2, 1/2 \)

\( \Lambda + K \)  \( I = 1/2 \)

The resulting \( R \) matrices for reactions of this type are found in table III, p.4.
We can test here the triangle relation \( \Delta (a, \sqrt{2}b, c) \geq 0 \). In particular we have \( c+a \geq \sqrt{2}b \) (see table III, p.4, for definitions of \( a, b \) and \( c \)), or, expressed in the cross-sections for the three reactions with different \( \Sigma \) particles,

\[
\sqrt{\frac{d\sigma}{d\omega} (\Sigma^+)} + \sqrt{\frac{d\sigma}{d\omega} (\Sigma^-)} \geq \sqrt{2} \sqrt{\frac{d\sigma}{d\omega} (\Sigma^0)}
\]

Last year this relation was reported to be violated; later experiments have, however, removed this discrepancy.

d) \( Y + N \rightarrow N + Y \)

For the \( \Sigma + N \) system \( I = 1/2, 3/2 \), while for the \( \Lambda + N \) system \( I = 1/2 \) is the only possibility. For the definition of the different \( R \) matrices for \( (Y, N) \) systems with different isospin see table III, p.5.

Possible reactions allowed for the system with \( I = 1/2 \) are:

\[
\begin{align*}
\Sigma + N & \rightarrow N + \Sigma \\
\Lambda + N & \rightarrow N + \Lambda
\end{align*}
\]

Therefore the \( R \) matrix for the \( I = 1/2 \) system can be written as

\[
R_1 = \begin{pmatrix} R_{\Sigma} & R_X \\ R_X & R_{\Lambda} \end{pmatrix}
\]

(Note that the subscript of \( R \) is \( 2I \)). The whole \( \Sigma + N \) scattering can now be described with the help of \( R_1, R_3 \) just like the \( T + N \) scattering. If \( R_X \) is zero, the
analogy is perfect. Note that

\[ R_X = (N \wedge | R \cdot (N \Sigma)_{\frac{1}{2}}) = ((N \Sigma)_{\frac{1}{2}} | R \cdot N \wedge) \]

because of time reversal invariance. We may diagonalize our \( R_1 \) matrix, which has as a base

\[
\begin{pmatrix}
(N \Sigma)_{\frac{1}{2}} \\
N \wedge
\end{pmatrix},
\]

into the form

\[
R_d = \begin{pmatrix}
R_- & 0 \\
0 & R_+ 
\end{pmatrix}
\]

where \( R_1 = U^{-1} R_d U \). Because of time reversal invariance the elements of the \( U \) matrix must be real (see Blatt and Weisskopf, Theoretical Nuclear Physics, and Blatt and Biedenharn, Rev. Mod. Phys. 24, 258 (1952)). \( U \) then has the unique form

\[ U = \begin{pmatrix}
\cos \xi & \sin \xi \\
-\sin \xi & \cos \xi
\end{pmatrix} \]

The base for \( R_d \) becomes

\[ U \begin{pmatrix}
(N \Sigma)_{\frac{1}{2}} \\
N \wedge
\end{pmatrix} = \begin{pmatrix}
\cos \xi (N \Sigma)_{\frac{1}{2}} + \sin \xi (N \wedge) \\
-\sin \xi (N \Sigma)_{\frac{1}{2}} + \cos \xi (N \wedge)
\end{pmatrix} \]

Note that \( \xi = 0 \) gives \( R_X = 0 \), which makes \( \Sigma + N \to N + \Sigma \) identical with \( \pi^+ + N \to N + \pi^- \).
e) \( K^- + N \rightarrow Y + \pi \)

The initial state has \( I = 0,1 \) which leads to \( I = 0,1 \) for the \((\Sigma^+, \pi^-)\) final state and \( I = 1 \) for the \((\Lambda^0, \pi^-)\) state. For the definition of the \( R \) matrices for these different systems, see table III, p.7. The coefficients are so chosen that they cancel the Clebsch-Gordon coefficients in the \( R \) matrices for the different processes which involve isospin mixtures.

The equalities in table III, p.8, can be verified easily by the reader. The equalities again allow the possibility of testing C.I. in our case of kaon absorption by nucleons.

To the next table on p.8, for \( K^- \) capture at rest, one can add the following list of kinetic energies for the hyperons:

\[
\begin{align*}
\Sigma^+ & : 14 \text{ MeV} \\
\Sigma^0 & : 14 \text{ MeV} \\
\Sigma^- & : 12 \text{ MeV} \\
\Lambda^0 & : 29 \text{ MeV}
\end{align*}
\]

Note for later reference the twice higher kinetic energy for the \( \Lambda^0 \) compared to those of the \( \Sigma \)'s, which will lead to an enhancement of the last reaction compared to the others by an enlarged phase-space factor.

The \( K^- \) capture at rest in hydrogen, as studied in hydrogen bubble chambers gives as the branching ratio:

\[
\left| \frac{R_1 + R_0}{R_0} \right|^2 : \left| R_0 \right|^2 : \left| R_1 + R_0 \right|^2 : \left| R \right|^2 \cdot \frac{\text{phase factor for } \Lambda}{\text{phase factor for } \Sigma} = 1:1:2
\]

Hence

\[
\left| R_0 \right|^2 : \left| R_1 \right|^2 : \left| R_1 + R_0 \right|^2 \cdot \frac{\text{phase factor for } \Lambda}{\text{phase factor for } \Sigma} = 1:3:1
\]
The relative phase $\phi$ between $R_0, R_1$ can be found by

$$|R_0|^2 = |R_1 + R_0|^2 = |R_0|^2 + |R_1|^2 - 2|R_0||R_1| \cos \phi$$

$$\cos \phi = \frac{1}{2} \Rightarrow \phi \approx 70^\circ$$

For the $K^- + n$ system the only possibility is $I = 1$. Using the results for protons we can predict for $K^-$ capture at rest by neutrons:

$$K^- + n \rightarrow \begin{cases} \Sigma^- + \pi^- & \frac{1}{2} \binom{2}{1} \\ \Sigma^0 + \pi^0 & \frac{1}{2} \binom{2}{1} \\ \Lambda^0 + \pi^- & \frac{1}{2} \binom{2}{1} \end{cases}$$

The factor 2 in front of the $R$ matrices comes from the two isospin possibilities of $K^- + p$, i.e. $I = 0, 1$ compared with $I = 1$ for neutrons. A rather indirect comparison of these results with $K^-$ capture data in deuterium and emulsions shows reasonable agreement.

\[ f) \ K^- + d \rightarrow Y + N \quad (f1) \]

$I = 1/2, \quad I_3 = -1/2$

The reaction is rare because the two baryons must carry away the whole $K^-$ rest mass and the large momenta required are not available in the deuteron wave function; in other words, the overlap of the momentum wave functions of the initial and final states of the baryon pair is not large enough to give an appreciable probability for
reaction (f1) compared to

\[ K^- + d \rightarrow Y + N + \pi^- \quad (f2) \]

If we study the \( K^- \) capture in emulsion nuclei, we find a rather large ratio \((f1)/(f2)\) due to the higher momentum components of the nucleons in complex nuclei and to the large reabsorption probability of pions in these nuclei.

g) \( K^- + d \rightarrow Y + N + \pi^- \)

The isospin assignment as given in table III, p.10, needs no further comment to the now-experienced reader.

We introduce here the following assumptions:

(i) A simple impulse approximation, i.e. we regard one of the initial nucleons as a spectator not taking an active part in the reaction.

(ii) No final state interaction.

This leads to:

<table>
<thead>
<tr>
<th>Final State</th>
<th>Impulse Approximation</th>
<th>C.I.</th>
<th>Predicted Ratios</th>
<th>Observed Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma^+ + \pi^- )</td>
<td>(-R_1 \cdot R_0) = (-2R_1 R_2 )</td>
<td>(2)</td>
<td></td>
<td>(1.6)</td>
</tr>
<tr>
<td>( \Sigma^0 + \pi^0 )</td>
<td>(R_0) = (R_1 - 2R_3)</td>
<td>(1)</td>
<td></td>
<td>(1.6)</td>
</tr>
<tr>
<td>( \Sigma^- + \pi^+ )</td>
<td>(R_1 \cdot R_0) = (3R_3)</td>
<td>(1)</td>
<td></td>
<td>(1.3)</td>
</tr>
<tr>
<td>( \Lambda^0 + \pi^0 )</td>
<td>(R_\Lambda) = (R_\Lambda)</td>
<td>(\frac{1}{4})</td>
<td></td>
<td>(0.6)</td>
</tr>
</tbody>
</table>

Here \(R_0\), \(R_1\), and \(R_\Lambda\) are the \( R \) matrices defined in table III, p.7.
Hence we can derive the following equalities:

\[ R_0^d = R^\Lambda \]
\[ R_1^d = \frac{R_1 + R_0}{3} \]
\[ R_1^d = \frac{2R_1 - R_0}{3} \]

Knowing the ratios between \( R_0 \), \( R_1 \) and \( R^\Lambda \) we can derive the predicted ratios as given in the above table.

As we can see, the result is not in agreement with experiment, proving the breakdown of our assumptions. It seems necessary, therefore, to take into account at least one of the final state interactions. Note that the peaks in the kinetic energy distribution of the \( \pi^- \) occur near 100 and 180 MeV, corresponding to \( \Sigma^- \) and \( \Lambda^- \) production respectively, while the baryons have much lower energy. From this we see that the pion moves much faster than the hyperon. This suggests as a first approximation including only the \((Y,N)\) final state interaction. Calling the two nucleons in deuterium \( N_1 \) and \( N_2 \), we can construct the following picture of \( \Lambda^- \) capture in deuterium, as a two-step process:

\[ K^- + N_1 \rightarrow Y' + \pi^- \quad (g1) \]
\[ Y' + N_2 \rightarrow Y + N_2' \quad (g2) \]

If \( Y' \) is a \( \Lambda^- \), process \((g2)\) cannot produce a \( \Sigma^- \) for lack of energy (it needs about 70 MeV kinetic energy), while if \( Y' \) is a \( \Sigma^- \) the scattering leads to different channels:

For \( I=1/2 \) \( \Sigma^- + N \rightarrow N + \Lambda^- \)
\( \Sigma^- + N \rightarrow N + \Sigma^- \)

Thus we can expect a substantial increase of \( \Lambda^- \)'s, which
II. Strong interactions.

(2) Experimental tests of charge independence (esp. for strange particles) (by Y. Yamaguchi)
### Table III

References:

(a) \( \pi - N \) scattering

<table>
<thead>
<tr>
<th>Process</th>
<th>R-matrix element</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^+ + p \to p + \pi^+ )</td>
<td>( R_3 )</td>
</tr>
<tr>
<td>( \pi^- + n \to n + \pi^- )</td>
<td></td>
</tr>
<tr>
<td>( \pi^0 + p \to p + \pi^0 )</td>
<td>( \frac{2R_3 + R_1}{3} )</td>
</tr>
<tr>
<td>( \pi^0 + n \to n + \pi^0 )</td>
<td></td>
</tr>
<tr>
<td>( \pi^+ + p \to p + \pi^- )</td>
<td>( \frac{R_3 + 2R_1}{3} )</td>
</tr>
<tr>
<td>( \pi^+ + n \to n + \pi^+ )</td>
<td></td>
</tr>
<tr>
<td>( \pi^- + p \to n + \pi^0 )</td>
<td>( \frac{\sqrt{2} (R_3 - R_1)}{3} )</td>
</tr>
<tr>
<td>( \pi^+ + n \to p + \pi^0 )</td>
<td></td>
</tr>
</tbody>
</table>
(b) \( N+N \rightarrow N+N+\pi \)

\[
\begin{array}{c}
I_N \\
I
\end{array}
\]

<table>
<thead>
<tr>
<th>( I )</th>
<th>1</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_N )</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>R-matrix</td>
<td>( R_{11} )</td>
<td>( R_{10} )</td>
<td>( R_{01} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
<th>Cross-section is proportional to</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p+p \rightarrow \pi^+ ) ( n+n \rightarrow \pi^- )</td>
<td>( \frac{1}{2} \left( R_{11} \right)^2 + \left( R_{10} \right)^2 )</td>
</tr>
<tr>
<td>( p+p \rightarrow \pi^0 ) ( n+n \rightarrow \pi^0 )</td>
<td>( \frac{1}{2} \left( R_{11} \right)^2 )</td>
</tr>
<tr>
<td>( p+n \rightarrow \pi^+ ) ( p+n \rightarrow \pi^- )</td>
<td>( \frac{1}{2} \left{ \frac{1}{2} \left( R_{11} \right)^2 + \frac{1}{3} \left( R_{01} \right)^2 \right} )</td>
</tr>
<tr>
<td>( p+n \rightarrow \pi^0 )</td>
<td>( \frac{1}{2} \left{ \left( R_{10} \right)^2 + \frac{1}{3} \left( R_{01} \right)^2 \right} )</td>
</tr>
</tbody>
</table>

References: van Hove, Marshak and Pais, Phys.Rev. 88, 1211 (1952)(L)
Notes:

(1) \[
\frac{p+p \rightarrow d+\pi^+}{p+n \rightarrow d+\pi^0} = \frac{\left| R_{10}(d) \right|^2}{\frac{1}{3} \left| R_{10}(d) \right|^2} = 2
\]
(isospin of \( d = 0 \))

(2) Production of neutral mesons

\[\sigma(pn \rightarrow \pi^0) = \frac{1}{2} \sigma(pp \rightarrow \pi^+) + \sigma(pn \rightarrow \pi^-) - \sigma(pp \rightarrow \pi^0)\]

If \( \sigma(pn \rightarrow \text{neutral meson}) > \left[ \sigma(pn \rightarrow \pi^0) \right] \)
expected from C.I., then:
a) C.I. is not valid
b) C.I. is still valid, but there exists another \( \pi^0 \) (neutral, \( I = 0 \)).

<table>
<thead>
<tr>
<th>I</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>R'</td>
<td>R'_0</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c|cc}
pp \rightarrow \pi^0 & R_1' & R'_0 \\
\hline
pn \rightarrow \pi^0 & \frac{1}{2} \left( | R_1' |^2 + | R'_0 |^2 \right) \\
\end{array}
\]

(3) \[
\frac{p+d \rightarrow He^+ + \pi^0}{p+d \rightarrow H^3 + \pi^+} = \frac{1}{8}
\]
\[H^3: I = \frac{1}{2}, \quad I_3 = -\frac{1}{2}\]
\[He^3: I = \frac{1}{2}, \quad I_3 = +\frac{1}{2}\]
(c) $\pi^+ N \rightarrow Y + K$

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>3/2</th>
<th>1/2</th>
<th>1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>final state</td>
<td>$\Sigma^+ + K$</td>
<td>$\Sigma^- + K$</td>
<td>$\Lambda + K$</td>
<td></td>
</tr>
<tr>
<td>R-matrix</td>
<td>$R_3$</td>
<td>$R_1$</td>
<td>$R_\Lambda$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
<th>$(f \mid R \mid i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ p \rightarrow K^+ + \Sigma^+$</td>
<td>$R_3 = a$</td>
</tr>
</tbody>
</table>
| $\pi^- p \rightarrow \left\{ \begin{array}{l} K^0 + \Sigma^0 \\ K^+ + \Sigma^- \\ K^+ + \Lambda^0 \end{array} \right.$ | $\frac{\sqrt{2}}{3} (R_3 - R_1) = b$  
|                  | $\frac{1}{3} (R_3 + 2R_1) = c$  
|                  | $-\sqrt{\frac{2}{3}} R_\Lambda$ |

**Note:**

$$3 \left\{ 2 |b|^2 + |c|^2 \right\} \geq |a|^2$$

$$\Delta (a, \sqrt{2} b, c) \geq 0$$
(d) $Y+N \rightarrow N+Y$  \hspace{1cm} (I = 3/2 and 1/2)

<table>
<thead>
<tr>
<th>I</th>
<th>3/2</th>
<th>1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma^+ + p \rightarrow p + \Sigma^+$</td>
<td>$\Sigma^- + n \rightarrow n + \Sigma^-$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^+ + n \rightarrow p + \Sigma^0$</td>
<td>$\Lambda + p$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^+ + n \leftrightarrow p + \Sigma^0$</td>
<td>$\Lambda + p$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^0 + p \leftrightarrow p + \Lambda^0$</td>
<td>$\Sigma^0 + n \leftrightarrow p + \Sigma^-$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^0 + n \leftrightarrow p + \Lambda^0$</td>
<td>$\Sigma^0 + n \leftrightarrow n + \Lambda^0$</td>
<td></td>
</tr>
</tbody>
</table>

| Process                  | $(f|R|i)$                  |
|--------------------------|----------------------------|
| $\Sigma^+ + p \rightarrow p + \Sigma^+$ | $R_3$ = a                  |
| $\Sigma^- + n \rightarrow n + \Sigma^-$ | $\frac{R_3}{3} + \frac{2R_\Sigma}{\sqrt{3}}$ = b |
| $\Sigma^+ + n \leftrightarrow \begin{cases} n + \Sigma^+ \\ p + \Sigma^0 \\ \Lambda + p \end{cases}$ | $\frac{\sqrt{2}}{3} (R_3 - R_\Sigma)$ = c |
| $\Sigma^0 + p \leftrightarrow p + \Lambda^0$ | $\frac{2R_3}{3} + \frac{R_\Sigma}{3}$ = d |
| $\Sigma^0 + n \leftrightarrow p + \Sigma^-$ | $-\frac{1}{\sqrt{3}} R_X$ |
| $\Sigma^0 + n \leftrightarrow n + \Lambda^0$ | $\frac{2R_3}{3} + \frac{R_\Sigma}{3}$ |
| $p + \Lambda \rightarrow p + \Lambda$ | $\frac{1}{\sqrt{3}} R_X$ |
| $n + \Lambda \rightarrow n + \Lambda$ | $\sqrt{\frac{1}{3}} R_X$ |

Note: $\Delta(a, b, \sqrt{2} c) \gg 0$
Notes:

(1) If $R_{\Lambda} = 0$; "$\Xi + N \rightarrow N + \Sigma'$" is the same as "$\pi + N \rightarrow N + \pi$".

$R_{\Lambda}$ gives rise to $(\Sigma' + N)_{I = \frac{3}{2}} \leftrightarrow (N + \Lambda)$.

($I = \frac{1}{2}$, $I_3 = \frac{3}{2}$):

<table>
<thead>
<tr>
<th>Initial</th>
<th>$\frac{\sqrt{2n} \Sigma^+ - p \Sigma^0}{\sqrt{3}}$</th>
<th>$\Lambda p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2n} \Sigma^+ - p \Sigma^0$</td>
<td>$R \Sigma$</td>
<td>$R_{\Lambda}$</td>
</tr>
<tr>
<td>$\Lambda p$</td>
<td>$R_{\Lambda}$</td>
<td>$R_{\Lambda}$</td>
</tr>
</tbody>
</table>

(2) $\Sigma + N \rightarrow N + \Sigma$, and $N + \Lambda$ (for $I = \frac{1}{2}$).

$\Lambda + N \rightarrow N + \Sigma$ is possible when the energy $E$ of $\Lambda$ is higher than the threshold $Q$.

For $E < Q$ one has only the elastic scattering:

$\Lambda + N \rightarrow N + \Lambda$, and $R_{\Sigma} = R_{\Lambda} = 0$
(e) $K^- + N \rightarrow Y + \pi^-$

<table>
<thead>
<tr>
<th>I</th>
<th>1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>final state</td>
<td>$\Sigma^+ + \pi$</td>
<td>$\Sigma^0 + \pi$</td>
<td>$\Lambda + \pi$</td>
</tr>
<tr>
<td>R-matrix</td>
<td>$2R_1$</td>
<td>$\sqrt{6}R_0$</td>
<td>$\sqrt{2}R_\Lambda$</td>
</tr>
</tbody>
</table>

| Process | $(f/R|i)$ |
|---------|---------|
| $K^- + p \rightarrow \left\{ \begin{align*} \Sigma^+ + \pi^- \\
\Sigma^0 + \pi^0 \\
\Sigma^0 + \pi^+ \\
\Lambda^0 + \pi^0 \end{align*} \right\}$ | $-R_1 + R_0 = a$ |
| $K^- + n \rightarrow \left\{ \begin{align*} \Sigma^0 + \pi^- \\
\Sigma^0 + \pi^0 \\
\Lambda^0 + \pi^- \end{align*} \right\}$ | $\sqrt{2}R_\Lambda = B$ |
Notes:

(1) \[
2 |A|^2 = |B|^2 \\
|a|^2 + |c|^2 + |d|^2 = 2(|b|^2 + |d|^2) \\
|a|^2 + |c|^2 + |d|^2 = 2(|b|^2 + |e|^2)
\]

(2) \(K^-\) capture at rest

\[
K^- + p \rightarrow \begin{cases} 
\Sigma^+ + \pi^- + 102 \text{ MeV} & 14 \text{ MeV} \\
\Sigma^0 + \pi^0 + 107 \text{ MeV} & 14 \\
\Sigma^- + \pi^+ + 95 \text{ MeV} & 12 \\
\Lambda^0 + \pi^0 + 182 \text{ MeV} & 29 
\end{cases}
\]

available "kinetic" energy
(f) $K^- + d \rightarrow Y + N$

$(I = \frac{1}{2}, I_3 = -\frac{3}{2})$

<table>
<thead>
<tr>
<th>final state</th>
<th>$\Sigma + N$</th>
<th>$\Lambda + N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-matrix</td>
<td>$R_{\Sigma}$</td>
<td>$R_{\Lambda}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
<th>$(f \mid R \mid i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^- + d \rightarrow \left{ \begin{array}{l} \Sigma^o + n \ \Sigma^- + p \ \Lambda^o + p \end{array} \right.$</td>
<td>$\begin{array}{l} \frac{1}{3} \sqrt{1} R_{\Sigma} \ -\frac{2}{3} \sqrt{2} R_{\Sigma} \ R_{\Lambda} \end{array}$</td>
</tr>
</tbody>
</table>

Problem:

$K^- + N + N \rightarrow N + Y$

based on charge independence.
\( (g) \quad K^- + d \rightarrow Y + N + \pi^- \quad (I = \frac{1}{2}, \quad I_3 = -\frac{1}{2}) \)

\[ I_B \quad I \]

<table>
<thead>
<tr>
<th>( I_B )</th>
<th>3/2</th>
<th>1/2</th>
<th>1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>final state</td>
<td>( \Sigma + N )</td>
<td>( \Sigma_i + N )</td>
<td>( \Lambda + N )</td>
</tr>
<tr>
<td>R-matrix</td>
<td>( 3\sqrt{2} R_2^d )</td>
<td>( 3 R_1^d )</td>
<td>( \sqrt{3} R_\Lambda^d )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>final state</th>
<th>( (f \mid R \mid i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma^+ + n + \pi^- )</td>
<td>(-2 R_1^d + R_3^d) = (a)</td>
</tr>
<tr>
<td>( \Sigma^0 + n + \pi^0 )</td>
<td>( R_1^d - 2 R_3^d ) = (b)</td>
</tr>
<tr>
<td>( \Sigma^- + n + \pi^+ )</td>
<td>( 3 R_3^d ) = (c)</td>
</tr>
<tr>
<td>( \Lambda^0 + n + \pi^0 )</td>
<td>( R_\Lambda^d ) = (A)</td>
</tr>
<tr>
<td>( \Sigma^0 + p + \pi^- )</td>
<td>( + \sqrt{2} (R_1^d + R_3^d) ) = (d)</td>
</tr>
<tr>
<td>( \Sigma^- + p + \pi^0 )</td>
<td>(- \sqrt{2} (R_1^d + R_3^d) ) = (e)</td>
</tr>
<tr>
<td>( \Lambda^0 + p + \pi^- )</td>
<td>(- \sqrt{2} R_\Lambda^d ) = (B)</td>
</tr>
</tbody>
</table>

**Notes:**

\[
2 \left| A \right|^2 = \left| B \right|^2
\]

\[
|a|^2 + |c|^2 + |e|^2 = 2 \{ \right| b|^2 + |d|^2 \}
\]

\[
|a|^2 + |c|^2 + |d|^2 = 2 \{ \right| b|^2 + |e|^2 \}
\]

\[
d = -e
\]
II. STRONG INTERACTIONS.

(3) Global symmetry.  

(4) Experimental test of restricted symmetry.  

Y. Yamaguchi
Global symmetry. *)

(a) Introduction. Last week we have seen how it is possible to establish several relations between R-matrices by means of the charge independence hypothesis. Its formulation was based on disregarding the mass differences within each isospin multiplets, and striking similarity among them. Here we have to neglect the electromagnetic interactions.

It is interesting to examine if there exist any other symmetry properties besides that of charge independence. For this purpose let us first examine the mass differences between the baryons.

These are tabulated on page 1 of Annex IV and the greatest mass difference occurs between \( \Sigma \) and \( N \) which is 34%. These mass splittings are certainly due to interactions which are much stronger than the electromagnetic interaction. But we make now the following daring approximation:

"Neglect the mass splitting of baryons!"

(One should note that this is an approximation 10 times worse than C I! Then there should exist a symmetry property more general than that of charge independence, in the interactions among baryons and between baryons and \( \pi^- \), K-mesons. This hypothetical symmetry is called the "global symmetry". Roughly speaking, it means, for example, \( \Upsilon^-N \) \( \pi^-\Xi \) and \( \Xi^-\Xi \) scattering should have the same R-matrix. Then first one has to see if all baryons can be described by similar charge wave functions.

As far as isotopic spin and strangeness are concerned, \( \Sigma \) and \( \Lambda \) would be precisely the same as the nucleon-\( \bar{K} \) system:

*) Notes collected by T. Yamagata and E. Zavattini.
Then within the approximation of global symmetry, the same \( R_t \) and \( R_s \) can be applied to describe the Y-N scattering;

Namely

<table>
<thead>
<tr>
<th>( i )</th>
<th>( i_3 )</th>
<th>Initial</th>
<th>Final</th>
<th>R-matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \Sigma^+_P )</td>
<td>( \Sigma^+_P )</td>
<td>( R_t ) (1)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( \Sigma^+_P )</td>
<td>( \Sigma^+_P )</td>
<td>( R_t ) (2)</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>( \gamma^0 n )</td>
<td>( \gamma^0 n )</td>
<td>( R_t ) (3)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>( \gamma^0 - (-\Sigma^+_P) n )</td>
<td>( \gamma^0 - (-\Sigma^+_P) n )</td>
<td>( R_s ) (4)</td>
<td></td>
</tr>
</tbody>
</table>

On the other hand from charge independence we have already seen that Y-N scattering would be described by 4 R-matrices

\[
R_3 \quad \text{and} \quad R_4 = \begin{pmatrix}
R^\Sigma & R_X \\
R_X & R^\Lambda
\end{pmatrix}
\]

The relationship between \( R_1 \), \( R_2 \) and \( R_t \), \( R_s \) may be derived in a following manner.

Let us use the following notations to express charge wave functions of \((N\Sigma)\) and \((N\Lambda)\) systems:

\[
\begin{align*}
(N\Sigma)_{1/2} & \quad I_{3/2} \quad \frac{1}{2} \\
(N\Sigma)_{1/2} & \quad I_{1/2} \quad \frac{1}{2} \\
(N\Lambda) & \quad I_{1/2} \quad \frac{1}{2}
\end{align*}
\]

Then

\[
\begin{align*}
p^\Lambda^0 &= (N\Lambda) \\
p^\Sigma^0 &= \sqrt{\frac{2}{3}} (N\Sigma)_{3/2} - \sqrt{\frac{1}{3}} (N\Sigma)_{1/2} \\
\gamma \Sigma^\mp &= \sqrt{\frac{1}{3}} (N\Sigma)_{3/2} + \sqrt{\frac{2}{3}} (N\Sigma)_{1/2}
\end{align*}
\]
\[ \Sigma^+ \sim p \bar{K}^0 \]
\[ \Sigma^0 \sim \frac{1}{\sqrt{2}} (p K^- + n \bar{K}^0) \quad \Lambda^0 \sim \frac{1}{\sqrt{2}} (p K^- - n \bar{K}^0) \]
\[ \Sigma^- \sim n K^- \]

So that we have the following analogy:

\[
\begin{pmatrix}
-\Sigma^+ \\
\gamma^0
\end{pmatrix} \sim \begin{pmatrix}
\rho \\
n
\end{pmatrix} \Gamma^0 \quad \text{where} \quad \gamma^0 \equiv \frac{\Lambda^0 - \Sigma^0}{\sqrt{2}}
\]

\[
\begin{pmatrix}
\Sigma^0 \\
\Sigma^-
\end{pmatrix} \sim \begin{pmatrix}
p \\
n
\end{pmatrix} \Gamma^- \quad \text{where} \quad \zeta^0 = \frac{\Lambda^0 + \Sigma^0}{\sqrt{2}}
\]

We see now: if we take \((-\Sigma^+, \gamma^0)\) and \((\Sigma^0, \Sigma^-)\) instead of \(\Sigma^+ \Sigma^0 \Sigma^-\) and \(\Lambda^0\), hyperons can be regarded as the same spinors as nucleons. Let us introduce a "new" isospin \(i\) and call it \(i\)-spin which should not be confused with \(I\) (the ordinary isospin). Let us suppose all baryons have the same \(i\)-spin \((= \frac{1}{2})\) and they are assigned as in the following table.

<table>
<thead>
<tr>
<th>(i)</th>
<th>(\frac{1}{2})</th>
<th>(\frac{1}{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma^0)</td>
<td>(\rho)</td>
<td>(-\Sigma^+)</td>
</tr>
<tr>
<td>(-\frac{1}{2})</td>
<td>(n)</td>
<td>(\gamma^0)</td>
</tr>
</tbody>
</table>

The values of \(i\) and \(I\) are the same for nucleons and \(\Sigma\)-particles while they are different for \(\Sigma\) and \(\Lambda\).

(b) \(YN\) scattering

In case of \(YN\) scattering, we write

\[ R_t \quad \text{as R - matrix for} \quad i = I = 1 \]

and \[ R_s \quad \text{"} \quad \text{for} \quad i = I = 0 \]
Using these expressions, (2) and (4) can be written as

\begin{align*}
(2) & = -\frac{\sqrt{2}}{3} (N \Sigma)_{3/2} - \frac{1}{2\sqrt{3}} (N \Sigma)_{1/2} + \frac{1}{2} (N \Lambda) \\
(4) & = \frac{\sqrt{3}}{2} (N \Sigma)_{1/2} + \frac{1}{2} (N \Lambda)
\end{align*}

(2) will become after the scattering

\[ R(2) = -\frac{\sqrt{2}}{3} (N \Sigma)_{3/2} \frac{1}{R_3} - \frac{1}{2\sqrt{3}} \left[ R_2 (N \Sigma)_{1/2} + R_X (N \Lambda) \right] \]

\[ + \frac{1}{2} \left[ R_\Lambda (N \Lambda) + R_X (N \Sigma)_{1/2} \right] \]

On the other hand, from the global symmetry we have

\[ R(2) = R_t (2) = -\frac{\sqrt{2}}{3} \frac{R_t}{R_3} (N \Sigma)_{3/2} - \frac{1}{2\sqrt{3}} \frac{R_t}{R_3} (N \Sigma)_{1/2} + \frac{1}{2} \frac{R_t}{R_3} (N \Lambda) \]

Equating the corresponding coefficients.

\[ R_t = R_3 \]

\[ R_t = R_2 - \sqrt{3} R_X \]

\[ R_t = R_\Lambda - \frac{1}{\sqrt{3}} R_X \]

Combining these equalities with those similarly derived from (4), one obtains

\[ R_3 = R_t, \quad R_1 = \begin{pmatrix} R_2 & R_X \\ R_X & R_\Lambda \end{pmatrix} = \begin{pmatrix} \frac{1}{4} (3 R_t + R_3) & \frac{\sqrt{3}}{4} (R_t - R_3) \\ \frac{\sqrt{3}}{4} (R_2 - R_3) & \frac{1}{4} (R_3 + 3 R_t) \end{pmatrix} \]
Diagonalization of $\mathbf{R}_1$ matrix may be performed by a $\mathbf{U}$-matrix

$$\mathbf{R}_1 = \mathbf{U}^{-1} (\mathbf{R}_1)_{\text{diagonal}} \mathbf{U}$$

$$\mathbf{U} = \frac{1}{2} \begin{pmatrix} \sqrt{3} & 1 \\ -1 & 1 \end{pmatrix}$$

$$(\mathbf{R}_1)_{\text{diagonal}} = \begin{pmatrix} \mathbf{R}_s & 0 \\ 0 & \mathbf{R}_t \end{pmatrix} \quad \psi_s$$

with the eigenfunctions $\psi_s$ and $\psi_t$ which are expressed as

$$\psi_s = \frac{1}{2} \left[ \sqrt{3} \left( \mathbf{N} \Sigma \right) \frac{1}{2} + \left( \mathbf{N} \Lambda \right) \right]$$

$$\psi_t = \frac{1}{2} \left[ - \left( \mathbf{N} \Sigma \right) \frac{1}{2} + \sqrt{3} \left( \mathbf{N} \Lambda \right) \right]$$

(c) $\mathbf{\Pi}$-$\mathbf{\Sigma}$ scattering

Let us further assume that pions have $\mathbf{i}$-spin 1, i.e., $\mathbf{i}$-spin = usual isotopic spin for pions. Then pion-hyperon system have total $\mathbf{i}$-spins $3/2$ and $1/2$. We can easily construct the table of $\mathbf{i}$-spin wave functions for our system;
\[
\begin{array}{c|c|c}
\text{i} & 3/2 & 1/2 \\
\hline
1/2 & -\Sigma^+\pi^+ & \\
\hline
1/2 & \frac{\gamma^0\pi^+ - \frac{\sqrt{2}}{\sqrt{3}} \Sigma^+\pi^0}{\sqrt{3}} = \frac{\lambda^0\pi^+ - \Sigma^0\pi^+ - 2\Sigma^+\pi^0}{\sqrt{3}} & \frac{\sqrt{2} \gamma^0\pi^+ + \frac{\sqrt{2}}{\sqrt{3}} \Sigma^+\pi^0}{\sqrt{3}} = \frac{\lambda^0\pi^+ + \Sigma^0\pi^+ + \Sigma^+\pi^0}{\sqrt{3}} \\
\hline
1/2 & \frac{\lambda^0\pi^0 - \Sigma^0\pi^-}{\sqrt{3}} = \frac{\lambda^0\pi^0 - \Sigma^0\pi^- - \Sigma^+\pi^-}{\sqrt{3}} & \frac{\gamma^0\pi^0 + \frac{\sqrt{2}}{\sqrt{3}} \Sigma^0\pi^-}{\sqrt{3}} = \frac{2\Sigma^0\pi^- + \lambda^0\pi^0 - \Sigma^0\pi^-}{\sqrt{3}} \\
\hline
3/2 & \gamma^0\pi^- = \frac{\lambda^0\pi^- - \Sigma^0\pi^-}{\sqrt{2}} & \\
\hline
\text{matrix} & R_3 & R_1 \\
\end{array}
\]

To get the relations among, \( R_2, R_1 \) and \( R^2, R^1, R^0 \) (see page 5 of the annex table IV) we proceed as in the example b), the results are given in the p. 6 of the Annex (table IV).

In the same page is given the component \( I_3 = 0 \) of the pure states \( i = \frac{3}{2}, i = \frac{1}{2} \).

(a) Mathematical formulation of Global symmetry (G.S.) and Restricted Symmetry (R.S.)

Let us introduce 3 "Pauli-spin" type operators \( \mathbf{I}, \omega \) and \( \mathbf{S} \) and assign to the elementary particles the eigenvalues of these operators as listed in table p. 7 of the annex.*

* This formulation was first introduced by:
M. Ida, Prog.Theor.Phys.17(1957);733 (L).
The relations between $\vec{I}$, $\vec{\omega}$, $\vec{S}$ and $\vec{N}$, (strangeness), $N$ (baryonic number) are:

$$\vec{I} = \vec{i} + \vec{\omega}$$

$$\vec{U} = S_3 = \frac{N + S}{2}$$

the charge of a particle is then given by

$$Q = i_3 + \omega_3 + S_3$$

Global symmetry is then obtained requiring the conservation of:

$$\Sigma \vec{i}, \quad \Sigma \vec{\omega}, \quad \Sigma \vec{S} \quad (G.S.)$$

separately.

We have seen that in formulating G.S. we have to disregard mass difference of the order of 34% (see table IV (a)), among baryons; a less stringent symmetry can be formulated - Restricted Symmetry* - asking the conservation of

(R.S.) $\Sigma \vec{i}, \Sigma \vec{\omega}, \Sigma S_3$ separately.

In this scheme we can give different masses to

$$ (p, n), \quad (\Xi^0, \Xi^-), \quad \text{and} \quad (\Sigma, \Lambda \sigma \Sigma^+, \gamma^0 \Xi^0 \Xi^-)$$

However the mass splitting of $\Sigma$ and $\Lambda$ is still ignored.

In page 8 of annex IV there is a table summarizing the conservation laws for various symmetries.

---

* A.Pais PR 110, 574 (1958)
Remarks to (b) and (c)

G.S. would imply the equalities:

\[ R_t, R_s \text{ for } N-N \text{ scattering are equal to } \]
\[ \text{those of } \gamma-N \text{ scattering,} \]

while R.S. does not give these equalities, (but still we can
describe the \( \gamma-N \) scattering in terms of \( R_t \) and \( R_s \)). Similar statement
can be applied to (c).

(4) Experimental test of restricted symmetry

\[ \kappa^- + p \rightarrow \gamma + \pi^- \]

From the tables page 9 of the annex IV the only channel
allowed by R.S. is:

\[ \kappa^- + \rho \rightarrow 2 \frac{\Sigma^- \pi^+ - \Sigma^0 \pi^0 - \Lambda^0 \pi^0}{\sqrt{6}} \]

from which one can predict the branching ratios listed on page 10. Furthermore if one takes into account the \( \Sigma \) and \( \Lambda \) mass differences (hence difference in the phase space factor) the reaction \( \kappa^- + \rho \rightarrow \Lambda^0 + \pi^0 \) should have even higher branching ratio.

Our prediction is not in good shape as compared with experimental information. However we may argue the following fact: in the R.S. we neglect the \( \Sigma-\Lambda \) mass difference. Hence the experimental test of R.S. should be done at higher energy regions where we can safely neglect \( \Sigma-\Lambda \) mass difference. Whereas our present experimental conditions do not satisfy this criterion. In this respect we are looking forward in seeing further experimental tests.
(2) K – N scattering

In page 10 of Annex IV, $i_2$, $i_3$, $\omega$, $\omega_3$ and $S_3$ of K-N system are listed. By restricted symmetry one predicts that

$$(K^+p | R | K^+p) = (K^+n | R | K^+n)$$
$$= (K^o n | R | K^o n)$$

and

$$(K^o n | R | K^o p) = 0$$

So the charge exchange scattering is forbidden. Experimentally at least high energy $K^+$ are known to suffer charge exchange scattering. But if that is $\sim 10\%$ of the ordinary scattering, it may be still alright.

(3) Y-K Associated production

R.S. predicts the following results

1. \[ \sigma(\pi^+p \rightarrow \Sigma^+ + K^+) = 0 \]
2. \[ \sigma(\pi^+p \rightarrow \Sigma^- + K^+) = 2 \sigma(\pi^+p \rightarrow \Lambda^0 + K^o) \]

Experimentally reaction[1] has been found to be of the same order of the reactions[2] which is much more than the 10\% deviation allowed by R.S; though the experiments available are for "low" energy of the $\pi$'s and the same criticism made in the case of (1) is valid here.
cross section

\[ \Lambda \]

\[ \Sigma \]

incident energy

This figure shows schematically the cross-sections of the \( Y-K \) production due to \( \pi - p \) collision. We see the "cusp"

behaviour, which demonstrates the importance of \( \Sigma - \Lambda \) mass difference. In other words we have not yet high enough energy to detect R.S.

(f) Weak K-interaction; Gell-Mann's global symmetry.

Let us now assume that \( \pi \)-interaction is strong and obeys the restricted symmetry as defined before, while \( K \)-interaction is relatively weak and obeys charge independence only. This corresponds to a generalization of Gell-Mann's global symmetry. (Gell-Mann\(^*\) assumed that pionic interactions are very strong and obey G.S. while kaonic interactions are relatively weak and obey C.I.)

Test of this symmetry: \( \kappa^- + p \rightarrow \gamma + \pi \).

The \( \pi - \gamma \) final system can be described by the following three independent states:

\[
\begin{align*}
I = 1 & \quad \psi_1 \quad \sqrt{3} \, A_1 \\
I_3 = 0 & \quad \psi_3 \quad \sqrt{6} \, A_3 \\
I = 0 & \quad \psi_0 \quad \sqrt{3} \, A_0
\end{align*}
\]

If we neglect the effect of $K^{-}$ meson to the final states and apply the perturbation theory only to $K^{-}$ meson absorption, we get the relation among the present case and the C.I. case

\[ R_1 = - (A_1 + A_3) \]
\[ R_0 = A_0 \]
\[ R_\Lambda = - A_1 + 2A_3 \]

when $R_1, R_0, R_\Lambda$ are $R$-matrices for I.S. states (annex-table III p.7,).

Furthermore we should have

\[
\begin{align*}
& \Sigma^+ + \pi^+ \quad |A_1 + A_3 + A_0|^2 \\
& \Sigma^0 + \pi^0 \quad |A_0|^2 \\
& \Sigma^+ + \pi^- \quad |1 - A_1 - A_3 + A_0|^2 \\
& \Lambda^0 + \pi^0 \quad |1 - A_1 + 2A_3|^2
\end{align*}
\]

Hence, e.g., we have

\[ [\omega(\Sigma^+) + \omega(\Sigma^-) - 4\omega(\Sigma^0)]^2 + 4\omega(\Sigma^0) \omega(\Lambda^0) > 4 \omega(\Sigma^+) \omega(\Sigma^-)^* \]

Experimentally we find that this inequality is very drastically violated. We may conclude that the present hypothesis seems to be ruled out (although we still have a criticism similar to that stated at the end of (1)).

---

* Amati & Vitale, Nuovo Cimento 1, 190 (58)
II. Strong interactions.

(3) Global symmetry.

(4) Experimental tests of restricted symmetry. (by Y. Yamaguchi)

TABLE IV
### Table IV

(a) Mass differences among baryons.

<table>
<thead>
<tr>
<th></th>
<th>( (\Lambda, N) )</th>
<th>( (\Xi, \Lambda) )</th>
<th>( (\Sigma, N) )</th>
<th>( (\Xi, N) )</th>
</tr>
</thead>
</table>
| \( \Delta m \)
| \( m \)         | 0.13              | 0.07              | 0.24            | 0.34           |

\[
\frac{\Delta m}{m} = \frac{m_\Lambda - m_N}{\frac{3}{8}(m_\Lambda + m_N)}, \ etc.
\]
(b) Hyperon-nucleon scattering.

\[ \text{baryons: } i = \frac{1}{2} \]

<table>
<thead>
<tr>
<th>( i_3 = \frac{1}{2} )</th>
<th>p</th>
<th>(-\Sigma^+)</th>
<th>( Z^0 )</th>
<th>( \Xi^0 )</th>
<th>( \bar{n} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_3 = -\frac{1}{2} )</td>
<td>n</td>
<td>( Y^0 )</td>
<td>( \Sigma^- )</td>
<td>( \Xi^- )</td>
<td>( \bar{p} )</td>
</tr>
</tbody>
</table>

\[ Y^0 = \frac{\Lambda^0 - \Sigma^0}{\sqrt{2}} \]

\[ Z^0 = \frac{\Lambda^0 + \Sigma^0}{\sqrt{2}} \]

Y-N scattering

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-matrix</td>
<td>( R_t )</td>
<td>( R_S )</td>
</tr>
</tbody>
</table>

Relation to C.I. case

\[ R_3 = R_t \]
\[ R_1 = \begin{pmatrix} R_\Sigma & R_X \\ R_X & R_\Lambda \end{pmatrix} = \begin{pmatrix} \frac{3R_s + R_t}{4} & \frac{\sqrt{3}(R_t - R_s)}{4} \\ \frac{\sqrt{3}(R_t - R_s)}{4} & \frac{R_s + 3R_t}{4} \end{pmatrix} \]

\[ R_1 = U^{-1}(R_1) \text{ diagonal} \]

Eigenfunction

\[ (R_1)_{\text{diagonal}} = \begin{pmatrix} R_s & 0 \\ 0 & R_t \end{pmatrix} \begin{pmatrix} \cdots & & \psi_s \\ \cdots & & \psi_t \end{pmatrix} \]

\[ U = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{3} + 1 \\ -1 \sqrt{3} \end{pmatrix} \]

\[ I = \frac{1}{2} \]

\[ \psi_t = \frac{-(N\Sigma)_{I=\frac{1}{2}} + \sqrt{3}(N\Lambda)}{2} \]

\[ \psi_s = \frac{\sqrt{3}(N\Sigma)_{I=\frac{1}{2}} + (N\Lambda)}{2} \]
Examples:

<table>
<thead>
<tr>
<th>I</th>
<th>R-matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>$\langle \psi^+ \rangle_{I_3=+\frac{1}{2}} = \frac{-\sqrt{2} \Sigma^+ n^+ \Sigma^0 p + 3(\Lambda p)}{2 \sqrt{3}}$</td>
</tr>
<tr>
<td></td>
<td>$\langle \psi^- \rangle_{I_3=-\frac{1}{2}} = \frac{\sqrt{2} \Sigma^+ n^- \Sigma^0 p + (\Lambda p)}{2}$</td>
</tr>
<tr>
<td>3/2</td>
<td>$\frac{\Sigma^+ n^+ \sqrt{2} \Sigma^0 p}{\sqrt{3}}$</td>
</tr>
</tbody>
</table>
(c) \( \pi - Y \) scattering

\( \pi : i = 1 \)

C.I. case

<table>
<thead>
<tr>
<th>I</th>
<th>( \pi + \Sigma \rightarrow \pi + \Sigma )</th>
<th>( R^{(2)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( \pi + \Sigma \rightarrow \pi + \Sigma )</td>
<td>( R^{(1)} = \begin{pmatrix} R_\Sigma &amp; R_X \ R_X &amp; R_\Lambda \end{pmatrix} )</td>
</tr>
<tr>
<td>1</td>
<td>( \pi + \Sigma \rightarrow \pi + \Sigma )</td>
<td>( R^{(1)} = \begin{pmatrix} R_\Sigma &amp; R_X \ R_X &amp; R_\Lambda \end{pmatrix} )</td>
</tr>
<tr>
<td>0</td>
<td>( \pi + \Sigma \rightarrow \pi + \Sigma )</td>
<td>( R^{(0)} )</td>
</tr>
</tbody>
</table>

Global description

\( \pi + Y \rightarrow Y + \pi \)

\[
\begin{array}{|c|c|c|}
\hline
i & 3/2 & 1/2 \\
\hline
\text{R-matrix} & R_3 & R_1 \\
\hline
\end{array}
\]

Examples:

\[
\begin{array}{l}
(\pi^+ + \Sigma^+ \rightarrow \Sigma^+ + \pi^+) \\
(\bar{\pi}^+ + Z^0 \rightarrow Z^0 + \pi^+) \\
\end{array}
\]

\( R_3 \)
Relation between these two descriptions

\[ R^{(2)} = R_3 \]

\[ R^{(1)} = \begin{pmatrix} \frac{2R_1 + R_3}{3} & \frac{\sqrt{2}(R_3 - R_1)}{3} \\ \frac{\sqrt{2}(R_3 - R_1)}{3} & \frac{2R_3 + R_1}{3} \end{pmatrix} \]

\[ R^{(0)} = R_1 \]

\[ R^{(1)} = U^{-1}(R^{(1)})_{\text{diagonal}} U \]

\[ (R^{(1)})_{\text{diagonal}} = \begin{pmatrix} R_1 & 0 \\ 0 & R_3 \end{pmatrix} \]

\[ U = \frac{1}{\sqrt{3}} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \]

eigenfunction

Example: \[ I_3 = 0 \]

\[ \Psi_1 = \frac{-\Sigma^+ \pi^- + \Sigma^- \pi^+ - \Lambda^0 \pi^0}{\sqrt{3}} \]

\[ \Psi_2 = \frac{-\Sigma^+ \pi^- + \Sigma^- \pi^+ + 2 \Lambda^0 \pi^0}{\sqrt{6}} \]
(d) Assignments of quantum numbers for strongly interacting particles. (global symmetry).

<table>
<thead>
<tr>
<th></th>
<th>$i$</th>
<th>$\omega$</th>
<th>$S$</th>
<th>$i_3$</th>
<th>$\omega_3$</th>
<th>$S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$\frac{3}{2}$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$K$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>$\bar{K}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$-\frac{1}{2}$</td>
</tr>
</tbody>
</table>

**Remarks**

\[ \overrightarrow{I} = \overrightarrow{i} + \overrightarrow{\omega} \]

\[ U = S_3 = \frac{N + S}{2} \]

\[ Q = i_3 + \omega_3 + S_3 \]
<table>
<thead>
<tr>
<th>symmetry</th>
<th>conservation laws</th>
</tr>
</thead>
<tbody>
<tr>
<td>global symmetry</td>
<td>$\Sigma^i$, $\Sigma^\omega$, $\Sigma^s$</td>
</tr>
<tr>
<td>restricted symmetry</td>
<td>$\Sigma^i$, $\Sigma^\omega$, $\Sigma^{s_3}$</td>
</tr>
<tr>
<td>charge independence</td>
<td>$\Sigma(i+\omega)$, $\Sigma^{s_3}$</td>
</tr>
</tbody>
</table>
(e) Experimental tests of restricted symmetry.

(i) \( K^- + p \rightarrow Y + \pi \)

\[
\begin{array}{|c|c|c|}
\hline
 & \omega \bar{\omega} & S_3 \\
\hline
K^- & 0 & -\frac{1}{2} \\
p & \frac{1}{2} & 0 \\
\hline
\end{array}
\]

\((i=\frac{1}{2}) \quad (\omega=\frac{1}{2})\)

\[
\begin{array}{|c|c|c|}
\hline
 & \omega \bar{\omega} & S_3 \\
\hline
\Sigma^+ & +\pi & \frac{3}{2} \text{ or } \frac{1}{2} \\
\Sigma^0 & +\pi & \frac{1}{2} \\
\Sigma^- & -\pi & -\frac{1}{2} \\
\hline
\end{array}
\]

\[
p + K^- \rightarrow \frac{\sqrt{2}(+\Sigma^+ - \Sigma^- + \pi^0 \pi^0)}{\sqrt{3}}
\]

\[= + \frac{1}{\sqrt{6}} \left\{ 2 \Sigma^- \pi^+ - \Sigma^0 \pi^0 - \Lambda^0 \pi^0 \right\}
\]
(ii) $K^+\!\!-\!N$ scattering

\[
\begin{array}{|c|ccccc|}
\hline
 & i & i_3 & \omega & \omega_3 & S_3 \\
\hline
K^+ + p & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \\
K^+ + n & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1 \\
K^0 + p & \frac{1}{2} & +\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 1 \\
K^0 + n & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 1 \\
\hline
\end{array}
\]

\[
\begin{align*}
(K^+ + p/R/K^+ + p) &= (K^+ + n/R/K^+ + n) \\
&= (K^0 + n/R/K^0 + n) \\
(K^+ + n/R/K^0 + p) &= 0
\end{align*}
\]
(iii) \( \pi^+ N \rightarrow Y + K \)

\[
\sigma^- (\pi^+ p \rightarrow \Sigma^+ K^+) = 0
\]

\[
\sigma^- (\pi^- p \rightarrow \Sigma^- K^+) = \sigma^- (\pi^- p \rightarrow \Sigma^0 K^0)
\]

\[
= 2 \sigma (\pi^- p \rightarrow \Sigma^0 K^0)
\]

\[
= 2 \sigma (\pi^- p \rightarrow \Lambda^0 K^0)
\]
(f) Assumption: Weak K-interaction

(global π interaction)

Example: \( K^- + p \rightarrow Y + \pi^- \)

<table>
<thead>
<tr>
<th>I</th>
<th>final state</th>
<th>R-matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \psi_1 = \frac{-\Sigma^+ \pi^- + \Sigma^- \pi^0 - \Lambda \pi^0}{\sqrt{3}} )</td>
<td>( \sqrt{3} A_1 )</td>
</tr>
<tr>
<td></td>
<td>( \psi_3 = \frac{2 \Lambda^0 \pi^0 - \Sigma^+ \pi^- + \Sigma^- \pi^0}{\sqrt{6}} )</td>
<td>( \sqrt{6} A_3 )</td>
</tr>
<tr>
<td>0</td>
<td>( \frac{\Sigma^- \pi^- - \Sigma^0 \pi^0 + \Sigma^+ \pi^-}{\sqrt{3}} )</td>
<td>( \sqrt{3} A_0 )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
R_1 &= -(\lambda_1 + \lambda_3) \\
R_0 &= A_0 \\
R &= -\lambda_1 + 2\lambda_3
\end{align*}
\]

\[K^+ + p \rightarrow \]

<table>
<thead>
<tr>
<th>( K^- ) capture at rest</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma^+ + \pi^- )</td>
</tr>
<tr>
<td>( \Sigma^0 + \pi^0 )</td>
</tr>
<tr>
<td>( \Sigma^- + \pi^+ )</td>
</tr>
</tbody>
</table>
\[ \left\{ \omega (\Sigma^+) + \omega (\Sigma^-) - 4 \omega (\Sigma^0) \right\}^2 \]
\[ + 4 \omega (\Sigma^0) \omega (\Lambda^0) \geq 4 \omega (\Sigma^+) \omega (\Xi^-) \]
II. STRONG INTERACTIONS.

(5) Experimental tests of global symmetry, using antihyperons.

B. d'Espagnat
(5) Experimental tests of global\(^{(\ast)}\)

**symmetry, using antihyperons**

**Part 1**: Cross-sections for production of hyperon-antihyperon pairs on the basis of restricted symmetry.

In the previous lectures the concepts of global symmetry and of restricted symmetry have been introduced. Three spin operators, namely \(\vec{I}, \vec{\omega}\) and \(\vec{a}\) have been defined (see pp. 71-72). The assignment of their eigenvalues to the various elementary particles is as on page 7 of Table IV. We shall now apply this formalism to the production of hyperon-antihyperon pairs.

Consider the following spinors in the previously introduced \(\vec{I}\)-space:

\[
\begin{align*}
A &= \begin{pmatrix} -\Sigma^+ \\ Y \end{pmatrix} \quad \frac{1}{2} \quad \begin{cases} +\frac{3}{2} \\ -\frac{3}{2} \end{cases} \\
B &= \begin{pmatrix} \Sigma^- \\ -\Sigma^+ \end{pmatrix} \quad \frac{1}{2} \quad \begin{cases} +\frac{1}{2} \\ -\frac{1}{2} \end{cases} \\
A' &= \begin{pmatrix} \bar{\Sigma}^+ \\ \bar{Y} \end{pmatrix} \quad \frac{1}{2} \quad \begin{cases} +\frac{3}{2} \\ -\frac{3}{2} \end{cases} \\
B' &= \begin{pmatrix} \bar{\Sigma}^- \\ -\Sigma^+ \end{pmatrix} \quad \frac{1}{2} \quad \begin{cases} +\frac{1}{2} \\ -\frac{1}{2} \end{cases}
\end{align*}
\]

From these, the spinors \(X, X'\):

\[
\begin{align*}
X &= \begin{pmatrix} A \\ B \end{pmatrix} \quad \frac{1}{2} \quad \begin{cases} +\frac{1}{2} \\ -\frac{1}{2} \end{cases} \\
X' &= \begin{pmatrix} B' \\ -A' \end{pmatrix} \quad \frac{1}{2} \quad \begin{cases} +\frac{1}{2} \\ -\frac{1}{2} \end{cases}
\end{align*}
\]

\((\ast)\) Notes collected by J.C. Sens and A. Zichichi
can be constructed in \( \omega \)-space. Now we write down the reaction:

\[
\text{ordinary particles} \rightarrow \text{H} + \overline{\text{H}} + \text{ordinary particles.} \quad (5.1)
\]

(\( \text{H,} \overline{\text{H}} \) stands for hyperon, antihyperon; ordinary particles are nucleons and \( \pi \)-mesons.) (1) can proceed through \( \pi \)-interactions only. Restricted symmetry requires separate conservation of \( \mathbf{I} \) and \( \mathbf{J} \). The \( K \) interactions (responsible for the mass differences) are weak relative to the \( \pi \)-interactions. Since \( \overline{\omega}^\text{H} = \frac{1}{2} \) and \( \overline{\omega}^\overline{\text{H}} = \frac{1}{2} \), \( \omega^\text{H+H} \) can have the values 1 or 0. If, however, \( \omega^{(\text{H+H})} \) is to be a conserved quantum number, then its value can only be 0, since ordinary particles have \( \omega = 0 \). As far as \( \mathbf{I} \) is concerned, with \( \text{H} \) and \( \overline{\text{H}} \) both having \( i = \frac{1}{2} \), the values \( i^{\text{H+H}} = 1 \) or 0 are both possible, independent of the number or kind of ordinary particles that go in or come out.

The hyperon pair wave function, for which the "good" quantum numbers are \( \overline{\omega}^{(\text{H})} + \omega^{(\overline{\text{H}})} = \omega^{(\text{H+H})} \) and \( \overline{\omega}^\frac{1}{2} \overline{\text{H}} + \omega^\frac{1}{2} \overline{\text{H}} = \omega^{\frac{1}{2}} \text{H+H} \), can be expressed in terms of the wave functions \( X \) and \( X' \) of the members of the hyperon pair, with \( X \) resp. \( X' \) written in a representation in which \( \overline{\omega}^{(\text{H})} \) and \( \omega^{(\overline{\text{H}})} \) resp. \( \overline{\omega}^{(\text{H})} \) and \( \omega^{(\overline{\text{H}})} \) are diagonal.\(^*\)

The hyperon pair wave function with \( \omega^{(\text{H+H})} = 0 \) (and so \( \omega^{\frac{1}{2}} \text{H+H} = 0 \)) is then given by

\[
\frac{1}{\sqrt{2}} \left( X^\frac{1}{2} \sigma X^\overline{\frac{1}{2}} - X'^\overline{\frac{1}{2}} \sigma X^\frac{1}{2} \right) \quad (5.2)
\]

(the superscripts on \( X \) and \( X' \) denote \( \omega^\text{H} \) and \( \omega^\overline{\text{H}} \), \( \sigma \) denotes some operator in \( i \)-space to be determined below).

\(^*\) The method is naturally the same as the one used to construct wave functions of a definite \( I \) (see lectures by Dr. Yamaguchi). A good analogy is perhaps the space part of the two-electron wave function of the Helium atom, for which \( L = \ell(1) + \ell(2) \) and \( L_\lambda = \ell_\lambda(1) + \ell_\lambda(2) \) are the "good" quantum numbers. This wave function is expressible in terms of the spherical harmonics (written in terms of \( \ell(1), m(1) \) resp. \( \ell(2), m(2) \)) of the individual electrons. There are, of course, many more examples.
These wave functions are in turn expressible in terms of wave functions $AA', BB'$ which are diagonal in $i^2$ and $i^3$, as $X^{\frac{2}{3}} = B'$, $X^{-\frac{2}{3}} = A'$, $X^\frac{2}{3} = A$, $X^{-\frac{2}{3}} = B$:

$$ (2) = \frac{1}{\sqrt{2}} (B'B + A'A). $$

For $\omega = 0$, $i = 1$ we have $i^3 = \pm 1, 0$:

$$ i^3 = +1 : \frac{1}{\sqrt{2}} (B^{\frac{1}{2}}B^{\frac{1}{2}} + A^{\frac{1}{2}}A^{\frac{1}{2}}) = \frac{1}{\sqrt{2}} (\Sigma^{-} Z - \overline{\Sigma}^{+}) \quad (5.3) $$

$$ i^3 = 0 : \frac{1}{\sqrt{2}} \left( \frac{B^{\frac{1}{2}}B^{-\frac{1}{2}} + B'^{-\frac{1}{2}}B^{\frac{1}{2}}}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} A^{-\frac{1}{2}}A^{\frac{1}{2}} + A'^{-\frac{1}{2}}A'^{\frac{1}{2}} $$

$$ = \frac{1}{\sqrt{2}} (\overline{\Sigma}^{-} \Sigma^{-} - \overline{Z}Z + \overline{\Sigma}^{+} \Sigma^{+} ) \quad (5.4) $$

$$ i^3 = -1 : \frac{1}{\sqrt{2}} (B'^{-\frac{1}{2}}B^{-\frac{1}{2}} + A'^{-\frac{1}{2}}A^{-\frac{1}{2}}) = \frac{1}{\sqrt{2}} (-\overline{Z}Z^{-} + \overline{\Sigma}^{+} \Sigma^{-}) \quad (5.5) $$

For $\omega = 0$, $i = 0$ we have $(i^3 = 0)$:

$$ \frac{1}{\sqrt{2}} \left( \frac{B^{\frac{1}{2}}B^{-\frac{1}{2}} - B'^{-\frac{1}{2}}B^{\frac{1}{2}}}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} A^{-\frac{1}{2}}A^{\frac{1}{2}} - A'^{-\frac{1}{2}}A'^{\frac{1}{2}} $$

$$ = \frac{1}{2} \left( \overline{\Sigma}^{-} \Sigma^{-} + \overline{Z}Z + \overline{\Sigma}^{+} \Sigma^{+} \right) \quad (5.6) $$

(the superscripts on $A, B, A', B'$ denote $i^3$).

With these wave functions one can now derive relations between cross-sections for the production of $H, \overline{H}$ pairs of specified total charge $Q$:

$$ Q = I^3 + U/2 \quad U = S + N \quad I^3 = i^3 + \omega^3 $$

for $H, \overline{H} : U = 0$. 

In reaction (1): $\omega = 0$, therefore $\omega_3 = 0$. So

$$Q = i_3.$$  

(5.7)

a) Total charge of $H_3\bar{H}$ pair $Q = \pm 2$.

In reaction (1) $i = 1, 0$; therefore $i_3 = \pm 1, 0$.

So from (3): $Q = \pm 1, 0$.

Conclusion: The pair with total charge $Q = \pm 2$ is not produced.

b) Total charge of $H_3\bar{H}$ pair $Q = \pm 1$.

The cases $Q = \pm 1$ follow by substituting

$$Y \text{ by } \frac{\Lambda - \Sigma^o}{\sqrt{2}} \text{ and } Z \text{ by } \frac{\Lambda + \Sigma^o}{\sqrt{2}}$$

in the wave functions for $i_3 = \pm 1$ above. One obtains:

$$Q = 1 : \frac{1}{2}(\Sigma^- \Lambda + \Sigma^o - \Lambda \Sigma^+ + \Sigma^o \Sigma^+)$$

$$Q = -1 : \frac{1}{2}(-\Lambda \Sigma^- - \Sigma^o \Sigma^- + \Sigma^+ \Lambda - \Sigma^+ \Sigma^o).$$

The cross-sections for production of the different pairs are therefore related as follows:

$$Q = 1 : \sigma(\Sigma^- \Lambda) = \sigma(\Sigma^- \Sigma^o) = \sigma(\Lambda \Sigma^+) = \sigma(\Sigma^o \Sigma^+)$$

$$Q = -1 : \sigma(\Lambda \Sigma^-) = \sigma(\Sigma^o \Sigma^-) = \sigma(\Sigma^+ \Lambda) = \sigma(\Sigma^+ \Sigma^o).$$

(5.8)

(5.9)

c) Total charge of $H_3\bar{H}$ pair $Q = 0$.

The case $Q = i_3 = 0$ is a linear combination of the two possible ways in which wave functions with $i_3 = 0$ can be constructed, namely the case $\bar{I} = 0$, $i_3 = 0$ and the case $\bar{I} = 1$, $i_3 = 0$ (always $\bar{\omega} = 0$). With coefficients $a$ and $b$ respectively, one obtains:

$$Q = 0 : \frac{1}{2}a(\Sigma^- \Sigma^- + \Sigma^+ \Sigma^+ + \Lambda \Lambda + \Sigma^o \Sigma^o)$$

$$+ \frac{1}{2}b(\Sigma^- \Sigma^- - \Sigma^+ \Sigma^+ - \Lambda \Sigma^o - \Sigma^o \Lambda).$$

(5.10)
\[
\sigma(\Sigma_o^\pm) = \sigma(\Sigma_o^\mp) \sim 1b_1^2 \quad (5.11)
\]
\[
\sigma(\Lambda \Lambda) = \sigma(\Sigma_o \Sigma_o) \sim 1a_1^2 \quad (5.12)
\]
\[
\sigma(\Sigma^+ \Sigma^+) \sim 1a - b_1^2 \quad (5.13)
\]
\[
\sigma(\Sigma^- \Sigma^-) \sim 1a + b_1^2 \quad (5.14)
\]
\[
\sigma(\Sigma^+ \Sigma^+) + \sigma(\Sigma^- \Sigma^-) = 2\left[ \sigma(\Lambda \Lambda) + \sigma(\Sigma_o \Sigma_o) \right] . \quad (5.15)
\]

Part 2: Possible experiments to test global and restricted symmetry(*)

1st test: H-chamber and \(\pi\)-meson beam

The relations above indicate that

\[
N(\Sigma^+ \Sigma^-) \text{ and } N(\Sigma^- \Sigma^+) \ll N(\Sigma^+ \Sigma^+) \text{ and } N(\Sigma^- \Sigma^-) \quad (5.16)
\]

If one bombards protons with \(\pi\)-mesons, all final states that contain two \(\Sigma\)'s, contain also an antinucleon (conservation of number of baryons) and, in addition, contain at least two \(K\)-mesons: therefore(**) they are much less frequent than the final states containing a pair \(\Sigma, \Sigma\) and there is little chance of distorting the statistics. This conjecture implies that, in these pairs, the number of \(\Sigma^+\) is equal to that of \(\Sigma^-\). The experiment is therefore reduced to selecting pairs of strange particles, of which neither one is a \(K\)-meson, and verifying that among them there are many more with total charge \(Q = 0\) than with \(Q = \pm 2\).

(*) This section is a translation of a part of CERN Report 58-26: Production d'Antihyperons et tests de la symétrie globale (B. d'Espagnat and J. Prentki), because of its close correspondence with the content of the lecture.

(**) For two reasons: phase-space and weakness of the \(K\)-interactions.
Note that it is essential to work with hydrogen and \( \pi \)-mesons: a proton beam on hydrogen or an arbitrary beam on complex nuclei could generate two \( \Sigma \)'s without producing antibaryons: it is thus probably that in that case many selected pairs contain, in fact, no antihyperon and that the statistics will be completely distorted.

**2nd_test : Arbitrary chamber**

With an arbitrary chamber one can still verify

\[
N(\overline{\Sigma}^{+}\Sigma^{-}) \ll N(\overline{\Sigma}^{+}\Sigma^{+}) .
\]

It is sufficient to select those pairs, for which one of the members decays according to the mode

\[
\overline{\Sigma}^{+} \rightarrow \overline{p} + \pi^{0} .
\]

Since one can identify the antiproton, this mode characterizes the antihyperon. From there on, this test is the same as the preceding one. Note that it is very important here to use a \( \pi^- \)-beam and to observe only reactions with three branches. In fact, of the two reactions:

\[
\begin{align*}
\pi^- + n &\rightarrow \overline{\Sigma}^+ + \Sigma^- + p \\
\pi^- + n &\rightarrow \overline{\Sigma}^+ + \Sigma^+ + n + \pi^- 
\end{align*}
\]

phase-space favours the first one. If the second one is found to be more abundant, then this will indicate the validity of the selection rule

\[
N(\overline{\Sigma}^{+}\Sigma^-) \ll N(\overline{\Sigma}^{-}\Sigma^+) \approx 0 .
\]

With a \( \pi^+ \)-beam the circumstances are much less favourable.
3rd_test: Arbitrary_chamber

One selects the cases corresponding to $Q = -1$.

Counting the number $n$ of $\Sigma^+ \to p + \pi^0$

and the number $q$ of $\Lambda \to \bar{p} + \pi^+$

(directly, or via $\Sigma^0$)

one must have

$$2n = \frac{3}{2} q.$$ 

This test is an application of (5.9) and our knowledge of the branching ratios of $\Sigma^+$ and $\Lambda$. It implies PC conservation.

4th_test: Hydrogen_chamber_and_\pi-meson_beam

One selects all the cases corresponding to $Q = \pm 1$.

One counts the number $r$ of events, containing a $\Lambda$(directly or via a $\Sigma^0$) and the number $s$, containing a $\bar{\Lambda}$ (directly or via a $\Sigma^0$); one must have

$$r \approx s.$$ 

This equality holds for $Q = 1$ and $Q = -1$ separately. This test is an application of (5.8) and (5.9). For the same reasons as in the first test, the use of hydrogen and $\pi$-mesons is necessary.

Similar kinds of experiments can be devised to check the relations pertaining to the case $Q = 0$.

Test of Global Symmetry Hypothesis

The hypothesis of global symmetry introduces an additional restriction compared to the hypothesis of restricted
symmetry: global symmetry implies that the production of $\Sigma^+$ shall be about equal in number to the production of $\bar{p}$. One may add, incidentally, that it also implied a production of $\Sigma^-$ about equal in number to the production of $\bar{n}$ (*). This opens the road to a number of possible tests. As an example, let us remark that if $s$ is the total number of $\Lambda \to \bar{p} + \pi^+$, produced directly or via $\Xi^0$, one must have (apart from phase-space factors)

$$\frac{3}{2} s \approx N(\bar{n}) + N(\bar{p})$$

$N(\bar{n}), N(\bar{p})$ being the number of antineutrons and antiprotons produced. This is a consequence of (5.8), (5.9), (5.11), (5.12) and (5.15). It is of interest to remark that this relation is valid whatever the total charge of the pair produced, and that its verification does not necessitate the identification of the other member of the pair. It seems therefore that one could attempt to verify this relation experimentally in a variety of ways.

* * *

(*) The $\Xi^-$ can be members of a pair of total charge $Q = 1$ or $0$. If $Q = 1$, one applies (5.3) and gets

$$N(\Xi^-Z) \approx N(\Xi^+ \bar{Y})$$

but, by virtue of the hypothesis of global symmetry, $\Xi^+$ and $Y$ can be replaced by $p$ and $n$, which proves

$$N(\Xi^-) = N(\bar{n}).$$

If $Q = 0$, one applies (5.4) and (5.6) and gets

$$N(\Xi^- \Xi^-) = N(\bar{Y} Y) = |a + b|^2.$$

The same argument then leads to $N(\Xi^-) = N(\bar{n})$.

(**) Note, however, (Lichtenberg) that the Pauli Principle brings some restrictions to this statement because, in the case of $\bar{p}$ production, the associated nucleon cannot occupy any already occupied state.
III. WEAK INTERACTIONS.

(1) Asymmetry in non-leptonic decays.

(2) The $|\Delta I| = \frac{1}{2}$ rule.  

(3) Experimental tests of $|\Delta I| = \frac{1}{2}$ rule.  
   (Branching ratios and asymmetries).

E. d'Espagnat
(1) **Asymmetry in non-leptonic decays** (*

Consider the decay $\Lambda \rightarrow p + \pi^-$. Assume first the $\Lambda$'s to be completely polarized ($p = 1$); the spin of the $\Lambda$ is known to be $\frac{1}{2}$. Define the $z$-direction as the direction in which the spin points. Define furthermore:

- $\psi^\frac{1}{2}$ = spin part of proton wave function with spin up;
- $\psi^{-\frac{1}{2}}$ = spin part of proton wave function with spin down;
- $Y^m_l(\theta, \varphi)$ = space part of proton wave function.

$s$- as well as $p$-waves can be admitted, in accordance with the requirement for conservation of total angular momentum. The wave function of the decaying system is given by:

$$A^s \psi^\frac{1}{2} Y^0_0(\theta, \varphi) + A^p \left( -\frac{1}{\sqrt{3}} \psi^\frac{1}{2} Y^0_1 + \frac{2}{\sqrt{3}} \psi^{-\frac{1}{2}} Y^{-1}_1 \right). \quad (1.1)$$

The coefficients in the second term are just the Clebsch-Gordan coefficients for the combination of a spin $\frac{1}{2}$ (proton) and an angular momentum $1$ ($p$-wave) into a resultant spin $\frac{3}{2}$ ($\Lambda$). As spins and isospins combine in exactly the same way, these coefficients can simply be taken from Table II ($\pi + N$ system).

Squaring the modulus of this expression and noting that

$$Y^0_0 = \frac{1}{\sqrt{4\pi}} ; \quad Y^0_1 = \frac{1}{\sqrt{4\pi}} \cos \theta ; \quad Y^1_1 = \frac{1}{\sqrt{8\pi}} \sin \theta e^{i\varphi}$$

---

(*) Notes collected by J.C. Sens and A. Zichichi
one has:

\[
\left( A^S Y_0^0 - A^P \frac{1}{\sqrt{3}} Y_1^0 \right) V_{1/2}^0 + A^P \sqrt{\frac{2}{3}} Y_1^0 V_{1/2}^{-1/2} \right|^2
\]

\[
= \left( A^S Y_0^0 - \frac{1}{\sqrt{3}} A^P Y_1^0 \right)^2 + \frac{2}{3} \left| Y_1^0 \right|^2 \left| A^P \right|^2
\]

\[
= \frac{1}{4\pi} \left[ \left| A^S \right|^2 + \left| A^P \right|^2 \sin^2 \theta \right]
\]

\[
= \frac{1}{4\pi} \left[ \left| A^S \right|^2 + \left| A^P \right|^2 - 2 \text{Re}(A^S A^P^*) \cos \theta \right] = a + b \cos \theta. \tag{1.2}
\]

Experimentally, one measures the quantity

\[
X = \frac{N_{\text{up}} - N_{\text{down}}}{\frac{1}{2}(N_{\text{up}} + N_{\text{down}})}.
\]

From

\[
N_{\text{up}} = \int_0^{\pi/2} (a + b \cos \theta) \sin \theta \, d\theta = a + \frac{b}{2},
\]

\[
N_{\text{down}} = \int_{\pi/2}^\pi (a + b \cos \theta) \sin \theta \, d\theta = a - \frac{b}{2}
\]

we have, together with (1.2):

\[
X = \frac{b}{a} = \alpha
\]

with

\[
\alpha = \frac{-2 \text{Re} A^S A^P^*}{\left| A^S \right|^2 + \left| A^P \right|^2} \tag{1.3}
\]

If the A's are not completely polarized (P \# 1) the measured quantity X is, of course, equal to \( \alpha P \). The measured value is \( \alpha P \approx 0.5 \). Experiments have also been done to determine the helicity of the proton emitted in the A decay. This helicity is related to \( \alpha \) and would thus provide information on the absolute value and the sign of \( \alpha \) (and not only of \( Pa \)). Unfortunately, they do not seem as yet to have provided reliable and clear-cut results.
(2) The $|\Delta \mathbf{I}| = \frac{1}{2}$ rule

Some years ago, a rule was proposed with the purpose of explaining the lifetime and branching ratios of the strange particle decays by postulating a common transformation property in isotopic spin space for the decay interaction Hamiltonian of each strange particle. In fact, this rule consists in the assumption that this Hamiltonian transforms like a spinor of rank $\frac{1}{2}$ in isotopic spin space. As a consequence, the isotopic spin of the final state must differ from the isotopic spin of the initial state by:

$$|I_f^* - I_i^*| = |\Delta \mathbf{I}| = \frac{1}{2}.$$

This rule is only applicable if no leptons are present in the final state of the decaying particle, since isotopic spin is not defined for leptons.
(3) Experimental tests of $|\Delta I| = \frac{1}{2}$ rule
(Branching ratios and asymmetries)

**A-decay**

We apply the $|\Delta I| = \frac{1}{2}$ rule to A decay: $A \rightarrow n + \pi$.

A has isotopic spin $0$

$n$ \hspace{1cm} $\frac{1}{2}$

$\pi$ \hspace{1cm} $1$

Final state: $\frac{1}{2}$ or $\frac{3}{2}$.

With the $|\Delta I| = \frac{1}{2}$ rule, only the state $\frac{1}{2}$ is allowed. The wave function of the final state with isotopic spin $\frac{1}{2}$ and charge zero is

$$\sqrt{\frac{1}{3}} \ n \ \pi^0 - \sqrt{\frac{2}{3}} \ p \ \pi^-$$

(3.1)

(see Table II, $\pi + N$ system).

Therefore the branching ratio $A \rightarrow \frac{(n\pi^0)}{(p\pi^- + n\pi^0)} = \frac{1}{3}$. The experimental values are in agreement with this prediction.

**K-decay**

$$K \rightarrow \pi + \pi$$

K has isotopic spin $\frac{1}{2}$

$\pi$ \hspace{1cm} $1$

Final state: 0, 1, 2.

With the $|\Delta I| = \frac{1}{2}$ rule, only the states 0 or 1 are allowed. Assume the K-spin to be zero. The total wave function of two identical bosons must be symmetric. The space part is symmetric (s-wave); the spin part is also symmetric (spin of $\pi$ is zero). Therefore, the isotopic spin part must be symmetric

(*) Notes collected by J.C. Sens and A. Zichichi
as well. So the state with $I = 1$ is forbidden, since it is antisymmetric. This leaves $I = 0$. The wave function of the final state with $I = 0$, consisting of two pions, is

$$\frac{\pi^+ \pi^- + \pi^0 \pi^0 + \pi^+ \pi^-}{\sqrt{3}}$$

(3.2)

from which we get (the square of each three amplitudes has here to be taken separately)

$$K^0 \rightarrow \frac{(\pi^0 \pi^0)}{(\pi^0 \pi^0) + (\pi^+ \pi^-)} = \frac{1}{3}$$

(3.3)

The experimental values are: .14 (Columbia); .03 (MIT); .33 (Berkeley). More experimental material is clearly needed.

The $K^+$ cannot decay into a state of $I = 0$, and simultaneously conserve charge. The $K^+$, however, does decay through a combination of EM and weak interactions, that allows $|\Delta I'| = \frac{3}{2}$. Whether the ratio $\tau_{K^+}/\tau_{K^0} \approx 100$ is large enough to be compatible with such an interpretation is a point which is still controversial. This effect may be linked to the large $\pi^+, \pi^0$ mass difference.

$$\Sigma \rightarrow N + \pi$$

(3.4)

$\Sigma$ has isotopic spin $\frac{1}{2}$

$\pi$ " " " 1

$\Sigma$ " " " 1.

The possible values of the isospin of a system $N + \pi$ are obviously $\frac{1}{2}$ and $\frac{3}{2}$. Here, therefore, if we stated the rule as simply $|\Delta I| = \frac{1}{2}$ (without arrow) we would have no limitation at all. If
we want the rule to give really some limitations, i.e. some new relations between the various measurable quantities, we must give it in the more restrictive form

$$|\Delta \vec{I}| = \frac{1}{2} \tag{3.5}$$

with arrow on $\vec{I}$. The meaning of this equation is that

$$\vec{I}_f = \vec{I}_f + \vec{I} \tag{3.6}$$

with

$$|\vec{I}| = \frac{1}{2} \tag{3.7}$$

i.e. that the initial isospin is the vector sum of the total isospin $\vec{I}_f$ of the final system plus a supplementary isospin $\vec{I}$ with eigenvalue $\frac{1}{2}$, which is not attached to any physical particle in the final system but which is rather, so to speak, a property of the interaction. As far as the combination of isospins is concerned we may, however, as is obvious from (3.6), treat this supplementary isospin $\vec{I}$ as if it were attached to some neutral particle $x$ which would be emitted together with $N$ and $\pi$ in the decay. The fact that this "particle" $x$ (spurion) has no physical existence, i.e. carries no energy-momentum, is obviously irrelevant for our present purpose since the problem of isospin combination is entirely disconnected from the dynamical properties of the system. The content of rule (3.5) is thus expressed by writing (3.4) as

$$\Sigma \rightarrow N + \pi + x$$

$$I = 1, \quad \frac{1}{2}, \quad 1, \quad \frac{3}{2}$$

$$\frac{1}{2} \text{ or } \frac{3}{2}$$

$$\text{or } 1$$

$$\tag{3.8}$$
and by then assuming conservation of isotopic spin. This is expressed by making the vector combinations of isospin as shown in (3.8); the three isospins on the right hand side should then obviously combine to an isospin 1, this being the isospin of the initial system.

The \( I_f = \frac{1}{2}, I_f = \frac{3}{2} \) wave functions of the \( N + \pi \) system we call \( \Psi_{I_f}^{I_f/3} \), specifically (see Table II, Yamaguchi's lecture),

\[
\begin{align*}
\Psi_{1/2}^1 &= \sqrt{\frac{2}{3}} \pi^+ - \sqrt{\frac{1}{3}} \pi^0 \\
\Psi_{1/2}^2 &= \sqrt{\frac{1}{3}} \pi^+ + \sqrt{\frac{2}{3}} \pi^0 \\
\Psi_{3/2}^{-1} &= \pi^-
\end{align*}
\]

(3.9)

(These are the only ones that are needed).

These wave functions should then be combined with the wave functions for \( i = \frac{1}{2} \) to get \( I_i = 1 \) and \( I_i = \pm 1 (\Sigma^+) \), or \(-1 (\Sigma^-)\).

The Clebsch-Gordan coefficients for combining \((I_f, I_f')\) and \((i = \frac{1}{2}, i_f)\) into \((I_i = 1, I_i)\) can be found in appropriate tables

<table>
<thead>
<tr>
<th></th>
<th>( I_f = \frac{1}{2} )</th>
<th>( I_f = \frac{3}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\Sigma^+))</td>
<td>1</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>((\Sigma^-))</td>
<td>0</td>
<td>(-\sqrt{2})</td>
</tr>
</tbody>
</table>

The isospin wave functions for \( \Sigma^+ \) and \( \Sigma^- \) decay are therefore
\[ \psi^+ = a \frac{1}{\sqrt{2}} \psi^K - \frac{1}{2} b \psi^A = \left( a \sqrt{2} \frac{1}{3} - \frac{1}{2} b \sqrt{\frac{1}{3}} \right) \pi^+ - \left( a \sqrt{2} \frac{1}{3} + \frac{1}{2} b \sqrt{\frac{1}{3}} \right) \pi^0 \] (3.10)

\[ \psi^- = -\sqrt{2} b \psi = \frac{1}{\sqrt{2}} b \pi^- \]

a and b being arbitrary amplitudes. The amplitudes for \( \pi^+ \), \( \pi^0 \) and \( \pi^- \) decays are therefore

\[ (\pi^+) A^+ = a \sqrt{2} \frac{1}{3} - \frac{1}{2} b \sqrt{\frac{1}{3}} \] (3.12a)

\[ (\pi^0) A^0 = -a \sqrt{2} \frac{1}{3} - \frac{1}{2} b \sqrt{\frac{1}{3}} \] (3.12b)

\[ (\pi^-) A^- = -\sqrt{2} b \] (3.12c)

Elimination of a and b gives the remarkable relation

\[ A^0 = \frac{A^- - A^+}{\sqrt{2}} \] (3.13)

In general, amplitudes such as \( A^+ \), \( A^0 \), \( A^- \) are complex numbers. It can be shown, however, that if PC invariance holds, the phases entering in the various decay amplitudes are just the usual \( N - \pi \) scattering phase-shifts. As \( \Sigma \) has ordinary spin \( \frac{3}{2} \) only the small \( s \) or \( p \) phase-shifts can enter, therefore we can, with a very good approximation, treat the \( A \)'s as real numbers.

At this stage we should take into account parity non-conservation. This has the effect that, just as in Section 1, for each decay mode there are two A's, \( A^s \) and \( A^p \). The decay rate in each mode is given by (1.2) integrated over all directions, i.e. by \( |A|^2 + |A^p|^2 \). We may therefore, for convenience, consider \( A^s \) and \( A^p \) as the x and y components of a vector \( \vec{A} \) and write the decay rate \( \omega \) simply as \( |A|^2 \). Then, using (3.13):
\[
\begin{align*}
\begin{cases}
  w^+ &= |\vec{A}^+|^2 \\
  w^- &= |\vec{A}^-|^2 \\
  w^o &= |\vec{A}^o|^2 = \frac{|\vec{A}^+|^2 + |\vec{A}^-|^2}{2} - \vec{A}^+ \cdot \vec{A}^-.
\end{cases}
\end{align*}
\] (3.14)

Now the experimental results

\[
\frac{\tau_{\Sigma^+}}{\tau_{\Sigma^-}} \approx \frac{1}{2}; \quad \tau^+ - \frac{\eta^+}{p\pi^0} \approx 1
\] (3.15)

show that

\[
w^+ \approx w^- \approx w^o.
\] (3.16)

(3.16) together with (3.14) gives

\[
|\vec{A}^+|^2 = |\vec{A}^-|^2
\] (3.17)

\[
\vec{A}^+ \cdot \vec{A}^- = 0
\] (3.18)

The "vectors" $\vec{A}^+$ and $\vec{A}^-$ are thus orthogonal to each other and of equal length, which means that

\[
\begin{align*}
  A^{+8} &= C \cos \beta, \quad A^{+p} = C \sin \beta \\
  A^{-8} &= -C \sin \beta, \quad A^{-p} = C \cos \beta
\end{align*}
\] (3.19)

$C$ and $\beta$ being arbitrary parameters. Then from (3.13)

\[
\begin{align*}
  A^{+8} &= \frac{C^{-\cos \beta - \sin \beta}}{\sqrt{2}}, \quad A^{+p} = \frac{C \cos \beta - \sin \beta}{\sqrt{2}}
\end{align*}
\]

the three asymmetry parameters are then, using (1.3),

\[
\begin{align*}
  n\pi^+ \cdot \alpha^+ &= -\sin 2\beta \\
  p\pi^0 \cdot \alpha^0 &= \cos 2\beta \\
  n\pi^- \cdot \alpha^- &= \sin 2\beta
\end{align*}
\] (3.20)
β being arbitrary. Thus the $|\Delta T| = \frac{1}{2}$ rule, together with the assumption of PC invariance, and taking into account the experimental results (3.16), predicts very definite relationships between the three asymmetry parameters $\alpha^+, \alpha^0$ and $\alpha^-$. Experimentally it is found that

$$\begin{align*}
\alpha^0 \ P(\Sigma^+) & \neq 0 \\
\alpha^+ \ P(\Sigma^+) & \approx 0 .
\end{align*}$$

(3.21)

This shows that $P(\Sigma^+) \neq 0$ and, therefore, that $\alpha^+ \approx 0$. The value of the β parameter is thus experimentally determined as $\beta \approx 0, \frac{\pi}{2}$. It then follows from (3.20) that

$$\begin{align*}
\alpha^+ & \approx 0 \\
\alpha^0 & \approx \pm 1 \\
\alpha^- & \approx 0 .
\end{align*}$$

(3.22)

These values seem to be consistent with the results of the latest experiments.

Remark I

Before the discovery of parity violation in weak interactions it was impossible to reconcile the $|\Delta T| = \frac{1}{2}$ rule with the experimental results (3.15). This is easily seen in Eqs. (3.17) and (3.18) for, if parity were conserved, the $\Lambda$'s should be treated as scalars and not as 2-component vectors and these equations would give $\Lambda^+ = \Lambda^- = 0$.

Remark II

An interesting problem is to know whether some connection could be established between the asymmetries $\alpha^0$ and $\alpha^+_\Lambda$, the latter being the asymmetry parameter in $\Lambda$ decay. Two approaches
have, up to now, been made in this direction. One of them essentially assumes that the $I_f = \frac{1}{2}$ amplitudes found in $\Sigma$-decay should be transposed to $\Lambda$-decay (with some small modification due to the different decay energies); it gives a value of $\alpha_\Lambda^+$ which is noticeably smaller than unity and moreover does not seem to correspond to simple symmetry properties in the Lagrangian. The other one is based on a formal connection between the decay wave function of $\Sigma$ and $\Lambda$ which follows closely the two-doublet splitting method of global symmetry: it predicts

$$\alpha_\Lambda^+ \approx -\alpha^0. \quad (3.23)$$

It corresponds, moreover, to a fairly simple and symmetrical Lagrangian. Relation (3.23) can, of course, only be checked when the signs of $\alpha^0$ and $\alpha^+$ are measured. This requires a measure of the longitudinal polarization of the protons emitted in the two decays.
III. WEAK INTERACTIONS.

(4) $\mathcal{K}$ invariance (elementary notions on)

($K_1^0$ and $K_2^0$ decays, etc.)

P. d'Espagnet
(4) **PC Invariance (elementary notions on)**
\[ (K^0 \text{ and } K^\pm \text{ decays, etc.}) \] (\#)

(a) **Definition of \( C \)**

We define the operator \( C \) as that operator which transfers a particle into its antiparticle. In order that general relations between, for example, charge, isospin and strangeness should also remain valid for antiparticles, it is easily seen that \( C \) should change the sign, not only of \( N \) (baryonic number) and \( Q \) (charge) but also of \( S \) (strangeness) and \( I_3 \) (3rd component of isospin). This then gives the following table:

<table>
<thead>
<tr>
<th>( N )</th>
<th>( S )</th>
<th>( I_3 )</th>
<th>( Q )</th>
<th>( N )</th>
<th>( S )</th>
<th>( I_3 )</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2</td>
<td>( \frac{1}{2} )</td>
<td>-1</td>
<td>-1</td>
<td>+2</td>
<td>( \frac{1}{2} )</td>
<td>+1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>( \pm 1 )</td>
<td>( \pm 1 )</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>( \Sigma^0 \to \Sigma^0 )</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>( \Lambda^0 \to \Lambda^0 )</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
<td>( p \to \bar{p} )</td>
<td>-1</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>( n \to \bar{n} )</td>
<td>-1</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>0</td>
<td>+1</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
<td>( K^+ \to K^- )</td>
<td>0</td>
<td>-1</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>0</td>
<td>+1</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>( K^0 \to K^0 )</td>
<td>0</td>
<td>-1</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
<td>( \pi^+ \to \pi^- )</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \pi^0 \to \pi^0 \equiv \pi^0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(*) Notes collected by J.C. Sens and A. Zichichi
(b) PC Conservation

Consider now the weak interactions through which the above-listed particles decay. As we have seen in the last lecture, the asymmetry $\alpha$ is given by:

$$\alpha = \frac{-2 \text{Re} A^*_s A_p}{|A_s|^2 + |A_p|^2}$$

If parity is conserved in the decay process, this asymmetry has to be exactly zero, because depending upon the parity of the initial state, one of the two amplitudes, $A_s$ or $A_p$, has to vanish. Therefore $\alpha = 0$.

On the other hand, if C-invariance holds, $A_s$ and $A_p$ are given by:

$$A_s = |A_s| e^{i(\delta_s + n\pi)}$$

$$A_p = |A_p| e^{i(\delta_p + \frac{2n+1}{2}\pi)}$$

where $\delta_s$ and $\delta_p$ are the phase-shifts of the final state; for instance, if the decaying particle is a $\Lambda^0$, the final state consists of a $\pi$ and a nucleon (*)

Experimentally we know that in this case these phase-shifts are fairly small. Therefore, if C-invariance holds

$$A_s \sim \text{real}$$

$$A_p \sim \text{imaginary}.$$

This means that $\alpha$ is very small.

(*) The phase-shifts to be inserted in formulae (4.2) are the $\pi$-nucleon scattering phase-shifts at 37 MeV/c.
Now we consider the case where PC-invariance holds (but P and C are separately violated). In this case
\[ A_{s} = |A_{s}| e^{i(\delta_{s} + n\pi)} \]
\[ A_{p} = |A_{p}| e^{i(\delta_{p} + n\pi)} . \]

In this case we see that \( n \) can be large.

Therefore, from the large experimental value of \( n \) we can infer that neither P invariance nor C invariance holds in weak interactions, but that these interactions can be invariant under the product PC of these two transformations. For reasons of simplicity it is usually assumed that this invariance holds.

(c) The \( K_{1}^{0} - K_{2}^{0} \) complex

It is well known that, apart from other details, the mass and the lifetime of a given particle must be exactly equal to that of its antiparticle (Lee, Oehme, Yang). For instance, the neutron and its antiparticle (the antineutron) must have the same masses and lifetimes.

The fact that we want to illustrate here is how a particle-antiparticle complex can be described in terms of two particles with different decay modes and also different lifetimes. This we shall not do in a rigorous manner because it would involve spending much time on the fundamental axioms of quantum mechanics, which is not really the purpose of these lectures. But we shall make some remarks which may help to make the statement look not quite so surprising as it does at first.
Before continuing further, we must remember that previous to the discovery of strange particles, all the known neutral particles could be classified into two categories according to their behaviour under the operator $C$ as defined above.

**The first class** includes all the neutral particles that have themselves as antiparticles. A well-known example of a particle belonging to this class is the $\pi^0$-meson. Another example is the quantum of the EM field, also having itself as antiparticle. As far as properties under $C$ are concerned, the difference between $\pi^0$ and $\gamma$ lies in the fact that the $\pi^0$-meson is eigenfunction of the operator $C$ with eigenvalue $+1$, while the $\gamma$-quantum is eigenfunction of the operator $C$ with eigenvalue $-1$. In the case of the EM field, the eigenvalue $-1$ has as physical significance the obvious fact that when positive and negative charges are interchanged the EM field changes sign, whereas in the case of $\pi^0$ the eigenvalue $+1$ can be understood from its decay into two $\gamma$'s. In field theory, particles belonging to this class are represented by "real" fields.

**The second class** includes all the neutral particles which have antiparticles different from themselves and for which a rigorous law exists that prohibits the virtual transition between particle-antiparticle states (both conditions simultaneously). A well-known member of this class is the neutron. The neutron can be distinguished from its antiparticle, the antineutron, by the sign of its magnetic moment. Moreover, the law that forbids the transition neutron-antineutron is the law of conservation of baryons, which is absolute. In field theory, particles belonging to this class are represented by "complex" fields (*).

(*). See Weisskopf: *Introduction to Field Theory*, Lecture VI (CERN), for the more limited case of charged fields.
After experimental data became sufficient to give a preliminary view on the structure of the strange particles' assembly, it appeared clear that the $K^0$-meson was a neutral particle that could not belong to any of the two above-mentioned classes. In fact, the antiparticle, the $\bar{K}^0$, had to have strangeness opposite to that of $K^0$ in order to permit a satisfactory understanding of the production phenomena. Therefore it cannot belong to the first class. On the other hand, virtual transitions between particle-antiparticle states are allowed via the weak interactions, that violate the strangeness-conservation law. Therefore, it cannot belong to the second class, either.

To describe the production-interactions we have to use the $K^0$-$\bar{K}^0$ formalism. Under the action of the operator $C$, $K^0$ goes into $\bar{K}^0$, namely

$$C |K^0\rangle = |\bar{K}^0\rangle.$$ 

Therefore, in a field theory the $K^0$-field is a "complex" field. Having available the $K^0$-$\bar{K}^0$ wave functions we can construct two linear combinations of them that are eigenfunctions of the operator $C$ with eigenvalue $+1$ and $-1$ respectively. These linear combinations are

$$K_1^0 = \frac{K^0 + \bar{K}^0}{\sqrt{2}},$$

$$K_2^0 = \frac{K^0 - \bar{K}^0}{\sqrt{2}}.$$ 

In fact

$$C K_1^0 = \frac{\bar{K}^0 + K^0}{\sqrt{2}} = K_1^0,$$

$$C K_2^0 = \frac{\bar{K}^0 - K^0}{\sqrt{2}} = -K_2^0.$$
The same trick could, of course, be applied to the formalism with which we describe the neutron-antineutron field. In fact, let $\Psi_n$ be the wave function of the neutron-field and $\overline{\Psi}_n$ the wave function of the $\overline{n}$-field. Starting from $\Psi_n$, $\overline{\Psi}_n$ it is always possible to construct

$$N^O_1 = \frac{\Psi_n + \overline{\Psi}_n}{\sqrt{2}}$$

$$N^O_2 = \frac{\Psi_n - \overline{\Psi}_n}{\sqrt{2}}$$

in perfect analogy with what we have done with the $K^O$-field. Now the point is that the eigenfunctions $N^O_1$, $N^O_2$ of the operator $C$ represent only the result of a mathematical trick and in order to describe nature we have always to go back to the description in terms of neutrons and antineutrons. This is due to the fact that the neutron-antineutron transitions are strictly forbidden by all kinds of known interactions: strong, electromagnetic and weak interactions. Therefore, starting with a given number $N$ of neutrons, after a certain time we do not have any possibility of finding a state which consists, for instance, of 80% neutrons and 20% antineutrons; what is even more significant, we have no possibility of having a final state that would be an eigenstate of $C$. This is why no physical significance can be attributed to the eigenfunctions $N^O_1$ and $N^O_2$ of the operator $C$. On the other hand, the virtual transitions $K^O \rightarrow \overline{K^O}$ can be induced by the weak interactions. Therefore it makes sense to construct mathematical combinations of $K^O$, $\overline{K^O}$ wave functions that are eigenfunctions of the operator $C$ with eigenvalue $\pm 1$. In fact we can classify the final states of the non-leptonic ($K^O - \overline{K^O}$) decay in terms of eigenstates of $C$ with eigenvalues $\pm 1$. This gives two channels through which the ($K^O - \overline{K^O}$) complex can decay. Assuming for a moment that $C$-invariance holds in weak interactions, the two decay-channels are associated with two different quantum numbers, namely $+1$, $-1$, ...
which are conserved in the decay process. Therefore there will
be final states allowed for $K^0_1$ but forbidden for $K^0_2$, and vice
versa. This means that as far as the decay process is concerned,
$K^0_1$ and $K^0_2$ will behave as different particles, having two distinct
lifetimes\(^*\). When the possibility of describing the $(K^0\overline{K}^0)$
complex in terms of two particles $\phi^0_1, \phi^0_2$ with different lifetimes
was first predicted, it was still believed that $P$ and $C$ invariances
held in weak interactions. Since the discovery of $P$ and $C$
violation in weak interactions it has been immediately realised
that these predictions are still valid provided $PC$ invariance
holds\(^{**}\). To illustrate this circumstance let us consider two
final states that consist of (a) two $\pi$-mesons, (b) three $\pi$-mesons.
Let us see what is the behaviour of the system with 2 $\pi$'s under
the operation $PC$. We shall picture this as follows:

If we apply $P$ to $\pi^+ \pi^-$ we get $=(-)^{2l}(\begin{array}{c} \pi^+ \\ \pi^- \end{array})$ $l$ = orbital angular
momentum

'' '' $C$ to $\pi^+ \pi^-$ '' '' = $=(-)^{2l}(\begin{array}{c} \pi^+ \\ \pi^- \end{array})$ in fact:
$Y_{2l} (-x) = (-)^{2l} Y_{2l} (x)$

'' '' $PC$ to $\pi^+ \pi^-$ '' '' = $=(-)^{2l}(\begin{array}{c} \pi^+ \\ \pi^- \end{array})$

Therefore a system with 2 $\pi$'s has $PC = +1$.

Take now a system with 3 $\pi$'s:

$\pi^+ \pi^- \pi^0$

$l$ = relative orbital angular
momentum of the two charged $\pi$'s.

$L$ = orbital angular momentum of the
$\pi^0$ relative to the barycenter of
the system $\pi^+, \pi^-$.

\(^{*}\) For this footnote, see Annex.

\(^{**}\) $PC$ always holds (so far!). See Wigner, Rev.Mod.Phys. 22, 255 (195
If we apply $P$ to $+\overrightarrow{e}^{-}\overleftarrow{L}$ we get $-(\cdot)^L(-)^L(\overrightarrow{e}^{-}\overleftarrow{L})$

To this sign contributes the intrinsic parities of the particles involved. Having 3 π's that are pseudoscalar the final sign is negative.

If we apply $C$ to $+\overrightarrow{e}^{-}\overleftarrow{L}$ we get $-\overrightarrow{e}^{-}+ = (\cdot)^L(\overrightarrow{e}^{-}\overleftarrow{L})$

Therefore the application of $PC$ corresponds to $= (-)^2(-)^L = (-)^L$.

We can see that a system of 2 π's always has $PC = +1$, whilst for a system of 3 π's to get $PC = +1$ it is necessary that the orbital angular momentum $L$ is $= 2n+1$. Therefore the $2\pi$-decay mode is allowed for that combination of $K^0$ and $\overline{K}^0$ which has $PC = +1$, namely (in the case where the K-meson is scalar (*) )

$$K_1^0 = \frac{K^0 + \overline{K}^0}{\sqrt{2}}.$$ 

For the $K_1^0$ the $3\pi$-decay mode is decreased because of the high angular momenta involved. On the other hand, the linear combination

$$K_2^0 = \frac{K^0 - \overline{K}^0}{\sqrt{2}}$$ has $PC = -1$ (if the K-meson is scalar (*)).

Therefore the $2\pi$-decay mode is forbidden to this particle.

For the $K_2^0$ the $3\pi$-decay mode is, of course, allowed and not decreased because of any high angular momenta. In fact, a system of 3 π's has $PC = -1$ if $L = 0$.

(*) If the K-meson is pseudoscalar the expressions for $K_1^0$ and $K_2^0$ have to be interchanged.
The situation we have described can be summarized as follows:

$K_1^0$ has $PC = +1$, therefore it decays usually into 2 $\pi$'s; also into 3 $\pi$'s, but only if $L = 1, 3, \ldots$ (Because $K$-spin = 0, $J = L$ therefore this decay is decreased by angular momentum barrier.)

$K_2^0$ has $PC = -1$, therefore it cannot decay into 2 $\pi$'s; it can decay into 3 $\pi$'s.

It is clear that $W(K_1^0 \to 2\pi) \gg W(K_2^0 \to 3\pi)$ because of the larger available phase-space. It is also obvious that $W(K_1^0 \to 3\pi) \ll W(K_2^0 \to 3\pi)$ because of the angular momenta involved.

Let us now examine what happens when we create a $K^0$-particle

at $t = 0$  \[ K_o(t=0) = \frac{K_1^0 + K_2^0}{\sqrt{2}} \]

at $t = t$  \[ K_o(t) = \frac{K_{01} e^{-\frac{t}{2\tau_1}} + K_{02} e^{-\frac{t}{2\tau_2}}}{\sqrt{2}} \]

Therefore having at $t = 0$ a state that has strangeness $+1$, after a given time $t$ this state has evolved versus a mixture of states with strangeness $+1$ and $-1$. The relative percentage of this mixture depends upon the time interval $t$. The $K^0$-component of this mixture will give phenomena of hyperon production. The process in which an initial state of definite positive strangeness suffers, after a certain time, conversion into a state of negative strangeness has been experimentally detected.
A comparison with some well-known processes involving polarized light may help in making the subject more intuitive, although such comparisons are never completely adequate and should therefore be handled with care.

Consider a beam of light which propagates in the z-direction and enters a birefractive medium. It is then split into two beams which are polarized perpendicularly to each other, say in the x and y directions. These two beams can be compared with the two possible final states, one pertaining to $C = +1$, the other pertaining to $C = -1$ which in our case are physically completely different from each other. (One contains $2\pi$ states and the other does not.) The entrance of the beam into the birefractive medium can be compared, roughly speaking, with the effect of the weak interaction: if we have a pure $K_1^0$ (incoming beam polarized along Ox) we get only the $C = +1$ pionic final state (outgoing beam polarized along Ox), whereas if we have a pure $K_2^0$ (incoming beam polarized along Oy) we get only the $C = -1$ pionic final state (outgoing beam polarized along Oy). Now the point is that, because of the strangeness conservation, we never create $K_1^0$ or $K_2^0$ but instead $K_0 = \frac{K_1^0 + K_2^0}{\sqrt{2}}$ (or $\overline{K}_0 = \frac{K_1^0 - K_2^0}{\sqrt{2}}$). For simplicity consider some production at fairly low energy, where only $K_0$ are produced. This obviously corresponds to an incoming light beam polarized along the direction which makes a $45^\circ$ angle with Ox and Oy in the xOy plane.

Now in view of what is going to happen to this polarized light beam, we know that we must split it mentally into two orthogonal vibrations directed along Ox and Oy ($\sim K_1^0$ and $K_2^0$) and that these two vibrations will have a completely different fate.

Before its entrance into the birefractive medium it is, of course, a matter of taste whether one describes the beam one way or the other, but after this entrance only the description by means of the two orthogonal vibrations retains a meaning. This corresponds to the fact that only the description in terms of $K_1^0$
and \( K_2^0 \) is appropriate for the whole of the decay process. The main differences between \( K \) decay and the model just given is, of course, the fact that the entrance into the birefractive medium is a discontinuous process, whilst the action of the weak interaction is, so to speak, continuous in time. This prevents us from finding in the model any complete analogy to the difference in \( K_2^0, K_2^0 \) lifetimes, expect the fact that, from the time of production onwards the fate of the two orthogonal waves could each be calculated independently of the presence of the other wave and are indeed quite different. There is thus no a priori reason that the behaviour of one wave should have any influence on that of the other wave. This is obviously a very different situation from the one when a given particle has two decay modes because then the decay rate into one channel clearly has an influence on the probability of the particle decaying into the other channels.

* * *

* * *
III. WEAK INTERACTIONS.

(5) Leptonic decays - Proposed $\Delta \rho/\Delta S = 1$ rule  B. d'Espagnat
(5) Leptonic Decays. Proposed $\Delta Q / \Delta S = 1$ Rule (*)

A difficulty in the present theory of weak interactions is the lack of a rule which governs both leptonic and non-leptonic decays of the strange particles.

For non-leptonic decays the rule

$$|\Delta T| = \frac{1}{2},$$

applied to the system as a whole, accounts for the observed phenomena.

For leptonic decays, Feynmann and Gell-Mann have proposed the rule

$$\Delta Q / \Delta S = 1$$

(5.2) for the strangeness-non-conserving current (**) .

Consider, for example, the recently discovered leptonic decay of the $\Lambda$

$$\Lambda \rightarrow p + e^- + \bar{\nu}$$

(5.3)

Baryonic current $\Lambda \rightarrow p$
Leptonic current $\nu \rightarrow e^-$, equivalent to the emission of $\bar{\nu} + e^-$. Fig. 1

(*) Notes collected by E.G. Michaelis.

(**) Abbreviated SNC-current.
For the SNC-current, i.e. the baryonic current, in this reaction

$$
\Delta Q = Q_p - Q_\Lambda = 1, \quad \Delta S = S_p - S_\Lambda = 1, \quad \frac{\Delta Q}{\Delta S} = 1.
$$

The assumption is that this rule is general.

For the three-body leptonic K-decays we apply the rule to the mesonic current, which is SNC in these decays. Denoting $e^\pm$ and $\mu^\pm$ by $L^\pm$ we find the following reactions:

\[
\begin{align*}
\text{Allowed} & \quad K^+ \to \pi^0 + L^+ + \nu & \Delta Q = -1 & \Delta S = -1 \\
& \quad K^- \to \pi^0 + L^- + \bar{\nu} & \Delta Q = +1 & \Delta S = +1 \\
\text{Forbidden} & \quad K^+ \to \pi^+ + \nu + \bar{\nu} & 0 & -1 \\
& \quad K^+ \to \pi^+ + L^+ + L^- & 0 & -1 \\
& \quad K^- \to L^- + \nu + \bar{\nu} & 0 & + \\
\text{etc.}
\end{align*}
\]

The forbidden reactions have not been observed. The rule (5.2) may also be applied to two-body leptonic decays, if we remember that "nothing" has zero charge and strangeness\(^(*)\). Hence

\[
\begin{align*}
K^+ \to L^+ + \nu & \quad \Delta Q = -1 & \Delta S = -1 \\
K^- \to L^- + \bar{\nu} & \quad \Delta Q = +1 & \Delta S = +1 \\
\end{align*}
\]

are allowed. The $\mu$-decays have been observed; the rarity of the electron decay can be explained in much the same manner as the rarity of $\pi \to e + \nu$ decay, i.e. by assuming $V$ and $A$ couplings (see later).

For neutral decays we must start with particles possessing a definite strangeness, i.e. the $K_0$ and $\bar{K}_0$. Then

\[
(*) \text{ If "nothing" had } S \neq 0 \text{ the rule of } S \text{ conservation in strong interactions would obviously have no meaning at all since we could arbitrarily modify the } S \text{ of one number by adding "nothing" to it!}
\]
\[ \Delta Q \quad \Delta S \]

**Allowed**  
\[ K_0 \rightarrow \pi^- + L^+ + \nu \quad -1 \quad -1 \quad (5.8) \]
\[ \bar{K}_0 \rightarrow \pi^+ + L^- + \bar{\nu} \quad +1 \quad +1 \quad (5.9) \]

**Forbidden**  
\[ K_0 \rightarrow \pi^+ + L^- + \bar{\nu} \quad +1 \quad -1 \]
\[ \bar{K}_0 \rightarrow \pi^- + L^+ + \nu \quad -1 \quad +1 \quad (5.10) \]
\[ K_0 \rightarrow L^+ + L^- \quad 0 \quad -1 \]
\[ \bar{K}_0 \rightarrow L^+ + L^- \quad 0 \quad +1 \]

From the allowed reactions (5.8) and (5.9) we can deduce decay probabilities for the $K_{01}$ and $K_{02}$, the particles actually observed, whose wave functions are given by

\[ K_{01} = \frac{K_0 + \bar{K}_0}{\sqrt{2}} \quad (5.11) \]

and

\[ K_{02} = \frac{K_0 - \bar{K}_0}{\sqrt{2}} \quad (5.12) \]

We assume that PC is conserved in the decay. Then the reactions (5.8) and (5.9) have the same amplitude $a$ and probability $a^2$.

Consider then the four processes, where $L$ is, for instance, an electron:

\[ K_{01} \rightarrow \pi^- + L^+ + \nu \quad K_{02} \rightarrow \pi^- + L^+ + \nu \quad (5.13) \]
\[ K_{01} \rightarrow \pi^+ + L^- + \bar{\nu} \quad K_{02} \rightarrow \pi^+ + L^- + \bar{\nu} \quad (5.14) \]

From (5.11) and (5.12) their amplitudes are respectively $\frac{a}{\sqrt{2}}$, $\frac{a}{\sqrt{2}}$, $\frac{a}{\sqrt{2}}$ and $-\frac{a}{\sqrt{2}}$ since only components obeying the $\Delta Q/\Delta S = 1$ rule contribute in each case. Therefore all four reactions have equal probabilities $a^2/2$. The same remark holds, of course, if $L$ is a muon, but no statement can be made regarding the branching
ratio of muon to electron decays. The equality of the partial decay rates in (5.13) and (5.14) could be tested.

Now consider again the process

\[ \Lambda \to p + e^- + \bar{\nu} . \]  \hspace{1cm} (5.3)

For the baryonic (SNC) current only we have obviously

\[ |\Delta I^B| = \frac{1}{2} \]  \hspace{1cm} (5.15)

since possible strong interactions between the baryons do not alter I. (This rule must not be confused with the usual case (5.1) where the expression applies to the whole process.)

If now (5.3) were the only fundamental weak process, then (5.15) would hold for any leptonic decay, applied of course only to the SNC current. We postulate that something of this kind happens. Then taking, for example, the processes

\[ K_+ \to \pi^0 + L^+ + \nu \]  \hspace{1cm} (5.4)

and

\[ K_0 \to \pi^- + L^+ + \nu \]  \hspace{1cm} (5.8)

which are permitted by the $\Delta Q/\Delta S = 1$ rule, we note that initially $I = \frac{1}{2}$, finally $I = 1$. As previously in the case of the rule (5.1) we use (5.15) by introducing a spurion $x$ of I-spin $\frac{1}{2}$ to preserve I-spin in the SNC current. The only difference here is that $x$ must be a charged spurion since the SNC current implies a change of charge. The non-leptonic part of the decays then becomes

\[ K \to \pi + x \]

with

\[
\begin{array}{ccc}
I & \frac{1}{2} & 1 \ & \frac{1}{2} \\
\hline
\frac{1}{2} & 
\end{array}
\]
Then the amplitudes of the two processes (5.4) and (5.8) are found from the table of Clebsch Gordan Coefficients for the $\pi + N$ system with $I = \frac{1}{2}$, $I_3 = \pm \frac{1}{2}$ (Yamaguchi, Table II, p. 2 of Annex), replacing $N$ by $x$, i.e. $p$ by 1 and $n$ by 0. One finds for the decay rates, i.e. the amplitudes squared,

$$w(K_0 \to \pi^- + L^+ + \nu) = 2w(K_+ \to \pi^0 + L^+ + \nu).$$  \hfill (5.16)

Comparing this with the previous results for the processes (5.8), (5.13) and (5.14)

$$w(K_0 \to \pi^- + L^+ + \nu) = a^2$$

$$w(K_{01} \to \pi^- + L^+ + \nu) = w(K_{02} \to \pi^- + L^+ + \gamma) = \ldots.$$  \hfill (5.13)

$$= a^{2/2}$$

we have

$$w(K_+ \to \pi^0 + L^+ + \nu) = w(K_{01} \to \pi^- + L^+ + \nu) = \ldots,$$

$L$ being the same lepton throughout.

Now the three-body leptonic decays are the predominant modes of the $K_{02}$. The probabilities of the $K_+$ decays (5.4), which are available from experiment, can therefore be used to estimate the lifetime of the $K_{02}$. The result is compatible with the present experimental value.

* 

Attempts to apply the $\Delta Q = \Delta S$ rule to non-leptonic decays have not been successful.

(i) The rule yields the correct branching ratio $2:1$ for the charged and neutral decay modes of the $\Lambda$, using first order
perturbation theory and applying the rule to the baryonic current $\Lambda p$ in each of the two diagrams of Fig. 2.

![Diagram](image)

(a) \hspace{2cm} (b)

*Fig. 2*

However, there is no justification for restricting the perturbation calculations to the first order. Strong interactions among the decay products may lead to higher order processes. These change the branching ratio in the case of $\Delta Q/\Delta S = 1$, whereas in the case of the usual $|\Delta I| = 1/2$ rule the ratio remains correct.

(ii) $K_{01}$ Decay. The $\Delta Q/\Delta S = 1$ rule does not yield the 2:1 ratio of charged to neutral decays, which is predicted by $|\Delta I| = 1/2$ and supported by some recent experiments. However, an experimental controversy still exists on this point.

(iii) In lowest approximation, the $\Delta Q/\Delta S = 1$ rule forbids $\Sigma^+$ decays since

\[
\begin{array}{ccc}
\Delta Q & \Delta S \\
\Sigma^+ \rightarrow n & -1 & +1 \\
\Sigma^+ \rightarrow p & 0 & +1
\end{array}
\]

while allowing $\Sigma^- \rightarrow n + \pi^-$. 
Yet experimentally the probabilities of the three modes are found to be equal. This is difficult to reconcile with the use of lowest order calculation for the $\Lambda$ decay. Moreover, according to the results of Cool et al., Berkeley, the behaviour of the $\Sigma$ is well described by the usual $|\Delta I| = \frac{1}{2}$ rule.

Conclusions

(i) The $\Delta Q/\Delta S = 1$ rule applies to leptonic decays of strange particles; it forbids decay modes that are not observed(*).

(ii) The rule gives an estimate of the $K_{\Sigma 2}$ lifetime compatible with experiment.

(iii) The $\Delta Q/\Delta S = 1$ rule does not apply to non-leptonic decays, which seem to obey the usual $|\Delta I| = \frac{1}{2}$ rule, applied to the process as a whole.

* * *

(*) It should be noticed that the rule strictly forbids $\Sigma^+ \rightarrow n + L^+ + \gamma$. If this mode of decay were confirmed, the rule should have to be abandoned or drastically modified.
III. WEAK INTERACTIONS.

(6) Experiments on neutral K-mesons. L. Ledermann
(6) Experiments on Neutral K-Mesons (*)

In early machine experiments it was shown that $\theta^0_1$-particles, decaying according to

$$\theta^0_1 \rightarrow \pi^+ + \pi^- + (Q \approx 214 \text{ MeV})$$

with a lifetime $\tau \approx 10^{-10}$ sec, were produced in association with hyperons, but it appeared that not enough particles of this kind were observed if associated production was the rule and if no other neutral K-particles existed. Even the occurrence of the neutral decay mode

$$\theta^0 \rightarrow 2\pi^0$$

could not account for the observations unless it occurred pre-dominantly.

A further difficulty arose in the case of neutral K-particles. If the $K_0$ differs from its antiparticle $\overline{K}_0$ then charge-conjugation yields

$$C|K_0\rangle = |\overline{K}_0\rangle .$$

But for the case of a decay into two pions the final state in each case would be $\pi^+ + \pi^-$ and we have, with zero spin $K_0$,

$$C|\pi^+ \pi^-\rangle = |\pi^+ \pi^-\rangle .$$

Therefore the final state is an eigenstate of $C$, but the initial state is not. Hence the decay would violate $C$-invariance.

To overcome these difficulties Gell-Mann and Pais postulated that $C$-invariance holds but that the observed neutral $K$'s are mixtures of eigenstates $C = +1$ and $C = -1$. It follows that two types of neutral $K$ must exist whose wave functions are respectively

$$K_{01} = \frac{K_0 + \overline{K}_0}{\sqrt{2}} \quad C = +1$$

$$K_{02} = \frac{K_0 - \overline{K}_0}{\sqrt{2}} \quad C = -1 .$$

(*) Notes collected by E.G. Michaelis and F.J.M. Farley.
If C-invariance holds the $K_0$ can decay into two pions, whereas the
$K_{02}$ cannot; it follows that the $K_{02}$ must have a longer lifetime
than the $K_0$. (After the discovery of the failure of C-invariance
the argument was shown to follow exactly with C replaced by CP where
P is the parity operator.)

It appears that the observed neutral K-particles therefore
do not possess a definite strangeness. On the other hand (at time
t = 0) the rule of strangeness-conservation demands that a K-particle
of definite strangeness is produced in an interaction like, say,

\[ \pi + P \to \Lambda + K_{0} \]

Here the K-particle must have S = +1, i.e. it must be

\[ K_{0} = \frac{K_{01} + K_{02}}{\sqrt{2}} \]

The different time dependence of $K_{01}$ and $K_{02}$ then destroy the purity
of the state with respect to the strangeness quantum number. It
follows that exactly half the neutral K-particles produced in con-
junction with $\Lambda$'s should have a long lifetime. In view of the pos-
sible neutral decay mode of the $K_0$, more than half the neutral K's
would escape observation in an experiment designed to discover the
production and decay of short-lived ($\sim 10^{-10}$ sec) particles, for
example Steinberger's bubble chamber experiment.

A cloud chamber experiment was therefore designed to see
the decay of these long-lived neutral particles. The salient features
of the experiment were

1) Target

3 BeV
protons \rightarrow \text{Lead collimator}

\text{Sweeping magnet}

\text{Cloud chamber}

Magnetic field
10,000 g.
We wish to look at neutral radiation. Therefore we use a collimator followed by a magnet to sweep away the charged particles.

The smallest distance available between target and cloud chamber was 6 m, equivalent to a lifetime of $3 \times 10^{-8}$ sec for optimum sensitivity for 100 MeV K-particles.

The emission angle from the target was 68°. This is a compromise. For high yield small angles are better, but the neutron energy must be kept low to avoid the production of strange particles and pion pairs in the chamber.

2) The beam was brought to the target near the immovable cloud chamber using 6 magnets.

3) There were about $10^6$ neutrons per picture which gave some recoil protons from the chamber wall. (N.B. Because of this neutron flux a bubble chamber would have been useless for this experiment.)

4) The direction of the incident beam was obtained by observing charged particle tracks with the sweeping magnet turned off.

5) V-particle decays were observed in the chamber. Transverse momentum was clearly not conserved among the visible particles, and the products were not coplanar with the incident beam. Therefore at least one unobserved neutral particle was emitted in the decay. The tracks were evenly distributed in the chamber, indicating a long lifetime.

6) Because the incident $K^0$ mesons had low energy (~ 100 MeV) it was possible to identify the secondaries in favourable cases by track density measurement using the tracks of recoil protons in the same picture for calibration. In this way it was established that
the secondaries are \( \pi, \mu \) or \( e \), but not \( K \)-mesons. In 150 events we found the following definitely identified events which, however, give no idea of the branching ratios because the selection is biased by the requirement that the tracks be of low momentum.

\[
\begin{align*}
\pi^+ e^- & \quad 10 \\
\pi^- e^+ & \quad 6 \\
\pi^+ \mu^- & \quad 10 \\
\pi^- \mu^+ & \quad 2 \\
\pi^+ \pi^- & \quad 2 \\
\pi^+ e^+ & \quad 12
\end{align*}
\]

+ neutral particle,

a total of 42 identified events.

Comparing with \( K^+ \) decay which gives

\[
\begin{align*}
\pi^+ \pi^0 & \quad 27\% \\
\mu^+ \nu & \quad 55\% \\
\pi^+ \pi^0 \nu & \quad 1.1\% \\
e^+ \pi^0 \nu & \quad 3.3\% \\
\pi^+ \pi^- \pi^+ & \quad 6\% \\
\pi^+ \pi^0 \pi^0 & \quad \text{of which the first two are forbidden for } K_\pi^0 \text{ (there is no neutral } \mu \text{ !??). We have therefore observed the neutral counterparts of the last four } K^+ \text{ decays, the only ones which are allowed for } K_\pi^0.
\end{align*}
\]

7) No decays to \( \pi^+ + \pi^- \) only were observed out of a total of 180 events. Therefore this decay mode is excluded to \( \sim \frac{1}{2} \% \) for \( K_\pi^0 \). Does this prove anything about CP invariance? This has
been discussed by Weinberg, and by Lee and Yang. If the only two-body decay of \( K^- \) is to \( \pi^+\pi^- \), then, independently of CP invariance, one can always find a linear combination of \( K^0 \) plus \( \bar{K}^0 \) (i.e., a \( K_2^0 \)-particle) which cannot decay to \( \pi^+\pi^- \). But the existence of the other decay mode to \( \pi^0\pi^0 \) reopens the argument. Now the question is: What is the relative phase of the reaction amplitudes for these two decay modes? Time (i.e., CP) invariance requires \( 0^\circ \), and from the experiment Weinberg decided that the phase is less than \( 6^\circ \). However, the phase may be zero automatically, e.g., owing to the operation of the \( \Delta I = \frac{1}{2} \) selection rule, in which case we learn nothing about \( T \) (or CP) invariance. (The rule implies a branching ratio of \( K_2^0 \) to \( \pi^+\pi^- \) or \( \pi^0\pi^0 \) of \( 1:2 \), and experimentally this is still controversial.)

8) Measurement of lifetime can be made by changing the distance between target and cloud chamber keeping the emission angle the same. As the cloud chamber was immovable this was done by moving the target. A target position inside the cosmotron was found which gave four times the flight path keeping the emission angle \( 68^\circ \). The neutron flux (measured by counting stars in the gas) was used for normalization of the intensity.

The measurements give the ratio of \( K_2^0 \)-meson flux at two distances. To convert to proper lifetime one must have an estimate of the energy spectrum. This was obtained (a) by calculation, which was found to be not critical as to the assumptions made about \( K \)-meson production and (b) experimentally from the few identified decays. These methods were in rough agreement and gave

\[
\tau \sim (8.1 \pm 3) \times 10^{-8} \text{ sec.}
\]

Theoretically, using the rules \( \Delta S = \Delta Q \) and \( \Delta I = \frac{1}{2} \) applied to baryonic current (i.e., (5.15)), one obtains \( (5 \pm 1) \times 10^{-8} \) sec which is reasonable agreement.
9) A great deal remains to be measured including the spectrum of the three-particle decays which depends theoretically both on the strong and the weak interactions, and the helicity of the $\mu$-meson. (In these decays the $\mu$ can determine its own helicity, cf. $\nu^+ \rightarrow \mu^+ + \nu$ where the $\mu$ helicity is fixed by the neutrino and is the opposite of the "natural" helicity of the $\mu^+$.)

10) Any strong interaction of the $K_2^0$ beam will change the mixture of $K^0$ and $\bar{K}^0$ and therefore introduce some $K_1^0$ contamination.

The particle is born as $K^0$, $S = +1$. Later it becomes $K_2^0$, a mixture of $S = +1$ or $-1$. Therefore any evidence of negative strangeness will be interesting. The intensity of the $S = -1$ component should change with distance.

Fry finds $\Sigma$ and $\Lambda^0$ particles in emulsion at large distances, confirming the presence of -ve strangeness, but we do not see double $\Lambda^0$ events near the target, as we would expect if $K_2^0$ is already present at this stage (see below).

The mass difference of $K_1^0$ and $K_2^0$ is related to the time dependence of $S = -1$ component.

For non-relativistic particles the wave functions are

\[
K_1^0 = \frac{1}{\sqrt{2}} (K_0 + \bar{K}_0) = a_1(0)\exp(-t/2\tau_1)\exp(-im_1t/\hbar)
\]
\[
K_2^0 = \frac{1}{\sqrt{2}} (K_0 - \bar{K}_0) = a_2(0)\exp(-t/2\tau_2)\exp(-im_2t/\hbar)
\]

where $\tau_1$ and $\tau_2$ are the two lifetimes, $m_1$ and $m_2$ are the two masses, and $a_1(0)$ and $a_2(0)$ are the initial amplitudes of the two components. Putting in the initial conditions $a_1(0) = a_2(0) = \frac{1}{\sqrt{2}}$, one finds for the intensity of the $\bar{K}_0$ component

\[
\bar{a}(t)^2 = \frac{1}{4} \left( 1 + \exp(-t/\tau_1) - 2 \cos \left( \frac{\Delta m^2 \tau_1}{\hbar} \right) \exp(-t/2\tau_1) \right).
\]
Similarly we find for the $K_0$ intensity:

$$a(t)^2 = \frac{1}{4} \left( 1 + e^{-t/\tau_1} + 2e^{-t/2\tau_1} \cos \frac{\Delta m}{\hbar} t \right).$$

These simple formulas do not take into account the means of observation. Case has discussed in more detail what happens if a converter plate is used either for the production of $K_2^0$ events ("regeneration") or for the observation of hyperons out of the $K_0$ component. In this case we must define the mean free paths for scattering and absorption of $K_0$ and $\bar{K}_0$. Thus due to nuclear interactions

$$K_0 : \quad \frac{da}{dt} = -\frac{a}{2\tau_s}$$

$$\bar{K}_0 : \quad a = \frac{1}{2} \left( \frac{1}{\tau_{\bar{s}}} + \frac{1}{\tau_s} \right).$$

(Note: Recall that the $K_0^0$ can only scatter elastically or by charge exchange, the $\bar{K}_0^0$ can also be absorbed.)

If these are included, coupled differential equations are obtained (see Case). The solutions are (in terms of $K_1^0$ and $K_2^0$ amplitudes $a_1$ and $a_2$):

$$a_1(t) = x_{11}e^{-\mu_1 t} + x_{12}e^{-\mu_2 t}$$

$$a_2(t) = x_{21}e^{-\mu_1 t} + x_{22}e^{-\mu_2 t}$$

where

$$\mu_{1,2} = +\frac{1}{4} \left( \frac{1}{\tau_1} + \frac{1}{\tau_a} + \frac{1}{2\tau_s} + \frac{1}{2\tau_{\bar{s}}} + \frac{i\Delta m}{\hbar} \right)$$

$$\pm \left\{ \frac{1}{16} \left( \frac{1}{\tau_1} + \frac{1}{\tau_a} + \frac{1}{2\tau_s} + \frac{1}{2\tau_{\bar{s}}} + \frac{i\Delta m}{\hbar} \right)^2 \right. \right.$$
\[ x_{21} = \left( \frac{1}{2 \tau_1} + \frac{1}{4 \tau_a} + \frac{1}{4 \tau_s} + i \frac{\Delta m}{\hbar} - \mu_1 \right) \frac{x_{11}}{\left( \frac{1}{\tau_a} - \frac{1}{\tau_s} \right)} \]

\[ x_{12} = \left( \frac{1}{2 \tau_a} + \frac{1}{4 \tau_s} - \mu_2 \right) \frac{x_{22}}{\left( \frac{1}{\tau_a} - \frac{1}{\tau_s} \right)} \]

and \( x_{11}, x_{22} \) are determined by the initial conditions.

\( \Delta m \) can be measured by finding fluctuations in the intensity of \( S = -1 \) component.

\( \Delta m \) is estimated to be \( 10^{-5} \) from the difference in the weak interaction couplings.

If \( \Delta m \) is large the \( S = -1 \) population will build up rapidly after the process \( \pi + N \rightarrow \Lambda^0 + K^0 \) and we would get \( N + \bar{K}^0 \rightarrow \Lambda^0 \) in the same cloud chamber plate, i.e. we should see two \( \Lambda^0 \) emerging from one event. The MIT Group do not see this and conclude that \( \Delta m < 10 \frac{\hbar}{\tau_1}. \)

Fitch is planning an experiment in which \( \bar{K}^0 \) is converted to \( K^- \) in an absorber. (Only \( \bar{K}^0 \) can give \( K^- \), as \( S = -1. \)) He plans to study this process at various distances from the target using a counter arrangement to select \( K^- \)-particles.

* * *

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IV. THEORETICAL.

IV₁. Some basic concepts of quantum mechanics. J.M. Jauch
IV. Some basic concepts of quantum mechanics.

The following is a brief résumé of the principles of Quantum Mechanics. The presentation is neither systematic nor complete. Instead we shall emphasize those subjects which are most important for the discussion of fundamental particle physics, as well as some of the more recent developments in the formulation of quantum mechanics.

(1) The space of state vectors.

In Q.M. the states of a physical system are represented by elements in a linear vector space called a Hilbert space.

In the elementary form of the theory (non-relativistic wave mechanics of a finite number of particles) these elements are realized as square-integrable functions $\psi(x)$ over a finite dimensional configuration space $(x)$. These are the Schrödinger wave functions. If the particles have spin, it is necessary to admit finite sets of functions in order to describe the internal degrees of freedom.

In the application of quantum mechanics to fundamental particle physics this particular representation of the state vectors is not always possible. It is then necessary to retain only the essential properties of the Hilbert space. Among these properties the linearity and the scalar product are the ones most often used explicitly (but they are not all!).
(a) Linearity.

State vectors \( \varphi \) can be multiplied with complex numbers and added together.

Thus if \( \lambda \) is a complex number we can form \( \lambda \varphi \), and if \( \varphi_1, \varphi_2 \) are two state vectors, we can form \( \varphi_1 + \varphi_2 \).

The latter operation represents the principle of superposition.

(b) The scalar product.

To any two elements \( \varphi_1 \) and \( \varphi_2 \) there corresponds a complex number, called the scalar product \( (\varphi_1, \varphi_2) \), with the properties

\[
(\varphi_1, \varphi_2) = (\varphi_2, \varphi_1)^* \\
(\varphi, \varphi_1 + \varphi_2) = (\varphi, \varphi_1) + (\varphi, \varphi_2) \\
(\varphi_1, \lambda \varphi_2) = \lambda (\varphi_1, \varphi_2).
\]

\[
\| \varphi \|^2 = (\varphi, \varphi) > 0 \text{ unless } \varphi = 0 \text{ (zero element of the vector space)}
\]

The last property is especially important. It is called the positive-definiteness of the scalar product.

In recent years attempts have been made to incorporate some of the non-local characteristics of field theory by relinquishing the positive-definiteness of the scalar product. They have not succeeded.
Remarks:

(i) The unrestricted use of the superposition principle is quite harmless in elementary wave mechanics but not so in more general systems. It always fails when we have superselection rules. A good example is the neutron-proton system. The neutron and the proton may be represented by two wave functions \( \psi_n \) and \( \psi_p \) but there is no state corresponding to the element \( \psi_n + \psi_p \). This means that there exists no physical arrangement by which one could prepare such a state. We shall later see other examples of such superselection rules.

(ii) The scalar product is directly related to the observable consequences of quantum mechanics. Indeed if \( \psi \) is the state of the system and \( \psi_i \) any other state, then

\[
P = \frac{|\langle \psi_i, \psi \rangle|}{\| \psi_i \| \| \psi \|}
\]

represents the probability of finding the system in the state \( \psi_i \) when before the measurement it was in the state \( \psi \).

(iii) The physical states are only represented by the elements \( \psi \), up to an arbitrary factor. If all elements are multiplied with a common factor the physical consequences remain unchanged. It is therefore customary to normalize the elements such that \( \| \psi \| = 1 \).
(iv) In Dirac's form of Q.M. improper elements are often introduced which cannot be normalized. The use of these elements leads to mathematical complications which force Dirac to a heuristic and non-rigorous mathematics. Fortunately the use of such elements can be entirely avoided, (v. Neumann's form of Q.M.) and therefore to some extent justified. Since the mathematical tools in v. Neumann's form of Q.M. are not generally available we shall have to use some of the heuristic methods of Dirac.

Observables and linear operators in the space $\mathcal{H}$ of state vectors are functions on certain subsets of $\mathcal{H}$ such that

(i) if $\varphi \in D$ then $\lambda \varphi \in D$ and

$$A \lambda \varphi = \lambda A \varphi$$

(ii) if $\varphi_1 \in D$ and $\varphi_2 \in D$ then $\varphi_1 + \varphi_2 \in D$ and

$$A(\varphi_1 + \varphi_2) = A\varphi_1 + A\varphi_2$$

A linear operator is self-adjoint if for every $\varphi_1, \varphi_2 \in D$

$$(\varphi_1, A\varphi_2) = (A\varphi_1, \varphi_2)$$

The importance of the self-adjoint linear operators in Q.M. is that they represent the measurements and hence the physical quantities of a system.
According to the basic principles of quantum mechanics every physical quantity \( \mathcal{A} \) is represented by some self-adjoint linear operator \( A \). If the system is in a state \( \varphi \) then the mean value of the quantity \( \mathcal{A} \) is given by the expectation value of \( A \) which is denoted and defined by

\[
\langle A \rangle_\varphi = (\varphi, A \varphi) \quad \text{for} \quad \| \varphi \| = 1 \quad (2.1)
\]

An element \( \varphi \) with the property

\[
A \varphi = \alpha \varphi \quad (2.2)
\]

where \( \alpha \) is a (real) number, is said to be an eigenvector. The number \( \alpha \) is an eigenvalue. If non-normalizable wave functions are admitted then we may assume that there exists a "complete" system of such eigenfunctions and eigenvalues. The set of the real eigenvalues of \( A \) is called the spectrum of \( A \).

The most important observables, we shall have to deal with, are listed in the following examples:

(a) The total energy \( H \), which is the 0-component of the total momentum operator \( P \) \( (P^0 = H = -P_0) \).

(b) The total angular momentum \( J_r \) \( (r = 1, 2, 3) \), with the properties

\[
\left[ J_1, J_2 \right] = i J_3, \quad \text{cycl.} \quad (2.3)
\]

(c) The particle number in the state \( s \) : \( N_s \). It is an operator with spectrum located at \( N_s = 0, +1, +2, \ldots \). The total particle number is obtained by summing over all the possible states of the particles

\[
N = \sum_s N_s \quad (2.4)
\]
(This summation may have to be interpreted as an integration unless one uses large boxes or similar devices).

(d) The charge in the state \( s \) is represented by an operator

\[
Q_s = N_s^{(+)} - N_s^{(-)}
\]

where \( N_s^{(+)} \) represents the particle numbers of positive and negative charge respectively. The total charge operator is then

\[
Q = \sum_s (N_s^{(+)} - N_s^{(-)}) = \sum_s Q_s \quad (2.5)
\]

while the total particle number in this case is

\[
N = \sum N_s = \sum (N_s^{(+)} + N_s^{(-)}) \quad (2.6)
\]

A quantum mechanical theory is only complete if a sufficiently large set of observables is determined. In practice this amounts to expressing this set in terms of some basic set of known operators. In wave mechanics (Dirac's form) without spin such a set is a set of canonical variables \( p \) and \( q \). In a field theory it is a set of field variables, at least this is the assumption which underlies a field theory, even though to this day nobody has succeeded in carrying this through in a consistent and satisfactory way.
(3) The various forms of the equations of motion.

The dynamical structure of the theory is contained in the Hamiltonian $H$, or more generally, in the energy momentum vector $P$. The operator $H$ determines the rate of change of the observable expectation values. Let

$$a(t) = (\varphi, A\varphi)$$

represent the expectation value of the fixed operator $A$ in the state $\varphi$. Its change in time is completely determined by the dynamical structure of the theory. This time variation may be produced by either a time variation of $\varphi$, or of $A$, or of both. Correspondingly, we have three possibilities:

(a) The Schrödinger picture:

$A$ is a constant operator $\varphi$ varies in time

(b) The Heisenberg picture:

$\varphi$ is a constant state vector, $A$ varies in time

(c) The interaction picture:

Both $\varphi$ and $A$ vary in time corresponding to a separation of the operator $H = H_0 + V$.

The connection between these three possibilities is as follows:
ad (a) In the Schrödinger picture the state vector satisfies a Schrödinger equation

\[ i \dot{\psi} = H \psi \]  \hspace{1cm} (3.1)

with the solution

\[ \psi_t = V_t \psi \]  \hspace{1cm} (3.2)

where \( \psi = \psi_0 \) and \( V_t = e^{-iHt} \) is the unitary transformation operator which transforms the state from one moment (in this case \( t=0 \)) to another \( t \).

ad (b) In the Heisenberg picture, the operator \( A(t) \) transforms in time according to

\[ A_t = V_t^* A V_t \]  \hspace{1cm} (3.3)

where \( V_t^* e^{iHt} = V_t^{-1} = V_{-t} \) is the adjoint of \( V_t \).

ad (c) In the interaction picture we have a separation of the total energy \( H = H_0 + V \) and \( A \) varies in time according to

\[ A(t) = U_t^* A U_t \left( U_t = e^{-iH_0 t} \right) \]  \hspace{1cm} (3.4)

while \( \varphi(t) \) satisfies the "Schrödinger equation in the interaction picture"

\[ i \dot{\varphi}(t) = V(t) \varphi(t) \]  \hspace{1cm} (3.5)

where

\[ V(t) = U_t^* V U_t \]  \hspace{1cm} (3.6)
with the solution

\[ \varphi(t) = U^*_t V_t \varphi \quad (\varphi(0) = \varphi) \quad (3.7) \]

We summarize this in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Schrödinger picture</th>
<th>Heisenberg picture</th>
<th>Interaction picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>State vector</td>
<td>( \varphi_t = V_t \varphi )</td>
<td>( \varphi = \text{constant} )</td>
<td>( \varphi(t) = V_t^* U_t )</td>
</tr>
<tr>
<td>Operator</td>
<td>( A = \text{constant} )</td>
<td>( A_t = V_t^* A V_t )</td>
<td>( A(t) = U_t^* A U_t )</td>
</tr>
</tbody>
</table>

**TABLE**

The time variation of state vectors and operators in Schrödinger, Heisenberg and interaction picture.

The expectation value of \( A \) always varies the same in all three cases. We may write it in the form

\[ i \dot{a}(t) = \langle \varphi, [A_t, H] \varphi \rangle \quad (3.8) \]
(4) Symmetry properties and symmetry transformations.

A symmetry of a physical system expresses itself in a symmetry transformation \( \Theta \) of the state vectors \( \varphi \). The following conditions define a symmetry transformation. A mapping of the space \( \mathcal{H} \) of state vectors onto itself is a symmetry transformation if

\[
\begin{align*}
(\text{i}) & \quad \Theta(\varphi_1 + \varphi_2) = \Theta \varphi_1 + \Theta \varphi_2 \\
(\text{ii}) & \quad |\langle \Theta \varphi_1, \Theta \varphi_2 \rangle| = |\langle \varphi_1, \varphi_2 \rangle| \\
(\text{iii}) & \quad \Theta \mathcal{H} = \mathcal{H} \Theta.
\end{align*}
\]

All three conditions have an immediate physical interpretation. The first expresses the invariance of the superposition principle, the second the invariance of the transition probability, and the third the invariance of the dynamical structure of the system.

From the first two conditions alone the following two mathematical consequences can be drawn:

(\(\alpha\)) Every symmetry transformation has an inverse \( \Theta^{-1} \).

(\(\beta\)) Every symmetry transformation is either unitary or antiunitary. In the unitary case the scalar product is invariant, in the antiunitary case it changes into the complex conjugate

\[
\begin{align*}
(\Theta \varphi_1, \Theta \varphi_2) &= (\varphi_1, \varphi_2) \quad \text{for unitary } \Theta \\
(\Theta \varphi_1, \Theta \varphi_2) &= (\varphi_2, \varphi_1) \quad \text{for antiunitary } \Theta.
\end{align*}
\]
Proof of (α)

Since the mapping Θ is onto K, we must only show that \( \varphi_1 \neq \varphi_2 \Rightarrow \Theta \varphi_1 \neq \Theta \varphi_2 \), so that to every \( \varphi \in K \) corresponds a unique element \( \Theta^{-1} \varphi \) of which \( \varphi \) is the image under \( \Theta \).

Assume now \( \psi = \varphi_1 - \varphi_2 \neq 0 \). Then

\[
\| \Theta \psi \|^2 = |(\Theta \psi, \Theta \psi)|^2 = (\psi, \psi) = \| \psi \|^2 \neq 0. \text{ by (i)''}
\]

Hence \( \Theta \psi \neq 0 \)

But \( \Theta \psi = \Theta (\varphi_1 - \varphi_2) = \Theta \varphi_1 - \Theta \varphi_2 \) \quad \text{by (i)}

Thus \( \Theta \varphi_1 = \Theta \varphi_2 \), q.e.d.

Proof of (β)

Let \( (\Theta \varphi_1, \Theta \varphi_2) = a' + ib' \)

\( (\varphi_1, \varphi_2) = a + ib \)

Condition (ii) gives \( a'^2 + b'^2 = a^2 + b^2 \), while from (i) we obtain \( a = a' \). Consequently \( b = i b' \), q.e.d.

Which of the two cases is needed for a given symmetry property depends on the type of symmetry. We shall see below that the time-reversal symmetry is always antiunitary, while all the other types of symmetry are necessarily expressed by unitary transformations.
IV. THEORETICAL.

IV₂. Field Theory. \hspace{1cm} J.M. Jauch
IV$^2$. Field theory.

(1) The creation and annihilation operators for bosons.

One of the basic properties of fundamental particle systems are the processes of creation and destruction of various particles in collision processes. Any theory of such particles must therefore contain in its basic ingredients a set of operators which describe these transition processes. These are the creation and annihilation operators. In the case of Bosons they are defined as follows.

Let $\varphi(n)$ be a state vector which represents the presence of exactly $n$ Bosons in a given fixed state $(n = 0, 1, \ldots \infty)$. We define the operators $a$ and $a^*$ by the following conditions

\[ a \varphi(n) = \sqrt{n} \varphi(n-1) \]  \hspace{1cm} (1.1)
\[ a^*(n) = \sqrt{n+1} \varphi(n+1) \]

One verifies the following simple consequences:

(\alpha) The operators $a$, $a^*$ satisfy the commutation rules

\[ [a, a^*] = I \quad \text{(unit operator)} \]  \hspace{1cm} (1.2)

(\beta) $a \varphi(0) = 0$ \quad (zero vector)  \hspace{1cm} (1.3)

\[ \text{The state } \varphi(0) \text{ represents the state with no particles present.} \]

(\gamma) $a^* a \varphi(n) = n \varphi(n)$  \hspace{1cm} (1.4)

The operator $N = a^* a$ is called the particle number.
The operators can be generalized to the case of more than one state: Let $n_s$ be the number of Bosons in the state $s$ and $\varphi(n_1, n_2, \ldots, n_s, \ldots)$ the state vector representing $n_1$ Bosons in state 1, $n_2$ Bosons in state 2, etc. Then we define

$$a_s(n_1, \ldots, n_s, \ldots) = \sqrt{n_s}(n_1, \ldots, n_s-1, \ldots)$$
$$a_s^*(n_1, \ldots, n_s, \ldots) = \sqrt{n_s+1}(n_1, \ldots, n_s+1, \ldots)$$

for all $s$.

The following consequences are immediately verifiable:

$$(\alpha) \quad \left[ a_s, a_{s'} \right] = \left[ a_s^*, a_{s'}^* \right] = 0$$
$$\left[ a_s, a_{s'}^* \right] = \delta_{ss'}$$

$$(\beta) \quad a_s \varphi(0,0,\ldots,0,\ldots) = 0$$

The state $\varphi(0,0,\ldots,0,\ldots)$ represents the vacuum.

$$(\gamma) \quad a_s^* a_s \varphi(n_1, \ldots, n_s, \ldots) = n_s \varphi(n_1, \ldots, n_s, \ldots)$$

The operator $N_s = a_s^* a_s$ is called the particle number in state $s$.

We obtain the basic variables of a Boson field if we let $s$ represent all the possible states of a particle, for instance the total momentum vector $k$ and whatever internal degrees of freedom $r$ may be present in addition. We write then $a_r(k)$.

We obtain a charged field if we introduce two different kinds of Bosons with $+$ charges respectively and correspondingly two different sets of operators $a_r(k), b_r(k)$.
The operators satisfy then the commutation rules

\[
\left[ a_r(k) , a_s(\ell) \right] = \left[ a_r^*(k) , a_s^*(\ell) \right] = 0 \\
\left[ a_r(k) , a_s^*(\ell) \right] = \delta_{rs} \delta(k-\ell).
\]

(1.9)

and a similar set for the b's.

The vacuum state \( \varphi_o \) is defined by

\[
a_r(k) \varphi_o = b_r(k) \varphi_o = 0 \quad \text{for all} \quad r,k
\]

(1.10)

and

\[
N_r^{(+)}(k) = a_r^*(k) a_r(k) \\
N_r^{(-)}(k) = b_r^*(k) b_r(k)
\]

(1.11)

represent respectively the number of particles with positive and negative charge in the state \( r,k \).

The particular case of a charged scalar field is obtained by restricting the index \( r \) to one value only. In this case it can be dropped. A field with both signs of charges and with neutral particles (pion field) can best be described by letting the index \( r \) stand for these values \( r = \pm 1, 0 \) corresponding to the three charge values. In the Boson case we shall only be concerned with charged or neutral scalar fields (this means spin zero particles).

We could have chosen any other set of states and defined corresponding operators. For instance it is often useful to refer to the eigenstates of total angular momentum. Instead of \( k \) we have for labels \( j, m, k \) representing respectively the values of the total angular momentum, its 3-component and the total momentum. There exists then a definite transformation formula from one set of operators to the other.
(2) The creation and annihilation operators for Fermions.

Fermions satisfy the Pauli principle and therefore every distinct state can be occupied at most with one particle. The state vector for a system of Fermions is therefore \( \varphi (n_1, n_2, \ldots, n_s, \ldots) \) where the numbers \( n_s = 0, 1 \). We define

\[
\begin{align*}
    a_s \varphi (n_1 \ldots n_r \ldots) &= \sigma_s n_s \varphi (n_1, \ldots, n_s^{-1}, \ldots) \\
    a_s^* \varphi (n_1 \ldots n_r \ldots) &= \sigma_s (1-n_s) (n_1, \ldots, n_s+1, \ldots)
\end{align*}
\]  
\( (2.1) \)

The factor \( \sigma_s \) is a sign factor determined by

\[
\sigma_s = (-1)^{\alpha_s} \quad \alpha_s = \sum_{t=1}^{s-1} n_t
\]

The sign factors are so chosen that these operators satisfy the anticommutation rules

\[
\begin{align*}
    \{ a_s, a_s' \} &= \{ a_s^*, a_s^* \} = 0 \\
    \{ a_s, a_s^* \} &= \sigma_{ss'}
\end{align*}
\]  
\( (2.2) \)

We obtain the basic variables of a Fermion field if we let \( s \) represent all the possible states of a Fermion particle, for instance the total momentum \( \mathbf{k} \) and the spin states (which are always present for a Fermion !) labelled for instance by \( r \). We write then \( a_r^{(k)} \). These operators satisfy then anticommutation rules

\[
\begin{align*}
    \{ a_r^{(k)}, a_s^{(\ell)} \} &= \{ a_r^{(k)}, a_s^{(\ell)} \} = 0 \\
    \{ a_r^{(k)}, a_s^{(\ell)} \} &= \delta_{rs} \delta (\mathbf{k} - \mathbf{\ell})
\end{align*}
\]  
\( (2.3) \)
Remark:

With equations such as (2.3) and (1.9) we have dropped any pretence of mathematical rigour. We shall not even try to define a continuous infinite product of Hilbert spaces as is implied in (1.9) or (2.3).

(3) Scalar and spinor field operators.

In classical language a "field" is a system of variables, depending on the space time coordinates \( x \), and describing the state of a physical system.

In quantum theory a "field" becomes a system of operators, describing the state of the system, and depending on the space time coordinates. These operators take the rôle of the basic system of operators such that any observable should be expressible as a function (or functional) of the basic set.

The operators may or may not themselves represent observables, we shall see that they never do for a spinor field.

We begin with a charged scalar field and define the field operators in terms of the creation and annihilation operators by setting

\[
\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{2\omega} (a(k)e^{ikx} + b^*(k)e^{-ikx})
\]

\[
\omega = \sqrt{k^2 + m^2} = k^0
\]
The motivation for the particular choice of normalization factor and integrand are to obtain normalized commutation rules and simple transformation properties under Lorentz transformations. With this normalization we have for instance

$$\left[ \phi(x), \phi^*(y) \right] = -i \delta(x-y) \quad (3.2)$$

$$x_0 = y_0$$

which allows us to interpret $\phi(x)$ as the canonically conjugate variables to $\phi^*(x)$.

Under a homogeneous Lorentz transformation $L$ which transforms the four-vector $x$ into $x' = Lx$ we have

$$\phi'(x') = \phi(x)$$

or

$$\phi'(x) = \phi(L^{-1}x) \quad (3.3)$$

Sometimes it is more convenient to employ the four-dimensional commutation rules

$$\left[ \phi(x), \phi^*(y) \right] = -i \Delta (x-y) \quad (3.4)$$

where

$$\Delta (z) = \frac{1}{(2\pi)^4} \int_{\mathbb{R}^2} \frac{ipz}{p^2 + m^2} d^4p$$

is the four-dimensional $\Delta$-function of Jordan and Pauli. The path of integration over the $p^0$-coordinate is to be defined in the complet $p^0$-plane as indicated in the figure ($p^0 = p^2 + m^2$)

```
Path of integration in (3.4)
```
The field functions satisfy the field equations:

\[(\Box - m^2)\phi = 0.\]  \hspace{1cm} (3.5)

In case of the spinor field we introduce first the spinor amplitudes of plane waves \(u\) and \(v\) defined in the following manner:

Let \(\gamma_\mu\) \((\mu = 0, 1, 2, 3)\) be a set of 4x4 matrices satisfying the anticommutation rules

\[\{ \gamma_\mu, \gamma_\nu \} = 2g_{\mu\nu}.\]  \hspace{1cm} (3.6)

Denote with \(k^\mu\) the contraction of the four vector \(k^\mu\) with \(\gamma_\mu\), thus \(k^\mu = k_\mu\gamma_\mu\) (summed over \(\mu\)). For each \(k\) we have a pair of homogeneous linear equations \(k^0 = \sqrt{k^2 + m^2}\)

\[(ik^\mu + m)u = 0\]  \hspace{1cm} (3.7)

\[(ik^\mu - m)v = 0\]

each of which defines (up to a factor) two linearly independent four component quantities \(u\) and \(v\). If it is necessary to distinguish the two solutions, we write an index \(u_\tau, v_\tau\) \((\tau = \pm)\).

We define the spinor field \(\psi(x)\) by

\[\psi(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \sum_\tau \left( a_\tau(k)u_\tau e^{ikx} + b_\tau^*(k)v_\tau e^{-ikx} \right).\]  \hspace{1cm} (3.8)

The operators \(a\) and \(b\) are the destruction operators for the particles and antiparticles respectively. Which of the two is particle and antiparticle is arbitrary and conventional.
(4) Dirac matrices.

We summarize here a few useful facts on the Dirac matrices \( \gamma_\mu \), which satisfy the commutation rules

\[
\{ \gamma_\mu, \gamma_\nu \} = 2 \gamma_\mu \gamma_\nu
\]  

(4.1)

It is easy to construct explicit representations of the \( \gamma_\mu \), for instance by setting (standard representation):

\[
\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \gamma_0 = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (i = 1, 2, 3)
\]

(4.2)

where

\[
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

are the Pauli spin matrices.

From a given representation one obtains a host of others by a similarity transformation with a non-singular matrix \( S \)

\[
\gamma'_\mu = S \gamma_\mu S^{-1}
\]

(4.3)

It is a mathematical theorem (proof: cf. Jauch-Rohrlich Appendix 2) that any other \( 4 \times 4 \) representation is obtained in this manner from the standard one.

There are three operations on the \( \gamma \) -matrices which occur often and which define three matrices \( A, B, \) and \( C \)

\[
A \gamma_\mu A^{-1} = -\gamma^*_\mu \\
B \gamma_\mu B^{-1} = \gamma_\mu \}
\]

(4.4)
The arbitrary factor in the definitions of these matrices can be chosen so that

\[ A^+ = A \]
\[ B = -B \]
\[ C^* C = 1 \] (4.5)

Under proper Lorentz transformations \( L \) which transforms a four vector \( x \) into \( x' = Lx \) the spinor field is to be transformed according to

\[ \psi'(x') = S \psi(x) \] (4.6)

where \( S \) is defined (up to a factor) by

\[ S^\mu \gamma_\mu = S^{-1} \gamma^\mu S \] (4.7)

The factor can be so normalized that the \( S(L) \) form a representation (up to \( \mathbb{1} \)) of the Lorentz group.

The following relations are consequences of these definitions

\[ S^+ A = A S^{-1} \] (4.8)
\[ S^* C = C S \] (4.9)

Furthermore we note for later use

\[ C^* u_r^* = v_r \] (4.10)
\[ C^* v_r^* = u_r \] (4.11)

The matrix \( A \) is used for the definition of the adjoint spinor

\[ \bar{\psi} = \psi^+ A \] (4.12)
It is to be considered a row, while \( \psi \) is a column. Relation (4.8) implies then that \( \overline{\psi} \psi \) transforms under \( L \)-transformation like a scalar. Similarly \( \overline{\psi} \Gamma_{\mu}^{\nu} \psi \) is a vector, \( \overline{\psi} \delta_{\lambda} \delta_{\eta} \psi \) a tensor of second rank, \( \overline{\psi} \delta_{\mu} \delta_{\nu} \delta_{\lambda} \psi \) a tensor of third rank and \( \overline{\psi} \delta_{\mu} \psi \) (\( \delta_{\mu} = \delta_{0} \delta_{1} \delta_{2} \delta_{3} \)) a pseudoscalar.

For later use the following fact is also useful: the trace of any product of \( \gamma \)-matrices is zero except for a multiple of the unit matrix.

Some simple consequences of this are

\[
\frac{1}{4} \text{Tr} \gamma_{1} \gamma_{2} = a_{1} \cdot a_{2} \\
\text{Tr} \gamma_{1} \gamma_{2} \cdots \gamma_{2n+1} = 0 \quad (4.13)
\]

\[
\frac{1}{4} \text{Tr} (\gamma_{1} \gamma_{2} \epsilon_{3} \gamma_{4}) = (a_{1} \cdot a_{2})(a_{3} \cdot a_{4}) - (a_{1} \cdot a_{3})(a_{2} \cdot a_{4}) + (a_{1} \cdot a_{4})(a_{2} \cdot a_{3})
\]

If the \( u \) and \( v \) are normalized in such a manner that

\[
\overline{u}_{r} u_{s} = \frac{m}{\mathcal{E}} \quad rs = -\overline{v}_{r} v_{s} \quad (4.14)
\]

then the \( (x) \) defined by (3.8) satisfy the commutation rules

\[
\{ (x), \overline{\psi}(y) \} = iS(x-y) \quad (4.15)
\]

\[
S(z) = (\mathcal{O} - m) \Delta(z) \quad (4.16)
\]

\[
\mathcal{O} = \partial^{\mu} \gamma_{\mu}
\]

The field-.vector functions \( \psi(x) \) satisfy the field equations

\[
(\mathcal{O} + m) \psi = 0 \quad (4.17)
\]
(5) Special spinor fields.

(a) Field with vanishing rest mass.

In the case of vanishing rest mass there are some special properties of the spinor field which may be noted.

For instance the definition of the spinor amplitudes (3.7) becomes identical for \( u \) and \( v \). Furthermore the relations (4.14) degenerate into

\[
\overline{u}_r u_s = \overline{v}_r v_s = 0 \quad (r, s = \uparrow) \tag{5.1}
\]

The field equations for a massless spinor are

\[
\Box \psi = 0 \tag{5.2}
\]
in place of (4.17).

(b) The Majorana field.

Consider a spinor field \( \psi \) satisfying the equation of motion

\[
(\Box + m)\psi = 0 \tag{5.3}
\]

and the transformation law under Lorentz transformations

\[
\psi'(x') = S \psi(x) \tag{5.4}
\]

The field \( \psi^c = c^* \psi^* \) has then the property that is too satisfies the same equation of motion and transformation law, that is

\[
(\Box + m)\psi^c = 0 \tag{5.3}^c
\]

\[
\psi^c'(x') = S \psi^c(x) \tag{5.4}^c
\]
Indeed, \((5.3)^c\) is an immediate consequence of the defining property and \((5.4)^c\) follows from \((4.9)\).

We obtain therefore with

\[
\psi = \psi_1 + \psi_2 \quad (5.5)
\]

\[
\psi_1 = \frac{1}{2} (\psi + \psi^c) \quad (5.6)_1
\]

\[
\psi_2 = \frac{1}{2} (\psi - \psi^c) \quad (5.6)_2
\]

An invariant splitting of the spinor field \(\psi\) into two fields which satisfy

\[
\psi_1^c = \psi_1 \quad (5.7)
\]

\[
\psi_2^c = -\psi_2 \quad (5.8)
\]

A field which satisfies \((5.7)\) is called a **Majorana field**.

If \(\psi\) is a Majorana field then it follows from \((4.10)\) that

\[
a_r(k) = b_r(k) \quad \text{for all } r \text{ and } k \quad (5.5)
\]

Thus we see:

In a Majorana field, the particles and antiparticles are identical.
(c) The Weyl-field. (Two-component spinor field)

We can ask the question: What linear transformations of the $\Psi$-field commute with the transformations $S$ associated with the Lorentz transformations?

The answer follows from (4.3): There are exactly two of the 16 $\gamma$-matrices which commute with $S$, viz. $I$ and $\gamma_5$. Hence any linear combination of them commutes also with $S$. The special linear combinations $\frac{1}{2}(1+i\gamma_5)$ are projections (and only those). Hence we can split every $\Psi$-field in a Lorentz invariant manner into two by setting

$$\Psi = \Psi_+ + \Psi_- \quad (5.10)$$

$$\Psi_\pm = \frac{1}{2} (1 \pm i\gamma_5) \Psi \quad (5.11)$$

Because $\gamma_5$ anticommutes with $\gamma_\mu$, the equation of motion appears as

$$\not\!D \Psi_+ + m \Psi_- = 0 \quad (5.12)$$

$$\not\!D \Psi_- + m \Psi_+ = 0$$

Under space reflections the subspaces $\Psi_+$ and $\Psi_-$ interchange.

Since the equation of motion mixes the two fields, the splitting has no physical significance unless the mass of the particle is identically zero. We arrive thus at the two-component neutrino field $\varphi$ for which, for instance

$$\not\!D \varphi = 0 \quad (5.13)$$

and

$$(1-i\gamma_5) \varphi = 0$$
This equation can also be cast into the equivalent form

\[ \partial_0 \varphi = \nabla \cdot \vec{\sigma} \varphi \\
(1 - i \gamma_5) \varphi = 0 \]  \hspace{1cm} (5.14)

with \( \gamma_1 \gamma_2 = i \gamma_3 \), ... etc.

In the special representation of the \( \gamma \)-matrices in which \( \gamma_5 \) is diagonal, that is

\[ \gamma_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \quad (i=1,2,3) \quad \gamma_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \]

\[ \gamma_5 = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

The equation (5.14) reduces to a two-component equation:

\[ \partial_0 \varphi = \nabla \cdot \vec{\sigma} \varphi \]  \hspace{1cm} (5.15)

the \( \vec{\sigma} \) being the ordinary 2\( \times \)2 spin matrices.

(d) The Pauli transformation.

We define as canonical transformation any transformation of a field \( \varphi \rightarrow \varphi' \) which leaves the commutation rules invariant. For instance the transformation \( \varphi \rightarrow \varphi' \) is canonical.
In the case of vanishing rest mass there exist additional canonical transformations. For instance the transformation

\[ \psi \rightarrow \psi \frac{1}{2} \psi \]

or more generally the transformation

\[ \psi \rightarrow e^{ir} \psi = (\cos \alpha + i \sin \alpha) \psi \]  \hspace{1cm} (5.16)

is canonical. This has been known for a long time.

Recently Pauli pointed out the existence of an additional transformation

\[ \psi \rightarrow \alpha \psi + \beta \sigma \psi^c \]  \hspace{1cm} (5.17)

which is also canonical provided

\[ |\alpha|^2 + |\beta|^2 = 1 \]

and

\[ m = 0 \]  \hspace{1cm} (5.18)

The interesting feature about this transformation is that it mixes the particles with the antiparticles.

The creation and annihilation operators are transformed under (5.17) according to

\[ a_r' = \alpha a_r - i \beta b_r \]

\[ b_s' = \alpha^* b_s - i \beta^* a_s \]  \hspace{1cm} (5.19)

In this form the canonical character of the transformation is easily recognized.

The existence of this transformation implies that the transition processes between states which do not distinguish the neutrino from the antineutrino (for instance double \( \beta \)-emission) the result can only depend on certain quadratic expressions of the coupling constants.
(e) The relation of the Majorana to the Weyl theory.

There exists a simple relation between the Majorana and the Weyl theory.

Let us consider a Majorana field $\psi$ so that $\psi^c = \psi$.

We can split this field with the projection

$$\psi = \psi_1 + \psi_2$$

(5.20)

$$\psi_1 = \frac{1}{2} (1 + i \gamma_5) \psi$$

(5.21)

$$\psi_2 = \frac{1}{2} (1 - i \gamma_5) \psi$$

From the field equation

$$\partial \psi + m \psi = 0$$

and $\psi^c = \psi$

we obtain after some simple algebra the coupled system

$$\partial_0 \psi_1 - \nabla \cdot \psi_1 + m \gamma_0 \psi_2 = 0$$

$$\partial_0 \psi_2 + \nabla \cdot \psi_2 + m \gamma_0 \psi_1 = 0$$

(5.22)

and

$$\psi_1^c = \psi_2$$

(5.23)

Here

$$\nabla_1 = -i \gamma_2 \gamma_3, \ldots \text{ cycl.}$$

Note that because of (5.23) the second equation is a consequence of the first. We can thus replace it by

$$\partial_0 \psi - \nabla \cdot \psi + m \gamma_0 \psi^c \psi^* = 0$$

(5.24)
This equation can easily be replaced by a two-component equation, for instance by choosing the special representation

\[ \gamma_k = \begin{pmatrix} \sigma_k & 0 \\ 0 & -\sigma_k \end{pmatrix}, \quad \gamma_\circ = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \]

in which

\[ c = c^* = \gamma_\circ \gamma_1 \sigma_3 = \sigma_2 \gamma_5 \]

\[ \gamma_\circ c^* = -\gamma_1 \sigma_3 = +i\sigma_2 \]

In this representation the system decomposes into two pairs of identical two-component systems \((\sigma = \text{two-component spin matrices})\)

\[ \partial_\circ \varphi - \vec{\sigma} \cdot \nabla \varphi + m i \sigma_2 \varphi^* = 0 \quad (5.25) \]

This system was first given by H. Jehle \((\text{Phys. Rev. 75, 1609 (1949)})\) and was further discussed by J. Serpe \((\text{Physica 18, 295 (1952)})\) and K.M. Case \((\text{Phys. Rev. 107, 307 (1957)})\).

The Weyl equation is now obtained by setting \(m=0\), but it may be observed that one can already write a two component equation for a finite mass provided one admits the process of complex conjugation. This is the essence of Case's paper.
STRANGE PARTICLE PHYSICS

IV. THEORETICAL.

IV3. Observables in field theory. J.M. Jauch
IV. Observables in field theory.

If the field is a complete description of a physical system it should be possible to express the observables in terms of the field variables, and there should exist a complete system of such observables. Whether this is true for the conventional field theory is an open question.

There are certain observables, however, for which one can construct the expressions in terms of the field variables already from the known transformation properties of the field operators. These are the observables associated with the mechanical quantities of the total system, such as the total momentum, total energy, and total angular momentum. It is also possible to separate in an unambiguous manner the spin from the orbital angular momentum. (cf. Jauch-Rohrlich, Section 1.10). We shall not do this here since we need only the total linear momentum and the charge operator and these can be obtained directly from the physical interpretation of the creation and annihilation operators.

(1) Total momentum for a scalar field.

We assume that we have a complex scalar field $\phi(x)$ representing spin-less particles of both signs of the charge.

We define as the total momentum operator the expression

$$P^\mu = \int d^3k \left( N_+(k) + N_-(k) \right) k^\mu$$  \hspace{1cm} (1.1)

We recall that $N_+(k) = b^*(k) b(k)$ represents the particle number operator of the positive particles and $N_-(k) = a^*(k) a(k)$ the corresponding operator for the negative particles. Thus (1.1) represents the sum of the contributions to the total momentum from all particles which can be present.
The expression (1.1) can also be written in terms of the field operators \( \psi(x) \). We have indeed up to a constant

\[
P^r = - \int \psi \, \partial^r \psi^* \, d^3x \quad (r = 1, 2, 3)
\]

\[
P^0 = \frac{1}{2} \int \left( \partial^\mu \psi^* \cdot \nabla \psi + \mu^2 \psi^* \psi \right) \, d^3x
\]

(1.2)

If we set the constant equal to zero we have defined the product of the non-commuting operators as "ordered products". This means: "In any quadratic expression of field operators substitute emission and absorption operators and write all emission operators on the left and absorption operators on the right". If the products in (1.2) are defined in this way then (1.2) is equivalent to (1.1).

We can interpret the integrands in (1.2) as the densities (in \( x \)-space) of the corresponding quantities. Thus

\[
\mathcal{H}(x) = \frac{1}{2} \left( \partial^\mu \psi^* \cdot \nabla \psi + \mu^2 \psi^* \psi \right)
\]

(1.3)

represents the energy density of the scalar field.

(2) The total charge for a scalar field.

The total charge operator is defined as

\[
Q = \int \left( N_+^*(k) - N_-^*(k) \right) \, d^3k
\]

(2.1)

Again the physical interpretation is obvious in this form. If the expression is transformed into \( x \)-space, it becomes

\[
Q = i \int \left( \dot{\psi}^* \phi - \dot{\psi} \phi^* \right) \, d^3x
\]

(2.2)
It is natural to generalize the integrand and denote

\[ J^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) \]

as the charge current density, so that \( J^0 \) equals the charge density. The four-current satisfies a conservation law

\[ \partial^\mu J_\mu = 0 \]

by virtue of the field equations.

(3) **Total momentum and charge for a spinor field.**

The procedure of obtaining the total momentum and charge operators for a spinor field is the same as for a scalar field: Find first the expression in momentum space, using the physical interpretation of the emission and absorption operators, and then transform into \( \mathbf{x} \)-space. The result:

\[ P^\mu = \sum_k \int d^3k \left( N_r^{(+)}(k) + N_r^{(-)}(k) \right) k^\mu \]  \hspace{1cm} (3.1)

\[ N_r^{(+)}(k) = b^*_r(k) b_r(k) \]

\[ N_r^{(-)}(k) = a^*_r(k) a_r(k) \]  \hspace{1cm} (3.2)

\[ P = + \int \overline{\psi} \gamma^0 \gamma^\mu \psi d^3x \]  \hspace{1cm} (3.3)

\[ Q = \sum \int d^3k \left( N_r^{(+)}(k) - N_r^{(-)}(k) \right) \]  \hspace{1cm} (3.4)

\[ Q = -i \int \overline{\psi} 0^0 \psi d^3x \]  \hspace{1cm} (3.5)
The current density is

\[ J^\mu = -i \overline{\psi} \gamma^\mu \psi \]  

(3.6)

It satisfies the conservation law

\[ \partial_\mu J^\mu = 0 . \]
IV. THEORETICAL.

IV\textsubscript{4}. The transformations $P.T.$ and $C$. 

J. M. Jauch
IV. The transformations P, T, and C.

(1) Symmetry transformations of state vectors and operators.

We interpret a symmetry transformation in the "active" sense, meaning that we consider it an actual change of a physical state of the system. As a result the expectation values of all operators change too and, insofar as they have a classical analogue, this change corresponds to the classical symmetry transformation.

Just as in the equation of motion we can express the change of a physical state either as a change of the state vector or a change of the operators corresponding to the Schrödinger or the interaction picture.

Let us start with the former. The symmetry transformation is then expressed as a unitary or antiunitary transformation operating on the state vectors

\[ f \rightarrow f' = \Theta f \]  \hspace{1cm} (1.1)

and leaving the operators unchanged. The expectation value

\[ a = (f, Af) \]  \hspace{1cm} (1.2)

of the operator \( A \) changes then according to

\[ a' = (f', Af') \]  \hspace{1cm} (1.3)

We ask now what are the corresponding transformations of the operators \( A \rightarrow A' \) such that they produce the same transformations of the expectation values?

We distinguish the two cases whether \( \Theta \) is unitary or antiunitary.
(i) $\Phi$ is unitary

From

$$a' = (f', A'f') = (\Phi f, A\Phi f) = (f, A'f)$$

we find

$$f, A'f) = (f, \Phi^{-1} A\Phi f) \quad (1.4)$$

By the process of polarization we can extend this to all pairs of matrix elements

$$f, A'g) = (f, \Phi^{-1} A\Phi g) \quad (1.5)$$

which means the operator relation

$$A' = \Phi^{-1} A \Phi \quad (1.6)$$

(ii) $\Phi$ is antiunitary

In this case we have also

$$(f, A'f) = (\Phi f, A\Phi f),$$

but because of the antiunitary character of $\Phi$, we obtain now

$$(f, A'f) = (A^* \Phi f, \Phi f) = (f, \Phi^{-1} A^* \Phi f)$$

and from this, as above, the operator relation

$$A' = \Phi^{-1} A^* \Phi \quad (1.7)$$

($A^*$ is the Hermitian conjugate of $A$)
Remarks:

(i) In case the operator $A$ is self adjoint (e.g. an observable) the two transformation laws of the operators (1.6) and (1.7) are identical in form. But we must always remember that in the second case $\Theta$ is an antiunitary operator.

(ii) In case $\Theta$ is antiunitary we can always decompose $\Theta = UJ$ where $U$ is unitary and $J$ a complex conjugation. If we define the complex conjugation of an operator by

$$A^* = J^{-1}AQJ$$

and

$$A = J^{-1}A^*J$$

then we have also instead of (1.7)

$$A' = U^{-1}A^*U$$

(1.7)

(2) Space Inversion.

(1) Scalar field.

We desire to find the transformation law of a scalar field operator which describes the transformation of space inversion. This is obtained by first considering the transformation law for state vectors derived from their physical interpretation and then transfer it to the transformation of the field operators.
Space inversion is defined as the transformation

\[ x_r \rightarrow x'_r = -x_r \quad (r=1,2,3) \]
\[ x_0 \rightarrow x'_0 = x_0 \]  \hspace{1cm} (2.1)

A particle with momentum \( k \) has in the transformed system the momentum \(-k\). Such a particle will be represented by a wave function

\[ \Omega_k = \begin{cases} 
  a(k)\Omega_0 \\
  b(k)\Omega_0 
\end{cases} \]  \hspace{1cm} (2.2)

where \( \Omega_0 \) is the vacuum state. Space inversion is thus represented by an operator \( \Pi \) which transforms \( \Omega_k \) into

\[ \Pi \Omega_k = \chi \Omega_{-k} \]  \hspace{1cm} (2.3)

In addition we require

\[ \Pi H = H \Pi \]  \hspace{1cm} (2.4)

and

\[ \Pi V_t = V_t \Pi \]
\[ (V_t = e^{-iHt}) \]  \hspace{1cm} (2.5)

(2.4) and (2.5) together imply that the transformation \( \Pi \) must be unitary.

Finally it is convenient to assume that the vacuum is invariant under any symmetry transformation:

\[ \Pi \Omega_0 = \Omega_0. \]

The square of the transformation \( \Pi \) is the identity transformation and therefore we have to require

\[ \chi^2 = 1 \]
\[ \chi = \pm 1 \]
We obtain therefore two cases ($\alpha = \pm 1$). The case $\alpha = +1$ is a scalar field and $\alpha = -1$ is a pseudoscalar field.

From the transformation law of the state vectors we obtain that for the operators (we discuss only the scalar case explicitly and add the pseudoscalar case at the end)

\[ \Gamma^{-1}a(\bar{k}) \Gamma = a(-\bar{k}) \]
\[ \Gamma^{-1}b(\bar{k}) \Gamma = b(-\bar{k}) \]  \hspace{0.5cm} (2.4)'

The field operators $\phi(x)$ transform then according to

\[ \phi(x) \rightarrow \phi'(x) = \Gamma^{-1}\phi(x) \Gamma \]

Substituting (2.4)' into (5.1) of section II, we find

\[ \phi'(x) = \phi(x') \]  \hspace{0.5cm} (2.5)'

This is the transformation law for scalar or pseudoscalar fields respectively.

Remarks:

(i) There is always an ambiguity in an overall phase factor for all state vectors. Therefore strictly speaking the assertion $\Gamma^2 = I$ cannot be derived but instead amounts to a definition of the arbitrary phase in the transformation operator. This ambiguity occurs for all the transformations $P$, $C$, and $T$.

(ii) The above-mentioned ambiguity is more serious when we have physical systems with superselection rules, for then the phase factors may be different in different superselection subspaces.
The transformation (2.5)' leaves unchanged the total energy and the total charge, while the current density reverses the sign.

(ii) Spinor field.

The space inversion transformation for a spinor field is more complicated because it involves, in addition to the operators \( a \) and \( b \), also a rearrangement of the spin components in such a way that \( a_+ (k) \) transforms into \( a_- (-k) \), etc. ... expressing the fact that momentum \( k \) reverses sign but spin does not under space inversion.

\[
\begin{align*}
\begin{array}{c}
\text{Transformation of momentum and spin under space inversion} \\
\end{array}
\end{align*}
\]

The space inversion \( \overline{1} \) for a spinor field must therefore satisfy

\[
\begin{align*}
\overline{1}^{-1} a_+ (k) \overline{1} &= a_- (-k) \\
\overline{1}^{-1} a_- (k) \overline{1} &= a_+ (-k) \\
\overline{1}^{-1} b_+ (k) \overline{1} &= b_- (-k) \\
\overline{1}^{-1} b_- (k) \overline{1} &= b_+ (-k)
\end{align*}
\]

(2.6)

in addition to (2.4) and (2.5).
We can transfer this transformation of the operators $a$ and $b$ on the transformation of the field operators by using

$$i \gamma_0 u_+(k) = u_-(k), \ldots \quad (2.7)$$

and similar formulae for the other spinor functions.

Substituting (2.7) into (3.8) of section II, we find the transformation law

$$\psi'(x') = i \gamma_0 \psi(x) \quad (2.8)$$

under space inversion.

With this transformation we can now evaluate the transformation property of the five bilinear covariants. The result is as follows: We denote the five sets of $\gamma$-matrices which enter in the definition of the covariants by $\gamma_t \; (t=1, \ldots, 5)$. (Thus $\gamma_1 = 1$, $\gamma_2 = \gamma_\mu$, etc.) and by $\gamma_t'$ the set which is obtained by replacing in each $\gamma_t$ the $\gamma_\mu$ by $\gamma'_\mu$. Then we have

$$\overline{\psi}'(x') \gamma_t \psi'(x) = \overline{\psi}(x') \gamma_t' \psi(x')$$

$$(t = 1, \ldots, 5)$$

$$x'_r = -x_r, \quad x'_o = x_o$$

(This is an immediate consequence of $\gamma' = \gamma i \gamma_0$ and the commutation properties of the $\gamma_\mu$.)

In particular the current vector $J_\mu$ transforms like an ordinary vector and $\overline{\psi} \gamma_5 \psi$ is a pseudoscalar.
(3) Time reversal.

We first show that in quantum mechanics the time reversal transformation must be antiunitary. Let $\varnothing$ be the transformation in question, satisfying the "dynamical symmetry condition":

$$\varnothing H \varnothing^{-1} = H \quad (3.1)$$

Let us denote with $a^\tau(t)$ the (classical) time reversal solution which is related to the transformed quantity $a'(t)$ by

$$a^\tau(t) = a'(-t) \quad (3.2)$$

For the example of classical canonical variables this notation is identified as

$$q^\tau(t) = q'(-t) = q(-t)$$
$$p^\tau(t) = p'(-t) = -p(-t)$$

For any functions of the $q$ and $p$ the symbols are defined correspondingly:

$$a(t) = \mathcal{F}(q(t), p(t))$$
$$a^\tau(t) = \mathcal{F}(q^\tau(t), p^\tau(t))$$
$$a'(t) = \mathcal{F}(q'(t), p'(t)) = \mathcal{F}(q(t), -p(t))\mathcal{J}.$$

The quantum mechanical analogue of the relation (3.2) is exactly the same, except that the quantities $a(t)$ are now interpreted as expectation values:

$$a^\tau(t) = (V_t \varnothing f, A V_t \varnothing f)$$
$$a'(t) = (V_t f, A' V_t f) = (\varnothing V_t f, A \varnothing V_t f)$$
We shall now show that (3.1) and (3.2) are incompatible if \( \Theta \) is unitary. Indeed if \( \Theta \) is unitary we have

\[
A' = \Theta^{-1} A \Theta
\]

and therefore

\[
a'(t) = (V_t f, A' V_t f) = (V_t f, \Theta^{-1} \Theta V_t f) = (\Theta V_t f, \Theta V_t \Theta f) = (V_t \Theta f, \Theta V_t \Theta f) = a^\tau(t)
\]

this contradicts (3.2).

On the other hand if \( \Theta \) is antiunitary then

\[
\Theta V_t = V_{-t} \Theta
\]

and therefore

\[
a^\tau(t) = a'(-t),
\]

in agreement with (3.2).

For the explicit construction of the time reversal transformation we start with the physical interpretation of the field operators and we separate the two cases of a scalar and a spinor field.

(i) **Scalar field.**

A scalar particle with momentum \( k \) is under time reversal transformed into a particle of the same charge and momentum \( -k \). The antiunitary transformation with this property satisfies

\[
\Theta \omega_k = \omega_{-k}
\]

\[
\Theta \omega_0 = \omega_0
\]
or
\[ a^\dagger(x) = a^*(k) = \dagger \Theta^{-1} a(k) \Theta, \]
and a similar equation for \( b(k) \).

In terms of the field operators we have therefore

\[ \phi^\dagger(x) = \Phi^\dagger(x') \]

(3.3)

The two cases correspond to the scalar and pseudoscalar case respectively. The \( \phi^x(x) \) is the complex conjugate field to \( \phi(x) \). \( \Phi \) is not Hermitian conjugate.

(ii) The spinor field.

The transformation must reverse direction of momentum and of the spin

\[ \begin{array}{c}
\uparrow k \\
\downarrow -k \\
\end{array} \]

Transformation of momentum and spin under time reversal

Hence we must have

\[ \Theta^{-1} a_{\tau}(k) \Theta = a_{\tau}^*(\mathbf{k}), \ldots \text{ etc.} \]

This transformation can be realized by defining the field transformation

\[ \psi(x) \rightarrow \psi'(x) = D \psi^x(x') \]

(3.4)

where \( \psi^x \) is the complex conjugate field,
\[ x'_0 = -x_0, \quad x'_i = x_i \quad (i=1,2,3) , \quad (3.5) \]

and \( D \) is the spinor matrix with the property

\[ \gamma'^\ast = D^{-1} \delta \mu D \quad (3.6) \]
\[ \gamma'_0 = -\gamma_0 \]
\[ \gamma'_i = \gamma_i \quad (i=1,2,3) . \]

An explicit expression of \( D \) in terms of \( C \) is

\[ D = \gamma_5 \gamma_0 C \quad (3.7) \]

from which we find incidentally the invariant relations

\[ D^\ast D = -I \quad (3.8) \]

and

\[ (A D)^\sim = -AD \quad (3.9) \]

A consequence of this is \( \psi''(x) = -\psi(x) \) and one can show that this leads to a superselection rule for the spinor field. It follows that the operator \( \psi \) can never represent an observable. cf. Jauch-Rohrlich p.95\(^7\).

With (3.8) and (3.9) one finds for the transformation of the bilinear covariants

\[ \bar{\psi}'(x) \Gamma_t \psi(x) = \bar{\psi}(x') \Gamma'_t \psi(x') \quad (t=1,\ldots,5) \]

where

\[ x'_0 = -x_0 \]
\[ x'_r = x_r \quad (r=1,2,3) \]

and \( \Gamma'_t \) is obtained from \( \Gamma_t \) by replacing each \( \gamma_\mu \) by \( \gamma'_\mu \).
(4) Charge conjugation.

The charge conjugation is a unitary transformation which interchanges the particles with the antiparticles. Thus for scalar fields

\[ \mathcal{L}^{-1} a(k) \mathcal{L} = b(k) = a^c(k) \]  \hspace{1cm} (4.1)

and for a spinor field

\[ \mathcal{L}^{-1} a_r(k) \mathcal{L} = b_r(k) = a^c_r(k) \hspace{1cm} (r = \uparrow) \]  \hspace{1cm} (4.2)

The corresponding field transformations are given by

\[ \phi(x) \rightarrow \phi^c(x) = \phi^*(x) \]  \hspace{1cm} (4.3)

\[ \psi(x) \rightarrow \psi^c(x) = c^* \psi^*(x) \]  \hspace{1cm} (4.4)

Under charge conjugation the bilinear covariants of the spinor field transform according to

\[ \overline{\psi}^c \mathcal{L} \psi^c = \sigma_t \overline{\psi} \mathcal{L} \psi \]  \hspace{1cm} (4.5)

where the sign factor \( \sigma_t \) has the following values for the five covariants:

<table>
<thead>
<tr>
<th>t</th>
<th>covariant</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S</td>
<td>+1</td>
</tr>
<tr>
<td>2</td>
<td>V</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>T</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>+1</td>
</tr>
<tr>
<td>5</td>
<td>P</td>
<td>+1</td>
</tr>
</tbody>
</table>

**TABLE**
Transformation of bilinear covariants under charge conjugation.
In the derivation of this result it is essential to define the bilinear covariants antisymmetrized with respect to the two factors. It is seen in particular that the current density reverses the sign under charge conjugation as is implied in the terminology "charge-conjugation".

\[ (\Psi \Gamma_t \psi)^* = (\overline{\psi} \Gamma_t \psi) \]  \hspace{1cm} (5.1)

For this to be valid we must define

\[ \Gamma_1 = 1, \quad \Gamma_2 = \{ i \sigma_{\mu} \}, \quad \Gamma_3 = \{ i \sigma_{\mu} \gamma_{\nu} \}, \quad \Gamma_4 = \{ \sigma_{5} \gamma_{\mu} \}, \quad \Gamma_5 = \gamma_{5} \]

We consider now local interactions which can be formed with a complex scalar and spinor field and determine the conditions for invariance under the transformations P, T, and C.

Only the following types of interactions will be explicitly discussed:

\begin{itemize}
  \item[(i)] Interactions of the form \[ \phi \overline{\Psi} \psi \]
  (including first order derivatives of \( \phi \))
  \item[(ii)] Interactions of the form \( (\overline{\Psi} \psi)(\overline{\Psi} \psi) \)
\end{itemize}
ad (i)

The general interaction of this type involves a term of the form

\[ L = \varepsilon_1 \phi \bar{\psi} \psi + \varepsilon_2 i \frac{\partial}{\partial \mu} \phi \bar{\psi} \gamma^\mu \psi \]

\[ + \varepsilon_1 \phi \bar{\psi} \gamma^5 \psi + \varepsilon_2 \phi \bar{\psi} \gamma^5 \gamma^\mu \psi \quad + \text{Herm. conj.} \]

(5.2)

in the Lagrange density.

We shall now determine separately the conditions for invariance of the theory under P, T, and C.

In order to simplify the notation we shall use the following convention: The transformed Lagrangian density at the transferred space time point is obtained by substituting the transformed field operators. The primes and the arguments of the field variables will be omitted. Invariance will then be verified if the transformed Lagrangian has the same form as the original Lagrangian.

\[ (\Box) \quad \text{Invariance of (5.2) under } P \]

The space reflection transforms (5.2) into

\[ L' = \varepsilon_1' \phi \bar{\psi} \psi + \varepsilon_2 i \frac{\partial}{\partial \mu} \phi \bar{\psi} \gamma^\mu \psi \]

\[ - \varepsilon_1' \phi \bar{\psi} \gamma^5 \psi - \varepsilon_2 \phi \bar{\psi} \gamma^5 \gamma^\mu \psi \quad + \text{H.C.} \]

(5.2)^P

The two signs correspond to the two possibilities whether \( \phi \) is a scalar or a pseudoscalar.

Invariance under \( P \) requires therefore

\[ \varepsilon_1' = \varepsilon_2 = 0 \quad \text{for } \phi \text{ scalar} \]

\[ \varepsilon_1 = \varepsilon_2' = 0 \quad \text{for } \phi \text{ pseudoscalar} \]

(5.3)
(3) Invariance under $T$. 

Since time reversal is antiunitary the transformation $T$ will replace $L_1$ by $L_1'x$ and the invariance requirement is $L_1 = L_1'$

$$L' = \pm \varepsilon_1^* \partial \bar{\varphi} \psi + \pm \varepsilon_2^* i \partial_{\mu} \bar{\varphi} \gamma^\mu \psi$$

$$+ \varepsilon_1'^* \partial \bar{\varphi} \gamma_5 \psi + \varepsilon_2'^* \partial_{\mu} \bar{\varphi} \gamma_5 \gamma^\mu \psi + H.C.$$  \hspace{1cm} (5.2)^T

Invariance under $T$ requires

$$\varepsilon_1 = -\varepsilon_1^*, \quad \varepsilon_2 = -\varepsilon_2^* \hspace{1cm} (5.4)$$

$$\varepsilon_1' = -\varepsilon_1'^*, \quad \varepsilon_2' = -\varepsilon_2'^*$$

for $\varphi$ scalar or pseudoscalar respectively.

(\text{8}) Invariance under $C$.

$$L' = \varepsilon_1^* \partial \bar{\psi} \psi + \varepsilon_2^* i \partial_{\mu} \bar{\psi} \gamma^\mu \psi$$

$$+ \varepsilon_1'^* \partial \bar{\psi} \gamma_5 \psi + \varepsilon_2'^* \partial_{\mu} \bar{\psi} \gamma_5 \gamma^\mu \psi$$ \hspace{1cm} (5.2)^C

Invariance under $C$ implies

$$\varepsilon_1 = \varepsilon_1^*, \quad \varepsilon_2 = \varepsilon_2^* \hspace{1cm} (5.5)$$

$$\varepsilon_1' = \varepsilon_1'^*, \quad \varepsilon_2' = \varepsilon_2'^*$$
We summarize the result in the following table by expressing the effects of $P$, $T$, and $C$ on the coupling constants.

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$T$</th>
<th>$C$</th>
<th>PTC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_1$</td>
<td>$\varepsilon_1$</td>
<td>$\varepsilon_1^*$</td>
<td>$\varepsilon_1$</td>
<td>$\varepsilon_1$</td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>$\varepsilon_2$</td>
<td>$\varepsilon_2^*$</td>
<td>$\varepsilon_2$</td>
<td>$\varepsilon_2$</td>
</tr>
<tr>
<td>$\varepsilon_1'$</td>
<td>$\varepsilon_1'$</td>
<td>$\varepsilon_1'^*$</td>
<td>$\varepsilon_1'$</td>
<td>$\varepsilon_1'$</td>
</tr>
<tr>
<td>$\varepsilon_2'$</td>
<td>$\varepsilon_2'$</td>
<td>$\varepsilon_2'^*$</td>
<td>$\varepsilon_2'$</td>
<td>$\varepsilon_2'$</td>
</tr>
</tbody>
</table>

**TABLE**

Transformation of the coupling constants under $P$, $T$, and $C$

ad (ii)

The most general interaction of this type invariant under proper Lorentz transformations is

$$L = \sum_{t=1}^{5} \bar{q}_t (\bar{\psi} \Gamma_t \psi)(\bar{\psi} \Gamma_t \psi) + \sum_{t=1}^{5} \bar{q}_t' i(\bar{\psi} \Gamma_t \gamma_5 \psi)(\bar{\psi} \Gamma_t \psi) + H.C.$$  

and it involves ten coupling constants $\varepsilon_t$, $\varepsilon_t'$. ($t=1, \ldots, 5$).
(α) **Invariance under P.**

The space inversion is equivalent to the transformation of the coupling constants

$$\varepsilon_t \rightarrow \varepsilon_t$$

$$\varepsilon'_t \rightarrow \varepsilon'_t$$

Thus invariance under P requires the vanishing of all $\varepsilon'_t$.

(β) **Time reversal T.**

This transformation can be expressed as the transformation

$$\varepsilon_t \rightarrow \varepsilon^*_t$$

$$\varepsilon'_t \rightarrow \varepsilon'^*_t$$

The invariance under T requires all the coupling constants to be **real**.

(γ) **Charge conjugation C.**

The corresponding transformation of the coupling constants is

$$\varepsilon_t \rightarrow \varepsilon^*_t$$

$$\varepsilon'_t \rightarrow \varepsilon'^*_t$$
We summarize the result in the table

<table>
<thead>
<tr>
<th>( \varepsilon_t )</th>
<th>P</th>
<th>T</th>
<th>C</th>
<th>PTC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_t )</td>
<td>( \varepsilon_t )</td>
<td>( \varepsilon_t^* )</td>
<td>( \varepsilon_t^* )</td>
<td>( \varepsilon_t )</td>
</tr>
<tr>
<td>( \varepsilon_t' )</td>
<td>( -\varepsilon_t' )</td>
<td>( \varepsilon_t' )</td>
<td>( -\varepsilon_t' )</td>
<td>( \varepsilon_t' )</td>
</tr>
</tbody>
</table>

**TABLE**

Transformation of coupling constants under P, T and C

(5) The PTC - Theorem.

A glance at the last columns in the last two tables reveals a special case of the PTC-theorem, which says that (under very general assumptions) the transformation PTC is always a symmetry transformation.

The special cases considered here are sufficient for most applications to fundamental particle physics. But the theorem is of much more general validity. (cf. W. Pauli, Essay dedicated to Niels Bohr, Pergamon Press, 1955, p. 30 ff.).
STRANGE PARTICLE PHYSICS

IV. THEORETICAL.

IV5. The mass and lifetime of antiparticles. J.M. Jauch
IV.5. The mass and lifetime of antiparticles.

(1) Formulation of the Question.

Every known particle has exactly the same mass as its antiparticle and if a particle is unstable its antiparticle is unstable too with the same lifetime. If the charge conjugation C is a symmetry transformation, this is a trivial consequence of the invariance of the Hamiltonian. When the symmetry with respect to C is violated (as we know it is in weak interactions) then the mass and lifetime of particles and antiparticles are still equal, but this is now a non-trivial consequence of the CPT - theorem. The equality of these quantities is therefore no test for the invariance under C, but it is a test of the validity of the CPT - theorem. This was first pointed out by Lee, Ohme and Yang (Phys. Rev. 106, 340 (1957)).

We shall derive and discuss here a slight generalisation of this result.

(2) Mass of Stable Particles.

Let $P^\mu$ be the total momentum operation of the system and denote by $\Theta$ the product of C, P, and T:

$$\Theta = CPT$$

(2.1)

We have shown that $T$ is always anti-unitary, while $C$ and $P$ are unitary. Consequently $\Theta$ is anti-unitary too.

The mass operator is defined as:

$$M = \sqrt{-P^\mu P^\mu} ;$$

(2.2)
It is self-adjoint and because of the CPT theorem

\[ \Theta M = M \Theta \]  \hspace{1cm} (2.3)

**Note**: The transformation of a general operator \( A \) under an anti-unitary transformation was proved to be

\[ A' = \Theta^{-1} A^* \Theta \]

if it is self-adjoint and invariant \( A^* = A \), and \( A' = A \), hence

\[ A = \Theta^{-1} A \Theta \]

The one-particle states are eigenstates of the operator \( M \) with eigenvalue \( m \):

\[ M \psi = m \psi \]  \hspace{1cm} (2.4)

We wish to show that if \( m \neq 0 \) then \( C \psi \) is an eigenstate of \( M \) too, with the same eigenvalue without assuming that \( C \) commutes with \( M \). The key of the proof consists in the observation that for one-particle states \( C \) can be written as a product of \( \Theta \) with a Lorentz transformation

\[ C \psi = \Lambda \Theta \psi \]  \hspace{1cm} (2.5)

If this were established we would have

\[ M C \psi = M \Lambda \Theta \psi = \Lambda M \Theta \psi = \Lambda \Theta M \psi = m \Lambda \Theta \psi \]

Therefore \( M C \psi = m C \psi \) which is the desired relation.
The problem is thus reduced to proving a relation (2.5). It is clearly sufficient to prove (2.5) for a complete orthogonal system. A general state of such a system is characterised by the momentum of the particle and its spin-component in its direction. Let us verify (2.5) graphically in the following way (we indicate \( \bigcirc \) for particle, \( \bigotimes \) for antiparticle)

Thus

while

We must show that there exists a Lorentz transformation \( \Lambda \) with the property

\[
\begin{align*}
\bigcirc & \quad \rightarrow \\
\bigotimes & \quad \rightarrow
\end{align*}
\]
This transformation is obtained by the following three steps

(i) Transform particle to rest ($m \neq 0$ is essential here!).

(ii) Rotate spin into opposite direction.

(iii) Transform particle back into same momentum state as the starting state.

The product of these three transformations is the desired Lorentz transformation $\gamma$. This finishes the proof of the equality of lifetimes for stable antiparticles.

(3) Mass of Unstable Particles.

The mass of an unstable particle can only be approximately defined. We assume that the particle is only weakly unstable, decaying only through the weak interaction. We write for the total mass operator

$$M = M_s + M_W$$

where $M_s$ is the mass operator in the absence of the weak interaction.

Let $\psi$ be an eigenstate of $M_s$

$$M_s \psi = m_0 \psi$$

It represents the zero th approximation of the unstable particle decaying through the weak interaction. The approximate mass value of the unstable particle is then given up, to first order in
the weak interaction, by

\[ m = (\psi, M \psi) = m_0 + (\psi, M_\omega \psi) \]

Let \( \varphi = C \psi \) be the state of the antiparticle. We find for its mass

\[ m' = (\varphi, M \varphi) = m_0 + (\varphi, M_\omega \varphi) \]

The two masses are equal if

\[ (\psi, M_\omega \psi) = (\varphi, M_\omega \varphi) \]

Using the result of the preceding section, we have

\[ (\psi, M_\omega \psi) = (C\psi, M_\omega C\psi) = (\omega \psi, M_\omega \omega \psi) \]

\[ = (\omega \psi, M_\omega \omega \psi) = (M_\omega \psi, \psi) = (\psi, M_\omega \psi) \]

Thus the two masses are the same.

Thus the two masses are the same. \( \omega \) is unitary

but it does not matter

Since \( M_\omega \) is self-adjoint

(4) Total Lifetimes of Charge Conjugate Particles.

We write for the S-operator

\[ S = I + R \] (4.1)

and for a general transition matrix element

\[ \langle \psi_f, R \psi_i \rangle = \langle f | R | i \rangle = \epsilon(E_f - E_i) M_{fi} \] (4.2)

\( M_{fi} \) is the "reduced" matrix element. The transition rate is
given by (see GR, part I of these notes)

\[ \Gamma_i \rightarrow f = \frac{1}{2\pi} \int_{E_f} S_f \left( E_i - E_f \right) \left| M_{fi} \right|^2 \]  \hspace{1cm} (4.3)

We shall now prove that the total lifetime of anti-particles is always the same, regardless of whether the theory is invariant under C. (If it is invariant, this statement is of course trivial!)

Evidently we need a relation between \( \langle f | R | i \rangle \) and \( \langle f | R | i \rangle \), where \( I \) and \( \tilde{f} \) stand for the charge conjugate initial and final states.

For the initial state \( i \) we assume a particle with total spin \( j \), \( z \)-component \( m \) and at rest. So we write

\[ \psi_i = |jm\rangle \]  \hspace{1cm} (4.4)

with the phase factors so adjusted that

\[ T |jm\rangle = |j-m\rangle \]  \hspace{1cm} (4.5)

We assume that it is a state of definite parity;

\[ P |jm\rangle = I(i) |jm\rangle \]  \hspace{1cm} (4.6)

where \( I(i) \) is the intrinsic parity of the initial state.

The final state \( f \) is characterised by total angular momentum \( j \), \( z \)-component \( m \) and by additional quantum numbers \( r \) (\( i \)-spin, orbital angular momentum, channel number for different decay modes and what-have-you). So we write

\[ \psi_f = |jmr\rangle \]  \hspace{1cm} (4.7)
Again phases shall be adjusted so that

\[ T \left| j m r \right> = \left| j \cdot m \ r \right> \]  \hspace{1cm} (4.8)

But now parity need not be conserved and so \( f \) may be a superposition of states with opposite parity

\[ \psi_f = \psi_f^+ + \psi_f^- \]  \hspace{1cm} (4.9)

The \( R \) may be decomposed correspondingly into part which commutes and one which anti-commutes with \( P \)

\[ R = R^+ + R^- \]  \hspace{1cm} (4.10)

\[ PR^+ P^{-1} = iR^+ \]  \hspace{1cm} (4.11)

For the matrix elements we have then

\[ (\psi_f^+, R \psi_i^-) = (\psi_f^+, R^+ \psi_i^-) + (\psi_f^-, R^- \psi_i^-) \]  \hspace{1cm} (4.12)

We introduce the following notation

\[ M_{fi}^+ \int (E_f - E_i) = R_{fi}^+ \]

\[ M_{fi}^+ = A \quad M_{if}^+ = A' \]

\[ M_{fi}^- = B \quad M_{if}^- = B' \]

and corresponding expressions for the charge conjugate matrix elements \( \bar{A}, \bar{B}, \bar{A}', \bar{B}' \).
The intrinsic parity of the final states is denoted by $I(f)$, so that
\[ P \psi_f^\dagger = \pm I(f) \psi_f^\dagger \] (4.13)

The CPT theorem gives the following relation
\[ \Lambda + \bar{E} = I(A' - B') \] (4.14)

where
\[ I = I(f) I(i) = \pm 1 \]

The proof follows the same pattern as the considerations of subsection 2:
\[ C \psi_i = I(i) \Lambda \psi_i \]
\[ C \psi_f^\dagger = \pm I(f) \Lambda \psi_f^\dagger \] (4.15)

where $\Lambda$ is a rotation around an axis \( \perp \) to the \( z \)-direction, by 180°.

Note:
With the phase factors as defined through Eqs. (4.5) and (4.8) the fact that $\Omega$ is anti-unitary, while $C$ is not, does not show up in (4.15). It would only do so for general linear combinations of such states. And it certainly does when we evaluate
\[ \langle \theta f | R | \theta i \rangle = (\theta \psi_f, R \theta \psi_i) = (\psi_f, \theta^{-1} R \theta \psi_i)^* \]
\[ = (\psi_f, R^* \psi_i)^* = (\psi_i, R \psi_f) = \langle i | R | f \rangle \]
\[ \text{CPT - theorem is used here.} \]
With this and (4.15) we find (4.14).

The special case \( f = i \) leads to the relation (after dropping a \( \delta \)-function) valid for the total reduced matrix element.

\[
M_{ii} = M_{ii}^{\dagger} \tag{4.16}
\]

On the other hand we find from the orthogonality condition of the \( -\)matrix

\[
\langle i' \mid R \mid i \rangle + \langle i \mid R \mid i' \rangle^* + \sum_f \langle i' \mid R^* \mid f \rangle < f \mid R \mid i \rangle = 0
\]

or, after dropping a \( \delta \)-function

\[
M_{ii} + M_i^* + \sum_f \delta(E_f - E_i) M_{fi} M_{fi}^* = 0
\]

Specializing to \( i' = i \) and using (4.16) we obtain

\[
\sum_f \left| M_{fi} \right|^2 \delta(E_f - E_i) = \sum_f \left| M_{fi}^{\dagger} \right|^2 \delta(E_f - E_i)
\]

This relation shows that the two lifetimes are the same for two antiparticles.

(5) Equality of Branching Ratios when Time-Reversal is a Symmetry.

Let \( f_1 \) and \( f_2 \) be two groups of final states, belonging to different decay modes. We define the branching ratio

\[
\rho = \frac{\sum_{f_1} \left| M_{f_1 i} \right|^2 \delta(E_{f_1} - E_i)}{\sum_{f_2} \left| M_{f_2 i} \right|^2 \delta(E_{f_2} - E_i)} \tag{5.1}
\]

and a corresponding expression for \( \bar{\rho} \).
Since the summation over the final states includes states of opposite parity, we have

$$\sum_{f_1} \left| M_{f_1} \right|^2 = \frac{1}{2} \sum_{f_1} \left( \left| A + B \right|^2 + \left| A - B \right|^2 \right)$$

$$= \sum_{f_1} \left( \left| A \right|^2 + \left| B \right|^2 \right)^2$$

In order to show the equality of the branching ratios we need to show that

$$\left| A \right|^2 + \left| B \right|^2 = \left| \bar{A} \right|^2 + \left| \bar{B} \right|^2$$

But because of Eq. (4.14) it suffices to show that

$$\left| A \right|^2 + \left| B \right|^2 = \left| A' \right|^2 + \left| B' \right|^2$$

(5.2)

Time reversal symmetry implies (principle of "detailed balance")!

$$\left\langle i \mid R \mid f \right\rangle = \left\langle T_f \mid R \mid T_i \right\rangle$$

(5.3)

We can get rid of the operator $T$ by observing that for the special initial state (with phase factors adjusted as in (4.5)) $T_i$ is equivalent to a rotation $i$ around a suitable axis in the $x$-$y$-plane.

$$T_i = x$$

For the same rotation we have

$$T_f = f$$

provided the state $f$ is defined as to satisfy (4.8).
So we have

$$\langle T_f R | T_i \rangle = \langle A_f R | A_i \rangle = \langle f R | i \rangle$$

(5.4)

or

$$A_r = A'_r, \quad B_r = B'_r \quad (r = 1, 2)$$

Since $I^2 = 1$ this leads immediately to the equality of the branching ratios

$$f = \bar{f}$$

It is clear that this result could be generalised to several decay channels.

In the derivation of this result, no use has been made of the fact that the decay interaction is weak.

If time-reversal is not a symmetry, then this result is in general no longer true. The branching ratios are different. In order to calculate the difference, more specific assumptions about the interaction are needed (see for instance S. Okubo, Phys. Rev. 109, 984 (1958)).
V. STRUCTURE OF WEAK INTERACTIONS IN SPACE-TIME.

(1) Introduction. Y. Yamaguchi
1. Introduction.

In this chapter we shall discuss the slow processes caused by weak interactions. To simplify the argument we shall accept in what follows:

(a) the two component neutrino theory,
(b) time reversal invariance, and
(c) lepton number conservation,

unless the contrary is explicitly mentioned. Arguments for reaching these three "empirical" facts are of course described below at the appropriate places.

In order to be consistent with the notation used in most references (particularly by Lee and Yang), we change our notation of Dirac matrices.

\[
\begin{align*}
\gamma_1, \gamma_2, \gamma_3 & : \text{ same as before,} \\
\gamma_4 &= i \gamma_0, \\
\gamma_5 &= \gamma_1 \gamma_2 \gamma_3 \gamma_4 \quad (= i \gamma_5 \text{ in Jauch's lecture})
\end{align*}
\]

Then one can conveniently choose the representation in which all \( \gamma_j \) \((j=1,2,3,4,5)\) are hermitian and

\[
(\gamma_j^\dagger \gamma_j)^2 = I. \quad \text{(unit matrix)}.
\] (1.2)

Consequently we have to use imaginary time \( x_+ = i x_0 = i t \), and imaginary energy \( p_+ = i E \). The scalar product \((ab)\) of two 4-vectors \( a_\mu, b_\mu \) is simply given by

\[
(ab) = \sum_{\mu=1,4} a_\mu b_\mu.
\] (1.3)

(We need not worry about the up and down position of the indices).
Before going into the detailed discussion of the leptonic processes, we would like to state a few remarks on (a) and (c), as well as on the Fierz identities.

(a) **Two component theory of the neutrino.**

This concept has already been introduced by Prof. Jauch. We just continue his arguments. One type of the two component neutrino field $\psi_\nu$ is described by

$$
(1 - \gamma_5) \psi_\nu = 0,
$$

or equivalently

$$
\begin{align*}
\gamma_5 \psi_\nu &= \psi_\nu, \\
(\gamma \sigma) \psi_\nu &= 0
\end{align*}
$$

From this set of equations one sees

$$
\frac{\partial}{\partial t} \psi_\nu = \vec{\sigma} \cdot \vec{\nabla} \psi_\nu, \tag{1.5}
$$

where

$$
2 \vec{\sigma} = -i \vec{\gamma} \times \vec{\gamma}
$$

$$
(\sigma_3 = -i \gamma_1 \gamma_2, \text{ etc.}) \tag{1.6}
$$

Let us investigate the positive frequency solution (corresponding to the neutrino $\nu$, not the **anti-neutrino** $\bar{\nu}$) of (1.5)

$$
\psi_\nu = u(q) e^{i \vec{q} \cdot \vec{r} - iEt}
$$

$$
E = |q|
$$

(1.7)

Inserting (1.7) into (1.4) one finds

$$
|q| u = - \vec{\sigma} \cdot \vec{q} u(q), \tag{1.8}
$$
namely the "spin" of the neutrino is anti-parallel to its motion. This is called that the neutrino $\nu$ is left-handed, or has negative helicity.

If one takes

$$
\begin{align*}
(1+\gamma_5)\psi_\nu &= 0, \\
\gamma_5 \psi_\nu &= -\psi_\nu,
\end{align*}
$$

(1.4')

instead of Eq. (1.4), then the neutrino turns out to be right-handed, or of positive helicity.

The anti-neutrino $\bar{\nu}$ can be imagined as a hole in the vacuum, completely filled with the negative energy states. And the helicity of the anti-neutrino is just opposite to that of the neutrino, namely the anti-neutrino described by

Eq. (1.4) is right
Eq. (1.4') is left

- handed.

<table>
<thead>
<tr>
<th>neutrino $\nu$</th>
<th>case (a)</th>
<th>case (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma \psi_\nu = 0$</td>
<td>$\gamma_5 \psi_\nu = \psi_\nu$</td>
<td>$\gamma_5 \psi_\nu = -\psi_\nu$</td>
</tr>
<tr>
<td>$1-\gamma_5 \psi_\nu = 0$</td>
<td>$1+\gamma_5 \psi_\nu = 0$</td>
<td></td>
</tr>
<tr>
<td>spin (L.H.)</td>
<td>$\uparrow$ spin (R.H.)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>anti-neutrino $\bar{\nu}$</th>
<th>case (a)</th>
<th>case (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma \psi_\nu = 0$</td>
<td>$\gamma_5 \psi_\nu = \psi_\nu$</td>
<td>$\gamma_5 \psi_\nu = -\psi_\nu$</td>
</tr>
<tr>
<td>$1-\gamma_5 \psi_\nu = 0$</td>
<td>$1+\gamma_5 \psi_\nu = 0$</td>
<td></td>
</tr>
<tr>
<td>spin (R.H.)</td>
<td>$\uparrow$ spin (L.H.)</td>
<td></td>
</tr>
</tbody>
</table>
In this two component theory one can distinguish $\nu$ and $\bar{\nu}$ by their helicity.

Eqs. (1.4) and (1.4') do not satisfy the invariance against space reflection $P$ and charge conjugation $C$. They are invariant for the combined operation $CP$. Therefore the CPT theorem guarantees that they are time reversal invariant. These statements can be easily verified using the $C, P, T$ operations explained by Jauch.

(b) Lepton number conservation.

We know so far four absolute conservation laws of:

(I) energy and momentum
(II) angular momentum
(III) charge, and
(IV) baryonic number

These are established soundly from our experience. We want to introduce here the fifth conservation law:

(V) the conservation of lepton number.

We call

$\bar{\mu}, e^-, \nu$ leptons
$\mu^+, e^+, \bar{\nu}$ anti-leptons.

For each lepton we assign the lepton number $n_\ell = +1$, for anti-leptons

<table>
<thead>
<tr>
<th>lepton</th>
<th>$\mu^-$ $\mu^+$ $\nu$</th>
<th>lepton number $n_\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>lepton</td>
<td>$e^-$</td>
<td>+1</td>
</tr>
<tr>
<td>anti-lepton</td>
<td>$e^+$ $\mu^+$ $\bar{\nu}$</td>
<td>-1</td>
</tr>
</tbody>
</table>

*) See F. Reines et al, Phys.Rev. 102, 609 (1956) (L).
the lepton number \(-1\). Our requirement \((V)\) can be formulated as

"the sum of the lepton number should not change in all sorts of reactions, including weak interactions".

Most conservatively one may regard \((V)\) as a working hypothesis. But so far we have no evidence against it.

The consequence of the lepton number conservation is as follows:

(a) allowed processes:

\[
\begin{align*}
\pi^- & \rightarrow \mu^- + \bar{\nu} \\
\pi^- & \rightarrow e^- + \bar{\nu} \\
K^- & \rightarrow \mu^- + \bar{\nu} \\
K^- & \rightarrow e^- + \bar{\nu} \\
K^- & \rightarrow \pi^- + (\mu^- + \bar{\nu}) \\
K^0 \rightarrow \{ \mu^+ + \pi^- + \nu \} \\
K^0 \rightarrow \{ \mu^- + \pi^+ + \bar{\nu} \} \\
n & \rightarrow p + e^- + \bar{\nu} \\
\bar{\nu} + p & \rightarrow n + e^+ \\
\mu^+ & \rightarrow e^+ + \nu + \bar{\nu} \\
\text{etc.}
\end{align*}
\]

The following reactions are allowed by the lepton number conservation but they are not seen yet.
\[ K_2^0 \rightarrow \mu^+ + \mu^- , \]
\[ \mu^+ \rightarrow e^+ + \gamma , \]
\[ \mu^+ \rightarrow e^+ + e^- + e^+ , \]
\[ \bar{\mu} + p \rightarrow p + e^- + e^+ . \]

(b) forbidden processes:
\[ \nu + p \rightarrow n + e^+ \]
\[ \mu^+ \rightarrow \begin{cases} e^+ + 2\nu \\ e^+ + 2\bar{\nu} \end{cases} \text{ etc.} \]

The double $\beta$-decay processes of nuclei have so small transition probability that they are impossible to detect.

(c) Fierz identities.

It is often useful to know the relation among the two different expressions for Fermi type interactions:

\[ (\bar{\psi}^a \Gamma \psi^b) (\bar{\psi}^c \Gamma \psi^d) \]

and

\[ (\bar{\psi}^a \Gamma \psi^d) (\bar{\psi}^c \Gamma \psi^b) . \]

To establish it, let us discuss important property of Dirac matrices. We use the following $\gamma^A (A=1, \ldots, 16)$:

\[
\begin{align*}
A = 1 & \quad : 1 \\
A = 2 \text{ to } 5 & \quad : \gamma_1, \gamma_2, \gamma_3, \gamma_4 \\
A = 6 \text{ to } 11 & \quad : i\gamma_2 \gamma_3, i\gamma_3 \gamma_1, i\gamma_1 \gamma_2, i\gamma_1 \gamma_4, i\gamma_2 \gamma_4, (=-\sigma_1) (=-\sigma_2) (=-\sigma_3) \\
A = 12 \text{ to } 15 & \quad : i\gamma_1 \gamma_2 \gamma_3, i\gamma_1 \gamma_2 \gamma_4, i\gamma_3 \gamma_1 \gamma_4, i\gamma_2 \gamma_3 \gamma_4.
\end{align*}
\]
We can easily see that

\[(\gamma^{(A)})^2 = I \quad (= \text{unit matrix})\]
\[\text{Sp } \gamma^{(A)} = 0 \quad (A \neq 1)\]  \hspace{1cm} (1.10)
\[\text{Sp}(\gamma^{(A)} \gamma^{(B)}) = \begin{cases} 4 & (A = B) \\ 0 & (A \neq B) \end{cases}\]

We can conveniently use the representation in which all \(\gamma^{(A)}\)'s are hermitian. Since \(\gamma^{(A)}\) \((A=1, \ldots, 16)\) are linearly independent, any 4x4 matrix \(X\) can be expressed as

\[X = \sum_{A=1}^{16} C_A \gamma^{(A)}\]  \hspace{1cm} (1.11)

From (1.10)

\[\text{Sp}(X \gamma^{(A)}) = 4C_A\]  \hspace{1cm} (1.12)

Hence

\[X = \sum_{A=1}^{16} C_A \gamma^{(A)}\]
\[= \sum_{A=1}^{16} \frac{1}{4} \text{Sp}(X \gamma^{(A)}) \gamma^{(A)}\]
\[= \frac{1}{4} \sum_A \sum_{\rho, \lambda} X_{\rho\lambda} \gamma^{(A)}_{\rho\lambda} \gamma^{(A)}\]

Taking \(\alpha\beta\) element of this expression, one finds

\[X_{\alpha\beta} = \frac{1}{4} \sum_{A=1}^{16} \sum_{\rho, \lambda - 1}^{4} X_{\rho\lambda} \gamma^{(A)}_{\rho\lambda} \gamma^{(A)}_{\alpha\beta}\]  \hspace{1cm} (1.13)
We choose now a special $X$:

$$X_{\alpha\beta} = \delta_{\alpha\sigma} \delta_{\rho\beta} = \begin{cases} 1 & (\alpha = \sigma, \beta = \rho) \\ 0 & \text{otherwise} \end{cases}$$

where

$$\delta_{\alpha\sigma} = \begin{cases} 1 & (\alpha = \sigma) \\ 0 & (\alpha \neq \sigma) \end{cases}$$

is the Kronecker's $\delta$.

Then Eq. (1.13) reduces to the following important relation:

$$\delta_{\alpha\sigma} \delta_{\rho\beta} = \frac{1}{4} \sum_{A=1}^{16} \gamma^{(A)}_{\rho\sigma} \gamma^{(A)}_{\alpha\beta}$$  \hspace{1cm} (1.14)

If $F$ and $G$ are arbitrary 4x4 matrices, one easily verifies

$$F_{\alpha\sigma} G_{\rho\beta} = \sum_{\alpha',\beta'} F_{\alpha'\alpha} \delta_{\alpha\sigma} \delta_{\rho\beta'} G_{\beta'\beta}$$

$$= \frac{1}{4} \sum_{A=1}^{16} \gamma^{(A)}_{\rho\sigma} (F \gamma^{(A)} G)_{\alpha\beta}$$  \hspace{1cm} (1.15)

Let us now introduce five Fermi-type interactions between four fields $\psi^a, \psi^b, \psi^c, \psi^d$:

$$L^{(S)}(a,b,c,d) = \left[ (\bar{\psi}^a \gamma^{(A)} \psi^b) (\bar{\psi}^c \gamma^{(A)} \psi^d) \right]_{A=1} + \text{h.c.}$$

$$= (\bar{\psi}^a \psi^b) (\bar{\psi}^c \psi^d) + \text{h.c.} \ldots \text{ scalar}$$

$$L^{(V)}(a,b,c,d) = \sum_{A=2}^{5} (\bar{\psi}^a \gamma^{(A)} \psi^b) (\bar{\psi}^c \gamma^{(A)} \psi^d) + \text{h.c.}$$

$$= \sum_{\mu=1}^{4} (\bar{\psi}^a \gamma^{\mu} \psi^b) (\bar{\psi}^c \gamma^{\mu} \psi^d) + \text{h.c.} \ldots \text{ vector.}$$
\[ L^{(T)}(a, b, c, d) = \sum_{A=5}^{11} \left( \bar{\psi}^a \gamma^{(A)} \psi^b \right) \left( \bar{\psi}^c \gamma^{(A)} \psi^d \right) + h.c. \]

\[ = \frac{1}{2} \sum_{j=1}^{4} \left( \bar{\psi}^a \sigma_{\mu \nu} \psi^b \right) \left( \bar{\psi}^c \sigma_{\mu \nu} \psi^d \right) + h.c. \]

where

\[ \sigma_{\mu \nu} = \frac{1}{2i} \left( V_{\mu} \gamma_{\nu} - V_{\nu} \gamma_{\mu} \right) \]

(note: \( \sigma_{23} = \sigma_1 \), \( \sigma_{31} = \sigma_2 \), \( \sigma_{12} = \sigma_3 \))

\[ L^{(A)}(a, b, c, d) = \sum_{A=12}^{15} \left( \bar{\psi}^a \gamma^{(A)} \psi^b \right) \left( \bar{\psi}^c \gamma^{(A)} \psi^d \right) + h.c. \]

\[ = \sum_{j=1}^{4} \left( \bar{\psi}^a \gamma_{\mu} \gamma^5 \psi^b \right) \left( \bar{\psi}^c \gamma_{\mu} \gamma^5 \psi^d \right) + h.c. \]

\[ \text{..... axial vector} \]

\[ L^{(P)}(a, b, c, d) = \left[ \left( \bar{\psi}^a \gamma^{(A)} \psi^b \right) \left( \bar{\psi}^c \gamma^{(A)} \psi^d \right) \right]_{A=16} + h.c. \]

\[ = \left( \bar{\psi}^a \gamma^5 \psi^b \right) \left( \bar{\psi}^c \gamma^5 \psi^d \right) + h.c. \]

\[ \text{..... pseudoscalar} \] (1.16)

If one uses Eq. (1.14), one can verify

\[ \left( \bar{\psi}^a \psi^b \right) \left( \bar{\psi}^c \psi^d \right) + h.c. \]

\[ = \sum_{\alpha \beta \rho \sigma} \left( \bar{\psi}_{\alpha} \gamma_{\rho} \psi_{\beta} \right) \left( \bar{\psi}_{\sigma} \gamma_{\omega} \psi_{\delta} \right) + h.c. \]

\[ = \frac{1}{4} \sum_{A=1}^{16} \sum_{\alpha \beta \rho \sigma} \left( \bar{\psi}^a \gamma^{(A)} \psi^b \right) \left( \bar{\psi}^c \gamma^{(A)} \psi^d \right) + h.c. \]

\[ = \frac{1}{4} \left[ L^{(S)}(a, d, c, b) + L^{(V)}(a, d, c, b) + L^{(T)}(a, d, c, b) \right. \]

\[ \left. + L^{(A)}(a, d, c, b) + L^{(P)}(a, d, c, b) \right] \] (1.17)
where $\dagger$ comes from the anticommutability among the various fields.

Similarly using Eq. (1.15) one can always express any $L^{(n)}(a,b,c,d)$ as a linear combination of $L^{(m)}(a,c,b,d)$:

$$L^{(n)}(a,b,c,d) = \sum_{m=S,V,T,A,P} f_{n,m} L^{(m)}(a,d,c,b), \quad (n=S,V,T,A,P)$$

(1.18)

where $f_{n,m}$ is given by

$$f_{n,m} = \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
1 & -\frac{1}{2} & 0 & \frac{1}{2} & -1 \\
\frac{3}{2} & 0 & -\frac{1}{2} & 0 & \frac{3}{2} \\
1 & \frac{1}{2} & 0 & -\frac{1}{2} & -1 \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2}
\end{pmatrix}$$

(1.19)

Remark (i)

When one talks about the type of the Fermi type interactions, it is very important to know what ordering ($L^{(n)}(a,b,c,d)$ or $L^{(n)}(a,d,c,b)$) is used there.

Remark (ii)

In the case of parity non-conserving Lagrangian in the order $(a,b,c,d)$

$$(\bar{\psi}^{(a)} \gamma_5 \gamma^{(A)} \psi^{(b)}) (\bar{\psi}^{(c)} \gamma^{(A)} \psi^{(d)})$$

(1.20)
one can use Eq. (1.15) to re-express it in the form of a linear combination of

\[
(\bar{\psi}^{(a)} \gamma^{(A')} \psi^{(d)}) \ (\bar{\psi}^{(a)} \gamma^{(A')} \psi^{(b)})
\]

(1.21)

Since \( \bar{\psi}^{(a)} \) does not move its position, one can replace \( \bar{\psi}^{(a)} \) in Eq. (1.16) by \( (\bar{\psi}^{(c)} \gamma^{5}) \) and the relation between (1.20) and (1.21) takes precisely the form of Eq. (1.18) with the same coefficients \( f_{m,n} \).

The relation (1.20) is very useful in the discussion of Fermi type interactions. We give a few applications of Eq. (1.20):

\[
(\bar{\psi}^{(a)} \gamma^{\alpha} (1+\gamma^{5}) \psi^{(b)}) \ (\bar{\psi}^{(c)} \gamma^{\alpha} (1+\gamma^{5}) \psi^{(d)})
\]

\[
= \pm (\bar{\psi}^{(a)} \gamma^{\alpha} (1+\gamma^{5}) \psi^{(d)}) \ (\bar{\psi}^{(c)} \gamma^{\alpha} (1+\gamma^{5}) \psi^{(b)})
\]

(1.22)

\[
(\bar{\psi}^{(a)} \gamma^{\alpha} (1-\gamma^{5}) \psi^{(b)}) \ (\bar{\psi}^{(c)} \gamma^{\alpha} (1-\gamma^{5}) \psi^{(d)})
\]

\[
= \pm (\bar{\psi}^{(a)} \gamma^{\alpha} (1-\gamma^{5}) \psi^{(d)}) \ (\bar{\psi}^{(c)} \gamma^{\alpha} (1-\gamma^{5}) \psi^{(b)})
\]

For \( \mu \)-decay interaction,

\[
(\bar{\psi}^{(\mu)} \gamma^{\alpha} (f_{V} - f_{A} \gamma^{5}) \psi^{(c)}) \ (\bar{\psi}^{(\nu)} \gamma^{\alpha} (1+\gamma^{5}) \psi^{(d)})
\]

\[
= - (f_{V} + f_{A}) \ (\bar{\psi}^{(\mu)} (1+\gamma^{5}) \psi^{(u)}) \ (\bar{\psi}^{(\nu)} (1-\gamma^{5}) \psi^{(c)})
\]

\[
+ \frac{f_{V} - f_{A}}{2} \ (\bar{\psi}^{(\mu)} \gamma^{\alpha} (1+\gamma^{5}) \psi^{(u)}) \ (\bar{\psi}^{(\nu)} \gamma^{\alpha} (1+\gamma^{5}) \psi^{(c)})
\]

(1.23)

One should remark that in the two component theory there is only one possible bilinear form in \( \bar{\psi}^{(\nu)} \) and \( \psi^{(\nu)} \),

\[
\bar{\psi}^{(\nu)} \gamma^{\alpha} (1 \pm \gamma^{5}) \psi^{(\nu)} \quad \text{for} \quad (1 \pm \gamma^{5}) \psi^{(\nu)} = 0
\]

\( \bar{\psi}^{(\nu)} (1 \pm \gamma^{5}) \psi^{(u)} \) or \( \bar{\psi}^{(\nu)} \sigma_{\alpha \beta}^{\nu} (1 \pm \gamma^{5}) \psi^{(\nu)} \) is identically zero.
V. STRUCTURE OF WEAK INTERACTIONS IN SPACE-TIME.

(2) $\Pi_{\ell_2}^i K_{\ell_2}$. ($\ell = \mu$ or $e$).

Y. Yamaguchi
2. \( \pi \ell_2, K \ell_2 \) \( (\ell = \mu \text{ or } e) \)

In this paragraph we shall discuss about the two body leptonic decay of the \( \pi^- \) and \( K^- \) mesons.

(a) **Empirical facts and kinematics.**

We list the two body decay processes of \( \pi^- \) and \( K^- \) mesons which are known so far:

\[
\pi^- \rightarrow \mu^+ + \nu \\
\pi^- \rightarrow e^+ + \bar{\nu}
\]

There should also exist the process

\[
K^- \rightarrow e^+ + \nu,
\]

but this has not yet been established.

The following processes \(^*\) are missing:

\[
K_2^0 \rightarrow \left\{ \begin{array}{c}
\mu^+ + \mu^- \\
e^+ + e^-
\end{array} \right.
\]

\(^*\) The corresponding decay process for \( \pi^0 \)-meson:

\[
\pi^0 \rightarrow e^+ + e^-
\]

must exist, since \( \pi^0 \) can decay into two photons via strong interactions and electromagnetic interaction, and thus there is a substantial probability that these photons form \( e^+e^- \) pairs:
We know that the decay rate \( \omega (\pi^0 \rightarrow 2\gamma) \gtrsim 10^{15} \text{sec}^{-1} \)
and thus
\[
\omega (\pi^0 \rightarrow e^+ + e^-) \\
\approx \omega (\pi^0 \rightarrow 2\gamma) \times \frac{1}{4\pi} \left( \frac{e^2}{4\pi} \right)^2 \left( \frac{m_e}{m_{\pi}} \right)^2 \\
\gtrsim 10^8 \text{sec}^{-1},
\]
where \( e^2/4\pi = 1/137 \).

**Kinematics:**

Let us write
\[
\pi^- \rightarrow \mu^+ + \nu
\]

<table>
<thead>
<tr>
<th>mass ( m_{\pi} )</th>
<th>( m_\mu )</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>momentum ( p ) ( q )</td>
<td>( p =</td>
<td>p</td>
</tr>
<tr>
<td>energy ( E ) ( \varepsilon )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We assume the pion is at rest. Then the energy-momentum balance
\[
\rightarrow \dot{p} + q = 0 \\
E + q = m_{\pi}
\]
gives

\[ E = \frac{m_{\pi}^2 + m_\mu^2}{2m_{\pi}} \]  
\[ p = q = \xi = \frac{m_{\pi}^2 - m_\mu^2}{2m_{\pi}} \]  

(2.1)

Numerical values are tabulated in the next table:

<table>
<thead>
<tr>
<th>process</th>
<th>E(MeV)</th>
<th>p(MeV/c)</th>
<th>Q-value (MeV)</th>
<th>mean lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^+ \to \mu^+ + \nu )</td>
<td>109.8</td>
<td>29.8</td>
<td>33.9</td>
<td>( 2.56 \times 10^{-8} ) sec</td>
</tr>
<tr>
<td>( \pi^+ \to e^+ + \nu )</td>
<td>69.8</td>
<td>69.8</td>
<td>139.1</td>
<td></td>
</tr>
<tr>
<td>( K^+ \to \mu^+ + \nu )</td>
<td>258.3</td>
<td>235.7</td>
<td>388.3</td>
<td>( 2.1 \times 10^{-8} ) sec</td>
</tr>
<tr>
<td>( K^+ \to e^+ + \nu )</td>
<td>247.0</td>
<td>247.0</td>
<td>493.5</td>
<td></td>
</tr>
</tbody>
</table>

Notes:

Here we have used \( m_{\pi} = 139.6 \ \text{MeV}, \ m_\mu = 105.7 \ \text{MeV}, \ m_K = 494.0 \ \text{MeV}, \ m_\nu = 0. \)

Q-value = available kinetic energy in the C.M. system.

It is useful to remember \( \beta = c = 1 \)

\[ m_\pi \approx 140 \ \text{MeV} \approx \frac{1}{1.4 \times 10^{-13} \text{cm}} \approx 2.1 \times 10^{23} \text{ sec}^{-1}. \]  
\[ (1 \ \text{MeV} \approx 1.5 \times 10^{21} \text{ sec}^{-1}) \]  

(2.2)

We have recently been informed by our CERN experimentalists that the branching ratio is

\[ \frac{\omega(\pi^+ \to e^+ + \nu)}{\omega(\pi^+ \to \mu^+ + \nu)} \approx 1.3 \times 10^{-4} \]
(b) Calculation of the $\pi - \mu$ decay rate.

In order to calculate the $\pi - \mu$ decay rate, we have to know the form of the decay interaction. One of the simplest choices will be to assume the local Yukawa type interaction among pion, muon and neutrino fields $\varphi_\pi$, $\psi_\mu$, $\psi_\nu$. If one accepts the two component theory, one has two possibilities (parity violating!):

$$V-A : \frac{eg}{m_\pi} \frac{\partial \varphi_\pi}{\partial x_\alpha} (\bar{\psi}_\mu \gamma_\alpha (1 + \gamma_5) \psi_\nu) \quad (2.3)$$

and

$$S-P : f \varphi_\pi (\bar{\psi}_\mu (1 + \gamma_5) \psi_\nu), \quad (2.4)$$

where $g$ and $f$ are the dimensionless coupling constant. $f$ and $g$ are real if time reversal invariance holds. There are no strong interactions among $\pi$, $\mu$ and $\nu$, so that one can use the equivalence theorem, which says the equivalence between (2.3) and (2.4). Namely one does the partial integration in the interaction Lagrangian $\int (2.3) \ (d^4x)$, then the Lagrangian density (2.3) is equivalent to

$$- \frac{eg}{m_\pi} \frac{\partial \varphi_\pi}{\partial x_\alpha} (\bar{\psi}_\mu \gamma_\alpha (1 + \gamma_5) \psi_\nu).$$

Now, making use of the Dirac equations for muon and neutrino

$$\frac{\partial}{\partial x_\alpha} \bar{\psi}_\mu \gamma_\alpha - m_\mu \bar{\psi}_\mu = 0,$$

$$\gamma_\alpha \frac{\partial \psi_\nu}{\partial x_\alpha} = 0,$$

one can establish the equivalence $^*)$

$$\frac{g}{m_\pi} \frac{\partial \varphi_\pi}{\partial x_\alpha} (\bar{\psi}_\mu \gamma_\alpha (1 + \gamma_5) \psi_\nu) \equiv - \frac{m_\mu}{m_\pi} \varphi_\pi (\bar{\psi}_\mu (1 + \gamma_5) \psi_\nu) \quad (2.5)$$

$^*)$ More generally, if $\psi_1$ and $\psi_2$ are the Dirac fields with mass $m_1$, $m_2$, one verifies the equivalence:
\[ \frac{\partial \varphi_\pi}{\partial x_\alpha} (\bar{\psi}_1 \gamma_\alpha \psi_2) = (\text{equivalent}) (m_2 - m_1) \varphi_\pi (\bar{\psi}_1 \psi_2), \]
\[ \frac{\partial \varphi_\pi}{\partial x_\alpha} (\bar{\psi}_1 \gamma_\alpha \gamma_5 \psi_2) = - (m_1 + m_2) \varphi_\pi (\bar{\psi}_1 \gamma_5 \psi_2). \]

The right hand side is precisely equal to (2.4) if \( f = - \frac{m_\mu}{m_\pi} g. \)
Hence we shall choose hereafter (2.3).

If one accepts a phenomenological local decay interaction (2.3), one can easily calculate the probability for the decay \( \pi^- \rightarrow \mu^- + \bar{\nu} \) at rest:

\[ \omega = 2 \pi \frac{g^2}{m_\pi} \left( \frac{1}{\sqrt{2m_\pi}} \right)^2 \left[ \sum_{\mu, \bar{\nu}} \left| \frac{u_\mu(p) (1 + \gamma_5)}{\gamma_q(q)} \right|^2 \right] \]
\[ \times \delta^4(k - p - q) \frac{d^3p d^3q}{(2\pi)^6} \] \hspace{1cm} (2.6)

where \( k, p, \) and \( q \) are the 4-momenta for \( \pi^-, \mu^- \) and \( \bar{\nu}, \) respectively. The quantity inside the square bracket \( \left[ \right] \) can be most easily evaluated by introducing the "Casimir" operators for the particle and antiparticle:

\[ \Lambda_{+}^{(\mu)}(p) = \frac{+ip \gamma + m_\mu}{2m_\mu}, \quad \Lambda_{+}^{(\nu)}(q) = \frac{+iq \gamma + m_\nu}{2m_\nu}. \] \hspace{1cm} (2.7)

For particles, one has

\[ \Lambda_{-}^{(\mu)}(p) u^{(\mu)}(p) = u^{(\mu)}(p), \quad \Lambda_{+}^{(\mu)}(p) u^{(\mu)}(p) = 0 \quad \text{for} \ \mu^- \]
\[ \Lambda_{-}^{(\nu)}(q) u^{(\nu)}(q) = u^{(\nu)}(q), \quad \Lambda_{+}^{(\nu)}(q) u^{(\nu)}(q) = 0 \quad \text{for} \ \nu \]
while for antiparticles

$$\Lambda_+^{(\mu)}(p) \nu^{(\mu)}(p) = \nu^{(\mu)}(p) \quad \Lambda_-^{(\mu)}(p) \nu^{(\mu)}(p) = 0 \quad \text{for } \mu^+$$

$$\Lambda_+^{(\nu)}(q) \nu^{(\nu)}(q) = \nu^{(\nu)}(q) \quad \Lambda_-^{(\nu)}(q) \nu^{(\nu)}(q) = 0 \quad \text{for } \bar{\nu}$$

Also notice that

$$\sum_r u_r^{(\mu)} \bar{u}_r^{(\mu)} = \frac{m_\mu}{E} \Lambda_-^{(\mu)}(p) = \frac{-i p \gamma + m_\mu}{2E}$$

$$\sum_s v_s^{(\nu)} \bar{v}_s^{(\nu)} = -\frac{m_\nu}{E} \Lambda_+^{(\nu)}(q) \Rightarrow \frac{-i q \gamma}{2E}$$

Then one immediately sees that

$$\left[ \right] = \text{Sp} \left[ (1+\gamma_5)(-k \gamma) \frac{-i p \gamma + m_\mu}{2E} (k \gamma) \right. \left. (1+\gamma_5) \right]$$

$$\times \left( \frac{-i q \gamma}{2E} \right)$$

$$= \text{Sp} \left[ 2(1-\gamma_5)(+k \gamma) \frac{-i p \gamma + m_\mu}{2E} (k \gamma) \frac{i q \gamma}{2} \right]$$

$$\left( \because (1-\gamma_5)^2 = 2(1-\gamma_5) \right)$$

$$= \text{Sp} \left[ 2(1-\gamma_5)(+\gamma k) \frac{-i p \gamma + m_\mu}{2E} i \frac{2(kq)}{2E} \right.$$

$$+2(1-\gamma_5)(+\gamma k) \frac{-i p \gamma + m_\mu}{2E} \left\{ -i \frac{q \gamma}{2E} (k' \gamma) \right\} \right]$$

$$= + \frac{2(kq)}{E} \text{Sp} \left[ (1-\gamma_5)(\gamma k) \frac{p \gamma + im_\mu}{2E} \right.$$

$$+ \text{Sp} \left[ 2(1+\gamma_5)(\gamma k)(\gamma k) \frac{-i p \gamma + m}{2E} \frac{-i(kq)}{2E} \right]$$

$$= (kk)$$
\[
\omega = 4 \frac{1}{E \varepsilon} (kq)(kp) + 2 \frac{(kk)}{E \varepsilon} (-pq) \\
= \frac{2}{E \varepsilon} \left[ 2(kq)(kp) - (kk)(pq) \right] \\
= \frac{2}{E \varepsilon} \left[ 2 \frac{m^2}{\pi} E \varepsilon - (-m^2)(q\vec{p} - E \varepsilon) \right] \\
= 2 \frac{m^2}{\pi} \left[ 1 - \frac{1}{E \varepsilon} \right]
\]

(Note: \( k = (0, im_\pi), \ p = (\vec{p}, iE), \ q = (\vec{q}, i\varepsilon), \ \vec{p} + \vec{q} = 0, \ |\vec{p}| = |\vec{q}| = \varepsilon \))

\[
\therefore \omega = 2\pi \frac{\varepsilon^2}{m_{\pi}} \int \left[ 2 \frac{1}{2m_{\pi}} \cdot 2 \frac{m^2}{\pi} \left(1 - \frac{p}{E}\right) \right] \\
\times \frac{d^3 p}{(2\pi)^3} G (E + \varepsilon - m_{\pi}) \\
= 2\pi \frac{\varepsilon^2}{m_{\pi}} \left(1 - \frac{p}{E}\right) \frac{4\pi p^2}{(2\pi)^3} \left(\frac{p}{E} + 1\right) \\
= \frac{\varepsilon^2}{4\pi} \frac{(E - p)}{m_{\pi}(E+p)} 4p^2
\]

Note that \( p/E = \) muon velocity in the rest system of parent \( \pi \).

Using

\[
p + E = \varepsilon + E = m_{\pi}, \quad p = \frac{m^2_{\pi} - m^2_{\mu}}{2m_{\pi}}, \quad E - p = \frac{m^2_{\mu}}{m_{\pi}},
\]

one finally obtains

\[
\omega = \frac{\varepsilon^2}{4\pi} \left(\frac{m^2_{\pi} - m^2_{\mu}}{m_{\pi}}\right)^2 \left(\frac{m_{\mu}}{m_{\pi}}\right)^2 m_{\pi} = \frac{1}{\text{(mean lifetime)}}
\]

(2.10)
From known $\pi^-$ lifetime, one can determine the weak coupling constant $g$

$$\frac{\alpha}{4\pi} = 1.76 \times 10^{-15}$$  \hspace{1cm} (2.11)

(for $\pi \rightarrow \mu + \nu$; $\frac{g}{m_\mu} \gamma_{\alpha} \left( \bar{\Psi}_\mu \gamma_\alpha \left( 1 + \gamma_5 \right) \Psi_\nu \right)$).

Remark (1):

The $\pi \rightarrow e^+ e^-$ decay rate can be obtained from Eq. (2.10) just by replacing $\frac{m_\mu}{m_e}$ by $m_e$.

If we assume the "universal V-A interactions"

$$\frac{1}{\hbar c} \frac{g}{m_\pi} \gamma_{\alpha} \left[ \left( \bar{\Psi}_\mu \gamma_\alpha \left( 1 + \gamma_5 \right) \Psi_\nu \right) + \left( \bar{\Psi}_e \gamma_\alpha \left( 1 + \gamma_5 \right) \Psi_\nu \right) \right]$$  \hspace{1cm} (2.12)

we obtain the ratio

$$\frac{\omega(\pi^+ \rightarrow e^+ + \bar{\nu})}{\omega(\pi^- \rightarrow \mu^- + \bar{\nu})} = \left( \frac{m_e}{m_\mu} \right)^2 \left( \frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right)^2$$  \hspace{1cm} (2.13)

$$= 1.29 \times 10^{-4}$$

The ratio (2.13) does not depend on the two component nature of the neutrino. Even if we take more general V-A mixture:

$$g \frac{g}{\alpha \gamma_{\alpha}} \left[ \left( \bar{\Psi}_\mu + \bar{\Psi}_e \right) \gamma_\alpha \left( a + b \gamma_5 \right) \Psi_\nu \right]$$

we still find the same ratio: (2.13).
Remark (2):
If one assumes the "universal S-P interactions"

$$f \psi_{\mu} (\overline{\psi}_{\mu} (1+\gamma_{5}) \psi_{\nu} + \overline{\psi}_{e} (1+\gamma_{5}) \psi_{\nu})$$ (2.14)

one finds

$$\frac{\sigma_{\pi^{-} \rightarrow e^{-} + \nu}}{\sigma_{\pi^{-} \rightarrow \mu^{-}} + \nu} = \left( \frac{m_{\pi}^{2} - m_{e}^{2}}{m_{\pi}^{2} - m_{\mu}^{2}} \right)^{2} = 5.53$$ (2.15)

This result sharply contradicts with experiment.

Remark (3):
K \ell\ell_{2} decay

The decay interactions:

$$\frac{g_{K}^{2}}{4\pi} \partial_{X_{\alpha}} \left[ (\overline{\psi}_{\mu} \gamma_{\alpha} (1+\gamma_{5}) \psi_{\nu}) + (\overline{\psi}_{e} \gamma_{\alpha} (1+\gamma_{5}) \psi_{\nu}) \right]$$ (2.16)

give

$$\sigma(K^{-} \rightarrow \mu^{-} + \nu) = \frac{g_{K}^{2}}{4\pi} \left( \frac{m_{K}^{2} - m_{\mu}^{2}}{m_{K}^{2} - m_{\mu}^{2}} \right)^{2} \left( \frac{m_{\mu}}{m_{K}} \right)^{2} m_{K}^{2}$$ (2.17)

$$\frac{g_{K}^{2}}{4\pi} \approx \frac{1}{12.5} \left( \frac{m_{K}}{m_{\pi}} \right)^{2} = 1.53 \times 10^{-15}$$ (2.18)

(see Eq.(2.11))

$$\frac{\sigma(K^{-} \rightarrow e^{-} + \nu)}{\sigma(K^{-} \rightarrow \mu^{-} + \nu)} = 2.57 \times 10^{-4}$$ (2.19)
On the other hand the universal S-P interactions

\[ e_K p_K \left( \bar{\psi}_\mu (1 + \gamma_5) \psi_\nu + \bar{\psi}_e (1 + \gamma_5) \psi_\nu \right) \]

give

\[ \frac{\omega (K^- \to e^- + \bar{\nu})}{\omega (K^- \to \mu^- + \bar{\nu})} = \left( \frac{m_K^2 - m_e^2}{m_K^2 - m_\mu^2} \right)^2 = 1.10 \]

This is also inconsistent with the experimental fact.
Remarks:

According to the CPT theorem, the lifetimes of $\pi^+$ and $K^+$ are precisely equal to those of $\pi^-$ and $K^-$, if the decay interaction is invariant for the combined operation (CPT). See the lecture IV 9 (1), given by Prof. Jauch. Decay interactions (2.3), (2.4), (2.12) are all (CPT)-invariant.

(c) Polarization of leptons.

(0,1) Introduction and elementary discussion.

It is now one of the classical examples of parity non-conservation that the muons from $\pi$-decay are polarized in high degree.

First we give a simple proof of the fact that the $\mu^-$-meson from $\pi^-$-decay must be polarized along its direction if the parity is not conserved in the $\pi^-\mu$ decay. We shall use non relativistic approximation to $\mu^-$-mesons, whose spin can be represented by Pauli-spin matrix $\sigma^\mu$. Parity non-conservation means that the $\pi^+$-decay amplitude $R_+ = (\mu^+\nu | R | \pi^+)$ ($\pi^+$ is at rest) consists of a scalar part and a pseudoscalar part:

$$R_+ = a + b \mathbf{\sigma} \mathbf{\hat{p}}$$

where $\mathbf{\hat{p}}$ is the momentum of the $\mu^+$-meson in the rest system of the parent $\pi^+$ and $a, b$ are the scalar functions of $\mu^+$ (or $\nu$) momentum and masses of particles relating this decay process.

The density matrix of $\mu^+$-meson is proportional to

$$R_+R_+^* = |a|^2 + |b|^2 + (ab^* + ba^*) \mathbf{\sigma} \mathbf{\hat{p}}$$

From this result we conclude that $\mu^+$ is polarized along $\mathbf{\hat{p}}$-direction ("longitudinal" polarization), and the degree of polarization is given by

$$\mathbf{\hat{p}} = \frac{b^* + ba^*}{|a|^2 + |b|^2}$$
Evidently
\[ -1 \leq P_{\mu^+} \leq +1 \]
and
\[ P_{\mu^+} = \frac{+1}{a} \quad \text{for} \quad a = \frac{+b}{\sqrt{a^2 + b^2}} \quad (2.23) \]

Using the CPT theorem, the decay amplitude \( R_- \) for \( \pi^- \rightarrow \mu^- + \bar{\nu} \)
can be obtained from (2.20) by operating "CPT" operation. Therefore we obtain for \( \pi^- \rightarrow \mu^- + \bar{\nu} \)

\[
\text{decay amplitude} \quad R_- = a - b \frac{\Psi}{P} \\
\mu^- - \text{polarization} \quad P = -\frac{a^* b + b^* a}{|a|^2 + |b|^2} \quad (2.22')
\]

We have learnt from experiments that \( \mu^- \)-polarization is nearly maximum:
\[ P_{\mu^-} \approx \frac{+1}{a} \quad \text{and therefore} \quad a \approx \frac{+b}{\sqrt{a^2 + b^2}}. \]

This result and information on nuclear \( \beta^- \)-decay \( \text{see next chapter (3)} \)
provide strong support for the two component theory of neutrino. If the neutrino obeys the two component theory, with \( \gamma_c \Psi_\nu = \Psi_\nu \), \( \nu (\bar{\nu}) \) is left-handed (right-handed).

Since \( \pi^- \) is spinless, the conservation of total angular momentum guarantees automatically that the \( \mu^+ (\mu^-) \) accompanied by \( \nu (\bar{\nu}) \) must be completely polarized along the direction \( -\frac{P}{\gamma} \) (\( +\frac{P}{\gamma} \)), or \( \mu^+ (\mu^-) \) is 100% left- (right-) handed.

Precisely the same argument can be applied for all other two body leptonic decay processes of spinless particles:

\[ \pi^- \rightarrow e^- + \nu, \quad K^+ \rightarrow \mu^+ + \nu, \quad K^- \rightarrow e^- + \nu. \]

Recently the (maximum) longitudinal polarization of \( \mu \) from \( K_{\mu_2}^+ \) decay has been found experimentally by Columbia and Chicago groups.
(c_2) **Straightforward calculation.**

For the sake of completeness, we shall check the above statement by a straightforward calculation. We must here introduce the longitudinal spin projection operators:

\[
\Sigma_{\pm} (\vec{p}) = \frac{\pm \vec{\sigma} \vec{p} \pm |\vec{p}|}{2|\vec{p}|},
\]

(2.24)

\[
\left\{ \Sigma_{\pm} (\vec{p}) \right\}^2 = \Sigma_{\pm} (\vec{p}), \quad \Sigma_+(\vec{p}) \Sigma_-(\vec{p}) = \Sigma_-(\vec{p}) \Sigma_+(\vec{p}) = 0
\]

(2.25)

where \( \vec{\sigma}_3 = -i \gamma_1 \gamma_2 \), etc.

For the leptons \((\mu^-, e^-, \text{and } \nu)\) we have

\[
\begin{align*}
\Sigma_+ (\vec{p}) u_+(p) &= u_+(p) \\
\Sigma_- (\vec{p}) u_+(p) &= 0 \\
\Sigma_-(\vec{p}) u_-(p) &= u_-(p) \\
\Sigma_+(\vec{p}) u_-(p) &= 0
\end{align*}
\]

for R.H. lepton (2.26)

(2.27)

where \( u_+(p) \) and \( u_-(p) \) represent the right-handed and left-handed leptons, respectively. While for the anti-leptons \((\mu^+, e^+, \text{and } \bar{\nu})\) we have

\[
\begin{align*}
\Sigma_- (\vec{p}) \bar{u}_+(p) &= \bar{u}_+(p) \\
\Sigma_+ (\vec{p}) \bar{u}_+(p) &= 0 \\
\Sigma_+ (\vec{p}) \bar{u}_-(p) &= \bar{u}_-(p) \\
\Sigma_- (\vec{p}) \bar{u}_-(p) &= 0
\end{align*}
\]

for R.H. anti-lepton (2.26')

(2.27')

for L.H. anti-lepton (2.27')
Notice the difference in signs between the cases for leptons and anti-leptons \( \sum \) see the next Table. Also it should be remarked that the "longitudinal polarization" is commutable with Dirac Hamiltonian \( H(\vec{p}) = \varepsilon_\sigma \vec{p} + \beta m = +i \gamma_4 \vec{p} \gamma^\tau + \gamma_4 m \):

\[
\left[ \Sigma_\pm(\vec{p}), \ H(\vec{p}) \right]_\tau = 0
\]  

(2.28)

<table>
<thead>
<tr>
<th>longitudinal spin projection operator</th>
<th>lepton</th>
<th>anti-lepton</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma_+(\vec{p}) = \frac{\vec{\sigma} \vec{p} + \vec{p} \cdot \vec{\sigma}}{2</td>
<td>\vec{p}</td>
<td>} )</td>
</tr>
<tr>
<td>( \Sigma_-(\vec{p}) = \frac{-\vec{\sigma} \vec{p} + \vec{p} \cdot \vec{\sigma}}{2</td>
<td>\vec{p}</td>
<td>} )</td>
</tr>
</tbody>
</table>

In other words, we can specify simultaneously the energy \( E \), linear momentum \( \vec{p} \) and the longitudinal spin orientation (the longitudinal polarization) of Dirac particles. This is the remarkable property of the longitudinal polarization \(^*)\). The transverse spin orientation fails to have such a property.

**Remarks:** \(^*)\)

We can only talk about the "expectation value" of transverse spin component for the particles with definite momentum and energy.

\(^*)\) These statements can be applied for particles with arbitrary spin.
If a particle is at rest (therefore rest mass \( \neq 0 \)), we can specify any spin orientation, because

\[
\left[ \mathcal{H}(e), \mathcal{H}(0) \right] = 0.
\]

(2.28')

If we can use the non-relativistic approximation, we can diagonalize the spin component with respect to arbitrary axis without any harm.

In the relativistic case, the concept of longitudinal polarization is not Lorentz invariant if the particle has finite rest mass. We have to specify therefore the Lorentz frame \( L \) in which we describe the longitudinally polarized particles. The degree of spin polarization in other Lorentz frame \( L' \) can be easily found by performing the Lorentz transformation from \( L \) to \( L' \). If particles have vanishing rest mass (like photon or neutrino), they have only two spin orientations irrespective of their values of spin \( (\geq \frac{1}{2}) \). These two spin orientations are nothing but the two possibilities (i.e., left or right-handed) of 100% longitudinal polarization, which has Lorentz invariant meaning.

Now we return to our problem. To calculate the \( \pi^- \mu^- \) decay probability with leptons of specified "helicity", we simply drop the summation over spin orientation in Eq. (2.6). We replace:

\[
\left. \begin{array}{c}
\lambda^\pm (\rho) \lambda^\pm (\rho') \rightarrow \frac{-i (\rho \gamma^5) + m_\mu}{2 \varepsilon} \mathcal{H} (\varepsilon) \mathcal{H} (\rho).
\end{array} \right\} (2.29)
\]

\[
\left. \begin{array}{c}
\lambda^\pm (\rho) \lambda^\pm (\rho') \rightarrow \frac{m_\mu}{2 \varepsilon} \mathcal{H} (\varepsilon) \mathcal{H} (\rho).
\end{array} \right\} (2.30)
\]
We see that \[\text{in Eq. (2.6) is now replaced by}\]

\[
\text{Sp} \left[ (1 - \gamma_5)(-\gamma_\nu) \frac{-i(p \gamma) + m_\nu}{2 \varepsilon} \frac{\pm \sigma \cdot p + |p|}{2 |p|} \right.

\[
\times (\gamma_\mu)(1 + \gamma_5) \left( -\frac{i q \gamma}{2 \varepsilon} \right) \frac{\pm \sigma \cdot q + |q|}{2 |q|} \left] \right.
\]

\[(2.31)\]

We notice the following identity

\[
(1 + \gamma_5)(q \gamma) = (1 + \gamma_5) \left\{ i \gamma_4 \left( -\frac{\gamma \cdot q}{2 \varepsilon} \right) + \varepsilon \right\}
\]

\[
= (1 + \gamma_5) \left\{ \gamma_5 \left( -\frac{\gamma \cdot q}{2 \varepsilon} \right) + \varepsilon \right\}
\]

\[
= (1 + \gamma_5) \left\{ i \gamma_4 \gamma_5 \left( i \gamma_4 \left( -\frac{\gamma \cdot q}{2 \varepsilon} \right) + \varepsilon \right) \right\}
\]

\[
= (1 + \gamma_4)(i \gamma_4) \left\{ -\left( -\frac{\gamma \cdot q}{2 \varepsilon} \right) + \varepsilon \right\}
\]

\[(2.32)\]

Therefore we obtain in general

\[
(1 + \gamma_5) \frac{\pm i(q \gamma)}{2 \varepsilon}
\]

\[
= (1 + \gamma_5)(\mp \gamma_4) \frac{-\sigma \cdot q + |q|}{2 |q|} = (1 + \gamma_5)(\mp \gamma_4) \left\{ \frac{-\sigma \cdot q + |q|}{2 |q|} \right\}
\]

\[
= (1 + \gamma_5) \frac{\pm i(q \gamma)}{2 \varepsilon} \frac{-\sigma \cdot q + |q|}{2 |q|}
\]

\[(\because \Sigma \left( -\frac{\gamma \cdot q}{2 \varepsilon} \right) = \Sigma \left( \frac{\gamma \cdot q}{2 \varepsilon} \right) \]

\[
1 |q| = \varepsilon
\]

and

\[
(1 + \gamma_5) \frac{\pm i(q \gamma)}{2 \varepsilon} \frac{\pm \sigma \cdot q + |q|}{2 |q|} = 0
\]
We conclude that only the L.H. \( \nu \) and R.H. \( \bar{\nu} \) can contribute to the decay process (also see the Table given above). This is what we should expect for the two component neutrino: \( \gamma' \frac{5}{4} \nu = \nu' \).

Therefore we need not use Eq. (2.30), but we can simply take

\[
\nu(q) \bar{\nu}(q') \rightarrow -\frac{i \frac{\gamma'}{2 \sigma}}{2 E}
\]

(2.30')

for two component anti-neutrino.

We are considering \( \pi \rightarrow \mu \) decay at rest. Since

\( \sigma \cdot p = -\frac{1}{2} i \hat{\gamma} \times \hat{q} \cdot p \) is commutable with \( \gamma' \frac{5}{4} \gamma' \frac{5}{4} \) and \( (\gamma' k) = \frac{i m}{m} \gamma' \frac{5}{4} \),

we can transform Eq. (2.31) as follows

\[
S_p \left[ \left( 1 - \gamma' \frac{5}{4} \gamma' \frac{5}{4} \right) \frac{-i P Y + m_\mu}{2 E} \right] \left( \frac{\sigma P + 1 P}{2 |P|} \right) \left( \frac{(\gamma' k)(1 + \gamma' \frac{5}{4})}{2} \left( -\frac{i \gamma' \frac{5}{4} q}{2 \sigma} \right) \right)
\]

\[
= S_p \left[ \left( 1 - \gamma' \frac{5}{4} \gamma' \frac{5}{4} \right) \frac{-i P Y + m_\mu}{2 E} \right] \left( \frac{\sigma P + 1 P}{2 |P|} \right) \left( \frac{(\gamma' k)(1 + \gamma' \frac{5}{4})}{2} \right)
\]

\[
\times \left( \frac{\gamma' \frac{5}{4} q + 1 q}{2 |q|} \right) \left( \frac{\gamma' \frac{5}{4} q + 1 q}{2 |q|} \right)
\]

\[
= S_p \left[ \left( 1 - \gamma' \frac{5}{4} \gamma' \frac{5}{4} \right) \frac{-i P Y + m_\mu}{2 E} \right] \left( \frac{\sigma P + 1 P}{2 |P|} \right) \left( \frac{(\gamma' k)(1 + \gamma' \frac{5}{4})}{2} \right) \left( \frac{\gamma' \frac{5}{4} q + 1 q}{2 |q|} \right)
\]

\[
\equiv (a)
\]
Using the relation \( \vec{p} = -\vec{q} \) (\( \pi^- \)-decay at rest !)

\[
(a) = \frac{\pm e^{-\beta} + |q|^2}{2 |p|^2} \cdot \frac{\pm e^{-\beta} + |q|^2}{2 |q|^2} = \frac{\pm e^{-\beta} + |p|^2}{2 |p|^2} \cdot \frac{\pm e^{-\beta} + |p|^2}{2 |p|^2}
\]

\[
= \left\{ \begin{array}{ll}
\frac{\sigma_{\bar{p} + |p|}}{2 |p|} & \text{for "+"} \\
0 & \text{for "-"}
\end{array} \right.
\]

This result tells that only the right-handed \( \mu^- \) (100\% polarization !) can be produced by \( \pi^- \)-decay.

Quite similarly \( \mu^+ \) from \( \pi^+ \)-decay can be shown to be 100\% left-handed:

\[
\begin{array}{c}
\mu^+ \\
\mu^-
\end{array} \quad \begin{array}{c}
\overrightarrow{p} \\
\overrightarrow{q}_\nu
\end{array} \quad \begin{array}{c}
\nu \\
\bar{\nu}
\end{array}
\]

\( \pi^+ \)-decay at rest \quad \( \pi^- \)-decay at rest

Remark (1):

The sign of longitudinal polarization of \( \mu^+ \) from \( \pi^+ \) decay is independent of the choice of decay interactions (2.3) or (2.4), since these two interactions are completely equivalent.

Remark (2):

So far we have assumed \( \nu \) is L.H. and \( \bar{\nu} \) is R.H. (\( \gamma_5 \psi_\nu = \psi_\nu \)) and we have found \( P = +1 \). If we take another two component neutrino theory \( \gamma_5 \psi_\nu = -\psi_\nu \) (\( \psi \) is R.H. and \( \bar{\psi} \) is L.H.), then the \( \mu^- \)-meson from \( \pi^- \)-decay will have opposite helicity:

\[
P \mu^+ = -1
\]

\( \sum \text{cf. also (1), Fig.1, p.47} \).
Remark (3):

Let us assume a decay interaction

\[ \varphi_\alpha \varphi_\pi \left( \overline{\psi}_\mu \gamma_\alpha (g_\nu + g_\lambda \gamma_5) \psi_\nu \right) \]

\[ = \frac{g_\nu + g_\lambda}{2} \varphi_\alpha \varphi_\pi \left( \overline{\psi}_\mu \gamma_\alpha (1 + \gamma_5) \psi_\nu \right) \]

\[ + \frac{g_\nu - g_\lambda}{2} \varphi_\alpha \varphi_\pi \left( \overline{\psi}_\mu \gamma_\alpha (1 - \gamma_5) \psi_\nu \right) \]  \hspace{1cm} (2.33)

The neutrino \( \psi_\nu \) in (2.33) is no more a two component field, but as is seen from the latter expression (2.33), \( \psi_\nu \) can be decomposed into 2 two-component fields:

\[ \psi_\nu = \frac{1}{\sqrt{2}} (\psi_+ + \psi_-), \quad \psi_\pm = \frac{1 \pm \gamma_5}{\sqrt{2}} \psi_\nu \]

\[ \begin{pmatrix} \psi_+ : \text{L.H. } \nu \\ \psi_- : \text{R.H. } \nu \end{pmatrix} \text{ see Fig.1, p. 4} \]

Since there are no interferences between \( \psi_+ \) and \( \psi_- \), one can conclude \( \int \text{cf. Remark (2), p. 30} \):

The probability of finding

\[ \begin{array}{c} \text{L.H. } \mu^+ \\
\text{R.H. } \mu^+ \end{array} \] from \( \pi^+ \) decay

is proportional to

\[ \begin{cases} |g_V + g_A|^2 \\
|g_V - g_A|^2 \end{cases} \]
Thus the polarization of $\mu^+$ along its momentum $p$ (in the rest system of $\pi^\pm$) is given by

$$\mathcal{P}_{\mu^\pm} = \pm \frac{|g_V - g_A|^2 - |g_V + g_A|^2}{|g_V - g_A|^2 + |g_V + g_A|^2}$$

(2.34)

$$= \pm \left( -\frac{2 \Re (g_V^* g_A)}{|g_V|^2 + |g_A|^2} \right)$$

($\mu^\pm$ polarization for the 4-component neutrino case, decay interaction is given by (2.33)). $g_V$ and $g_A$ are real if (2.33) is time reversal invariant and vice versa.

$$|\mathcal{P}_{\mu^\pm}| = 1 \quad \text{for} \quad g_V = \pm g_A$$

Conversely, if the muon polarization is maximum $|\mathcal{P}_{\mu}| = 1$, one can conclude $g_V = \pm g_A$ and thus the time reversal invariance for $\pi^-\mu$ decay and neutrino obeys the two component theory.
V. STRUCTURE OF WEAK INTERACTIONS IN SPACE-TIME,

(3) Beta radioactivity. B. d'Espagnat
3. Beta Radioactivity. (by B. d'Espagnat)

(a) Introduction.

We shall make only a brief summary of this enormous subject. From experiment we shall take only what is essential for our purpose, i.e. for determining without ambiguity the "V-A" form of the interaction. An extensive use will be made of semi-quantitative physical arguments. This means that instead of the long calculations which are necessary for deriving the exact form of e.g. angular correlation we shall predominantly use simple considerations in order to find whether for instance these correlations are predominantly forward or backward. It is hoped that in this way the natural sequence of ideas will become more transparent.

It will be assumed that in all possible forms of $\beta^-$ decay it is always the same kind of neutrino which is emitted (this is equivalent to the conservation of lepton number). On the other hand, neither the 2-component theory of the neutrino nor PC invariance will have to be assumed from the start. Indeed the (at least approximate) validity of these concepts will be - within the limits of experimental errors of course - derived from the experimental data.

In order to make the sequence of ideas easier to follow, let us first state it without any proof or comment

a) From the angular distribution of electrons from polarized nuclei and from the measurement of longitudinal polarization of the electrons it is deduced that:

with $V$ and $A$ couplings one should, in the Hamiltonian, introduce the neutrino field $\psi^\nu$ through the combination $(1 + \gamma_5) \psi^\nu$. Or in other words, the neutrino emitted by $\beta^+$ emitters has negative helicity, the antineutrino emitted by $\beta^-$ emitters has positive helicity;

with $S$, $T$, $P$ couplings exactly the reverse is true.
b) From observations on the neutrino helicity (G.T. couplings) and on the electron-neutrino angular correlation (F. couplings) it is then deduced that A and V couplings are present and not T and S.

c) From a study of the neutron lifetime and of the electron and neutrino angular distribution in the decay of polarized neutrons it is deduced that the V and A coupling come in in the approximate relative amounts \( V = 1.2 \, A \).

(b) Notations and formulae.

In what follows the index \( k \) runs from 1 to 3 while greek indices run from 1 to 4. We set

\[
\begin{align*}
\kappa &= c = 1 \\
\sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
\sigma_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\
\sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\end{align*}
\]

four by four matrices are often written in a compact notation, e.g.

\[
\begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ means } \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}
\]

\[
\begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ means } \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}
\]

\[
\bar{\sigma}_1 \quad \text{means} \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}
\]

etc.
We denote by $p = \sqrt{p_1^2 + p_2^2 + p_3^2}$ the absolute value of the momentum (positive number). The notations of Dirac matrices are the same as those of Yamaguchi, above, but two explicit representations of the matrices are used. In both of them all $\gamma_\mu$ are unitary and hermitian. Then
\[ \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4 \] (1)
is also unitary and hermitian. In both of them the Dirac equation is then written as
\[ \gamma_\mu \frac{\partial \psi}{\partial x_\mu} + m \psi = 0 \] (2)

1st representation:

This one is denoted by $\gamma$ without a dash. It is
\[ \gamma_k = i \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad \gamma_\nu = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \] (3)

writing $\psi = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} e^{i(p \cdot x - Et)}$, $u_1$, $u_2$ being 2 two-component spinors, the Dirac equation reduces to
\[ (E-m) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = -\frac{\gamma}{E-m} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \]
\[ (E+m) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = -\frac{\gamma}{E+m} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \] (4)

which, it should be stressed, are not independent (one is obtained from the other through multiplication by $\gamma_5$). Either one gives
\[ u_2 = \frac{-\gamma}{E+m} u_1 \] (5)

It shows that $u_2$ vanishes for $p \to 0$ : for non relativistic velocities $u_2$ is much smaller than $u_1$. $u_2$ is called the "small component".
2nd representation:

Let us make the change of representation

\[ \psi' = S^{-1} \psi \]
\[ \xi'_\mu = S^{-1} \xi_\mu \]
\[ S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \]

(6) (7)

which of course leaves the Dirac equation invariant

\[ \left( \gamma'_\mu \frac{\partial \psi'}{\partial x_\mu} + m \psi' \right) = S^{-1} \left[ \xi'_\mu S S^{-1} \frac{\partial \psi}{\partial x_\mu} + m \psi \right] = S^{-1} \left[ \xi'_\mu \frac{\partial \psi}{\partial x_\mu} + m \psi \right] = 0 \]

one finds

\[ \gamma' = -\gamma_k = i \begin{pmatrix} -\sigma_k \\ \sigma_k \end{pmatrix}, \quad \gamma'_5 = \gamma_5 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \gamma'_5 = \gamma_5 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \]

(8)

this is therefore the representation which is adapted to the 2 component neutrino theory (see Jauch's lectures). Setting

\[ \psi' = \varphi' e^{i \left( \mathbf{p} \cdot \mathbf{x} - Et \right)}; \quad \varphi' = \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} \]

(9)

the Dirac equation reduces to

\[ \overrightarrow{\sigma} \mathbf{p} \cdot \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} - E \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} + m \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = 0 \]

(10a)

\[ -\overleftarrow{\sigma} \mathbf{p} \cdot \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} - E \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} + m \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = 0 \]

(10b)

If \( m=0, \ E=\mathbf{p} \) (positive energy) two special solutions are

\[ \varphi_+ = \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = \frac{1+\gamma^5}{2} \varphi' \quad \text{with} \quad \overrightarrow{\sigma} \mathbf{p} \cdot \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = -\mathbf{p} \cdot \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} \]

(11)

and

\[ \varphi_- = \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = \frac{1-\gamma^5}{2} \varphi' \quad \text{with} \quad \overleftarrow{\sigma} \mathbf{p} \cdot \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = \mathbf{p} \cdot \begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} \]

(12)
\( \Phi^+_1 \) and \( \Phi^-_1 \) correspond to cases (a) and (b) in Fig. 1 of Dr. Yamaguchi’s lectures, i.e.

\[
\begin{align*}
\Phi^+_1 &= \frac{1 + \gamma_5^1}{2} \Phi^1 \quad \text{has helicity } -1 \\
\Phi^-_1 &= \frac{1 - \gamma_5^1}{2} \Phi^1 \quad \text{has helicity } +1
\end{align*}
\]

If an interaction Hamiltonian \( h \) which describes the absorption of a neutrino contains the 4 component neutrino field \( \Psi_\nu \) only in the combination \( (1 + \gamma_5^1) \Psi_\nu \), it is clear that the wave function of this neutrino is necessarily of the type \( \Phi^+_1 \): this means that such a Hamiltonian can absorb only neutrinos with helicity \(-1\). This same Hamiltonian \( h \) only creates of course antineutrinos with helicity \(+1\). \(^*)\)

\(^*)\) This is obvious in the language of the hole theory: creating an antineutrino is the same thing as absorbing a neutrino in a negative energy state \( (E = -p) \). For such a particle the \( \Phi^+_1 \) solution corresponds (see 10b) to

\[
\begin{align*}
\not{p} u^+_1 &= p u^+_1 \quad \text{thus corresponds to spin parallel to momentum.}
\end{align*}
\]

The hermitian conjugate term to \( h \) (which is always simultaneously present) then creates neutrinos with helicity \(-1\) and absorbs antineutrinos with helicity \(+1\).

In the case that we have to do with a relativistic particle of non zero mass (for instance a high energy electron) it is intuitively clear that the foregoing results will remain true to the extent that \( m \) can be neglected in comparison with the energy \( E \) of the particle. The proof requires some calculation \(^**)\)

\(^**)\) Again - if \( m \neq 0 \) - (10a) and (10b) are not independent. Let us look for a solution corresponding to a spin parallel or antiparallel to the momentum. The solution we are looking for must be an eigenfunction of the equation
\[ \left( \vec{\sigma} \cdot \vec{p} \right) \Phi' = \lambda \Phi' \]

i.e. it must satisfy to
\[ \vec{\sigma} \cdot \vec{p} \ u'_1 = \lambda u'_1 \]
\[ \vec{\sigma} \cdot \vec{p} \ u'_2 = \lambda u'_2 \]

\( \lambda \) being an eigenvalue that has to be determined. From (10) and these equations:
\[ (E - \lambda) \ u'_2 = m u'_1 \]
\[ (E + \lambda) \ u'_1 = m u'_2 \]

this gives
\[ (E^2 - \lambda^2) \ u'_1 = m^2 u'_1 \]

therefore if \( u'_1 \neq 0 \), \( \lambda^2 = E^2 - m^2 = p^2 \), \( \lambda = \pm p \). Expressing the smaller component in terms of the larger one we have the two (positive energy) solutions

\[ \begin{pmatrix} u'_1 \\ m \rho \ u'_1 \end{pmatrix} \] \hspace{1cm} \text{with} \hspace{1cm} \vec{\sigma} \cdot \vec{p} \ u'_1 = -p u'_1 \hspace{1cm} (\lambda = -p) \hspace{1cm} \text{(spin antiparallel to momentum)}

\[ \begin{pmatrix} m \rho \ u'_2 \\ u'_2 \end{pmatrix} \] \hspace{1cm} \text{with} \hspace{1cm} \vec{\sigma} \cdot \vec{p} \ u'_2 = p u'_2 \hspace{1cm} (\lambda = p) \hspace{1cm} \text{(spin parallel to momentum)}

which reduce to \( \Phi'_+ \) and \( \Phi'_- \) if \( m \to 0 \). One sees that the difference between the first solution and \((1 + \gamma_5) \psi_c \) is of the order \( \frac{m c}{E} \) if \( m_c \ll E \). This means that a Hamiltonian involving \((1 + \gamma_5) \psi_c \) and its hermitian conjugate has a natural tendency to absorb and create electrons with a negative helicity: indeed with a helicity which tends to \(-1\) when \( \frac{m c}{E} \to 0 \) i.e. when \( \frac{E}{c} = \frac{p}{E} \to 1 \). More detailed calculations show that in \( \beta \) decay this helicity is just \(-\frac{\gamma}{c}\).
(c) Statement of the problem.

The fundamental assumptions are

i) that the interaction hamiltonian of $^\beta$ decay is quadrilinear in $\psi_p \, \psi_n \, \psi_e$ and $\psi_{\nu}$

ii) that whatever term of the hamiltonian is responsible for a particular $^\beta$ decay, the mass zero particle emitted together with a negative electron is always the same. We call this particle antineutrino (in order to have lepton number conservation).

Then the most general Lorentz invariant hamiltonian is

$$
\sum C_i (\psi_p^+ \, \gamma_p \, O_i \, \psi_n \, \gamma_e \, \gamma_e \, O_i \, \psi_{\nu}) + C_i' (\psi_{\nu}^+ \, \gamma_{\nu} \, O_i \, \psi_n \, \gamma_e \, \gamma_e \, O_i \, \psi_{\nu}) + \text{h.c.} \tag{13}
$$

where $C_i, \, C_i'$ are 10 arbitrary complex numbers and where

$$
0_1 = 1, \quad 0_2 = \gamma_\mu, \quad 0_3 = -i (\gamma_\mu \, \gamma_\nu - \gamma_\nu \, \gamma_\mu), \quad 0_4 = i \gamma_\mu \, \gamma_5, \quad 0_5 = \gamma_5 \tag{14}
$$

(notice $0_1^+ = 0_1$ ; + means hermitian conjugate) *)

*) It is not necessary to add terms with $(0_1 \, \gamma_5) \, (0_1)$, nor with $(0_1 \, \gamma_5) \, (0_1 \, \gamma_5)$ since these reduce to those already in (13) and would therefore simply modify the definition of the $C_i, \, C_i'$. This is easily checked.

$\psi_e$ is the field corresponding to $e^-$, i.e. $\psi_e$ absorbs $e^-$ (and creates $e^+$), $\psi_e^+$ creates $e^-$ (and absorbs $e^+$).

In the days when parity was thought to be conserved, one of course only wrote the terms with $C_i$. These five interaction terms ($i=1, \ldots, 5$) were given the names $S, V, T, A, P$ respectively.
Now the definition has somewhat changed. Writing (13) as *)

\[ \sum_i C_i \left[ (\psi_\nu^+ Y_i O_i \psi_\nu)(\psi_\nu^+ Y_i O_i (1 + \frac{C_i}{C_i'} \delta_r) \psi_\nu^+) + h.c. \right] = 0. \]  

(15)

one calls S, V, T, A, P the five straight brackets (all of them thus including the \((1 + \frac{C_i}{C_i'} \delta_r)\) factor). We shall see first that \(\frac{C_i}{C_i'}\) is determined by experiments to be \(\approx +1\) in all cases (thus giving experimental support to the 2 component neutrino theory with \(1 + \delta_5\) neutrino with helicity \(-1\), antineutrino with helicity \(+1\), see above). We shall then specify the only non zero \(C_i\) to be \(C_2\) and \(C_4\), both nearly real and having comparable magnitude and opposite signs (V-A coupling).

**

Before analysing the results of the different experiments we should briefly recall the distinction between Fermi and Gamow-Teller transitions. This distinction is based on a non-relativistic approximation for the nucleons. Let us therefore study the non-relativistic approximations to

\[ \psi_\nu^+ Y_i O_i \psi_\nu \]

using for the \(Y\) matrices the representation (3). We get with

\[ \psi_\nu = \begin{pmatrix} u_1^+ \\ u_2^+ \end{pmatrix}, \quad \psi_\nu^+ = \begin{pmatrix} u_1^+ \\ u_2^+ \end{pmatrix} \]

and using the result (5) that \(u_2\) is of order \(v/c\) relative to \(u_1\):

\[ \psi_\nu^+ Y_i O_i \psi_\nu = u_1^+ u_1^n - u_2^+ u_1^n = u_1^+ u_1^n + \text{order } \frac{v}{c}. \]  

(16)

\[ \begin{align*}
\psi_\nu^+ Y_i Y_j \psi_\nu &= i u_1^+ \sigma_k u_2^+ u_2^n + i u_2^+ \sigma_k u_1^n = \text{order } \frac{v^2}{c^2} \\
\psi_\nu^+ Y_i \psi_\nu &= u_1^+ u_1^n + u_2^+ u_1^n = u_1^+ u_1^n + \text{order } \frac{v^2}{c^2}. 
\end{align*} \]  

(17)

*) It should perhaps be pointed out that expressions

\[ (\psi_i^* \gamma_i \psi_j \psi_k \psi_l \psi_m) \]

and

\[ (\psi_i^* \gamma_i \psi_j \psi_k \psi_l) \]

are identical, by virtue of (6). We have therefore the freedom of putting or removing dashes (i.e. changing the representation).
\[
\begin{align*}
\left\{ \begin{array}{l}
-i \Psi^+_{k,i,j} \chi_i \chi_j \Psi_n = \Psi^+_{1,1} \sigma^x \Psi_n + \text{order } \frac{\nu}{c} \\
-i \Psi^+_{k,i,j} \chi_i \chi_j \Psi_n = \text{order } \frac{\nu}{c} \\
i \Psi^+_{k,i,j} \chi_i \chi_j \chi_j \Psi_n = -i \Psi^+_{1,1} \sigma^x \Psi_n + \text{order } \frac{\nu}{c} \\
i \Psi^+_{k,i,j} \chi_i \chi_j \chi_j \Psi_n = \text{order } \frac{\nu}{c} \\
\end{array} \right.
\end{align*}
\]

(18) \hspace{1cm} i,j,k \text{ cycl. perm.}

(19)

(20)

Neglecting terms of order \( \nu/c \) this table shows that:

S and V interactions involve only \( \Psi^+_{1,1} \Psi_n \). This corresponds to no spin change in the nucleon system; therefore the e, \( \nu \) system can be said to carry away essentially no angular momentum. These transitions are called Fermi (F) transitions. For nuclei they correspond to the selection rule \( \Delta J = 0 \).

Notice moreover that, to this approximation, only the product of the 4th components contributes to the V interaction.

T and A interactions involve only \( \Psi^+ \sigma \Psi_n \). This corresponds either to a spin flip (components \( \sigma_1, \sigma_2 \)) with the e, \( \nu \) system taking away one unit of angular momentum, or to a non spin flip (component \( \sigma_3 \)) but with the e, \( \nu \) system still carrying away one unit of angular momentum according to the vector combination

\[
\left( \begin{array}{c}
J = \frac{1}{2} \\
J_3 = +\frac{1}{2}
\end{array} \right) \Rightarrow \left( \begin{array}{c}
J = \frac{1}{2} \\
J_3 = +\frac{1}{2}
\end{array} \right) + \left( \begin{array}{c}
J = 1 \\
J_3 = 0
\end{array} \right)
\]

in. spin \hspace{1cm} \text{final p spin} \hspace{1cm} \text{angular momentum carried away}
These transitions are called Gamow-Teller (G.T) transitions. For nuclei they correspond to the selection rule $\Delta J = \pm 1, 0$ with no $0 \rightarrow 0$.

It is not a straightforward matter to distinguish between a Fermi transition and a G.T. transition with $\Delta J = 0, J \neq 0$. Sometimes, however, this is possible (for values for mirror nuclei). We shall not enter here into such considerations.

(d) Experiments on the asymmetry of electrons from oriented nuclei, and on electron polarization: $1 + \gamma_5$ associated with $\gamma_e$.

Let us consider for instance the famous experiment with polarized Co$^{60}$

$$\text{Co}^{60} \rightarrow \text{Ni}^{60} + e^- + \bar{\nu}$$

Co$^{60}$ has spin 5, Ni$^{60}$ has spin 4. This is therefore a G.T. transition, either A or T. Let us take the z axis in the direction of the Co spin. The experiment shows that the electrons are preferentially emitted downward. Now simply from conservation of angular momentum in the z direction (see Fig.1) the electron must have its spin pointing upward. Therefore the electrons are preferentially emitted with a negative helicity.

<table>
<thead>
<tr>
<th>initial state</th>
<th>final state</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \uparrow ] Co</td>
<td>either [ \bar{\nu} ] or [ \uparrow ] Ni</td>
</tr>
<tr>
<td>[ \uparrow ] Co</td>
<td>[ \uparrow ] Ni</td>
</tr>
<tr>
<td>[ \uparrow ] Ni</td>
<td>[ \uparrow ] e</td>
</tr>
<tr>
<td>[ \uparrow ] e</td>
<td>[ \uparrow ] \bar{\nu}</td>
</tr>
</tbody>
</table>

Fig.1

(single arrow : momentum; double arrow : spin).
This result is confirmed by all experiments that have been done to measure the helicity of the electrons and positrons. These have been done for both G.T. and Fermi transitions, using a large number of nuclei, and the result is that within 10% errors,

the helicity of $e^-$ is $-\frac{v}{c}$

" " " e^+ is $+\frac{v}{c}$

**Interpretation**

From what was said in the first section this means that the interaction hamiltonian of $\beta$ decay contains $\psi_\psi$ only in the combination

$$(1 + \gamma_5) \psi_e$$

in other words its most general form is

$$\sum_i C_i (\psi^+_\psi \chi_i S_i \psi_e) (\gamma^+_\gamma (1+\gamma_5) \gamma_i S_i \gamma_e) + h.c. \quad (21)$$

Commuting $1+\gamma_5$ through $\gamma_4^0 \chi_i$ and remembering that

$$\gamma_5 \gamma_4^0 \chi_i = \gamma_4^0 \chi_i \gamma_5 \quad \text{for } i = 2,4$$

$$= -\gamma_4^0 \chi_i \gamma_5 \quad \text{for } i = 1,3,5$$

we obtain the general form (15) with

$$\frac{C_i}{C_i} = +1 \quad \text{for } i = 2,4 \quad (V \text{ and } A)$$

$$-1 \quad \text{for } i = 1,3,5 \quad (S,T \text{ and } P) \quad (22)$$

We may notice here that these experiments are compatible with the 2 component neutrino theory provided that the coupling is either a $V,A$ mixture ( $\gamma$, helicity = -1) or a $S,T,P$ mixture ( $\gamma$, helicity +1). As, however, we do not want to assume the 2 component neutrino theory we may still at this point have all five couplings.
(e) Experiments on the neutrino polarization and experiments on electron-neutrino angular correlation: $V_A$ coupling.

The next step is obviously to try and measure directly the helicity of the neutrino. This is the well-known ingenious experiment

$$\text{Eu}^{152} + e^- \rightarrow \text{Sm}^{152*} + \gamma$$

$$\downarrow$$

$$\text{Sm}^{152} + \gamma$$

Eu$^{152}$ has spin 0 while Sm$^{152*}$ has spin 1 (Sm$^{152}$ has spin 0). This is therefore a pure G.T. transition (A or T). The important point of the experiment is that they can select those $\gamma$ rays which move opposite to the direction of the neutrino and measure their polarization.

Let us take as our $z$ axis the (known) direction of motion of the neutrino. The orbital angular momentum along Oz is zero since all particles move along this line: along Oz we have thus simply to balance the spin angular momenta in the initial and final state. Now it is found experimentally that the spin of the $\gamma$ along Oz is +1; this value added to the $z$ component of the neutrino spin (Sm$^{152}$ has spin 0) should give the ($\frac{1}{2}$) $z$ component of the electron spin (Eu$^{152}$ has spin 0). Clearly this is possible only if the $z$ component of the neutrino spin is $-\frac{1}{2}$, which means that the helicity of the neutrino is $-1$, or in other words that the Hamiltonian which is responsible for this transition contains $\gamma_5$ through the combination

$$(1 + \gamma_5) \psi_\nu$$

Comparing this with the results obtained from the previous experiments we deduce that the interaction is A and not T.
If one could do the same thing for a Fermi transition one would similarly be able to choose between $S$ and $V$ (remember that $P$ transitions are very small). This however is very difficult for practical reasons, so that the discrimination between $S$ and $V$ has indeed been made using a different approach, namely the electron-neutrino angular correlation (recoil experiments). The argument in that case is as follows:

For a Fermi transition we know that the electron-neutrino system carries away no angular momentum. Let us consider a situation where $e$ and $\bar{\nu}$ propagate along the same line (same or opposite direction). Then the orbital angular momentum along this line is essentially zero and therefore the spin components of $e$ and $\bar{\nu}$ along that direction must be opposite. Now we know already from the first category of experiments (electron polarization) that emitted electrons have negative helicity, or in other words that $\psi_e$ appears only in the combination

$$(1 + \gamma_5) \psi_e$$

which means that the $S$ and $V$ interactions contain the factors

$$\psi_e^+ (1 + \gamma_5) \gamma_\mu \psi_\nu = \psi_e^+ \gamma_\mu (1 - \gamma_5) \psi_\nu \quad S \quad (23)$$

$$\psi_e^+ (1 + \gamma_5) \gamma_\mu \gamma_\nu = \psi_e^+ \gamma_\mu (1 + \gamma_5) \psi_\nu \quad V \quad (24)$$

respectively.

Therefore if the interaction were $S$ the neutrino field should appear under the combination

$$(1 - \gamma_5) \psi_\nu$$

which would mean neutrino with positive helicity, or antineutrinos with negative helicity. $e$ and $\bar{\nu}$ (and similarly $e^+$ and $\nu$ )
should therefore have the same helicity which means that, their spins being antiparallel, they should propagate preferentially in opposite directions (negative correlation). If the interaction is $V$ clearly the reverse is true. Experimentally $A^{35}$, which from its ft value is expected to be a nearly pure Fermi transition between mirror nuclei, gave a positive correlation: this shows that the Fermi transition is $V$ and not $S$. It has therefore been shown experimentally that $\beta$ decay is essentially $V$ and $A$.

\[(f) \text{ Experiments on the neutron lifetime} \]
\[
\frac{|C_A|}{|C_V|}.
\]

A precise knowledge of the neutron lifetime has recently been acquired. This lifetime can of course be computed in terms of $C_A$ and $C_V$.

On the other hand $^{\text{14}}O$ decays by a transition which is a $0 \rightarrow 0$ transition (therefore pure Fermi) and moreover a transition between two members of the same isotopic spin multiplet: this means that the initial and final nuclear wave functions are essentially the same except for known numerical factors (Clebsch-Gordan coefficients) and that therefore the nuclear part of the transition matrix can be evaluated. The mean lifetime of $^{\text{14}}O$ therefore gives $|C_V|$ and comparing with the neutron lifetime one gets

\[
\frac{|C_A|^2}{|C_V|^2} = 1.42 \pm 0.08 \quad \text{(25)}
\]

\[
\frac{|C_A|}{|C_V|} = 1.2 \quad \text{(26)}
\]
(g) Experiments on the up-down asymmetry of electrons and neutrinos in the decay of polarized neutrons.

\[ C_A = -1.2 \, C_V. \]

The last step in the analysis is to determine the relative phase of \( C_V \) and \( C_A \). Let us recall that (15) now reduces, according to the previous analysis, to

\[
C_V \left( \gamma_4 \gamma_\mu \psi_n \right) \left( \gamma_e^+ \gamma_4 \gamma_\mu \left( 1 + \gamma_5 \right) \psi_\nu \right) + \]

\[
+ C_A \left( \gamma_p^+ \gamma_4 \gamma_\mu \gamma_5 \psi_n \right) \left( \gamma_e^+ \gamma_4 \gamma_\mu \gamma_5 \left( 1 + \gamma_5 \right) \psi_\nu \right)
\]

\[
= \left( \gamma_p^+ \gamma_4 \gamma_\mu \left( C_V - C_A \gamma_5 \right) \psi_n \right) \left( \gamma_e^+ \gamma_4 \gamma_\mu \left( 1 + \gamma_5 \right) \psi_\nu \right)
\]

whose non relativistic limit (for nuclear particles) is according to (17) and (19)

\[
C_V \left( \bar{u}_p^+ \bar{u}_n \right) \left( \gamma_e^+ \left( 1 + \gamma_5 \right) \psi_\nu \right)
\]

\[
- i \, C_A \left( \bar{u}_p^+ \gamma_k \bar{u}_n \right) \left( \gamma_e^+ \gamma_4 \gamma_k \left( 1 + \gamma_5 \right) \psi_\nu \right)
\]

k = 1, 2, 3

as regards the second brackets in each of these terms it is convenient for our purpose to make the change of representation (6), (7) which gives

\[
\gamma_e^+ \left( 1 + \gamma_5 \right) \psi_\nu = 2 \, \bar{u}_{e1}^+ \psi_{\nu 1}
\]

\[
- i \, \gamma_e^+ \gamma_4 \gamma_k \left( 1 + \gamma_5 \right) \psi_\nu = 2 \, \bar{u}_{e1}^+ \gamma_k \psi_{\nu 1}
\]

in terms of the 2 components spinors \( \psi_{\nu 1} \), \( \psi_{\nu 1} \) which correspond to negative (\( -\frac{\gamma_5}{\gamma_c} \)) helicity.
Let us now consider the decay
\[ n \to p + e + \bar{\nu} \]
starting from a neutrino which is completely polarized along the \( z \) direction ("upward" direction). In order to have a qualitative view of what happens while at the same time avoiding long calculation, let us focus our attention on the cases where all spins and momenta are colinear. Then, remembering that because of the \( V,A \) structure of the Hamiltonian \( e \) has negative helicity and \( \bar{\nu} \) has positive helicity (see above) we have only to consider the three cases of fig. (2) where the spins have been balanced along \( O_2 \).

\[
\begin{array}{cccccc}
  n & p & e & \bar{\nu} & \text{amplitude} \\
\end{array}
\]

\[
\text{(see below)}
\]

a)
\[
\begin{array}{ccc}
  \uparrow & \downarrow & \uparrow \\
\end{array}
\]
\[2C_A \]

b)
\[
\begin{array}{ccc}
  \uparrow & \uparrow & \downarrow \\
\end{array}
\]
\[C_V + C_A \]

c)
\[
\begin{array}{ccc}
  \uparrow & \downarrow & \uparrow \\
\end{array}
\]
\[C_V - C_A \]

fig. (2)

Case a) of fig. (2) corresponds to a spin flip of the nucleon, i.e. to \( k = 1,2 \) in (29) (because \( \sigma_1 \) and \( \sigma_2 \) are antidiagonal). From (29 and 31) we see that the amplitude is \( 2C_A \).

Cases b) and c) of fig. (2) correspond to a non spin flip, i.e. to both the first terms and the term with \( k = 3 \) of (29). As \( U_n = ( ) \) these two terms can be condensed into
\[(U_{1p}^+ U_{1n}^-) (U_{e1}^+ (c_v + c_A \sigma_3^e) U_{e1}^-)\]

now in order to distinguish between cases b) and c) it is sufficient to
look at the effect of \(\sigma_3^e\) on U_{e1}^+ (as an exercise one can then verify that
for \(\bar{V}\) everything comes out all right, if, for instance, one treats it as a
hole in negative energy states). In case b), \(U_{e1}^+ \simeq {}^1\), therefore
\(\sigma_3^e U_{e1}^+ \simeq U_{e1}^+\) and the amplitude is \(\asymp c_v + c_A\). In case c), \(U_{e1}^+ \simeq {}^0\) and
the amplitude is \(\asymp c_v - c_A\).

**Electron asymmetry.**

The probability for the electron going upward is \(|c_v - c_A|^2\) (case (c))
while the probability for the electron going downward is \(|2c_A|^2 + |c_v + c_A|^2\)
(cases (a) + (b)). Therefore:

\[
\begin{align*}
\text{if } c_v &\asymp c_A \text{ the asymmetry is maximum;} \\
\text{if } c_v &\asymp -c_A \text{ the asymmetry is } 0
\end{align*}
\]

(both probabilities having
then equal magnitudes.)

**\(\bar{V}\) asymmetry.**

The probability for \(\bar{V}\) going upward is \(|2c_A|^2 + |c_v - c_A|^2\)
(cases (a) and (c)), while the probability for \(\bar{V}\) going downward is
\(|c_v + c_A|^2\) (case (b)). Therefore:

\[
\begin{align*}
\text{if } c_v &\asymp c_A \text{ the asymmetry is } \asymp 0 \\
\text{if } c_v &\asymp -c_A \text{ the asymmetry is maximum.}
\end{align*}
\]

Thus, a study of either the electron or the \(\bar{V}\) asymmetry with respect
to n polarization gives information about the relative magnitude and phase
of \(c_v\) and \(c_A\). Both experiments have indeed been by the same authors with
the result that
the electron asymmetry is very small, while the neutrino asymmetry is very near to the maximum. From what we have just seen these two results show that $C_V$ and $C_A$ have roughly equal magnitudes and that their phases differ by roughly $180^\circ$ or combining these results with the previous one, that

$$C_A \approx -1.2 C_V$$  \hspace{1cm} (32)

(h) Conclusion

Carrying (32) into (28) one finds that this long sequence of beautiful experiments leads to the simple result that the $\beta$ decay Hamiltonian has the form

$$C \left( \psi_p^+ \gamma_4 \gamma_\mu (1+\gamma_5) \psi_n \right) \left( \psi_e^+ \gamma_4 (1+\gamma_5) \psi_\nu \right) - \text{h.c.} \hspace{1cm} (33)$$

the absolute value of $C$ being $^{0_{14} \text{ experiment}}$

$$C = 1.4 \times 10^{-49} \text{ erg cm}^3$$  \hspace{1cm} (34)

The phase of $C$ is in fact not observable so that we may just as well take $C$ real.

An important corollary to the result (32), or (33), is PC invariance: to the extent that $C_A/C_V$ is a truly real number, the criteria given by Professor Jauch in previous lectures show that PC invariance holds.*). The statement is therefore that no deviation from PC invariance has been detected by experiment.

*) if $C_V$ were taken as complex, one could always compensate this by introducing complex factors in the definition of charge conjugation.
V. STRUCTURE OF WEAK INTERACTIONS IN SPACE-TIME.

(4) Muon decay.

B. d'Espagnat
4. Muon Decay. (by B. d'Espagnat)

(a) Assumptions.

Much less experimental data are available in muon decay than in beta decay. Therefore it is not possible here to proceed in a purely deductive way, as was done, more or less, in the foregoing chapter. Instead of deducing the two component neutrino formalism from the experimental data we shall here be obliged to assume its validity from the start. We shall then check that it is indeed compatible with the experimental data we know and, moreover, we shall use these data, as in beta decay, in order to determine the values of the coupling constants.

Here the assumptions are therefore:

A) conservation of lepton number (as before).

B) validity of the two component neutrino theory.

More precisely, this second assumption means: the interaction hamiltonian is such that only neutrinos with helicity $\pm 1$ can be created.

(b) The alternative $\nu + \bar{\nu}$

or $\nu + \bar{\nu}$. General form of the Hamiltonian.

The alternative

In muon decay there are altogether four leptons, $\mu$, $e$, $\nu$, $\bar{\nu}$ and, making always the convention that the $e^-$ is a lepton, we still have two possibilities of satisfying the rule of lepton conservation. The first one is to treat $\mu^-$ as a lepton and to write ($l$ = leptonic number)
\[ \ell^- \rightarrow e^- + \nu + \bar{\nu} \quad (\text{then} \quad \ell^+ \rightarrow e^+ + \bar{\nu} + \nu) \]
\[ \ell = 1 \quad 1 \quad 1 \quad 1 \]
\[ \ell^- = -1 \quad -1 \quad -1 \quad 1 \quad 1 \]

while the second one is to treat \( \mu^- \) as an antilepton, i.e. \( \mu^+ \) as a lepton, and therefore to write

\[ \mu^- \rightarrow e^- + \bar{\nu} + \nu \quad (\text{then} \quad \mu^+ \rightarrow e^+ + \nu + \bar{\nu}) \]
\[ \ell = -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad 1 \quad 1 \quad \]
\[ \ell^+ = +1 \quad -1 \quad +1 \quad +1 \quad +1 \quad +1 \quad +1 \quad +1 \quad +1 \]

(1) and (2) are indeed completely different hypotheses - and not simply two different ways of writing - as can be seen from the fact that in (1) the two neutral particles are different, while in (2) they are identical. (2) was, for a time, considered as rather attractive because it provided a general principle for excluding unobserved processes such as

\[ \mu^- \rightarrow e^- + e^+ + e^- \]

(3)
simply through the use of the law of lepton conservation. Hypothesis (2), however, is now abandoned because, together with the two-component neutrino assumption, it leads to a shape of the spectrum which is contradicted by experiment, as we shall see in the next section.

**General form of the Hamiltonian.**

Returning then to hypothesis (1) (non-identical neutrinos) let us write down the most general 4 fermion interaction hamiltonian which is compatible with it and also with the two-component neutrino theory.

Clearly, if we want to write such a general expression, the first question which arises is: in what order shall we write the four different fields? This is of course a general problem with four fermion interactions and one could also have mentioned it in the case of beta decay.
There, however, we had two baryons and two leptons and it was therefore a very natural choice to group these two pairs into the two brackets, as was done. Because of the existence of the Fierz identities, which were proven in Professor Yamaguchi's lectures, this choice does not affect the generality of what we wrote. Here the situation is exactly the same in the sense that we can choose a definite ordering without weakening the generality of our expressions, but it is different in the sense that, having four leptons at our disposal, there is not a particular choice which looks a priori more natural than the others. Indeed two possible choices immediately come to our attention. The first one is to use the ordering

\[(\mu, \nu) (e, \nu)\]  \hspace{1cm} (4)

parallel to the ordering

\[(p, n) (e, \nu)\]

used in the $\beta$ decay formalism. The second one is to use the ordering

\[(\mu, e) (\nu, \nu)\]  \hspace{1cm} (5)

which groups the two neutrino fields in the same bracket.

It turns out that for later purpose it is more convenient to use the second ordering. The neutrino bracket must then, under our assumption, contain $\psi_\nu^+$ and $\psi^+_\nu$ only in the combination \((1+\gamma_5)\psi_\nu\) and $\psi^+_\nu(1+\gamma_5)$, i.e. it must be of the general form

\[\psi^+_\nu(1+\gamma_5) \gamma_4 \, 0_1 \, (1+\gamma_5) \psi_\nu\]  \hspace{1cm} (6)

however, with $i=1,3,5$, $\gamma_5 \gamma_4 \, 0_1 = - \gamma_4 \, 0_1 \gamma_5$ and therefore with $i=1,3,5$

\[(1+\gamma_5) \gamma_4 \, 0_1 (1+\gamma_5) = \gamma_4 \, 0_1 (1-\gamma_5)(1+\gamma_5) = \gamma_4 \, 0_1 (1-\gamma_5^2) = 0\]
We are thus simply left with \( i=2,4 \) i.e. with \( V \) and \( A \) couplings and, because of \((1+\gamma_5)^2 = 2(1+\gamma_5)\) the most general Hamiltonian for the \( V, \bar{V} \) hypothesis is, under our assumptions

\[
(\psi_e^+ \gamma_4 \gamma_\mu (G_V - G_A \gamma_5) \psi_\mu^-) (\psi^+_\nu \gamma_4 \gamma_\mu (1+\gamma_5) \psi^-_\mu)
\]

It should be stressed that the fact that we are left with \( V \) and \( A \) couplings only is nothing absolute, in the sense that it depends on the particular choice of ordering we made. If this ordering is changed, by means of a Fierz transformation applied on (7), then in general other coupling types will also appear.

One could of course similarly write down the Hamiltonian for the \( V + \bar{V} \) hypothesis (identical neutrinos), but this will not prove necessary for the discussion which follows.

\[
(c) \text{ Experiments on the shape of the spectrum}
\]

\[\rho \neq 0 \text{ gives } V + \bar{V}.\]

The "elementary" shape of the spectrum

From perturbation theory one has

\[
\text{Probability per unit time } \propto |\text{Matrix Element}|^2 \times \text{density of final states}
\]

If these expressions are calculated for all values of the electron momentum \( p_e \), the shape of the electron spectrum obviously results. Let us first consider the simplified case where the matrix element is assumed not to depend on \( p_e/p_\nu \) (this we call by convention the "elementary" shape), and let us investigate the \( p_e \) dependence of the second factor.
We have

\[ +p_{\gamma_1} + p_{\gamma_2} = -p_e \]  \hspace{1cm} (8)

from momentum conservation and

\[ p_{\gamma_1} + p_{\gamma_2} = m_\mu - E_e = 2a \]  \hspace{1cm} (9)

from energy conservation \((E_e = \sqrt{m_\mu^2 + p_e^2})\). (8) means that with \(p_{\gamma_1}, p_{\gamma_2}\) and \(p_e\) we can construct a triangle, as shown in Fig.1. (9) then means that, for \(p_e\) fixed, point A of Fig.1 moves on an ellipsoid of revolution around \(p_e\), whose foci are B and C.

![Fig.1]

The ellipse itself has as parameters

- \(a\), half large axis
- \(b = \sqrt{a^2 - \frac{p_e^2}{4}}\), half small axis

For fixed \(p_e\) the density of states is obtained by varying the available neutrino energy \(2a\), by counting the number of "states" A available in the range of energy \(2a\), \(2a + d(2a)\), and by dividing by \(d(2a)\). This means evaluating the small volume enclosed between the two ellipsoids \(2a\) and \(2a + d(2a)\) and dividing by \(2a\) or, in other words, taking the differential

\[ \frac{dV}{d2a} \]
where $V$ is the volume of the ellipsoid

$$V = \frac{4}{3} \pi a b^2 = \frac{4}{3} \pi a (a^2 - \frac{p_e^2}{q})$$

thus

$$\text{density of states for fixed } p_e \quad \mathcal{L} \frac{dV}{dz} = \frac{2\pi}{3} (\frac{a^1}{q} - \frac{p_e^2}{q})$$

$$= 2\pi \left( \frac{m_e - \epsilon_e}{q} \right)^2 - \frac{2}{3} \frac{p_e^2}{q}$$

being not interested in the lower energy end of the spectrum we can make the approximation $m_e = 0$, $\epsilon_e = p_e$: then the density of states for $p_e$ comprised between $p_e$ and $p_e + dp_e$ is

$$\text{density of states } \propto \frac{dV}{dz} \propto p_e^2 dp_e \propto p_e^2 (m_e^2 - 2m_e p_e + \frac{2}{3} p_e^2) dp_e$$

(10)

This therefore is the "elementary" shape of the spectrum. On the other hand, the maximum momentum $W$ of the electron corresponds to a flat triangle in Fig. 1, i.e. to $p_{v1} + p_{v2} = p_e$ which means from (9):

$$p_e = W = m_e / 2$$

(11)

Rewriting (10) in terms of $p_e$ and $W$ we get

$$P(p_e) \propto p_e^2 (W - p_e) + \frac{1}{6} \frac{p_e^2}{W}$$

(12)

This shows that the "elementary" spectrum does not go smoothly to zero towards the high momenta but presents a discontinuity at the higher end ($p_e = W$). In the true spectrum this discontinuity will of course only be present if the matrix element does not go to zero at the high energy end $p_e = W$. A study of the high energy end of the spectrum therefore gives a very important test of whether the matrix element goes to zero or not at this energy.
In a more elaborate theory the fact has to be taken into account that no 4 fermion interaction gives a matrix element really independent of \( p_e, p_\nu \). This modifies slightly the spectrum (12) which becomes

\[
P(p_e) \propto p_e^2 \left[ w - p_e + \frac{2}{3} \rho (4p_e - 3w) \right]
\]

\( \rho \) being a function of the various coupling strengths. The presence of a discontinuity, i.e. the existence of a non vanishing matrix element at the high energy end is then expressed by \( \rho \neq 0 \).

**Discarding the hypothesis of identical neutrinos**

The high energy end of the spectrum obviously corresponds to both neutrinos going away in the same direction, which is opposite to that of the electron. (See Fig. 2).

![Fig. 2](image)

If both neutrinos are identical (2 neutrinos or 2 antineutrinos) and if the 2 component neutrino assumption is valid, then their spins are obviously parallel and therefore the spin wave function of the two neutrinos system is necessarily the symmetrical combination corresponding to total spin one. As, moreover, the two neutrinos are identical fermions, their total wave function must be antisymmetrical. This then means that, in momentum space, the momentum part of the wave function must be antisymmetrical. The neutrinos, however, have no interaction with each other and therefore their wave function is simply a product \( \Psi(p_{\nu_1})\Psi(p_{\nu_2}) \) of the individual plane-wave wave functions. The only antisymmetrical combination that one could think of would therefore be

\[
\Psi(p_{\nu_1})\Psi(p_{\nu_2}) - \Psi(p_{\nu_2})\Psi(p_{\nu_1})
\]

(14)
which is identically zero. This means that the matrix element, under our present assumptions, goes to zero at the high energy end of the spectrum or in other words, that \( \rho = 0 \).

Experimentally, one finds \( \rho \approx \frac{2}{3} \). The hypothesis of identical neutrinos should therefore be discarded.

(d) **Experiments on the electron angular distribution and polarization: \( G_V = G_A \).**

We are now left with the theory of non identical neutrinos, i.e. with (7) as the most general hamiltonian.

In order to determine \( \frac{G_V}{G_A} \) one can use experiments on the electron angular distribution in the decay of a (polarized) muon. The calculation of this angular distribution in terms of \( G_V \) and \( G_A \) is tedious, but, as far at least as the highest energy electrons are concerned, we may find the qualitative trend as follows:

Let us consider muons at rest, completely polarized in the z direction, and let us evaluate the relative probabilities of the highest energy electrons \( (p_e = W = \frac{m}{2}) \) going away downward (case (1)) and upward (case (2)). \( \text{[see Fig.2]} \). In each case the neutrinos have to go away parallel to each other and therefore, one of them being a \( \nu \) and the other one a \( \bar{\nu} \), with opposite spins: the z component of the total neutrino spin is therefore zero and the spin of the electron is, in both cases, parallel to the muon spin.

\[
\begin{array}{ccc}
\text{initial} & \text{case (1)} & \text{case (2)} \\
\uparrow & \bar{\nu} & \uparrow \\
\mu^+ & \uparrow & \uparrow \\
\downarrow & \nu & \downarrow \\
\end{array}
\]

*Fig.3*
Let us now calculate explicitly the first bracket of (7), using the 1st representation of the $\gamma$ matrices (described on p.36). The muon being at rest one has

$$\psi^-_\mu = \begin{pmatrix} u \\ 0 \end{pmatrix} \text{ with } u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (15)$$

On the other hand

$$\psi^-_e = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \text{ with } u_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ (spin up)} \quad (16)$$

and

$$u_2 = \frac{-\vec{P}_e}{m_e + E_e} u_1 \approx -\frac{p_e}{p_e} u_1 = -\frac{p_e}{p_e} \sigma_j u_1 = -\frac{p_e}{p_e} u_1 = \begin{cases} u_1 \text{ case (1)} \\ -u_1 \text{ case (2)} \end{cases} \quad (17)$$

Formulae (3) on p.36 give moreover

$$\psi^+_{e^-} Y_i Y_k Y_{\mu} = (u_i^+, u_i^+) \begin{pmatrix} i \sigma_k \\ \end{pmatrix} \begin{pmatrix} u \\ 0 \end{pmatrix} = u_2^+ \cdot \sigma_k \cdot u$$

$$\psi^+_{e^-} Y_i Y_k Y_{\mu} = (u_i^+, u_i^+) \begin{pmatrix} 1 \\ \end{pmatrix} = u_i^+ \cdot u$$

$$\psi^+_{e^-} Y_i Y_k Y_{\mu} = (u_i^+, u_i^+) \begin{pmatrix} i \sigma_k \\ \end{pmatrix} \begin{pmatrix} u \\ 0 \end{pmatrix} = u_i^+ \cdot \sigma_k \cdot u$$

$$\psi^+_{e^-} Y_i Y_k Y_{\mu} = (u_i^+, u_i^+) \begin{pmatrix} 1 \\ \end{pmatrix} \begin{pmatrix} u \\ 0 \end{pmatrix} = u_i^+ \cdot u$$
In case (1) \((u_2 = u_1)\), (7) thus gives
\[
\begin{align*}
(G_V - G_A) (u_1^+, u_1) (\psi_1^+ & \gamma_{15}, \gamma_{15} (1 + \gamma_5) \psi_1) + \\
+ (G_V - G_A) (u_1^+, u) (\psi_1^+ \gamma_4 \gamma_4 (1 + \gamma_5) \psi_1)
\end{align*}
\] (18)

whereas in case (2), \((u_2 = -u_1)\), (7) gives
\[
\begin{align*}
(-G_V - G_A) (u_1^+, \sigma_{15}, u) (\psi_1^+ & \gamma_{15}, \gamma_{15} (1 + \gamma_5) \psi_1) + \\
+ (G_V + G_A) (u_1^+, u) (\psi_1^+ \gamma_4 \gamma_4 (1 + \gamma_5) \psi_1)
\end{align*}
\] (19)

If \(G_V = G_A\), the theory thus predicts a maximum forward negatron asymmetry whereas if \(G_V = -G_A\) it predicts a maximum backward negatron asymmetry. If we look again at Fig. 3 we see that this is consistent with the general idea that if \(\psi_1^+\) appears in the combination \(\psi_1^+ (1 + \gamma_5)\) \((G_V = -G_A)\) negatrons like to be emitted with negative helicity, whereas if \(\psi_1^+\) appears in the combination \(\psi_1^+ (1 - \gamma_5)\) \((G_V = G_A)\) negatrons like to be emitted with positive helicity. For positrons one should reverse all these results.

**Remark I**

In practice it is easier to work with electrons of all energies and not just with electrons of the highest energy. In this case the explicit calculations show that the asymmetry is reduced to \(1/3\) of its maximum value.

**Remark II**

What is in fact known is not the direction of the muon spin but that of the muon momentum. If, however, one believes in lepton conservation also in \(\pi, \mu\) decay, one may be deduced from the other.
The experiments show asymmetries which agree with \( G_V = - G_A \). These results are further supported by direct measurements of the electron helicity which give a negative one for \( \beta^- \) and a positive one for \( \beta^+ \).

(e) Conclusion.

With \( G_V = - G_A \), Eq. (7) takes the form

\[
G_V \left( \psi_e^+ \gamma_\nu \gamma_\mu (1 + \gamma_5) \psi_\mu^- \right) \left( \psi_\nu^+ \gamma_\mu \gamma_\nu (1 + \gamma_5) \psi_\mu \right) \tag{20}
\]

As in \( \beta \) decay the fact that \( G_A / G_V \) is real guarantees invariance under PC (or under time reversal). In order to compare with \( \beta \) decay one should, in fact, change the ordering of the four fermion fields. This, as explained in paragraph (b), is achieved by means of a Fierz transformation.

In general, i.e., for an unspecified value of \( G_V / G_A \), such a transformation would lead to the appearance of \( S \) and \( P \) terms. With \( G_V = - G_A \), however, we are in the remarkable case that exchanging \( \psi_e^+ \) and \( \psi_\nu^+ \) leaves the expression invariant. In other words, (20) can also be written

\[
G_V \left( \psi_\nu^+ \gamma_\nu \gamma_\mu (1 + \gamma_5) \psi_\mu^- \right) \left( \psi_e^+ \gamma_\mu \gamma_\nu (1 + \gamma_5) \psi_\nu \right) \tag{21}
\]

The parallelism with the \( \beta \) decay Hamiltonian obtained above is quite remarkable: all the more so as the coupling constants \( G_V \) and \( C_V \) of \( \beta \) decay are indeed quite close to each other.
V. STRUCTURE OF WEAK INTERACTIONS IN SPACE–TIME.

(5) Primary weak interactions. Y. Yamaguchi
5. Primary weak interactions. (by Y. Yamaguchi)

(a) Introduction.

Processes which are induced by weak interactions shall be called weak processes. Decay processes (except $\Sigma^0$ and $\pi^0$ decays) and $\mu^-$ capture by nuclei are the weak processes. (Most of) weak processes are regarded as the first order effect in the weak interactions.

In this section we shall give some general discussions on weak interactions (briefly WI). First of all, we are interested in the primary (or fundamental) WI's. This is not a trivial question; because if we assume a particular WI in order to describe, say, a specific decay process then we shall also find other weak processes, which are induced by this WI and the effects of strong interactions (briefly SI).

Example (i) The $\beta$-decay interaction of Fermi type:

\[
\begin{array}{c}
\nu \\
\rightarrow \\
p \\
\end{array}

\begin{array}{c}
e^- \\
\rightarrow \\
n \\
\end{array}

\begin{array}{c}
\bar{\nu} \\
\rightarrow \\
\bar{\nu}
\end{array}

\]

is assumed to be primary. Then the $\pi^- e$ decay is derived from this WI plus SI:

Example (ii)

The $\pi^- e$ decay is assumed to be primary; then $\beta$-decay would be a derived phenomenon:

\[
\begin{array}{c}
\nu \\
\rightarrow \\
\pi^- \\
\end{array}

\begin{array}{c}
e^- \\
\rightarrow \\
n \\
\end{array}

\begin{array}{c}
\bar{\nu} \\
\rightarrow \\
\bar{\nu}
\end{array}
\]
We must now ask: which alternative is better: (i) or (ii)? Or more generally, we must try to find what are the primary WI. Our discussions are restricted according to the unavoidable situation that we have no satisfactory way of treating the SI. Nevertheless, we can use some criteria to be imposed on the primary WI and show that the Fermi type interactions appear to be the primary WI. The criteria to be used here are as follows:

(I) Primary WI should be capable of explaining as many weak processes as possible

(II) Compatibility with experimental data, and so on

Several WI's show remarkable universality: they *) are V-A type and their **) vector coupling constants are approximately equal. In order to explain the latter property, we shall use the analogy to the electro-magnetism and introduce a concept of conserved weak vector current \( J \). Tests of such an idea also briefly discussed in (c).

*) Perhaps except pionic decays of strange particles.

**) Perhaps except leptonic decays of strange particles.
Finally, the third mysterious property of the WI:

"WI are the product of charge changing currents",
suggests that usual WI might be the second order effects due to intermediary charged bosons. Such a possibility will be pursued in (d).

(b) Fermi interactions as primary WI.

In this subsection we shall try to determine the primary WI.

(b1) Muon decay.

First of all, let us discuss the $\mu$-e decay:

$$\mu \to e + \nu + \bar{\nu}.$$ 

There are involved no strongly interacting particles and there are no complications due to SI. The $\mu$-e decay interaction *) of Fermi type

$$G_{\mu}^{\text{dec}} \frac{1}{\sqrt{2}} \left( \bar{\psi}_\nu \gamma^5 \psi_\mu \right) \left( \bar{\psi}_e \gamma^5 \psi_\nu \right) + \text{h.c.}, \quad (5.1)$$

introduced in subsection V(4), must be simply regarded as the primary WI for $\mu$-e decay. If we use the $\mu$-e decay life $2.2 \times 10^{-6}$ sec, we obtain

$$G_{\mu}^{\text{dec}} = 1.4 \times 10^{-49} \text{ erg cm}^3$$

or

$$G_{\mu}^{\text{dec}} = 1.0 \times 10^{-5}/M^2$$

for $\hbar = c = 1$ and $M = \text{proton mass}$

*) $G_{\mu}^{\text{dec}}/\sqrt{2} = G_V$, cf. Eq. (21) of V(4), p. 64.

$$\bar{\psi}_e = \psi_e^+ \gamma^\mu \text{e}^\mu.$$
(b2) Decay of ordinary particles.

(A) $\beta$-decay and $\pi^-e$ decay.

Let us begin our discussion with the $\beta$-decay and $\pi^-e$-decay \[\text{see examples (i) and (ii) given in (a)\textsuperscript{7}. According to V(3), the}\]

$\beta$-decay can be written phenomenologically as *)

$$\frac{G_\beta}{\sqrt{2}} (\bar{\psi}_p \gamma_\alpha (1+\gamma_5) \psi_n)(\bar{\psi}_e \gamma_\alpha (1+\gamma_5) \psi_\nu) + \text{h.c.} \quad (5.2)$$

$$\lambda = - \frac{\text{(axial vector constant)}}{(\text{vector constant})} = 1.2,$$

while the phenomenological $\pi^-e$ decay interaction was given by

$$\frac{G_\pi}{m_\pi} \frac{e}{\alpha} (\bar{\psi}_e \gamma_\alpha (1+\gamma_5) \psi_\nu) + \text{h.c.} \quad (5.3)$$

\[\text{see (2.3) of V(2), p.167.}\]

Our problem is to see which of the alternatives (i) or (ii) given in (a) is more satisfactory.

Let us assume $\pi^-e$ decay interaction to be fundamental (alternative (ii)). Then the neutron decay process would be given by the diagram:

\[\text{SI} \rightarrow \pi^- \uparrow \text{WI (5.3)}\]

*) Note that \[\text{cf. (33) of V(3), p.51}\]

$$0 = \frac{G_\beta}{\sqrt{2}}$$

$$\Psi_4^+ = \bar{\Psi}$$

$\alpha$ is a dummy suffix, and if $\alpha$ appears twice it means the sum \[\sum_{\alpha=1}^4\].
The pion nucleon vertex can be represented by *)

$$\sqrt{2} \ f(\bar{u}_p \gamma_5 u_n)$$

where \( f \) is the (renormalized) pion-nucleon coupling constant \((\pi^2/4\pi \approx 15)\), and the matrix element for the \( \beta \)-decay process of the first order in WI is given by **)

$$\sqrt{2} i \ f \ \frac{g_\pi}{m_\pi} (\bar{u}_p \gamma_5 u_n) \ \frac{1}{m_\pi^2 + q^2} (\bar{u}_e (q_\mu) (1 + \gamma_5) u) \quad (5.4)$$

where \( q \) is the 4-momentum of virtual pion, and \( m_\pi \) = pion mass. This is the pseudoscalar variant of \( \beta \)-decay interaction. We have found neither Fermi nor Gamow-Teller interactions. We now conclude that case (ii) cannot explain the \( \beta \)-decay.

On the other hand, if we assume the \( \beta \)-decay interaction of V-A type \( \overline{\text{See } (5.2')} \) to be the primary WI (case (i)), then we can predict the \( \pi^- \rightarrow e^- \mu^+ \) decay via:

\[
\begin{array}{c}
\pi^- \\
(\overline{\mu} \\
\rightarrow \\
\nu)
\end{array}
\]

*) This is because:
(i) the initial and final nucleons are "real", and
(ii) the momentum \( q \) carried out by the virtual pion (= momentum taken by leptons) is very small.

**) \((5.4)\) is the Fourier transform of the Yukawa force — exchange of one pion! — between the nucleon and the lepton.
Thus we can say that alternative (i) is good, and the Fermi type interaction for $\beta$-decay should be regarded as (one of) the primary WI's.

Remark:

The $\beta$-decay process contains the strongly interacting particles and is expected to suffer the effect of SI. We cannot take the phenomenological $\beta$-interaction (5.2) as the primary weak interaction. It is commonly believed (though not yet proven) that the fundamental $\beta$-decay interaction is given by:

$$
\frac{G_0}{\sqrt{2}} (\bar{\psi}_p \gamma^5 \psi_n) (\bar{\psi}_e \gamma^5 \psi_n) + \text{h.c.}
$$

(5.2')

and the effect of SI changes the "bare" (or original) coupling constants to "renormalized" ones:

\begin{align*}
\text{vector:} & \quad G_0 \rightarrow G_\beta \\
\text{axial vector:} & \quad -G_0 \rightarrow -\lambda G_\beta
\end{align*}

Such an effect is called "renormalization effect". Experimentally we know that

$$
G_\beta = 1.4 \times 10^{-49} \text{ erg cm}^3
$$

and find that the equality holds

$$
G_\beta = G^{\text{dec}} \mu
$$

to a surprisingly high accuracy. Thus we are inclined to accept the equality

$$
G^{\text{dec}} \mu = G_\beta = G_0.
$$

We must search the reason for it. A possible way of understanding this equality is discussed in (c). We do not yet understand why $\lambda$ is so close to 1 ($\lambda \approx 1.2$).

*) $G_0$ is real for the time reversal invariance. Since the SI is invariant against the time reversal, $G_\beta$, $-\lambda G_\beta$ would also be real if $G_0$ is real.
\( \beta \)-decay (5.2') and \( \mu \)-decay (5.1) are the first example of "universality" of weak Fermi interactions (or universal Fermi interactions).

**\( \pi \)-e decay.**

We come back to the discussion of \( \pi \)-e decay. The matrix element of \( \pi \)-e decay can be obtained from (5.2'):

\[
\langle 0 \mid \frac{G^0}{\sqrt{2}} (\bar{v}_\pi \gamma^\alpha (1+\gamma^5)\psi_n) \mid \pi^- \rangle 
\times (\bar{u}_e \gamma^\alpha (1+\gamma^5)u_\nu)
\]

where \( \mid \pi^- \rangle \) is the one physical pion state and \( \mid 0 \rangle \) is the physical vacuum state. These two states must be exact eigenstates as far as SI is concerned. The pion is spinless, and \( (\bar{v}_\pi \gamma^\alpha (1+\gamma^5)\psi_n) \) behaves as vector for proper Lorentz transformations. Hence the matrix element

\[
\frac{G^0}{\sqrt{2}} \langle 0 \mid \bar{v}_\pi \gamma^\alpha (1+\gamma^5)\psi_n \mid \pi^- \rangle
\]

must be a vector, and the only vector we can construct out of the one pion state is the 4-momentum \( q^\alpha \) of the pion. In this way we can conclude

\[
\frac{G^0}{\sqrt{2}} \langle 0 \mid \bar{v}_\pi \gamma^\alpha (1+\gamma^5)\psi_n \mid \pi^- \rangle = q^\alpha \times \frac{1}{\sqrt{2} \omega} \times (\text{scalar function of } q) = \epsilon
\]

where \( \omega = \sqrt{m^2 + q^2} \) = pion energy, \( 1/\sqrt{2} \omega \) comes from the normalization of the pion field \( \Sigma \) (see (3.1) of IV_2(3)\). Since \( q^\alpha q_\alpha = q^2 = -m^2 \), the last factor is a purely numerical constant \( (= \epsilon) \). The relation between \( \epsilon \) and the phenomenological decay constant \( g_\pi \) in (5.3) is given by

\[
\epsilon = i \frac{g_\pi}{m_\pi}
\]
Furthermore, it should be remarked *)

\[ \langle 0 | \overline{\psi}_p \gamma_\alpha (1 + \gamma_5) \psi_n | \pi^- \rangle \]

\[ = \langle 0 | \overline{\psi}_p \gamma_\alpha \gamma_5 \psi_n | \pi^- \rangle \]

This is because of the parity conservation in SI, positive parity of the vacuum and negative (intrinsic) parity of the pion (the pion is pseudoscalar). The vector part (\( \overline{\psi}_p \gamma_\alpha \psi_n \)) does not contribute to the leptonic decay of pions.

**Remark:**

The calculation of \( \pi^- \rightarrow e^- \) decay matrix element according to the mechanism

\[ \begin{array}{c}
S, I \\
\downarrow \\
W I \\
\downarrow
\end{array} \]

\[ \overline{\psi} \quad \psi \\
\downarrow \\
\pm \\
\quad e^- \\
\quad \nu \\
\quad \pi^-
\]

is very difficult. In the lowest order perturbation calculation, the matrix element diverges**). If one uses the cut-off method (cut-off momentum \( \Lambda \approx M(\text{nucleon mass}) \)) to avoid the infinity, one finds a

\[ \langle 0 | \overline{\psi}_p \gamma_\alpha \gamma_5 \psi_n | \pi^- \rangle \propto q_\alpha \\
\downarrow \text{axial} \downarrow \psi \\
\downarrow \text{vector} \quad \text{pseudoscalar} \quad \text{vector} \\
0^+ \\
1^+ \\
0^- \\
1^- \\

***) \( \xi \) in (5.4) is logarithmically divergent. The \( \pi^- \rightarrow e^- \nu \) decay rate is

\[ \frac{1}{2\pi^4} m_\pi (g^0 m_\pi^2) \frac{e^2}{4\pi} \left( \frac{m_\Lambda}{M} \right)^2 \left( \frac{M}{m_\pi} \right)^2 \]

\[ \times \frac{(m_\pi^2)}{m_\pi} \left( 1 - \frac{m_\pi^2}{m_\pi^2} \right)^2 \]

\[ \left( \frac{\Lambda}{M} \right)^2 \approx 15 \]
decay rate, which is not far from the empirical value. Recently
the dispersion technique has been applied to calculate the \( \pi^{-2} \)
decay rate, which is in surprisingly good agreement with the experi-
mental decay rate *).

\\((\text{B})\) \( \mu^-\) -capture and \( \pi^-\) - \( \mu^-\) decay.

Quite analogous statements can be made for the \( \mu^-\) -capture
interaction and the \( \pi^-\) - \( \mu^-\) decay. We merely summarize the con-
clusion. The primary WI responsible for \( \mu^-\) -capture by nuclei is
given by

\[
\frac{G^0}{\sqrt{2}} \left( \bar{\nu}_p \gamma_\alpha(1+\gamma_5)\bar{\nu}_n \right) \left( \bar{\nu}_\mu \gamma_\alpha(1+\gamma_5)\bar{\nu}_\nu \right) + \text{h.c.} \tag{5.6'}
\]

After the "renormalization" due to SI, the \( \text{phenomenological}\) \( \mu^-\) -capture
interaction would be given approximately by **

\[
\frac{G_\mu}{\sqrt{2}} \left( \bar{\nu}_p \gamma_\alpha(1+\gamma_5)\bar{\nu}_n \right) \left( \bar{\nu}_\mu \gamma_\alpha(1+\gamma_5)\bar{\nu}_\nu \right) + \text{h.c.} \tag{5.6}
\]

since the effect of SI to nucleon part is common to \( \mu^-\) -capture inter-
action and the \( \beta^-\) -interaction. This result (5.6) is consistent with

\( *\) M.L. Goldberger, Rev. Mod. Phys. 31, (59) 797.

\( **\) The larger momentum transfer for the nucleon is involved
in the \( \mu^-\) -capture than for the nuclear \( \beta^-\) -decay. Here we have
neglected the "momentum" dependence of \( G_\beta \) and \( \lambda \),
see (c).
\( \mu^- \)-capture data. The \( \pi^-/\mu^- \) decay is induced by (5.6') and the SI.

\[ \pi^- \rightarrow \mu^- + \bar{\nu} \]

(5.6') is the second example of universal Fermi interaction.

Remark.

The universality is supported from the branching ratio
\( (\pi \rightarrow e^+\nu) / (\pi \rightarrow \mu^+\nu) \); the matrix elements for \( \pi^- \rightarrow \left[ \begin{array}{c} \alpha^- \\ \mu^- \end{array} \right] + \bar{\nu} \) is given by

\[
\langle 0 | \frac{g^0}{\sqrt{2}} \left( \bar{\psi}_p \gamma_\alpha (1+\gamma_5) \psi_n \right) | \pi^- \rangle
\]

\[
\times \left( \bar{u}_e \gamma_\alpha (1+\gamma_5) u_\nu \right) \left( \bar{u}_\mu \gamma_\alpha (1+\gamma_5) u_\nu \right)
\]

(5.7)

where the "nuclear" matrix element \( \langle 0 | \ldots | \pi^- \rangle = g^0 \times 1/\sqrt{2\omega} \) is common to \( \pi^e_2 \) and \( \pi^\mu_2 \) by the "universal" of WI. (Eqs. (5.2') and (5.6')). The form of matrix element is precisely the same as that for the phenomenological \( \pi^e_2 \) decay interactions (5.3) \((= (2.12) \ p.20, \ V(2))\). Therefore the branching ratio (2.13) - which is perfectly in agreement with experiment - follows from (5.7).
(b3) **Discussion.**

We have reached the "universal" Fermi interaction for the primary weak interactions $\Gamma(5.1)$, $(5.2')$ and $(5.6')$. These WI can be summarized in the form

$$\frac{G^0}{\sqrt{2}} (J^{sc}_\alpha + J^\ell_\alpha) (J^{sc}_\alpha + J^\ell_\alpha)^*$$  \hfill (5.8)

where

$$J^{sc}_\alpha = (\bar{\psi}_p \gamma_\alpha (1+\gamma_5) \psi_n) \text{ (strangeness conserving)}$$ \hfill (5.9)

$$J^\ell_\alpha = J^\mu_\alpha + J^e_\alpha \text{ (leptonic)}$$ \hfill (5.10)

$$J^\mu_\alpha = (\bar{\psi}_\nu \gamma_\alpha (1+\gamma_5) \psi_\mu)$$

$$J^e_\alpha = (\bar{\psi}_\nu \gamma_\alpha (1+\gamma_5) \psi_e)$$

and $^*$ means hermitian conjugate. (5.8) contains more terms than the simple sum

$$(5.1) + (5.2) + (5.6')$$

However, the additional terms (which are still WI) do not make any harm for us.

$J_\alpha$'s shall be called the weak current. $J_\alpha$ includes the change of electric charge by one: $\Delta Q = +1$. Sometimes we shall talk about the weak vector current $J^{(v)}_\alpha$ and the weak axial

*\) Experimental tests of these terms (e.g., $\nu + e \rightarrow e + \nu$), if possible, are very interesting.
vector current \( J_\alpha^{(A)} \); for example

\[
J_\alpha^{sc(v)} = \left( \overline{\psi}_p \gamma_\alpha \psi_n \right)
\]

\[
J_\alpha^{sc(A)} = \left( \overline{\psi}_p \gamma_\alpha \gamma_5 \psi_n \right)
\]

Evidently

\[
J_\alpha = J_\alpha^{(v)} + J_\alpha^{(A)}
\]

**Remark (1).**

We have just said that the weak currents has the property \( \Delta Q = +1 \). If the weak currents with \( \Delta Q = 0 \), e.g.

\[
(\overline{\psi}_\mu \gamma_\alpha (1+\gamma_5) \psi_e), \quad (\overline{\psi}_e \gamma_\alpha (1+\gamma_5) \psi_e)
\]

are accepted, their product

\[
(\overline{\psi}_\mu \gamma_\alpha (1+\gamma_5) \psi_e)(\overline{\psi}_e \gamma_\alpha (1+\gamma_5) \psi_e)
\]

would cause the decay process

\[
\mu^+ \rightarrow 2e^- + \bar{e}^+
\]

which has never been observed. The property \( \Delta Q = +1 \) is postulated for the weak current in order to forbid such unwanted decay processes \((\overline{\nu}(1) (a) p, \gamma)\).
Remark (2).

The expression (5.9) for $J^S_{\alpha}$ is not complete. More complete form of $J^S_{\alpha}$ will be given in (C).

It is interesting to generalize the $\mathcal{W}I$ (as the product of weak currents) to include the decay processes of strange particles. Decay of strange particles seems to obey the law

$$\Delta S = \text{(change of strangeness)} = \pm 1.$$

Hence, it is natural to introduce the weak currents, one of whose example is given by

$$\left( \bar{\psi}_p \gamma_{\alpha} (1 + \gamma_5) \psi \right).$$

(Such a weak current shall be called the \textit{Strangeness-non-conserving weak current} $J^\text{Snc}_{\alpha}$) and to construct the weak interaction as the products of weak currents

$$\frac{G}{\sqrt{2}} J^\text{Snc}_{\alpha} (J^\ell_{\alpha})^* + \bar{\psi} \gamma_5 \psi \tag{5.11}$$

and

$$\frac{G''}{\sqrt{2}} J^\text{Snc}_{\alpha} (J^S_{\alpha})^* + \bar{\psi} \gamma_5 \psi \tag{5.12}$$

The former induces the leptonic decay of the strange particles, while the latter can be used qualitatively at least to describe the non-leptonic decay (see section (8)). Since we shall discuss the strange particle decays in subsequent sections, we shall merely give a few remarks about (5.11).
Remark (3).

Since we do not observe the decay

$$K_1^0 \rightarrow \mu^+ + \mu^-$$

we must restrict $j_{\alpha}^{SNC}$ to those with $\Delta Q = +1$. Furthermore, let us restrict $j_{\alpha}^{SNC}$ so as to be consistent with the rule $\Delta S/\Delta Q = +1$.

$$j_{\alpha}^{SNC} = (\bar{\psi}_p \gamma_\alpha (1+\gamma_5) \psi_{\Lambda}) + (\bar{\psi}_\Lambda \gamma_\alpha (1+\gamma_5) \psi_\Sigma^\pm) + \ldots .$$

<table>
<thead>
<tr>
<th>$S$</th>
<th>0</th>
<th>-1</th>
<th>-1</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

We sometimes restrict $j_{\alpha}^{SNC}$ to be isospinor. Then we obtain the $|\Delta F| = \frac{1}{2}$-rule for the SNC-current $j_{\alpha}^{SNC}$.

Then all statements made in III(5) can be applied also to our WI (5.11).

Remark (4).

The Fermi interaction (5.11) for leptonic-decay of strange particles, appears to be fundamental. The reason is as follows: Let us suppose that the primary weak interaction for $K^-\pi^0$ decay is the Yukawa type (2.16) $\bar{\psi}(2)p,2\bar{\psi}(2)$ $\bar{\psi}(2)$. Then the leptonic-decay of the $\Lambda$-hyperons can be explained by the diagram

![Diagram]

*) Do not confuse with the over all $|\Delta \frac{1}{2} + \frac{1}{2}|$ law
(cf. the example (ii) in (a)). However, we cannot derive the \( K_{\ell 3}^+ \) decay

\[
K^+ \rightarrow \pi^0 + e^+ + \nu,
\]

Because the \( K_{\ell 3}^+ \) decay proceeds as follows

\[
\begin{align*}
K^+ & \rightarrow \pi^0 + K^+ + \nu \quad \text{SI} \\
& \uparrow \quad \uparrow \\
& \quad \nu \\
& \quad e^+ \quad \text{WI} \\
& (2.16)
\end{align*}
\]

But the parity conservation in SI, the pseudoscalar of \( \pi^0 \), spinless \( K \) tell the matrix element

\[
K^+ \rightarrow \pi^0 + K^+
\]

is identically zero. Therefore, we must introduce a direct \( K_{\ell 3} \) decay interaction besides the direct \( K_{\ell 2} \) decay interaction of Yukawa type. The situation becomes very complicated. On the contrary, if we accept the Fermi type weak interaction (5.11), we can understand the existence of all leptonic decay processes:
We have thus a more unified explanation based on (5.11).

**Important Remarks.**

The primary interactions (5.8) and (5.11) are by no means established. They are conjectured as very plausible. Even for the simplest and best known WI of $\mu - e$ decay (5.1), we can only say that it is **not inconsistent** with our experimental information.

(c) Conserved weak vector current.

In the preceding subsection, we have reached more or less "primary" weak interactions and noticed that the equality of vector constants

$$G_{\mu}^{\text{dec}} \approx G_{\beta} \approx G_{\mu}$$

We have conjectured even further

$$G_{\beta} \approx G^0$$

In this subsection we develop an explanation for it.

(c1) General discussion.

In this subsection we shall make full use of the analogy between WI and the electromagnetic theory to find many useful results.

*) Unfortunately all consequences of conserved weak vector current are hard to detect by experiment.
Let us begin with the electromagnetic case. Leptons have no SI (except electromagnetic coupling), while baryons and mesons have SI. In spite of this difference, $\mu^+$, $\epsilon^+$ and $p$, $\pi^+$, $k^+$, $\Sigma^+$ have all the same electric charge. We can say that the total amount of electric charge carried on any particle is not affected by the presence of SI. This noble feature of electric charge is a consequence of the conservation law of the electric charge. This conservation law can be expressed as

$$\partial_\alpha j^\alpha = 0$$  \hspace{1cm} (5.13)

where

$$\partial_\alpha = \partial/\partial \chi_\alpha$$

and $e j^\alpha$ is the electric current (4-vector). It should be noted that the charge of elementary particles is simultaneously the coupling strength of the electromagnetic interaction.

Let us restrict our discussion within the strongly interacting particles. Then $j^\alpha$ is given by

$$j^\alpha = i \left[ \bar{N} \gamma_\alpha \frac{\tau^+}{2} N + \bar{\Sigma} \gamma_\alpha \rho_3 \Sigma + \bar{\Xi} \gamma_\alpha \frac{\tau^-}{2} \Xi \right.$$

\hspace{1cm} $$+ \bar{x} \rho_3 \partial_\alpha \bar{n} + \bar{K}^* \frac{1+\tau_3}{2} \partial_\alpha K - \bar{\bar{K}}^* \frac{\tau^-}{2} \partial_\alpha \bar{K} \left. \right]$$  \hspace{1cm} (5.14)

where we have used abbreviated notation.
\[ N = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \psi_+^\Sigma \\ \psi_0^\Sigma \\ \psi_-^\Sigma \end{pmatrix}, \quad \Xi = \begin{pmatrix} \psi_+^\Xi \\ \psi_0^\Xi \\ \psi_-^\Xi \end{pmatrix} \]

\[ \eta = \begin{pmatrix} \varphi_{\pi^+} \\ \varphi_{\pi^0} \\ \varphi_{\pi^-} \end{pmatrix}, \quad K = \begin{pmatrix} \varphi_{K^+} \\ \varphi_{K^0} \\ \varphi_{K^-} \end{pmatrix} \]

\[ \varphi_{\pi^+} = (\varphi_{\pi^-})^* \]

\[ \varphi_{K^-} = (\varphi_{K^+})^* \]

\[ \varphi_{K^0} = (\varphi_{K^0})^* \]

\[ \frac{\tilde{\tau}}{\tau} (\tau_1, \tau_2, \tau_3) = 2 \times 2 \text{ isospin matrices }^{(*)} \]

\[ \frac{\rho}{\rho} (\rho_1, \rho_2, \rho_3) = 3 \times 3 \text{ isospin matrices }^{(**)} \]

\[ \psi_+^\Sigma \] is the field operator for \( \Sigma^+ \) (annihilates \( \Sigma^+ \) and creates \( \Sigma^+ \)), and so on. As is seen from (5.14), \( j_{\alpha}^{\Sigma} \) consists of two parts: the isoscalar part \( j_{\alpha}^{S} \) and the third component \( j_{\alpha}^{(3)} \) of the isovector part \( j_{\alpha}^{V} \). Let us further assume the charge independence for the meson-baryon system. (We neglect the effect of electromagnetic coupling and the mass differences of \( p-n \), \( \pi^\pm - \eta^0 \) etc.

---

\[ (*) \text{ See (2.3) I(2), p.6. This is used for the nucleon, } \Sigma \text{ and } K, \tilde{K} . \]

\[ (**) \text{ See (2.15) I(2), p.8. This is used for } \pi \text{ and } \Sigma . \]
to the expression (5.14)). We can now split the conservation law (5.13) into two parts

$$\partial_\alpha j^{(3)}_\alpha = 0$$  \hspace{1cm} (5.15)
$$\partial_\alpha j^{S}_\alpha = 0$$

because $j^{(3)}_\alpha$ and $j^{S}_\alpha$ have the different isospin character. By the charge independence, we must not only have (5.15), but also

$$\partial_\alpha j^{V}_\alpha = 0$$

In particular we find the conservation law valid in the charge independent approximation

$$\partial_\alpha (j^{(1)}_\alpha + ij^{(2)}_\alpha) = 0$$  \hspace{1cm} (5.16)

Let us try to write down explicitly $j^{(1)}_\alpha + ij^{(2)}_\alpha$.

Evidently

$$j^{V}_\alpha = i \int \bar{N} \gamma^\alpha \bar{2} \not{\Sigma} N + \bar{\Sigma} \gamma^\alpha \not{\rho} \Sigma + \bar{\Sigma} \gamma^\alpha \not{\bar{\rho}} \Sigma$$

$$+ \pi^* \not{\rho} \gamma^\alpha \not{\pi} + K^* \not{\rho} \gamma^\alpha K - K^* \not{\bar{\rho}} \gamma^\alpha \not{\bar{K}}$$
hence *)

\[ j^{(1)}_\alpha + ij^{(2)}_\alpha = \frac{1}{2} \left( \bar{\Psi}_P \gamma_\alpha \Psi_n \right) + \sqrt{2} \left( \bar{\Psi}_Z \gamma_\alpha \Psi_0 \right) \]

\[ - \sqrt{2} \left( \bar{\Psi}_Z \gamma_\alpha \Psi_1 \right) + \left( \bar{\Psi}_z \gamma_\alpha \Psi_+ \right) \]

\[ + \sqrt{2} \left( \phi_n^* \gamma_\alpha \phi_n \gamma_\alpha \phi_n^0 \partial_\alpha \phi_n^+ \right) \]

\[ + \left( \phi_K^* \gamma_\alpha \phi_K^0 \gamma_\alpha \phi_K^+ \right) \]

(5.17)

We now define the complete (strangeness conserving) weak vector current \( J^{\text{Sc}}_\alpha \) by

\[ J^{\text{Sc}}_\alpha = \frac{1}{2} j^{(1)}_\alpha + j^{(2)}_\alpha \]

(5.18)

We have, of course, the conservation law

\[ \partial_\alpha J^{\text{Sc}}_\alpha = 0 \]

(5.19)

Let us compare the electromagnetic interaction

\[ - e j^{(3)}_\alpha A_\alpha \]  
(or more appropriately \(- e j^{(3)}_\alpha A_\alpha\))

*) For example,

\[ \left( \vec{N} \gamma_\alpha \frac{\vec{r}_1 + \vec{r}_2}{\alpha} \vec{N} \right) = \left( \vec{N} \gamma_\alpha \Psi_\alpha \right) \]
and the weak interaction (vector part)

\[ \frac{g^0}{\sqrt{2}} j^{\alpha} \rightleftharpoons (j^{\alpha}_\nu)^+ \]

There are perfect analogues between them

\[ -e \rightarrow g^0/\sqrt{2} \]
\[ j^{(3)}_\alpha \rightarrow j^\alpha_{\Sigma} \phi (\nu) \]
\[ A^\alpha \rightarrow \frac{1}{i} (j^{(\ell)}_\alpha)^+ \]

in the charge independent approximation. The charge conservation law guarantees that the coupling constant \( e \) does not change by the influence of SI. Likewise, the conservation law (5.19) tells us that weak coupling constant \( g^0 \) does not alter by the presence of SI. More explicitly, we have just established the relation

\[ g^0 = G^\beta = G^\cap_{\lambda} \]

Combine this with \( g_{\lambda}^0 = G_{\lambda}^{\text{dec}} \), we obtain

\[ G^\beta = G^\cap_{\lambda} = G^{\text{dec}}_{\lambda} = g^0 \]  \hspace{1cm} (5.20)
In contract to the case of e, the electric charge, the equality of weak charge (5.20) is not the absolute law, but is valid under the assumption of charge independence. (5.20) is supposed to be valid within \( \sim 1\% \).

It is important to remark that the vector constant for the \( \beta \)-decay of complex nuclei is still equal to \( G_0 \) as long as the approximation of charge independence is valid. Therefore the \( f^\ell \)-value of \( 0^{14}_\ell \) (allowed, Fermi transition) can be used to evaluate \( G_\beta \), if the deviation from the charge-independence and the electromagnetic corrections are ignored (these effects could not be very important for such light nuclei as \( 0^{14}_\ell \)).

**Remark (1).**

One may think about the possibility of conserved axial vector current *)

\[
\partial_\alpha J^\text{Sc(A)}_\alpha = 0 \quad (5.21)
\]

However, this is not acceptable. First of all it is rather difficult (though not impossible) to construct such axial vector current. (notice that

\[
\partial_\alpha (\bar{\psi}_p \gamma_\alpha \gamma_5 \psi_n) = 0
\]

while

\[
\partial_\alpha (\bar{\psi}_p \gamma_\alpha \gamma_5 \psi_n) = 2M (\bar{\psi}_p \gamma_5 \psi_n) \neq 0
\]

*) In contract to the conserved vector current, the conserved axial vector current does not imply the equality of weak axial vector constants.
for the case of free fields). Secondly, (5.21) predicts that the \( \pi \ell_2 \) decay is forbidden. (proof): We notice the relation

\[
\gamma_\alpha J^S_c(A) = i \int P_\alpha, J^S_c(A) \int
\]

\[
= i (P_\alpha J^S_c(A) - J^S_c(A) P_\alpha)
\]

where \( P_\alpha \) is the total 4-momentum (operator). We take the matrix element of \( \gamma_\alpha J^S_c(A) \) between the vacuum and the one pion state

\[
\langle 0 / \gamma_\alpha J^S_c(A) / \pi \rangle = - i k_\alpha \langle 0 / J^S_c(A) / \pi \rangle
\]

where we have used

\[
P_\alpha / 0 = 0
\]

\[
P_\alpha / \pi = k_\alpha / \pi
\]

\[
k_\alpha = \text{momentum of the pion (c-number)}
\]

From (5.21) we shall find

\[
\langle 0 / J^S_c(A) / \pi \rangle = 0
\]

i.e., the \( \pi - \mu \) and \( \pi - e \) decay matrices (5.7) vanish identically.
Remark (2).

Possibility of

\[ \gamma_\alpha J_\alpha^{\text{Snc}(\nu)} = 0 \]  \hfill (5.22)

Evidently the charge independence is not sufficient to prove (5.22). Notice that

\[ \gamma_\alpha (\bar{\psi}_p \gamma_\alpha \psi_\Lambda) = (m_\Lambda - m_N)(\bar{\psi}_p \psi_\Lambda) \neq 0 \]

for the free particles. If the global symmetry (poorer approximation than the charge independence) is used, all baryon masses can be regarded as equal. In such a case, we can have the approximate rule (5.22) provided \( J^{\text{Snc}(\nu)} \) is appropriately constructed.

(\text{c2}) Direct consequence of conserved weak vector current.

The vector part

\[ \frac{G^3}{\sqrt{2}} J_\alpha^{\text{Snc}(\nu)} (J_\alpha^\nu)^+ \approx \kappa' \zeta. \]

of the WI

\[ \frac{G^3}{\sqrt{2}} J_\alpha^{\text{Snc}} (J_\alpha^\nu)^+ \approx \kappa' \zeta. \]
has been completely determined by the new definition (5.18) of \( J^{SC}_\alpha(v) \). If we find a rule to construct \( J^{SC}_\alpha(A) \), we can write down a complete WI for leptonic weak processes of strongly interacting particle with \( \Delta S = 0 \). The common form of WI, (5.1), (5.2') and (5.6'), suggests that the vector part \( (\bar{\Psi}_a \gamma_\alpha \Psi_b) \) should be replaced by \( (\bar{\Psi}_a \gamma_\alpha (1+\gamma_5) \Psi_b) \). We shall accept this simple prescription \(^*)\). Then a complete \( J^{SC}_\alpha \) would be written as

\[
J^{SC}_\alpha = (\bar{\tau}_p \gamma_\alpha (1+\gamma_5) \psi_n) \\
+\sqrt{2} \left\{ (\bar{\tau}_e \gamma_\alpha (1+\gamma_5) \psi_e) + (\bar{\tau}_e \gamma_\alpha (1+\gamma_5) \psi_e^-) \right\} \\
+ (\bar{\tau}_e \gamma_\alpha (1+\gamma_5) \psi_e^-) \\
+ \sqrt{2} \left( \phi_{\pi^+}^* \partial_\alpha \phi_{\pi^-} - \phi_{\pi^+} \partial_\alpha \phi_{\pi^-}^* \right) \\
+ \left( \phi_{K^+}^* \partial_\alpha \phi_{K^-} - \phi_{K^+} \partial_\alpha \phi_{K^-}^* \right)
\]

(5.23)

Since pions and kaons are spinless, it is impossible to construct axial vector currents bilinear in meson fields.

\(^*)\) There exists many attempts to explain this (or similar) rule. But none of them seem to be convincing enough. We shall tentatively assume this rule for the moment. (however, see Remark (2) p. 91 ).
The weak interaction

\[ \frac{G^0}{\sqrt{\alpha}} J_\alpha^{SC} (J_\alpha^0)^+ + \bar{\kappa}, \mathcal{C}. \]  \hspace{1cm} (5.24)

predicts the new \( \beta \)-decay processes of mesons

\[ \bar{\pi}^+ \rightarrow \pi^0 + e^+ + \nu \]
\[ \kappa^0 \rightarrow \kappa^+ + e^- + \bar{\nu} \]
\[ \bar{\kappa}^0 \rightarrow \kappa^- + e^+ + \nu \]

These \( \beta \)-decays are solely due to \( J_\alpha^{SC}(\nu) \) and their decay rates can be predicted with confidence:

<table>
<thead>
<tr>
<th>( \beta )-decay</th>
<th>assumed mass difference</th>
<th>partial lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\pi}^+ \rightarrow \pi^0 + e^+ + \nu )</td>
<td>( m_{\pi^+} - m_{\pi^0} = 4.6 \text{ MeV} )</td>
<td>( 2.4 \text{ sec} )</td>
</tr>
<tr>
<td>( \kappa^0 \rightarrow \kappa^+ + e^- + \bar{\nu} )</td>
<td>( m_{\kappa^0} - m_{\kappa^+} = 3.9 \text{ MeV} )</td>
<td>( \approx 10 \text{ sec} )</td>
</tr>
</tbody>
</table>

These \( \beta \)-decay rates of mesons are the unambiguous tests of the conserved weak vector current \( *) \). Unfortunately the rates are too

\( *) \) Other \( \beta \)-decay theories will also lead to the decay rates of this order of magnitude, but not to the same number.
small to be successfully detected.

Remark (1).

Our weak interaction (5.24) also predicts the $\beta$-decay of hyperons

$$\Sigma^- \to \Sigma^0 + e^- + \bar{\nu}$$

$$\Xi^- \to \Xi^0 + e^- + \bar{\nu}$$

Their decay rates depend not only on the vector part, (which is reliable because of no renormalization effect) but also on the axial vector part. $J_\alpha^{\mathcal{C}(A)}$ has two sources of ambiguities:

(i) The form of $J_\alpha^{\mathcal{C}(A)}$ cannot be determined with confidence.

(ii) Even though $J_\alpha^{\mathcal{C}(A)}$ is fixed by a prescription, the renormalization effect due to SI is still unknown.

If we tentatively grant that the renormalized axial vector constants for the $\beta$-decay of hyperons are not very different from $-\lambda G_\beta = -1.2 G_\beta$, we find the decay lifetimes of $\Sigma^- \to \Sigma^0 + e^- + \bar{\nu}$ and $\Xi^- \to \Xi^0 + e^- + \bar{\nu}$ to be of the order of (0.1 sec.)

Remark (2).

We can think of other $\beta$-decay of $\Sigma$-hyperons with no strangeness change

$$\Sigma^\pm \to \Lambda^0 + e^\pm \pm \nu$$

(5.25)
In the static limit, the conserved weak vector current \( J^\text{Sc}(\nu) \) does not contribute to these \( \beta \)-decay *), because \( J^\text{Sc}(\nu)^4 \) is a component of the "isovector current" and conserves total isospin of the system consisting of strongly interacting particles (no Fermi transition). To predict the lifetimes of (5.25), we must know first of all the precise form of \( J^\text{Sc}(\Lambda) \), which we do not know. For example, \( J^\text{Sc}(\Lambda) \) may contain the term

\[
(\bar{\psi}_\Sigma^+ \gamma_\alpha \{ \gamma^5 \}_{1\alpha} \psi_\Lambda) \quad \text{and} \quad (\bar{\psi}_\Lambda \gamma_\alpha \{ \gamma^5 \}_{1\alpha} \psi_\Sigma^-)
\]

for the even/odd relative \( \Sigma - \Lambda \) parity. This possibility is not considered in (5.23). If we tentatively take the phenomenological WI

\[
\frac{G^F}{\sqrt{2}} (\bar{\psi}_\Lambda \gamma_\alpha \{ \gamma^5 \}_{1\alpha} \psi_\Sigma^-) (J^\text{N}_\alpha )^+ + \text{h.c.}
\]

we find the partial mean life \( \sim 1 \times 10^{-6} \) sec. for \( \Sigma \rightarrow \Lambda + e^- + \bar{\nu} \)**).

*) There exists the weak magnetism due to \( J^\text{Sc}(\nu) \).

**) Notice that the combined effect of

\[
\{ \text{SI} \quad \text{WI: (5.24)} \quad \text{with} \quad J^\text{Sc}_\alpha \quad (5.23) \}
\]

can give rise to the \( \beta \)-decay \( \Sigma \rightarrow \Lambda + e + \nu \)

whose lifetime will be different from \( 1 \times 10^{-6} \) sec.
Weak magnetism.

Once the conserved weak vector current has been accepted, a new definite correction term (weak magnetic term) for the nuclear matrix element in the weak leptonic processes (β-decay and μ-capture) can immediately be derived. To see this, it is again convenient to use an analogy of weak to electromagnetic interactions. We shall do this for the single nucleon case.

Let us write down the matrix element of the electric current operator between the one nucleon states with momenta \( p' \) and \( p \):

\[
\langle \text{nucleon, } p' \mid e \tilde{j}_\alpha \mid \text{nucleon, } p \rangle = i e \bar{u}_f (p') \left( \frac{1}{2} \left[ \tau_3 F_1^V(q^2) + F_1^S(q^2) \right] \gamma_\alpha \right.
\]

\[+ \frac{1}{2} \left[ \tau_3 \frac{\mu_p - \mu_n}{2M} F_2^V(q^2) + \frac{\mu_p + \mu_n}{2M} F_2^S(q^2) \right] \gamma_\alpha \gamma_\beta \sigma_{\alpha \beta} \left. \right] u_i (p),
\]

(5.26)

where

\[
\sigma_{\alpha \beta} = \frac{1}{2i} (\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha)
\]

\[q = p - p' = 4 \text{ momentum transfer}
\]

\[M = \text{proton mass}
\]
\[ \mu_p = 1.8 \] and \[ \mu_n = -1.9 \] are the static anomalous magnetic moments of proton and of neutron in nuclear magneton, \[ u_\alpha \] and \[ u_\bar{\alpha} \] are the Dirac spinor for the final and initial nucleons. \( F \)'s are the electromagnetic form factors of the nucleon and the function of \[ q_0^2 = q_\alpha q_\bar{\alpha} \cdot \] All \( F \)'s are normalized to 1 at \[ q_0^2 = 0. \]

\[ F(0) = 1 \]

As has been remarked in (cl), the weak vector current \( j^{Sc(\nu)}_\alpha \) is closely related to the isovector part of \( j_\omega^d \) (see (5.18)). Particularly if the charge independence is assumed, two matrix elements

\[
\langle \text{proton, } p' / j^{Sc(\nu)}_\alpha / \text{neutron, } p \rangle
\]

\[
\langle \text{nucleon, } p' / j^{(3)}_\alpha / \text{nucleon, } p \rangle = \text{isovector part of (5.26)}
\]

must be equal (apart from a trivial numerical factor);

\[
\langle \text{proton, } p' / j^{Sc(\nu)}_\alpha / \text{neutron, } p \rangle
\]

\[
= \bar{u}_p(p') \left[ F_1 V(q_0^2) \gamma_\alpha + \frac{\mu_p - \mu_n}{2M} F_2 V(q_0^2) \sigma_{\alpha\beta} q_\beta \right] u_n(p)
\]

\[ p^{\text{isolam}} \]

\[ \text{neutron} \]

According to Hofstadter,

\[
F_1 V(q^2) \approx F_2 V(q^2) \approx 1 - \frac{1}{6} \frac{q^2}{(2m_\pi)^2} + \ldots
\]
The second term in (5.27) is the so called weak magnetism. This form of weak magnetism is a consequence of the conserved weak vector current *). Moreover (5.27) shows that the phenomenological vector coupling constant \( G_\beta \) of nuclear \( \beta \)-decay (5.2) is given by

\[
G_\beta = G^\circ F_1^V(q_0^2)
\]

(5.28)

Since \( q_0^2 \) is very small in nuclear \( \beta \)-decay, the equality holds to a very good approximation. (No renormalization for the vector constant).

**Estimation of weak magnetism in complex nuclei** can also be done. Let us assume that we know the matrix element \( \langle f | (M.D.) | i^\gamma \rangle \) for the M.D. transition of certain nucleus. We now consider the definite \( \beta \)-decay:

\[
i^\beta \rightarrow f
\]

where the two initial nuclear states \( i^\gamma, i^\beta \) belong to the same isomultiplet. In such an occasion, weak magnetism in \( i^\beta \rightarrow f \) is equal to \( \langle f | (M.D.) | i^\gamma \rangle \) (apart from a trivial factor) by the charge independence **)

\[
\begin{array}{c}
\gamma \\
n \end{array} \rightarrow \begin{array}{c}
\gamma \\
n \end{array} = (i^\gamma \text{ and } i^\beta \text{ belong to the same isomultiplet})
\]

*) If the weak vector current is not conserved, the coefficient of weak magnetic term will be different from that given in (5.27).

**) In fact the same final state for \( \gamma \)- and \( \beta \)-decay is not essential here. If two final states are members of the same isomultiplet, the same argument can be made.
Weak magnetic term contributes to Gamow-Teller transition, but not to Fermi transition.

A fine example is given by:

\[
\text{Spin parity}
\begin{array}{c}
1^+ \\
0^+
\end{array}
\xrightarrow{\beta^-} \xleftarrow{\text{isomultiplet}} \xrightarrow{\beta^+} \\
E_{12} C^{12} N_{12} \xleftarrow{\text{isosinglet}}
\]

The weak magnetism for $\beta^\pm$-decay can be estimated from the M.D. transition ($C^{12*} \rightarrow C_{12} + \gamma$) rate.

The weak magnetic term contains a factor

\[
\frac{g_0}{M_j}
\]

where $g_0 = (\text{electron momentum}) + (\text{neutrino momentum})$ for nuclear $\beta$-decay.

and $g_\varphi = (\text{neutrino momentum})$, for $\mu$-capture.

In both cases $g_\varphi$ is not too big, so that $F(q^i)$'s can be set equal to 1. The effect of the weak magnetism is rather small in the nuclear $\beta$-decay, while it would be considerable in the $\mu^-$-capture case. Experimental detection of weak magnetism is very important for testing the present theory.
The effect of weak magnetism for the $\beta^+$ energy spectra of the allowed Gamow-Teller transition is given by the correction factor

$$(1 + \frac{6}{5} \alpha E_e)$$

(5.29)

(for $\beta^+$ decay)

where $E_e$ is the energy of $\beta$-rays, and $\alpha$ can be evaluated from the MD transition rate. For example, $\alpha$ is estimated to be

$$\alpha \approx \frac{2}{\hbar} \text{ for the } \beta^+ \text{ decay of the nuclei } ^{B_{12}}_N \rightarrow ^{C_{12}}_N$$

It should be noted the sign difference for $\beta^+$-decay in (5.29). This is due to the following reason: The main term is due to the axial vector term (even under the charge conjugation), while the correction term (weak magnetism) comes from the vector term (odd under the charge conjugation).

(c4) Induced pseudoscalar term.

This subject is not connected to the weak vector current, but it is discussed here for the sake of completeness.

*) (5.29) should be applied to the $\beta$-spectra from unpolarized nuclei, in which no parity violation effects are seen.

By the same reason, the Fierz term in $\beta$-ray energy spectra differs in sign for $\beta^+$-decay.
We discuss here the contribution of \( J^S_{\alpha}(A) \) to the weak leptonic processes (\( \beta \)-decay and \( \mu^- \) capture). By the invariance ground, the relevant matrix element

\[
<\text{proton}, p' | \frac{G^0}{\sqrt{2}} J^S_{\alpha}(A) | \text{neutron}, p> (\bar{u}_f(p_f) \gamma_\alpha (1 + \gamma_5) u_v(p_v))
\]

can be written as

\[
\frac{G^0}{\sqrt{2}} \left( \bar{u}_p(p') \left\{ \lambda(q^1) \gamma_\alpha \gamma_5 + \sigma(q^2) q_\alpha \gamma_5 \right\} u_n(p) \right) \times \left( \bar{u}_f(p_f) \gamma_\alpha (1 + \gamma_5) u_v(p_v) \right)
\]

(5.29)

where

\[
q = p - p' = p_f + p_v
\]

\( p_f \) and \( p_v \) are the momenta for the lepton (\( \mu \) or \( e \)) and the antineutrino. \( \lambda \) and \( \sigma \) are the invariant function of \( q^2 \). \( q^2 \) is rather small. We may well approximate \( \lambda \) and \( \sigma \) by their values \( \lambda(0) \) and \( \sigma(0) \) at \( q^2 = 0 \). The renormalized axial vector constant is given by \(-\lambda(0) G^0\). Empirically we know \( \lambda(0) \approx 1.2 \).

The second term can be transformed as follows

\[
\frac{G^0}{\sqrt{2}} \sigma \left( \bar{u}_p \gamma_5 u_n \right) \left( \bar{u}_f(p_f) \gamma_\alpha (1 + \gamma_5) u_v(p_v) \right)
\]

\[
= i \frac{G^0}{\sqrt{2}} m_f \left( \bar{u}_p \gamma_5 u_n \right) \left( \bar{u}_f(p_f) (1 + \gamma_5) u_v \right)
\]

(5.30)
where the Dirac equations

\[(\gamma \ell \nu) \Psi_\nu = 0\]

\[\bar{\Psi}_\ell (\gamma \rho_\ell) = i m_\ell \bar{\Psi}_\ell\]

\[(q_0 = p_\ell + p_\nu)\]

have been used. (5.30) shows that the axial vector current \(J^{\text{Sc}}(\lambda)\) induces the effective pseudoscalar interaction. The effective pseudoscalar constant is

\[G_p = m_\ell \sigma(\lambda)\]

(5.31)

Remark.

Modification of the axial vector constant is due to the SI. An example of the corresponding Feynman Diagram is given by
The induced pseudoscalar term is also the effect of SI. Its main contribution comes from

The strong vertex can be expressed using the renormalized pion-nucleon coupling constant $f$ and the weak vertex may be replaced by the empirical $\pi f_2$ decay interaction (4.3), with $g_\pi$ given by (2.11). In this way we can avoid the explicit use of weak Fermi interaction (where $j_{\alpha}^{\text{SC(A)}}$ is not known with certainty) in the weak vertex. Then the matrix element is precisely given by (5.4). Moreover, assuming $q_0^2 \ll m_\pi^2$ and using the Dirac equations for leptons, the relevant matrix element turns out to be

$$i \sqrt{2} m_\ell f \frac{g_{\pi}}{m_\pi^3} \left( \bar{u}_p \gamma_5 u_n \right) \left( \bar{u}_e \left( 1 + \gamma_5 \right) \nu_\nu \right)$$

Hence the effective pseudoscalar constant is given by

$$| G_P | \approx \left| 2 f \frac{m_\pi}{m_\pi^3} g_\pi \right|$$

$$\sim 10 \left| G_P \right| \frac{m_\ell}{m_\mu} \quad (5.31')$$
where
\[ \frac{1}{\sqrt{4\pi}} \approx 15 \]  
\[ \frac{\alpha}{4\pi} \approx 1.8 \times 10^{-15} \]

have been used.

The effective pseudoscalar constant is proportional to the charged lepton mass (see (5.31) for $\mu$-decay and $\mu^-$ for $\mu^-$ capture). The estimation (5.31') shows that the pseudoscalar term is not important in nuclear $\beta$-decay but it is not negligible for the $\mu^-$-capture.

(d) Intermediate charge boson.

(d1) Introduction.

We have seen that the primary WI is a form of the product of weak currents, which induces always the change of electric charge (see (b3)). Why do the neutral weak currents not appear? A simple explanation for it may be given by the following hypothesis: The weak Fermi interactions (5.8), which we have up to now thought as the primary interactions, are not quite fundamental, but they are the second order effects due to intermediary charged boson $B^\pm$.

For example, we now suppose the $\pi^\pm\mu$ and $\mu^-\pi^-$ decays as the following graph.
We can immediately see the following properties for $B$

(i) $B^\dagger$ is the vector (spin 1), since it is coupled to the weak currents.

(ii) There are no neutral $B$'s and that is why no neutral weak currents appear.

The fundamental weak coupling would now be

$$F \bar{\Phi}_\alpha \left( J_{\alpha}^0 + J_{\alpha}^{SC} + \ldots \right) + \text{h.c.}, \quad (5.32)$$

where $\bar{\Phi}_\alpha$ is the field operator for $B$. Usual weak processes are the second order effects of (5.32). The coupling constant $F$ is roughly the square root of the weak Fermi constant $G^0$; more specifically, it can be shown that

$$G^0 \approx \sqrt{2} \frac{F}{m_B^2} \quad (5.33)$$

provided the mass $m_B$ of $B$ is "large".

\textbf{Demonstration of (5.33).}

We calculate the $\mu - e$ decay matrix element in two ways:

\textbf{From the Fermi interaction (5.1)}

$$\frac{G^0}{\sqrt{2}} \left( \bar{u}_\nu \gamma^\alpha (1 + \gamma^5) u_\mu \right) \left( \bar{u}_e \gamma^\alpha (1 + \gamma^5) u_\nu \right)$$
From the second order perturbation applied to (5.32)

\[ F^2 \frac{\left( \bar{\nu}_\nu \gamma_\alpha \left( 1 + \gamma_5 \right) \nu \mu \right) \left( \bar{\nu} \gamma_\alpha \left( 1 + \gamma_5 \right) \nu \bar{\nu} \right)}{m_B^2 + q_0^2} \]

where \( q_0^2 \) is the momentum of virtual \( B \).

Equating these two results, we immediately find (5.33), provided \( B \) is massive;

\[ m_B^2 \gg q_0^2 \]

First order processes in the weak interaction (5.32) must have, in general, intermediate transition rates between those of fast reactions (due to SI) and weak processes (second order in (5.32)). If the \( B \) is lighter than the K-meson \( (m_B < m_K) \), the \( K^+ \) would immediately decay into \( B \)

\[ K^+ \rightarrow B^+ + \gamma \quad \text{or} \quad B^+ + \pi^0 \quad \text{etc.} \]

Therefore we must suppose

\[ m_B > m_K \] (5.34)

The \( B \) can have many varieties of decay modes
\[ \mathcal{B}^+ \rightarrow \begin{cases} 
\mu^+ + \nu \\
e^+ + \nu \\
p^+ + \pi^0 \\
p^0 + \mu^+ + \nu \\
k^+ + \pi^0 \\
e^+ \end{cases} \]

The lifetime of \( \mathcal{B} \) would be \( \lesssim 10^{-7} \) sec.

\textbf{(d2) Test of existence of} \( \mathcal{B} \).

If the strongly interacting particles are involved in the weak process, the SI causes so much complicated effect, that the effect of \( \mathcal{B} \) can hardly be noticed. \( \mu \)-decay processes would only be cases among many weak processes, where one might hope to see the effect of \( \mathcal{B} \).

\((i)\) \( \mu \rightarrow e^+ + \nu + \bar{\nu} \)

If the \( \mu - e \) decay proceeds via intermediary \( \mathcal{B} \)

\[ \begin{array}{c} \bar{\nu} \\ \mathcal{B} \end{array} \xleftrightarrow[\nu]{e^+} \]

the characteristics (described in \( \mathcal{V}(4) \)) of \( \mu - e \) decay are slightly modified from those predicted from the point (or local) interaction of Fermi type (5.1). In particular, the energy spectrum of electrons
is no more of Michel form (eq. (13), V(4), p. 59). However, if it is approximated by the Michel form in the spirit of least square fit, the effective Michel parameter \( \rho \) turns out to be

\[
\rho \approx \frac{3}{4} + \frac{1}{3} \left( \frac{m_\mu}{m_B} \right)^2
\]

provided \( m_B^2 \gg m_\mu^2 \). Accurate experiment on \( \mu - e \) decay is interesting in this connection.

(ii) \( \mu \rightarrow e + \gamma \)

Next we discuss the process

\[
\mu \rightarrow e + \gamma
\]

If \( B \) does not exist and the local Fermi interaction (5.1) is the fundamental WI, the radiative decay of muons is forbidden (in the first order in WI). On the other hand, if \( B \) exists, (5.36) is "allowed" (in the second order in (5.32); i.e., effectively in the first order in the Fermi interaction)
Unfortunately the decay matrix element for (5.36) is logarithmically divergent and the decay rate \( \mu \rightarrow e + \gamma \) does depend upon the mass \( m_B \) and the cut-off momentum \( \Lambda^* \). Hence the clear cut statement can hardly be made. Experiment shows the branching ratio

\[
\frac{\omega(\mu \rightarrow e + \gamma)}{\omega(\mu \rightarrow e + \nu + \bar{\nu})} < 2 \times 10^{-6}
\]

(iii) photoproduction of \( B^- \).

\( B^+ \) is charged and can be produced in pair by the energetic photons:

\[
\gamma' + \text{(nucleus as the momentum absorber)} \rightarrow (\text{nucleus})^* + B^+ + B^-
\]

The final nucleus must absorb a substantial recoil momentum and nuclear excitations or reactions would follow. Cross-section for such pair creation is roughly \( (m_e/m_B)^2 \) times smaller than that of

*) See P. Meyer and G. Salzman, to be published.

The experimental upper limit on \( \omega(\mu \rightarrow e + \gamma) \) suggests that \( B \) does not exist, or \( B \) is very massive.
electron pair creation (well above the threshold). $B^+$ is extremely short-lived and sometimes decays into $\mu + \nu$ or $e + \nu$. Therefore $B$-pair creation can be established by observing $\mu^+ \mu^-$ pair.

Remark.

We must accept here the so-called minimal electromagnetic interaction, which excludes the direct $\mu - e - \gamma$ interaction

\[
\begin{pmatrix}
\bar{\Psi}_\mu \\
\sigma_{\alpha \beta} \Psi_e
\end{pmatrix}
(\varepsilon_\alpha A_\rho - \partial_\rho A_\alpha)
\]

\[
\rightarrow
\begin{array}{c}
e \\ \gamma
\end{array}
\]

\[
\mu
\]

The energy dependence (threshold) and angular spread of apparent ($\mu^+ e^-$) pair production via $B^+$ decay must be useful to eliminate the possible contamination of normal ($\mu^+ \mu^-$) or ($e^+ e^-$) pairs produced by photons.
V. STRUCTURE OF WEAK INTERACTIONS IN SPACE-TIME.

(6) $K_{e3}$-decays. (by Y. Yamaguchi)
6. **$K_{l3}$-decay.**

**INTRODUCTORY NOTES**

Two-body leptonic decays of $K$-mesons have already been discussed in $\Gamma(2)$. We would like to discuss here the three-body leptonic decays of $K$-mesons (briefly, $K_{l3}$; $\ell = \mu$ or $e$).

\[
K^- \rightarrow \pi^0 + (\mu^+)^+ \bar{\nu} \tag{6.1}
\]

and

\[
K_{12}^0 \rightarrow \pi^+ + (\mu^-)^- \bar{\nu} \tag{6.2}
\]

The relations between $K_{l3}^+$ and $K_{l3}^0$ decay modes have been discussed in III(5) based on the "$|\Delta \bar{Y}| = \frac{1}{2}$ rule for the strangeness changing current" $J^S_{\bar{Y}}$. Hence our discussions given below shall be restricted solely to those of $K_{l3}^+$.  

**Remarks.**

Charge changing $J^S_{\bar{Y}}$ ($\Delta Q = +1$) could be a linear combination of the following terms:

| Class | Characteristic | $|\Delta \bar{Y}|$ for the "current" | Examples |
|-------|----------------|-----------------------------------|-----------|
| (a)   | $\Delta Q = +1$ | 0                                  | $(p, \Lambda)$ ($\Lambda$, $\Xi^-$) |
|       | $\Delta S = +1$ |                                    |           |
| (b)   | $\Delta Q = +1$ | $\frac{1}{2}$ and $3/2$           | $(p, \Xi^0)$ ($\eta^+$+$\Sigma^-$) ($\Xi^+$,$\Xi^0$) |
|       | $\Delta S = +1$ |                                    | ($\Sigma^0$,$\Xi^-$) |
| (c)   | $\Delta Q = +1$ |                                    | ($\Sigma^+$,$\Xi^0$) ($\Xi^0$,$\Xi^-$) |
|       | $\Delta S = -1$ |                                    |           |
| (d)   | $\Delta Q = +1$ |                                    | $(p, \Xi^0)$ ($\eta$, $\Xi^-$)      |
|       | $\Delta S = +2$ |                                    |           |
where \((p, \lambda)\) is an abbreviation of \((\bar{\gamma}_p \gamma_\alpha (1+\gamma_5) \psi_\lambda)\), etc.

We notice again:

(i) If \(\Delta Q/\Delta S = +1\) for \(j^\text{SNC}_\alpha\) is assumed, then \(j^\text{SNC}_\alpha\) should \underbrace{\text{not contain the terms of the classes (c)}}_{(\Delta Q/\Delta S = -1)} \text{ and (d)} \,(\Delta Q/\Delta S = +\frac{1}{2}).

(ii) \(j^\text{SNC}_\alpha\) is assumed to obey \(\Delta Q/\Delta S = +1\) and to be isospinor. (Then it follows the \("/\Delta T/ = \frac{1}{2}\) rule-for-the-weak current") The terms in the class(a) satisfy these conditions. The terms in the class (b) must appear in a definite combination; e.g.

\[
\frac{1}{\sqrt{3}} \int \left( \bar{\psi}_p \gamma_\alpha \psi_\Sigma^0 \right) + \sqrt{2} \left( \bar{\psi}_n \gamma_\alpha \psi_\Sigma^- \right) \mathcal{J} \left( \gamma_\alpha = \gamma_\alpha (1+\gamma_5) \right) \quad (6.3)
\]

(note that \(\frac{+(-\bar{\psi}) \Sigma^0 - \sqrt{2} \bar{n} \Sigma^-}{\sqrt{3}}\) is the charge wave function for the total isospin \(\frac{1}{2}\) (II(1)). Hence (6.3) is isospinor).

*** *** ***

Quite a lot of theoretical works have been one on \(K_3\), while so little experimental information exist on this process.

Let us now assume the weak interaction, Eq. (5.11). Then the great advantage of making theoretical analysis can be found from the following fact: the \(K_3\) matrix element

\[
\langle \pi^0 | \frac{G}{\sqrt{2}} j^\text{SNC}_\alpha | K^+ \rangle \tilde{\mathcal{J}}_\lambda (p) \gamma_\alpha (1+\gamma_5) \psi_\nu (q)
\]

can be brought into the form \(\mathcal{J}\) see Eqs. (11) \sim (15). \[\frac{1}{N^* \text{Im} m_K} \tilde{\mathcal{J}}_\lambda (p) \int f(\omega) \psi_\nu (q) + \text{Im} g(\omega) \mathcal{J} (1+\gamma_5) \psi_\nu (q)\]
where $\ell^+$ (charged lepton) is either $\mu^+$ or $e^+$, $m_\ell =$ lepton mass, $p$, $q$, $p_K$ are the 4-momenta of $\ell$, $\nu$ and the K-meson, and $f$ and $g$ depend only on the pion energy $\omega$ measured in the rest system of the K-meson. This form of the matrix element serves to simplify the discussions. For example, once the pion energy $\omega$ is fixed, $f$ and $g$ are just numbers, and the energy distribution of leptons or the $\ell - \nu$ angular correlation is uniquely predictable from (6.3). Thereby one finds many fine tests of the assumed WI.

Remarks.

$J^{SNC}$ is simply written as $J_\omega$ in this section, since it is used so many times. The description given below presents more or less a review paper on theoretical analyses of $K_{e3}$ decays, and considerable parts of discussions already given in (5) are repeated.
6. \( K_{\mu 3} \) decay. (By Y. Yamaguchi)

(a) Introduction.

The \( K_{\mu 3} \) decay ("\( \kappa \)-meson") was found by C. O'Csallaigh \(^1\) in 1951 in the photographic emulsion exposed to the cosmic radiation. In fact this was the second decay mode of \( K \)-mesons which we had happened to know. (The first decay mode of \( K \)-meson was found by the Bristol group (1949) \(^2\); \( K_{\pi 3} \) decay, or "\( \tau \)-meson".) It may be rather surprising that \( K_{\mu 3} \) and \( K_{\pi 3} \) have small branching ratios (see the Table given on p.317) and these were discovered first. However, \( K_{\mu 3} \) and \( K_{\pi 3} \) are such unique decay processes among other decay modes of various particles, including \( K \)-mesons, that they are quite clearly distinguished from the others.

Up to now we know the following 3 body leptonic decay modes of \( K \)-mesons:

\[
K^+ \rightarrow \begin{cases} 
\mu^+ + \nu + \pi^0 \quad (K_{\mu 3}) \\
\mu^- + \nu + \pi^0 \quad (K_{e 3}) 
\end{cases} \tag{1}
\]

\[
K_{1,2}^0 \rightarrow \begin{cases} 
\mu^- + \nu + \pi^+ \\
\mu^- + \nu + \pi^+ \end{cases} \tag{2}
\]

Relations between (1) and (2) depend on the choice of weak interactions. In fact, such relations have already been discussed by Dr. d'Espagnat in this series of lectures, based on the "new" \( \left| \Delta I \right| = \frac{1}{2} \) laws \(^3\), so that we shall not discuss them here. We shall restrict, in this chapter, our discussion to those of \( K_{\mu 3}^+ \) and \( K_{e 3}^+ \).
(i) experimental data

(i_a) \( K^+ \)

lifetime = \((1.224 \pm 0.013) \times 10^{-8}\) sec.

branching ratios:

| \( K^+_{\mu3} \) | \( 4.0 \pm 0.8\% \) | \( 1.9 \pm 0.4\% \) |
| \( K^+_{e3} \) | \( 4.2 \pm 0.4\% \) | \( 3.3 \pm 1\% \) |

| Reference | Gell-Mann, Rosenfeld | Franzinetti, Morpurgo |

(i_b) \( K^0 \)

We do not have enough information except that we have established the existence of \( K^0_{\mu3}, K^0_{e3} \) decay modes 5).

(ii) kinematics (K\(^+\) meson at rest)

\( \mu \) or \( e \) shall be referred to as \( \ell \). We call:

\[
\begin{align*}
K^+ & \rightarrow \pi^0 + \ell^+ + \nu \\
\text{mass} & \\
\text{momentum} & \\
\text{energy} & \\
4\text{-momentum} & 
\end{align*}
\]

\[
\begin{align*}
m_K & \quad \omega_k \quad \ell_p \quad \nu_q \\
0 & \quad \vec{k} \quad \vec{p} \quad \vec{q} \\
m_\pi & \quad \omega \quad \ell \quad \nu \\
\vec{P}_K & \quad \vec{k} \quad \vec{p} \quad \vec{q}
\end{align*}
\]
Limit on energies:

\[ m_\pi \leq \omega \leq W_\pi = \frac{m_K^2 + m_\pi^2 - m_\nu^2}{2m_K} \]

\[ m_\ell \leq \epsilon \leq W_\ell = \frac{m_K^2 + m_\ell^2 - m_\pi^2}{2m_K} \]  \quad (4)

**neutrino** \[ 0 \leq q \leq W_\nu = \frac{m_K^2 - (m_\pi + m_\ell)^2}{2m_K} \]

When one fixes the pion energy \( \omega \), one finds the following restriction on lepton and neutrino energies:

\[ \frac{m_K - \omega - k}{2} + \frac{m_\ell^2}{2(m_K - \omega - k)} \leq \epsilon \leq \frac{m_K - \omega + k}{2} + \frac{m_\ell^2}{2(m_K - \omega + k)} \]  \quad (5)

\[ \frac{m_K (W_\pi - \omega)}{m_K - \omega + k} \leq q \leq \frac{m_K (W_\pi - \omega)}{m_K - \omega - k} \]

and the range of \( \Theta_\ell_\pi \) and \( \Theta_\nu_\pi \) are given by

\[ -1 \leq \cos \Theta_\ell_\pi \leq +1 \]  \quad \text{for } m_\pi \leq \omega < \omega^* \]

\[ -1 \leq \cos \Theta_\ell_\pi \leq 0 \]  \quad \text{for } \omega = \omega^* \]

\[ -1 \leq \cos \Theta_\ell_\pi \leq -\sqrt{1 - \frac{m_K (W - \omega)}{k m_\ell}} \]  \quad \text{for } \omega^* < \omega \leq W_\pi \]

\[ -1 \leq \cos \Theta_\nu_\pi \leq +1 \]  \quad \text{for } m_\pi \leq \omega < W_\pi \]  \quad (6)

(7)
where

\[
\omega^* = \frac{(m_K - m_\ell)^2 + m_\pi^2}{2(m_K - m_\ell)}
\]

(8)

Furthermore, one introduces

\[
k_{\text{max}} = \sqrt{\frac{W_\pi^2 - m_\pi^2}{2}}
\]

\[
p_{\text{max}} = \sqrt{\frac{W_\ell^2 - m_\ell^2}{2}}
\]

(9)

We give numerical values of these important quantities in the following table:

<table>
<thead>
<tr>
<th></th>
<th>(k_{\text{max}}) MeV/c</th>
<th>(W_\pi) MeV</th>
<th>(p_{\text{max}}) MeV/c</th>
<th>(W_\ell) MeV</th>
<th>(\omega^*) MeV</th>
<th>(Q) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K^+)</td>
<td>215.3</td>
<td>254.1</td>
<td>215.2</td>
<td>239.8</td>
<td>217.6</td>
<td>253.26</td>
</tr>
<tr>
<td>(\mu^+)</td>
<td>228.5</td>
<td>265.4</td>
<td>228.5</td>
<td>228.5</td>
<td>265.2</td>
<td>358.45</td>
</tr>
</tbody>
</table>

where we have used the mass values:

\[
K^+ = 494.0 \text{ MeV/c}^2
\]

\[
\pi^0 = 135.0
\]

\[
\mu^+ = 105.7
\]

\[
e^+ = 0.511
\]

and \(Q\) is the available kinetic energy in the C.M. system (in which the \(K^+\) is at rest).
(b) Decay amplitude.

To write down the decay interaction, or equivalently the decay amplitude, we assume:

(i) The $K^+$-meson has spin 0.

(ii) Decay modes are:

\[
K^+ \rightarrow \begin{cases} 
\mu^+ + \nu + \pi^0 \\
\nu^+ + \nu + \pi^0
\end{cases}
\]

\[
K_1^0, K_2^0 \rightarrow \begin{cases} 
\mu^- + \bar{\nu} + \bar{\pi}^+ \\
\bar{\nu} + \bar{\nu} + \bar{\pi}^-
\end{cases}
\]

(iii) The neutrino obeys the two component theory.
(The neutrino $\nu$ is assumed to be left-handed).

(iv) The lepton-neutrino pair interacts via "point interaction" of the type $V-A$.

These four assumptions are either already established or sound very plausible. Hence our $K_{23}$ decay process would look like

![Diagram](image)

We may give two models of such decay mechanism:
Model I

We assume a Fermi type interaction between hyperon-nucleon and lepton-neutrino \(^4\),\(^5\),\(^6\). The strong interactions convert a K-meson into a hyperon-nucleon pair and the latter transforms into \(\ell + \nu\) via Fermi interaction, together with \(\pi^0\)-emission.

\[
\begin{array}{c}
p \\
\pi^0 \\
\Lambda^0 \\
\ell^+ (\mu^+ \nu \epsilon^+) \\
\nu
\end{array}
\]

(an example of \(K^+\)-meson decay into \(\pi^0 + \mu^+\) (or \(\epsilon^+\)) + \(\nu\).)

Model II

We assume the existence of an intermediate boson:

First we shall choose the model I. (Model II will be discussed later.) Then one can write the decay interaction as follows:

\[
\frac{G}{\sqrt{2}} \quad j_\alpha j_\alpha + \text{h.c.}
\]

where

\[
J_\alpha = (\bar{\psi}_\mu \gamma_\alpha (1 + \gamma_5) \psi_\nu) + (\bar{\psi}_e \gamma_\alpha (1 + \gamma_5) \psi_\nu),
\]

\[
J_\alpha = \text{strangeness changing current}
\]

\[
= (\bar{p} \gamma_\alpha (1 + \gamma_5) \Lambda + \ldots) + \ldots
\]

and \(G\) is the coupling constant.
Then the matrix element for the $K^0\rightarrow\bar{\nu}_\mu\nu$ decay is given by

\[
\mathbf{\bar{\nu}_\mu}(p) \gamma_\alpha(1+\gamma_5) u_\nu(q) \langle \pi^0 | \frac{G}{\sqrt{2}} J_\alpha | K^+ \rangle \tag{11}
\]

where $|\pi^0\rangle$ and $|K^+\rangle$ are the eigenstates of "strong" Hamiltonian

($|\pi^0\rangle$ is the one pion state with 4-momentum $k$ while $|K^+\rangle$ is the one $K^+$ state at rest). From the invariance ground one can write \cite{7,8}

\[
\langle \pi^0 | \frac{G}{\sqrt{2}} J_\alpha | K^+ \rangle = \frac{1}{\sqrt{2}\omega} \frac{1}{2m_K} \left[ (p_{K^+})_\alpha c + (p_{K^-})_\alpha d \right] \tag{12}
\]

where $c$ and $d$ are the function of the Lorentz invariant momentum transfer:

\[
(k-p_K)^2 = -(m_K^2 + m^2 - 2m_K\omega) \tag{13}
\]

Notice that $c$ and $d$ depend neither on lepton momentum $p$ nor on neutrino momentum $q$ separately; i.e., they just depend on

\[
(k-p_K)^2 = (p+q)^2.
\]

Inserting (12) into (11) and using the free Dirac equations for lepton and neutrino:

\[
\mathbf{\bar{\nu}_\mu}(p)(\gamma^\mu) = i m_\mu \mathbf{\bar{\nu}_\mu}(p) \\
(\gamma^\mu) u_\nu(q) = 0 \tag{14}
\]

together with the energy-momentum balance $p_K = k+p+q$ one finally obtains:
\[(11) = \frac{1}{\sqrt{4m_K^2/\omega}} \bar{v}_\ell(p) \left[ 2c(\gamma \cdot p)(1 + \gamma^5) + \text{im} \epsilon \cdot (c-d)(1 + \gamma^5) \right] u_\nu(q). \]

\begin{align*}
&= \frac{1}{\sqrt{4m_K^2/\omega}} \bar{v}_\ell(p) \left[ f(\gamma \cdot p)(1 + \gamma^5) + \text{im} g(1 + \gamma^5) \right] u_\nu(q) \quad (15)
\end{align*}

where
\[
\begin{aligned}
f &= 2c \\
g &= c-d
\end{aligned}
\]

are again the functions of \(-(k-p_K)^2 = m_K^2 + m_\pi^2 - 2m_K\omega\). Since there are no strong interactions between \(\pi^0, K^+, \ell^+\) and \(\nu\), the use of the free Dirac Eq. (14) is completely legitimate, and one can verify that \(f\) and \(g\) (\(c\) and \(d\)) must be real if the time reversal invariance holds for the decay interaction (10).

One can easily calculate the energy spectrum or the polarization of lepton from the matrix element (15). To calculate the energy spectrum or decay probability we must know the explicit \(\omega\)-dependence of \(f\) and \(g\). (\(\omega = \text{pion energy}\)). Now the simplest choice of \(f\) and \(g\) will be:

\[
\begin{aligned}
f &= \text{const} \\
g &= \text{const.}
\end{aligned}
\quad (16)
\]

Roughly speaking, this corresponds to the case in which we can neglect the "non-locality" due to hyperon-nucleon pair; i.e., we replace

\[
\begin{array}{c}
\pi^0 \\
K^+ \\
\ell^+ (\mu^+ \text{ or } e^+) \\
\nu
\end{array}
\]
by the "local" (or point) interaction

Another simple choice of f and g will be given by a simplification of model II. Namely, we assume the existence of charged intermediate boson \( B^* \) and furthermore imagine that the non-locality due to the \( K^+ \pi^0 B^\pm \) vertex (which is due to the baryon pairs) is small compared to the non-locality due to the "exchange" of \( B^\pm \), so that we shall neglect the former but keep only the latter. In other words, we shall approximate

by

both are point interactions.

*) A charged vector boson hypothesis seems to be successful at least for all leptonic decay processes. A possible evidence against it might be provided from the experimental branching ratio

\[
\frac{w(\mu^+ \rightarrow e^+ \gamma)}{w(\mu^+ \rightarrow e^+ \nu + \overline{\nu})}
\]

It is an open question whether this hypothesis is consistent with non-leptonic decay processes of hyperons and K-mesons or not.
If \( 2 \cdot (\text{baryon mass}) \gg (\text{B-mass}) = m_B \), then such an approximation would be permissible. In this case we can introduce phenomenologically weak local interactions:

\[
\epsilon_I^B \alpha \left( \pi_0 \partial_\alpha \phi_K - \phi_K \partial_\alpha \pi_0 \right) + \text{h.c.} \tag{17}
\]

together with

\[
\epsilon_I^B \alpha \left( \overline{\psi} \gamma^{(1+\gamma_5)} \psi + \overline{\psi} \gamma^{(1+\gamma_5)} \phi \right) + \text{h.c.} \tag{18}
\]

and we can find the \( K_{\mu 3} \) or \( K_{e3} \) matrix element by the standard perturbation theory. The result will be again written in the form of Eq. (15) with

\[
f = 2 \left( \frac{\epsilon_I^B}{{m_B}^2} \right) \frac{m_B^2}{m_B^2 - m_K^2 - m_{\pi}^2 + 2m_K \omega} \tag{19}
\]

\[
g = + \left( \frac{\epsilon_I^B}{{m_B}^2} \right) \frac{m_B^2 + m_K^2 - m_{\pi}^2}{m_B^2 - m_K^2 - m_{\pi}^2 + 2m_K \omega}
\]

We also notice that: In the local limit \( m_B^2 \gg m_K^2 \), the effective \( K_{\nu 3} \) decay interaction may be written as

\[
G' \left[ \frac{\phi_0}{\partial_\alpha} - \frac{\phi_K}{\partial_\alpha} - \phi_0 \right] \times \left\{ (\overline{\psi} \gamma^{(1+\gamma_5)} \psi) + (\overline{\psi} \gamma^{(1+\gamma_5)} \phi) \right\} + \text{h.c.} \tag{20}
\]

\[
G' = \epsilon_I^B \sqrt{m_B^2}
\]

whose matrix element for \( K_3 \) decay is precisely given by (15) with

\[
f \approx 2G' \quad \text{or} \quad G' \approx \frac{G'}{2} \tag{21}
\]
(c) Energy spectra of decay products, polarization of leptons.

Owing to the energy-momentum balance there are only two independent variables among 6 quantities:

\[ \omega, \ p, \ q, \ \Theta_{\ell\pi}, \ \Theta_{\nu\pi}, \ \Theta_{\ell\nu} \ (= 2\pi - \Theta_{\ell\pi} - \Theta_{\nu\pi}) \]

Hence we shall consider particularly the following cases:

a) distribution of \( \omega \) and \( \Theta_{\ell\pi} \)

b) " " \( \omega \) and \( \Theta_{\nu\pi} \)

Moreover, let us discuss in this section:

c) energy spectrum of \( \pi^0 \)

d) " " lepton

e) longitudinal polarization of lepton

in this order.

We can easily calculate the decay probability from (15):

\[
W(K^+) = 2\pi \times \frac{1}{d m_K \omega} \times 2 \left[ m_K^{2} |f|^2 \left\{ 1 + \frac{\tilde{p} \cdot \tilde{q}}{\tilde{e}} \right\} - (f^* g + g^* f) m_{\ell} \frac{m_K}{\omega} \right.
\]

\[
+ m_{\ell}^{2} |g|^2 \left\{ 1 - \frac{\tilde{p} \cdot \tilde{q}}{\tilde{e}} \right\} \right] \]

\[
\times \delta^{\rightarrow (k+p+q)} \delta^{\rightarrow (\omega+e+q-n_K)} \frac{d^3 k d^3 p d^3 q}{(2\pi)^6}
\]

where \( \tilde{p}, \tilde{q} \) is the scalar product of 3-momenta \( \tilde{p} \) and \( \tilde{q} \). As was stated before, time reversal invariance guarantees that \( f \) and \( g \) are real. The interference term is an analogue of the Fierz term in the allowed \( \beta \)-decay spectra.
Once again we notice that unless we integrate over the pion energy $\omega$, our results are quite general under the assumptions (i) $\sim$ (iv). ($f$ and $g$ depend on $\Omega$!). To perform $\omega$-integration we must fix a special form of $f$ and $g$, for which we choose either (16) or (19).

a) Distribution of $\omega$ and $\Theta_{\ell\pi}$.

We can easily calculate ($\omega - \Theta_{\ell\pi}$) distribution from Eq.(22)(11)-15)

$$\frac{d^2 w(K_{\ell 3})}{d\omega dcos\Theta_{\ell\pi}} = \frac{1}{4(2\pi)^3 m_K} \times \frac{k \cdot p^2}{p + k \cos \Theta_{\ell\pi} \frac{p}{\epsilon} + \frac{p + k \cos \Theta_{\ell\pi}}{q}}$$

$$\times \left[ m^2_K |f|^2 \left\{ 1 - \frac{p}{\epsilon} \frac{(p + k \cos \Theta_{\ell\pi})}{q} \right\} \right]$$

$$- \frac{m^2}{\epsilon} m_K (f^* g + g^* f) \frac{m^2}{\epsilon}$$

$$+ \frac{m^2}{\epsilon} |g|^2 \left\{ 1 + \frac{p}{\epsilon} \frac{(p + k \cos \Theta_{\ell\pi})}{q} \right\}$$

(23)

where $Q = m_K - \omega - \epsilon$. For $K_{e3}$, the approximation $m_e = 0$ can be used so that (23) can be simplified as follows:
\[
\frac{d^2 w(K_{e3})}{d\omega d\cos\theta_{e\pi}} = \frac{k(m_K - \omega)^2(1 - \chi^2)^2}{4(2\pi)^3(1 + \chi \cos\theta_{e\pi})^4} \\
\times \left[ m_K^2 \left| f \right|^2 \chi^2 \sin^2\theta_{e\pi} + \lim_{m_e \to 0} (m_e g) \right]^2(1 + \chi \cos\theta_{e\pi})^2 \right] \\
\text{(24)} \\
\text{(for } K_{e3} \text{ with approximation)}^*) \\
\text{\{m_e = 0\}} \\
\text{\{m_e = 0\}}
\]

where

\[
\chi = \frac{k}{m_K - \omega}
\]

depend on the pion energy, and the interference term can be neglected.

If one uses the approximation (16): \( f = \text{const}, \ g = \text{const} \), one can find the \( (\ell \pi) \) angular correlation irrespective of pion energy. We shall illustrate two extreme cases:

\[
\begin{align*}
\text{f = const, \ (g = 0) ... "V"} \\
\text{g = const, \ (f = 0) ... "S"}
\end{align*}
\]

which we shall refer to hereafter as "Vector" and "Scalar" cases, respectively:

\[\ell - \pi \text{ angular correlation}
\]

*taken from ref.\textsuperscript{15}*)

*) Here we have kept the term \( (m_e g) \), because sometimes we want to see what happens when \( m_e g \to \text{finite for } m_e \to 0 \).
b) **Distribution of $\omega$, $\epsilon$.**

This can be found from Eq. (22)\(^8\),\(13\)

\[
\frac{d^2w(K\ell\rho)}{d\omega d\epsilon} = \frac{1}{2(2\pi)^3m_K} \left[ m^2_K |f|^2 \left\{ 4q\epsilon - 2m_Kw_\pi + 2m_K\omega \right\} 
- m_\ell m_K (g^*f + f^*g) 2m_\ell q + m_\ell^2 |g|^2 (2m_K)(W_\pi - \omega) \right] \tag{25}
\]

where

\[ q = m_K - \omega - \epsilon . \]

**Remark:**

These results - (24) and (25) - can be used as an experimental check of our assumptions (i) $\sim$ (iv).

c) **Energy spectrum of $\Pi^0$**

is given by \(^{13}\),\(^{15}\):

\[
\frac{dw}{d\omega} = \frac{2m_K}{(2\pi)^3} \frac{k(W_\pi - \omega)^2}{\lambda^2} \left[ m^2_K |f|^2 \left\{ \frac{(m_K - \omega)^2}{\lambda^2} \left( \frac{m_K W_\pi - \omega}{\lambda^2} \right) 
- \frac{2}{3} \frac{m_K(W_\pi - \omega)}{\lambda^4} k^2 - 1 \right\} 
- m_\ell m_K (f^*g + g^*f) \frac{m_\ell(m_K - \omega)}{\lambda^2} + m_\ell^2 |g|^2 \right] \] \tag{26}

where

\[ \lambda^2 = m^2_K + m^2_\pi - 2m_K\omega = -(k-p_K)^2 \]
If one assumes (16'), one finds the $\pi^0$-spectrum:

\begin{equation}
\frac{d\sigma}{d\omega} = \frac{1}{(2\pi)^3} \left[ m_K^2 \left| f \right|^2 \left( \frac{m^2_{\pi}}{3m_K} + \lim_{m_e \to 0} \left( m_e g \right) \right) \right] (27)
\end{equation}

It is interesting to see the effect of intermediate boson; namely if one accepts (19), one finds (note that $m_e g \to 0$)\(^{10}\)

\begin{equation}
\frac{d\sigma}{d\omega} = (\text{const}) \left( \frac{m_B^2}{m_B^2 - m_K^2 - m_{\pi}^2 + 2m_K\omega} \right)^2 (28)
\end{equation}

If $m_B^2 - m_K^2 - m_{\pi}^2$ is comparable to $2m_K\omega$, one can easily check this energy spectrum (28), i.e., the effect of an intermediate charged boson.
d) Energy spectrum of (charged) lepton.

To derive this we have to perform the integration over pion energy $\omega$, and we must know the $\omega$-dependence of $f$ and $g$. For simplicity we take very special case (16'). The results are shown in the following figures:\(^{11}\)

\[
\begin{align*}
\text{(taken from ref. }^{11})
\end{align*}
\]

e) Longitudinal polarization of the (charged) lepton.

Here again we have to specify $f$ and $g$. Using (16') we obtain: \(^{13},^{14},^{16}\)

\[
S: \quad \text{(long. pol. of } \ell^-) = \mp \frac{-(m_K-2 \varepsilon) p}{(m_K-2 \varepsilon) \varepsilon + m_p^2}
\]

\[
V: \quad \text{(long. pol. of } \ell^+) = \mp \frac{m_K p}{m_K \varepsilon - m_p}
\]
These results for $K^+_\mu^3$ are illustrated in the next figure:

(taken from ref. 16)

In the case of $K^e_\mu^3$ one can use the approximation $m_e = 0$ and the electron behaves precisely like the two component neutrino. Therefore:

$$
\begin{align*}
S: & \quad \text{e}^+ \text{ is left-handed} \\
V: & \quad \text{e}^+ \text{ is right-handed}
\end{align*}
$$

Furuchi et al. 17) have shown that the lepton polarization along $k$-direction (\(\pi^0\) momentum) is also sensitive to the choice of interactions.

Also notice that \((\text{long.pol. of } l^-) = -(\text{long.pol. of } l^+).\)

f) Test of time reversal.

In the $K_{e3}$ decay process one can construct odd quantities in time reversal; e.g.

$$
\sigma \cdot [k \times p]
$$

So that one can test the time reversal invariance in $K_{e3}$ process 18).
(d) Further discussions.

a) Strength of effective coupling constant.

Let us evaluate the strength of the effective coupling constant of $K_{e3}$ decay interaction. This will be most easily done if one chooses a very special decay interaction (20). If one uses the $K_{e3}$-decay rate given by Gell-Mann - Rosenfeld:

$$w(K_{e3}) \approx 3.4 \times 10^6 \text{ sec}^{-1}$$

then one finds $^9$)

$$G' \approx 1.1 \times 10^{-6}/(\text{nucleon mass})^2$$  \hfill (33)

This should be compared with the Fermi coupling constant

$$G_{\beta} = 1.0 \times 10^{-5}/(\text{nucleon mass})^2$$  \hfill (34)

for the nuclear $\beta$-decay. We also notice $^9)$ that

$$\frac{w(K_{e3}^+)}{w(K_{\mu3}^+)} = 1.55$$  \hfill (35)

for the local weak interaction (20).

We may tentatively conclude that the coupling constant $G$ in the Fermi type decay interaction (10) is

$$|G| \approx \frac{1}{\sqrt{2}} \left| \frac{1}{10} \frac{f_\pi}{f_K} G_{\beta} \right|$$  \hfill (36)

where $f_\pi$ and $f_K$ are the coupling constants for the strong interactions $\pi-N-N$ and $K-Y-N$. If $f_K^2/4 \pi \approx 1$, $f_\pi^2/4 \approx 15$

$$|G| \approx \frac{1}{3} |G_{\beta}|$$  \hfill (36')
Although this argument is not quite reliable, the weak coupling constant for (10) /i.e., (strangeness changing current) (lepton current) might be smaller than the other (universal) weak coupling constants for nuclear $\beta$-decay, $\mu$-decay, $\mu$-capture by a nucleon, etc. The rarity of the $\beta$- and $\mu$-decay modes of hyperons \(^{19}\) as compared to predictions from the "universal V-A" interactions, \(^{6,20}\) if true, could also be such an indication. But at the moment we do not want to state anything definite. One may even take another extreme point of view : (36') is not far from the universality of the coupling constants $G \approx \frac{G}{\sqrt{3}}$.

b) Conserved current. *)

Feynman and Gell-Mann \(^{6}\) have proposed that the strangeness conserving vector current $J_{SC}^V$ is conserved if the electro-magnetic interactions (and the mass splits among iso-multiplet) are neglected. This is a quite interesting suggestion since the vector coupling constants are not renormalized by any strong interactions (except electro-magnetic interactions) and thereby one has the universality (equality) of all weak vector coupling constants.

Let us now ask what conclusions we can draw if we assume the conserved strangeness changing current $J_\alpha$ in Eq. (10). We divide $J_\alpha$ into two parts: vector part $J_\alpha^V$ and axial part $J_\alpha^A$.

$$ J_\alpha = J_\alpha^V + J_\alpha^A $$

(37)

*) A more detailed explanation of this item will be given later.
Now we assume either

\[ \partial_\alpha J^V_\alpha = 0 \]  \hspace{1cm} (38) 

or

\[ \partial_\alpha J^A_\alpha = 0 \]  \hspace{1cm} (38')

or both. For the moment we shall not discuss how to construct such conserved currents, but we shall simply see what follows from (38), (38').

(i) First we discuss two body decay of \( K \)

\[ K^+ \rightarrow \ell^+ + \nu \quad (\ell^+ = \mu^+ \text{ or } e^+) \]

Assuming decay interaction (10), we find the following matrix element:

\[ \frac{g}{\sqrt{2}} \bar{\nu}_\ell (p) \gamma_\alpha (1 + \gamma_5) \langle 0 | J^\alpha | K^+ \rangle u_\nu (q) \]  \hspace{1cm} (39)

where \( |0\rangle \) is the true vacuum state (for the "strong" Hamiltonian). If \( K^+ \) is scalar, then \( \langle 0 | J^\alpha | K^+ \rangle = \langle 0 | J^V_\alpha | K^+ \rangle \). If \( K^+ \) is pseudoscalar, then

\[ \langle 0 | J^\alpha | K^+ \rangle = \langle 0 | J^A_\alpha | K^+ \rangle \]

Furthermore the assumption (38) or (38') guarantees

\[ \langle 0 | J^V_\alpha | K^+ \rangle = 0 \text{ for scalar } K^+ \]  \hspace{1cm} (40)

or

\[ \langle 0 | J^A_\alpha | K^+ \rangle = 0 \text{ for pseudoscalar } K^+ \]  \hspace{1cm} (40')

i.e., \( (K^+ \rightarrow \ell^+ + \nu) \) is forbidden \(^{21}\).
(ii) Next we shall discuss $K_{L3}$. We see that

$$\langle \pi^0 | J_{\alpha}^V | K^+ \rangle = \begin{cases} 
\langle \pi^0 | J_{\alpha}^V | K^+ \rangle & \text{for same } (\pi^0, K^+) \text{ parity} \\
\langle \pi^- | J_{\alpha}^A | K^+ \rangle & \text{for opposite } (\pi^0, K^+) \text{ parity.}
\end{cases}$$

See Eq. (11). If we assume (38) or (38') we can conclude 7):

$$(p_K - k)_\alpha \langle \pi^0 | J_{\alpha}^V | K^+ \rangle = 0$$

$$\therefore \quad p = \frac{m_K^2 - m_\pi^2}{(m_K - m_\pi)^2} C$$

$$= \frac{2}{m_K^2 + m_\pi^2 - 2m_K \omega} C \quad (41)$$

where we have used the notation given in Eq. (12). From (41) and (25) we can calculate the $K_{L3}$ decay rate 7):

$$w(K_{L3}) = \frac{1}{192 \pi^3 m_K^3} \int_{m_\pi^2}^{m_K^2} d \lambda \left[ \lambda^2 - (m_K + m_\pi)^2 \right] \left[ \lambda^2 - (m_K - m_\pi)^2 \right]^{-1/2}$$

$$\times \left[ \frac{m_\ell^2}{\lambda^2} + \frac{m_\ell^6}{\lambda^6} \right]. \quad (42)$$

where $\lambda^2 = m_K^2 + m_\pi^2 - 2m_K \omega$. 
Evidently we have in general:

\[ \frac{w(K_{e3})}{w(K_{\mu3})} > 1 \]

For example, if we take \( C = \text{const} \)

\[ \frac{w(K_{e3})}{w(K_{\mu3})} = 2.5 \]

(cf. experimentally this ratio is

\[ \frac{3.3 \pm 1}{1.9 \pm 0.4} \]

and is consistent with Eq. (43).\)

(iii) So far we did not care about the possibility of constructing such conserved currents. We want now to consider this problem. As an introduction, let us remind the axial vector part of strangeness conserving current \( J_{SC}^A \). We can of course construct \( J_{SC}^A \) with the property \( \varphi_\alpha (J_{SC}^A)_\alpha = 0 \), which, however, gives results which are not consistent with experiments.\) Furthermore, to construct conserved \( J_{SC}^A \) we must introduce a rather artificial device (introduce, say, Schwinger's \( \sigma \)-meson (scalar and isoscalar).\) From these arguments we do not want to postulate

\[ \varphi_\alpha (J_{SC}^A)_\alpha = 0. \]

Let us come back to our strangeness changing current \( J_\alpha \). As was discussed in (i), \( K_{e2} \) process is forbidden by \( \varphi_\alpha J_\alpha = 0 \). Hence we should not accept (at least) either (36) or (36') depending on the \( K^+ \)-parity. Here remains also the problem how to construct \( J_\alpha^V \) or \( A \) such that \( \varphi_\alpha J_\alpha^V \) or \( A = 0 \). Since the hyperon mass is considerably heavier than nucleon mass, we cannot assure

\[ \varphi_\alpha J_\alpha^V = 0 \] (44)

just by using C.I., as was done by Feynman and Gell-Mann for the strangeness conserving current. If we use global symmetry (in the
sense that we neglect all baryon mass difference and \( K-\pi \) mass difference), we can very easily construct the conserved vector current \( J_\alpha^V \). We may therefore accept (44) to the extent that the global symmetry is valid. On the other hand, the construction of \( J_\alpha^A \) with property \( \partial_\alpha J_\alpha^A = 0 \) is more complicated (though not impossible) even when we assume global symmetry. As was the case of \( J_{S,C}^A \), we shall not postulate \( \partial_\alpha J_\alpha^A = 0 \). We may further appeal to the "principle of simplicity" to reject the possibility \( \partial_\alpha (J_{SC}^A) = 0 \) and \( \partial_\alpha J_\alpha^A = 0 \). If \( \pi^0 \) and \( K^+ \) have the same "parity" then \( K_{\mu 2} \) and \( K_{\mu 2} \) are allowed and the arguments given in (ii) can be accepted within the approximation in which (44) can be regarded as valid ("global symmetry" approximation).

*) The same argument has independently been given by S. Okubo, to be published.
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* * * * * * *
V. STRUWE OF WEAJ INTERACTIONS IN SPACE-TIME.

(7) Leptonic decays of hyperons. Y. Yamaguchi
7. Leptonic decays of hyperons.

Leptonic decay processes of hyperons can be classified according to the change of strangeness, $\Delta S$.

(i) $\Delta S = 0$

Examples:

\[ \Sigma^- \rightarrow \Sigma^0 + e^- + \bar{\nu} \]
\[ \Sigma^+ \rightarrow \Lambda^0 + e^+ + \nu \]

(ii) $|\Delta S| = 1$

Example:

\[ \Lambda \rightarrow p + e^- + \bar{\nu}, \]

(iii) $|\Delta S| = 2$

Example:

\[ \Xi^- \rightarrow n + e^- + \bar{\nu}. \]

We have already discussed (i) in V(b). We shall discuss here (ii) and (iii).

Leptonic decays of the class (ii) are supposed to be induced by the primary weak interaction (5,11)

\[ \frac{G^f}{\sqrt{2}} j_{\alpha}^{SNC} (j_{\alpha}^\ell)^+ + h.c. \quad (7.1) \]

$j_{\alpha}^{SNC}$ is not a conserved current and the renormalization effect exists even for the vector constant. Furthermore, renormalization of axial vector constant, a so-called weak magnetism, and the induced pseudoscalar term $\varepsilon \xi c$ must be present. Since the form of $j_{\alpha}^{SNC}$ is not known, these four effects can hardly be discussed in any reliable manner.

First of all, let us remind the reader of the proposed rule $\Delta Q/\Delta S = \pm 1$ introduced in III(5). This rule, if accepted as of general validity for leptonic decays provides a restriction to the form of $j_{\alpha}^{SNC}$ see V(6)' and correspondingly a selection rule. $j_{\alpha}^{SNC}$ can be a linear combination of the following terms:

\[ (\bar{\Psi}_p \gamma_\alpha (1 + \gamma_5) \Psi_\Lambda ) \quad (\bar{\Psi}_n \gamma_\alpha (1 + \gamma_5) \Psi_{\Xi^-} ) \]
\[ (\bar{\Psi}_\Lambda \gamma_\alpha (1 + \gamma_5) \Psi_{\Xi^-} ) \quad \varepsilon \xi c. \quad (7.1') \]
but it cannot contain \( \bar{V}(6) \bar{7} \):

\[
\psi + \gamma_\alpha (1 + \gamma_5) \psi_n \quad \cdots \quad (\Delta Q/\Delta S = -1)
\]

Therefore we can establish the selection rules:

(a) allowed:

\[
\lambda \rightarrow p + e^- + \bar{\nu}, \quad \Sigma^- \rightarrow p + \left( \frac{e^-}{\mu^-} \right) + \bar{\nu}, \quad \Xi^- \rightarrow \Lambda + \left( \frac{e^-}{\mu^-} \right) + \bar{\nu}
\]

(b) forbidden:

\[
\Sigma^+ \rightarrow n + \left( \frac{e^+}{\mu^+} \right) + \nu
\]

\[
\Xi^0 \rightarrow \Sigma^- + e^+ + \nu
\]

(\(\ast\) \(\Delta Q/\Delta S = -1\))

Up to now, no exceptions to this rule have been found.

To describe the leptonic decay of the hyperon \((B_\perp \rightarrow B f + l + \nu)\) we assume a phenomenological weak interaction

\[
\frac{G}{\sqrt{2}} \left( \bar{\psi} f \gamma_\alpha (1 + \gamma_5) \psi \right) \left( \frac{l}{\alpha} \right) + h.c. \quad (7.2)
\]

where we have ignored the weak magnetism and the induced pseudoscalar effect \(\alpha^c\) for simplicity \(\ast\). Then the decay rate can be written in Michel's form

\[
\frac{1}{\tau} = \frac{(G)^2}{\sqrt{2}} \frac{\lambda^5}{3(2\pi)^3} \left\{ \left( \lambda^2 + 1 \right) F_1 + \frac{\lambda^2 - 1}{2} F_2 \right\} \quad (7.3)
\]

\(\ast\) The relative strength of \(G\) for each hyperon case does depend on the precise choice of \(J^SNC\).
\[ F_1 = \frac{1}{8} (1+u^2+v^2)(1-5u^2-8v^2+u^4+v^4)w + \frac{1}{2} u^2 v^2 (7-2u^2-2v^2)w + \frac{3}{2} u^4 (1-v^4)A + \frac{3}{2} v^4 (1-u^4)B \]  

\[ F_2 = \frac{1}{2} v \left[ (1-5u^2+10v^2+v^4-2u^4-5u^2 v^2)w + 6u^4 (1-v^2)A - 6v^2 \left\{ (1-u^2)^2+v^2 \right\} B \right] \]  

(7.4)

where we have used:

| (initial baryon) \rightarrow (final baryon) + (charged lepton) + (neutrino) |
|---|---|---|---|
| mass | \( M \) | \( m \) | \( \mu \) | 0 |

and

\[ u = \frac{\mu}{M}, \quad v = \frac{m}{M}, \quad w = \sqrt{\frac{1+u^4 v^4 - 2u^2 - 2v^2 - 2u^2 v^2}{2}} \]

\[ A = \ln \left( \frac{w+u^2 - v^2}{2u} \right), \quad B = \ln \left( \frac{w+1+u^2 - u^2}{2v} \right) \]

If \( \mu = 0 \), which is a reasonably good approximation for \( \beta \)-decay of hyperons, (7.4) can be very much simplified:

\[ F_1 = \frac{1}{16} (1-v^4)(1-8v^2+v^4) + \frac{3}{2} v^4 \ln \left( \frac{1}{v} \right) \]

\[ F_2 = \frac{1}{4} v(1-v^2)(1+10v^2+v^4) - 3v^3(1+v^2) \ln \left( \frac{1}{v} \right) \]  

(7.4') 

(for \( \mu = 0 \))
This is a well-known result, which can also be used for the $\mu$-decay
($\mu \rightarrow e + \bar{\nu} + \nu$; here $M$ and $m$ should be, of course, the $\mu$- and
electron masses, respectively).

If we take $\lambda = 1$, we find the decay rates:

<table>
<thead>
<tr>
<th>decay process</th>
<th>decay rate in sec$^{-1}$</th>
</tr>
</thead>
</table>
| $\Lambda^0 \rightarrow \{ \begin{array}{l}
  p + e^- + \bar{\nu} \\
  p + \mu^- + \nu
\end{array} \}$ | $\frac{5.9 \times 10^7}{9.4 \times 10^6}$ |
| $\Sigma^- \rightarrow \{ \begin{array}{l}
  n + e^- + \bar{\nu} \\
  n + \mu^- + \nu
\end{array} \}$ | $\frac{3.5 \times 10^8}{1.6 \times 10^6}$ |
| $\Xi^- \rightarrow \{ \begin{array}{l}
  \Lambda^0 + e^- + \bar{\nu} \\
  \Lambda^0 + \mu^- + \nu
\end{array} \}$ | $\frac{1.2 \times 10^8}{3.3 \times 10^7}$ |

where $G_\mu$ is the $\mu \rightarrow e$ decay constant [see (5.1)]. Comparison with
experimental information about $\Lambda$ and $\Sigma$ hyperons shows that

$$\left( \frac{g}{G_\mu} \right)^2 \lesssim \frac{1}{10}. $$

Therefore the concept of universal Fermi interaction does not seem to be
applicable to the leptonic decay process with $|\Delta S| = 1$.

**Remark.**

As has been mentioned in the preceding section V(6), the $K^0_L$ decay
rate, calculated from the weak Fermi interaction (7.2), also suggests $|G|$
is considerably smaller than $|G_\mu| \neq |G_\beta|$.

*) The difference between $G$ and $G_\mu = G_\beta$ could be due to the
renormalization effect, but such a possibility seems to be unlikely.
Finally, the leptonic decay of the class (iii) should be mentioned. If \( J_{SN C}^{\omega} \) contains the following term:

\[
(\bar{\nu}_n \gamma_\mu (1 + \gamma_5) \nu_{\bar{\Xi}^-})
\]

(7.5)

the weak interaction (7.1) will cause the new types of leptonic decay:

\[
\Xi^- \rightarrow n \left( e^-, \mu^- \right) + \bar{\nu}
\]

(7.6)

For example, the phenomenological interaction

\[
\frac{G^{(2)}}{\sqrt{2}} (\bar{\psi}_n \gamma_\mu (1 + \gamma_5) \psi_{\Xi^-}) (J^{(l)}_\alpha)^t + h.c.
\]

will predict the decay rate

\[
10^{-2} \times 10^9 \text{ sec}^{-1} \times \left( \frac{G^{(2)}}{G_{\mu}} \right)^2 \text{ for } \Xi^- \rightarrow n + e^- + \bar{\nu}
\]

For example, the leptonic decay could be the leptonic decay. The check of such leptonic decays of \( \Xi^- \) -hyperons is extremely interesting for obtaining valuable information about the weak current \( J_{SN C}^{\omega} \).

Remarks.

If the weak current \( J_{SN C}^{\omega} \) with \( \Delta Q = +1 \) obeys the law \( \Delta Q/\Delta S = 1 \), \( J_{SN C}^{\omega} \) should not contain the term like (7.5). In such a case the leptonic decays (7.6) are forbidden.
V. STRUCTURE OF WEAK INTERACTIONS IN SPACE-TIME.

(8) Non-leptonic decays of hyperons. B. d'Espagnat
8. Non leptonic decays of hyperons.

Introduction

In view of the wide success of the V-A hypothesis it certainly seems very attractive to make the same assumption as regards non leptonic weak interactions. (Structure and coupling constants). This is all the more the case as we have already the necessary currents at our disposal: there are the currents $J^{sc}_K$ and $J^{snc}_K$ used previously for the description of $\beta$ decay and the leptonic decays of $K$ and hyperons. In order to get a primary weak interaction which qualitatively at least - can account for the non leptonic decays of hyperons and $K$ mesons it is sufficient to write down the product

$$J^{snc}_K (J^{sc}_K)^*$$

and this, obviously, seems the most natural thing to do. $J^{snc}_K$ would, in this approach, be some linear combination of the terms $(7\cdot1')$ $(\Delta Q/\Delta S=1$ currents).

It is not yet possible to rule out definitely - on the basis of a sound mathematical proof - the possibility that $(5\cdot12)$ would indeed be a good quantitative description of the truth, and indeed many physicists still hope that $(5\cdot12)$ will finally prove to be correct and that, in this way, universality will be saved. It turns out, however - as was already mentioned in III·5- that, at the present time, the theory described by $(5\cdot12)$ seems to meet with serious objections. Let us review these once again with some details.

a) A first objection is that, even in leptonic decays, universality does not seem to be completely satisfied - as far at least as magnitudes of coupling constants are concerned. This is because the $\beta$ decay of $\Lambda$ and $\Sigma$ occur much less frequently than a straight forward calculation would predict. This can be due of course to renormalization effects of the weak interaction vertex due to strong interactions but it is then not understood why such effects would be large here while they are so small as far as $J^{sc}_K$ is concerned. Also the $\Sigma^+ \to n + e^+ + \nu$ decay, if confirmed, will be a strong objection to the $\Delta Q/\Delta S=1$ assumption.
b) A second objection is that (5.12) together with (7.1') predicts no \( \Sigma^+ \rightarrow n^+ \pi^+ \) decay in lowest order, while it does predict \( \Sigma^- \rightarrow n^+ \pi^- \) decay in lowest order. Again, the \( \Sigma^+ \) decay could be due to higher order corrections. But the fact that \( \Sigma^+ \) and \( \Sigma^- \) decay rates are quite similar then means that such higher order corrections are overwhelmingly important.

c) The same conclusion is reached through observation of the fact that the asymmetry parameters of \( \Sigma^+ \rightarrow n^+ \pi^+ \) and \( \Sigma^+ \rightarrow p^+ \pi^0 \) are experimentally quite different, whereas in lowest order (5.12) predicts them to be the same. There, moreover, one would have to assume that these higher order corrections which one does not know how to calculate would have very different effects on \( \Sigma^+ \rightarrow n^+ \pi^+ \) and on \( \Sigma^+ \rightarrow p^+ \pi^0 \) in spite of the obvious similarity between these two processes.

Concluding, one may say that, although (5.12) looks very attractive from the point of view of universality, and although it cannot be disproved by a comparison with experimental facts, it nevertheless does not seem, at the present time, to be a useful description of these facts. It implies an assumption about the importance of higher order effects which, though it is quite acceptable in itself, leads to a complete deadlock as far as comparison of theory and experiment is concerned.

There are therefore good reasons to take seriously the alternative approach described in III,3 in spite of the fact that it implies some breakdown of universality. Let us then try to write down explicitly the interaction hamiltonian in this other approach. As we are now admittedly giving up all attempt to unify all weak interactions it is just as well, and simpler, to look for a hamiltonian of the so called "Yukawa type", which describes the \( \Sigma \rightarrow N^+ \pi^- \) decays as direct processes.

As far as ordinary, spacetime, properties are concerned we may still keep the requirement that this hamiltonian is a mixture of vector (\( \gamma^\mu \)) and pseudovector (\( \gamma^\mu \sigma^\nu \)) couplings. As regards isotopic spin, the \( \Delta T = \frac{1}{2} \) rule of III2 and III,3 (not to be confused with the \( |\Delta T| = \frac{1}{2} \) rule-applied-to-strangeness-non-conserving-current" mentioned in III,5 and in V,7) tells us that the only
acceptable combination are of the types

\[ \bar{N} x \sum \vec{\pi} \quad \text{and} \quad \frac{1}{3} \left[ \bar{N} \sum x \times \sum \right] \vec{\pi} \ (8.1a, b) \]

where \( N = \left( \frac{P}{n} \right) \), \( x = \left( \frac{0}{1} \right) \) (spurion); and \( \sum \) and \( \vec{\pi} \) are the isotopic vectors pertaining to \( \sum \) and \( \pi \). These two expressions are isoscalars and therefore guarantee isotopic spin conservation, spurion included. As the spurion \( x \) has \( i = \frac{1}{3} \) they thus guarantee that \( \left| \Delta \vec{I} \right| = \frac{1}{3} \), spurion excluded.

Then the most general hamiltonian is

\[ \bar{N} x i\gamma_\mu \left( c + d \gamma_5 \right) \sum \frac{\partial \vec{\pi}}{\partial \chi_\mu} + \left[ \bar{N} \sum x \gamma_\mu \left( e + f \gamma_5 \right) \times \sum \right] \frac{\partial \pi}{\partial \chi_\mu} \]

\( c, d, e, f \) being arbitrary parameters. This expression has now to be fitted to the experimental data in such a way as to give the correct asymmetries and branching ratios of \( \sum \) decays. This is easily done by writing down explicitly \( (8.1a, b) \) in terms of \( \sum^+, \sum^-, \pi^+, \pi^-, \pi^0 \), \( p \) and \( n \) and the result is

\[ g \left\{ \bar{N} x i\gamma_\mu \left( 1 + \lambda \gamma_5 \right) \sum \frac{\partial \vec{\pi}}{\partial \chi_\mu} \pm \left[ \bar{N} \sum x \gamma_\mu \left( 1 - \lambda \gamma_5 \right) \sum \right] \frac{\partial \pi}{\partial \chi_\mu} \right\} \ (8.2) \]

with \( \lambda = \pm 1.2 \).

This value of \( \lambda \) is comparable to the value of \( \lambda \) which is found in \( \beta \) decay and the fact that it is different from one might similarly be attributed to renormalisation effects. But the new feature which appears is that the theory is not purely V-A but involves a peculiar combination of V-A and V+A, each of these two possible structures in ordinary space-time being associated with one of the two possible structures in isotopic spin space. There is up to now no theory which accounts for this fact.

Another way of writing \( (8.2) \) is to introduce the notations

\[ \left( \sum^1, \sum^2 \right) = \left( \sum^0/\sqrt{2}, \sum^+ \right) = \frac{1}{\sqrt{2}} \sum \]

\[ \left( \pi^1, \pi^2 \right) = \left( \pi^0/\sqrt{2}, \pi^+ \right) = \frac{1}{\sqrt{2}} \pi \]

\[ \left( \bar{N}^1, \bar{N}^2 \right) = \left( 0, p \right) \]

\[ \left( \bar{N}^1, \bar{N}^2 \right) = \left( 0, n \right) \]
(8.2) then takes the form

\[ \mathcal{N}_\alpha \beta \gamma \mu \frac{\partial}{\partial x_\mu} \sum_\gamma \beta \gamma \mu \delta \gamma \mu \frac{\partial}{\partial x_\mu} \sum_\alpha \gamma \alpha \] (8.4)

or a form derived from (8.4) by exchanging \( y_\mu \) and \( y_\mu \gamma \) (depending on the choice \( \pm \) in (8.2)).

While (8.4) is a simple transcription of (8.2) in different notations it can tentatively be used to try and relate \( \Sigma \) decay and \( \Lambda \) decay. For this purpose it is sufficient to replace (8.3) by the new definition

\[ \left( \begin{array}{c} \Sigma_4^+ \\
\Sigma_1^+ \\
\Sigma_2^+ \\
\Sigma_3^+ \end{array} \right) = \left( \begin{array}{c} \frac{\Lambda + \Sigma_0}{\sqrt{\lambda}} \\
\Lambda - \Sigma_0 \sqrt{\lambda} \end{array} \right) \] (8.5)

already used in the "restricted symmetry" theory of strong interactions (cf. II). Then (8.4) also includes terms representing \( \Lambda \) decay. Of course the same strong interactions which split the \( \Lambda \) and \( \Sigma \) masses should also somewhat modify the values of \( g_\Lambda, \lambda_\Lambda \) in relation to \( g_\Sigma, \lambda_\Sigma \); it is however the philosophy underlying the present approach that these changes are not very great so that an estimation of the ratio of \( \Sigma \) and \( \Lambda \) lifetimes and the ratio of \( \Sigma^+ \to \rho^0 + \pi^0 \) and \( \Lambda \to p + \pi^- \) asymmetry parameters is possible. As regards the former one finds an agreement within reasonable limits. As regards the latter it turns out that an opposite sign is predicted for the asymmetry parameters of these two decays *) , **) .

*) It is perhaps worthwhile recalling in this connection that, if the \( | \Lambda \rangle \equiv \frac{1}{2} \) rule is valid, the asymmetry parameters of \( \Lambda \to p + \pi^- \) and of \( \Lambda \to n + \pi^0 \) must be equal.

**) We give here the relevant formulae for asymmetries and lifetimes. Their proof is elementary.

Let \( g_\Lambda, \lambda_\Lambda, \Lambda_\Lambda, \lambda_\Sigma, \lambda_\Sigma \) be the renormalized constants. \( g_\Sigma, \lambda_\Sigma \) are just the constants appearing in (8.2); \( g_\Lambda, \lambda_\Lambda \) are roughly speaking the same constants, only slightly modified by renormalization effects, and the hamiltonian for \( \Lambda \) decay, as extracted from (8.4) is

\[ g_\Lambda \mathcal{N}_\Lambda x \gamma \mu (1 - \lambda_\Lambda) \gamma \mu \Lambda_\Lambda \frac{\partial}{\partial x_\mu}. \] (Footnote continued on next page →)
This however is not the only possible way in which \( \Sigma \) and \( \Lambda \) decay can be connected. For instance it has been suggested that, in the strong interactions, a "symmetry clash" comes in, in the sense that (8.5) would be adapted to the description of strong pion interactions and

\[
F(m) = \frac{1}{16\pi} \frac{i}{m^3} \left( m - m_N \right) \left( (m + m_N)^2 - m^2 \right)^{3/2} \left( (m - m_N)^2 - m^2 \right)^{1/2}
\]

\[
U(m) = \frac{m + m_N}{m - m_N} \left( \frac{m^2 - m_N^2}{(m + m_N)^2 - m^2} \right)
\]

If the + sign is chosen in (8.2) the partial decay rates

\[
\begin{align*}
\Gamma(\Sigma \rightarrow n \pi^+) &= \Gamma(\Sigma \rightarrow \rho \pi^0) = \Gamma(\Sigma \rightarrow n \pi^-) = \Gamma(\Lambda \rightarrow \rho \pi^0) = \Gamma(\Lambda \rightarrow n \pi^-) = \Gamma(\Lambda \rightarrow p \pi^-)
\end{align*}
\]

are given by \((\frac{\Gamma}{\Gamma} = c = 1)\)

\[
\begin{align*}
\Gamma(\Sigma \rightarrow n \pi^+) &= \frac{1}{4} g_N^2 \frac{F(m_N)}{F(m_\Sigma)} \\
\Gamma(\Sigma \rightarrow \rho \pi^0) &= 2 g_N^2 \left( 1 + \lambda_N \frac{U(m_N)}{U(m_\Sigma)} \right) \frac{F(m_N)}{F(m_\Sigma)} \\
\Gamma(\Sigma \rightarrow n \pi^-) &= \frac{1}{4} g_N^2 \lambda_N \frac{U(m_N)}{U(m_\Sigma)} \frac{F(m_N)}{F(m_\Sigma)} \\
\Gamma(\Lambda \rightarrow \rho \pi^0) &= 2 g_N^2 \left( 1 + \lambda_N \frac{U(m_N)}{U(m_\Lambda)} \right) \frac{F(m_N)}{F(m_\Lambda)} \\
\Gamma(\Lambda \rightarrow n \pi^-) &= g_N^2 \left( 1 + \lambda_N \frac{U(m_N)}{U(m_\Lambda)} \right) \frac{F(m_N)}{F(m_\Lambda)} \\
\Gamma(\Lambda \rightarrow p \pi^-) &= g_N^2 \left( 1 + \lambda_N \frac{U(m_N)}{U(m_\Lambda)} \right) \frac{F(m_N)}{F(m_\Lambda)}
\end{align*}
\]

(If the - sign is chosen in (8.2) the formulae for \( \Gamma^+ \) and \( \Gamma^- \) have to be interchanged). As for the asymmetry parameters \( \alpha \), their absolute values are given by

\[
|\alpha(\Sigma \rightarrow \pi^0)| = \frac{12 \lambda_N \frac{U(m_N)}{U(m_\Sigma)}}{1 + \lambda_N \frac{U(m_N)}{U(m_\Sigma)}}
\]

\[
|\alpha(\Lambda \rightarrow \pi^0)| = \frac{12 \lambda_N \frac{U(m_N)}{U(m_\Lambda)}}{1 + \lambda_N \frac{U(m_N)}{U(m_\Lambda)}}
\]

Their signs are opposite (in this version of the theory: see below) and the sign of \( \alpha(\Lambda) \) is such that \( \lambda_N > 0 \) corresponds to an emitted proton polarized parallel to its momentum.

If, due to renormalization effects, the values of \( g_\Lambda / g_\Sigma \)

\[
\lambda_\Lambda / \lambda_\Sigma
\]

are allowed to differ from unity through a factor of 20% to 30%, the formulae above still give a fairly precise evaluation of the \( \alpha(\Lambda) \), but the estimation of \( T_\Sigma / T_\Lambda \) becomes rather poor. Within these broad limits there is no contradiction with the experimental values.
$$\begin{pmatrix}
\frac{-\Lambda + \Sigma^0}{\sqrt{2}} & \Sigma^+ \\
\Sigma^- & \frac{-\Lambda - \Sigma^0}{\sqrt{2}}
\end{pmatrix}$$

(8.6)

would be adapted to the description of strong K-interaction. (This hypothesis allows for a \( \Lambda, \Sigma \) mass differences). If such a view is adopted one can also think of generalizing (8.4) by substituting (8.3) in its first term and (8.6) in its second term. This would lead to asymmetry parameters for \( \Lambda \) and for \( \Sigma^+ \rightarrow p + n^0 \) having the same sign.

Another possibility is to analyse the \( \Sigma \) decay in terms of \( I = 3/2 \) and \( I = 1/2 \) waves as was done in III.3, and to apply the coefficients of the \( I = 1/2 \) waves to the \( \Lambda \) case. This leads to an asymmetry parameter for \( \Lambda \) having the same sign as that of \( \Sigma^+ \rightarrow p + n^0 \) but, contrary to what is the case in the previous assumption, a magnitude significantly different from one.